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Some properties of vector sum and scalar multiplication of soft sets over a linear space

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Abstract. The aim of this paper is to define the vector sum and scalar multiplication of soft sets over a linear spaces and construct some useful propositions.

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1. Introduction

Advances in science and technology have made our modern society very complex and hence uncertainties are occurring increasingly in decision making process. To deal with uncertainties in decision making process, L.A. Zadeh[17] introduced the notion of fuzzy set in 1965. In his pioneering work, he has defined the algebraic operations on fuzzy set like union, intersection, complement etc. Later on many research works [5, 6, 16] have been done on this field. In 2001, Ismat Beg [3] constructed the sum and the scalar multiplication of fuzzy sets to define a fuzzy linear space. Infact, uncertainties are also being tackled by the theory of probability, fuzzy set, rough set etc.

All these concepts have some inherent difficulties. To over come a few of such difficulties, D. Molodtsov [12] introduced the notion of soft set in 1999. Thereafter so many research works[1, 2, 7, 8, 9, 11, 13, 14, 15] have been done with this concept in different disciplines of mathematics.

In this paper, the sum and scalar multiplication of soft sets over a linear space are being defined. Then we have established some propositions concerning the above said notions.

2. Preliminaries

Throughout the work, U refers to an initial universe, E is the set of parameters, $P(U)$ is the power set of U and $A \subseteq E$.

Definition 2.1 ([10])**.** Let f be a mapping of X into Y and μ be a fuzzy subset of X. The image $f(\mu)$ of μ is the fuzzy subset of Y defined by, for $y \in Y$

$$f(\mu)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu(x) & \text{if } f^{-1}(y) \neq \varnothing, \\ 0 & \text{otherwise.} \end{cases}$$

Definition 2.2 ([3])**.** Let X be a vector space over K where K denotes either a real or complex numbers. Let $\mu_1, \mu_2, \cdots, \mu_n$ be the fuzzy subsets of X, then $\mu_1 \times \mu_2 \times \cdots \times \mu_n$ is a fuzzy subset μ of X^n defined by

$$\mu(x_1, x_2, \cdots, x_n) = \min\{\mu_1(x_1), \mu_2(x_2), \cdots, \mu_n(x_n)\}.$$

If $f : X^n \to X$ is defined by $f(x_1, x_2, \cdots, x_n) = x_1 + x_2 + \cdots + x_n$, then the fuzzy set $f(\mu)$ in X is called the sum of fuzzy sets $\mu_1, \mu_2, \cdots, \mu_n$ and it is denoted by $\mu_1 + \mu_2 + \cdots + \mu_n$.
For a fuzzy subset μ of X and a scalar $t \in K$, we denote $t\mu$ the image of μ under the map $g : X \to X$, $g(x) = tx$.

Definition 2.3 ([4])**.** A soft set F_A on the universe U is defined by the set of ordered pairs $F_A = \{(e, F_A(e)) : e \in E, F_A(e) \in P(U)\}$, where $F_A : E \to P(U)$ such that $F_A(e) = \phi$ if e is not an element of A.
The set of all soft sets over (U, E) is denoted by $S(U)$.

Definition 2.4 ([4])**.** Let $F_A \in S(U)$. If $F_A(e) = \phi$, for all $e \in E$, then F_A is called a empty soft set, denoted by Φ. $F_A(e) = \phi$ means that there is no element in U related to the parameter $e \in E$.

Definition 2.5 ([4])**.** Let $F_A, G_B \in S(U)$. We say that F_A is a soft subsets of G_B and we write $F_A \sqsubseteq G_B$ if and only if
(i) $A \subseteq B$,
(ii) $F_A(e) \subseteq G_B(e)$ for all $e \in E$.

Definition 2.6 ([4])**.** Let $F_A, G_B \in S(U)$. Then F_A and G_B are said to be soft equal, denoted by $F_A = G_B$, if $F_A(e) = G_B(e)$ for all $e \in E$.

Definition 2.7 ([4])**.** Let $F_A, G_B \in S(U)$. Then the soft union of F_A and G_B is also a soft set $F_A \sqcup G_B = H_{A \cup B} \in S(U)$, defined by

$$H_{A \cup B}(e) = (F_A \sqcup G_B)(e) = F_A(e) \cup G_B(e) \text{ for all } e \in E.$$

Definition 2.8 ([4])**.** Let $F_A, G_B \in S(U)$. Then the soft intersection of F_A and G_B is also a soft set $F_A \sqcap G_B = H_{A \cap B} \in S(U)$, defined by

$$H_{A \cap B}(e) = (F_A \sqcap G_B)(e) = F_A(e) \cap G_B(e) \text{ for all } e \in E.$$

Definition 2.9. Let F_A be a soft set over (U, E) and $f : E \to E$. Then $f(F_A)$, a soft set over (U, E), is defined by

$$f(F_A)(e) = \begin{cases} \cup_{e' \in f^{-1}(e)} F_A(e') & \text{if } f^{-1}(e) \neq \varnothing, \\ \varnothing & \text{otherwise.} \end{cases}$$

3. Vector sum and scalar multiplication of soft sets

In this section, we denote 0 and 1 as the zero element and unity of the field respectively. Also the zero vector of the linear space is denoted by 0 which can easily separated from the zero element of the field.

Definition 3.1. If F_A and G_B are two soft sets over (U, E), then their product $F_A \times G_B$ is a soft set over $(U, E \times E)$ and it is defined by

$$(F_A \times G_B)(e_1, e_2) = F_A(e_1) \cap G_B(e_2), \text{ for all } (e_1, e_2) \in E \times E.$$

Example 3.2. Let $U = \{a, b, c, d, e\}$, $E = \{e_1, e_2, e_3, e_4, e_5\}$, $A = \{e_1, e_2\}$ and $B = \{e_2, e_3, e_4\}$. Also let F_A and G_B be two soft sets over (U, E) defined by
$F_A(e_1) = \{a, b, c\}$, $F_A(e_2) = \{d, e\}$
and $G_B(e_2) = \{b, d, e\}$, $G_B(e_3) = \{a, b\}$, $G_B(e_4) = \{c, d, e\}$. Then
$(F_A \times G_B)(e_1, e_2) = \{b\}$,
$(F_A \times G_B)(e_1, e_3) = \{a, b\}$,
$(F_A \times G_B)(e_1, e_4) = \{c\}$,
$(F_A \times G_B)(e_2, e_2) = \{d, e\}$,
$(F_A \times G_B)(e_2, e_3) = \varnothing$,
$(F_A \times G_B)(e_2, e_4) = \{d, e\}$ and
$(F_A \times G_B)(e_i, e_j) = \varnothing$ if $(e_i, e_j) \in (E \times E) - (A \times B)$.

Definition 3.3. Let U be a universal set and E be a usual vector space over $\mathbb{R}$ or $\mathbb{C}$ and $F_{A_1}, F_{A_2}, \cdots, F_{A_n}$ be soft sets over (U, E) and $f : E^n \to E$ be the function defined by $f(e_1, e_2, \cdots, e_n) = e_1 + e_2 + \cdots + e_n$. Then the vector sum $F_{A_1} + F_{A_2} + \cdots + F_{A_n}$ is defined by

$$F_{A_1} + F_{A_2} + \cdots + F_{A_n} = f(F_{A_1} \times F_{A_2} \times \cdots \times F_{A_n}).$$

That is, for each $e \in E$,

$$\begin{aligned}
&(F_{A_1} + F_{A_2} + \cdots + F_{A_n})(e)\\
&= f(F_{A_1} \times F_{A_2} \times \cdots \times F_{A_n})(e)\\
&= \cup_{(e_1, e_2, \cdots, e_n) \in f^{-1}(e)} (F_{A_1} \times F_{A_2} \times \cdots \times F_{A_n})(e_1, e_2, \cdots, e_n)\\
&= \cup_{(e_1, e_2, \cdots, e_n) \in f^{-1}(e)} \{F_{A_1}(e_1) \cap F_{A_2}(e_2) \cap \cdots \cap F_{A_n}(e_n)\}.
\end{aligned}$$

Example 3.4. Let universal set $U = \mathbb{R}^2$, the parameter set E= the real vector space $\mathbb{R}$ and $f : \mathbb{R}^2 \to \mathbb{R}$ be the function defined by $f(e_1, e_2) = e_1 + e_2$.
Also let $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and F_A, G_B be two soft sets over (U, E) defined by

$$F_A(e) = \begin{cases} \{(x, y) \in \mathbb{R}^2 : \ 2x + 3y = e\} & \text{if } e \in A, \\ \varnothing & \text{otherwise} \end{cases}$$

and

$$G_B(e) = \begin{cases} \{(x, y) \in \mathbb{R}^2 : \ 4x + 7y = e\} & \text{if } e \in B, \\ \varnothing & \text{otherwise.} \end{cases}$$

Now $1 + 3 = 4, 1 + 4 = 5, 2 + 3 = 5, 2 + 4 = 6, 3 + 3 = 6$ and $3 + 4 = 7$.
Then the vector sum of F_A and G_B is $F_A + G_B$, where

$$\begin{aligned}
(F_A + G_B)(4) &= \cup_{(e_1, e_2) \in f^{-1}(4)} \{F_A(e_1) \cap G_B(e_2)\}\\
&= F_A(1) \cap G_B(3) = \{(-1, 1)\},
\end{aligned}$$

$$\begin{aligned}(F_A+G_B)(5) &= \cup_{(e_1,e_2)\in f^{-1}(5)}\{F_A(e_1)\cap G_B(e_2)\}\\ &= \{F_A(1)\cap G_B(4)\}\cup\{F_A(2)\cap G_B(3)\}\\ &= \left\{\left(-\frac{5}{2},2\right),\ \left(\frac{5}{2},-1\right)\right\},\\ (F_A+G_B)(6) &= \cup_{(e_1,e_2)\in f^{-1}(6)}\{F_A(e_1)\cap G_B(e_2)\}\\ &= \{F_A(2)\cap G_B(4)\}\cup\{F_A(3)\cap G_B(3)\}\\ &= \{(1,0),\ (6,-3)\},\\ (F_A+G_B)(7) &= \cup_{(e_1,e_2)\in f^{-1}(7)}\{F_A(e_1)\cap G_B(e_2)\}\\ &= F_A(3)\cap G_B(4) = \left\{\left(\frac{9}{2},-2\right)\right\}\end{aligned}$$

and

$$(F_A+G_B)(e)=\varnothing,\text{ if } e\in\mathbb{R}-\{4,5,6,7\}.$$

Definition 3.5. If U is a universal set, E is a usual vector space over $\mathbb{R}$ or $\mathbb{C}$, t is a scalar and $g : E \to E$ is a mapping defined by $g(e) = te$, then the scalar multiplication tF_A of a soft set F_A is defined by $tF_A = g(F_A)$. That is, for $e \in E$,

$$tF_A(e) = g(F_A)(e) = \cup_{e'\in g^{-1}(e)}F_A(e').$$

Proposition 3.6. *Let U be a universal set, E be a usual vector space over $\mathbb{R}$ or $\mathbb{C}$, t be a scalar and F_A be a soft set over (U, E). Then*

$$tF_A(e)=\begin{cases}F_A(t^{-1}e) & if\quad t\neq 0,\\ \varnothing & if\quad t=0\quad and\quad e\neq 0,\\ \cup_{e\in E}F_A(e) & if\quad t=0\quad and\quad e=0.\end{cases}$$

Proof. Case(i): Suppose $t \neq 0$ and let $e \in E$. Then

$$tF_A(e) = g(F_A)(e) = \cup_{e'\in g^{-1}(e)}F_A(e') = \cup_{e'=t^{-1}e}F_A(e') = F_A(t^{-1}e).$$

Case(ii): Suppose $t = 0$.

Subcase(a). If $e(\neq 0) \in E$, then $tF_A(e) = \cup_{e'\in g^{-1}(e)}F_A(e') = \varnothing$. [as $t = 0$, $e = g(e') = 0$. But $e \neq 0$. Thus there exists no such e'.]

Subcase(b). If $e = 0$, then $tF_A(e) = \cup_{e'\in g^{-1}(e)}F_A(e')$.

Let $e' \in E$. Then $0e' = 0$, that is, $g(e') = 0$ or $e' \in g^{-1}(0) = g^{-1}(e)$. Thus, $g^{-1}(e) = E$. So $tF_A(e) = \cup_{e'\in E}F_A(e')$. □

Example 3.7. Let the Universal set U = the set of all integers, The parameter set E = the set of all real numbers, A =the set of all positive real numbers. Also let F_A be a soft set defined by

$$F_A(e)=\begin{cases}\{[e],[e]+1,[e]+2,\cdots\} & \text{if } e\in A,\\ \varnothing & \text{otherwise}\end{cases}$$

and $t \in \mathbb{R}$. Then, by the proposition 3.6, we now find tF_A.

If $t \neq 0$, then

$$tF_A(e)=F_A(t^{-1}e)=\begin{cases}\{[t^{-1}e],[t^{-1}e]+1,[t^{-1}e]+2,\cdots\} & \text{if } t^{-1}e\in A,\\ \varnothing & \text{if } t^{-1}e\notin A.\end{cases}$$

If $t = 0$ and $e \neq 0$, then $tF_A(e) = \varnothing$.

If $t = 0$ and $e = 0$, then $tF_A(e) = \cup_{e\in E} F_A(e) = \cup_{e\in A} F_A(e)$ =the set of all natural numbers including zero.

Proposition 3.8. *Suppose E_1 and E_2 are linear spaces over the same field $\mathbb{R}$ or $\mathbb{C}$ and $f : E_1 \to E_2$ is linear mapping. Then for any soft sets F_A and G_B over (U, E_1) and for the scalar t,*

(1) $f(F_A + G_B) = f(F_A) + f(G_B)$,

(2) $f(tF_A) = tf(F_A)$.

Proof. (1) Let the range of f be M and $w \in E_2$. Put $P = f(F_A + G_B)(w)$ and $Q = (f(F_A) + f(G_B))(w)$.

Case(i): If $w \in E_2 \setminus M$, then $f^{-1}(w) = \varnothing$. Thus $P = \varnothing$. Further, if $w = w_1 + w_2$, then at least one of w_1 and w_2 is not in M as f is linear. Thus, either

$$f(F_A)(w_1) = \cup_{e\in f^{-1}(w_1)} F_A(e) = \varnothing$$

or

$$f(G_B)(w_2) = \cup_{e\in f^{-1}(w_2)} G_B(e) = \varnothing.$$

So

$$Q = (f(F_A) + f(G_B))(w) = \cup_{w=w_1+w_2}(f(F_A)(w_1) \cap f(G_B)(w_2)) = \varnothing.$$

Hence, in this case, $P = Q$.

Case(ii): If $w \in M$ and $p \in P$, then $p \in \cup_{e\in f^{-1}(w)}(F_A + G_B)(e)$, that is, $p \in (F_A + G_B)(e)$ for some $e \in f^{-1}(w)$. That is, $p \in \cup_{e=e_1+e_2}(F_A(e_1) \cap G_B(e_2))$. Thus there exist $e_1, e_2 \in E$ with $e = e_1 + e_2$ such that $p \in F_A(e_1) \cap G_B(e_2)$.

On one hand,

$$Q = (f(F_A) + f(G_B))(w) = \cup_{w=w_1+w_2}(f(F_A)(w_1) \cap f(G_B)(w_2)).$$

Since $w = f(e) = f(e_1 + e_2) = f(e_1) + f(e_2)$,

$$f(F_A)(f(e_1)) \cap f(G_B)(f(e_2)) \subseteq Q.$$

Then,

$$\{\cup_{r_1\in f^{-1}(f(e_1))} F_A(r_1)\} \cap \{\cup_{r_2\in f^{-1}(f(e_2))} G_B(r_2)\} \subseteq Q.$$

Thus $F_A(e_1) \cap G_B(e_2) \subseteq Q$, that is, $p \in Q$. So, $P \subseteq Q$.

For the reverse, we take $q \in Q$. Then $q \in \cup_{w=w_1+w_2}(f(F_A)(w_1) \cap f(G_B)(w_2))$. Thus, there exist w_1, $w_2 \in M$ with $w = w_1 + w_2$ such that

$q \in f(F_A)(w_1) \cap f(G_B)(w_2)= \{\cup_{e_1\in f^{-1}(w_1)} F_A(e_1)\} \cap \{\cup_{e_2\in f^{-1}(w_2)} G_B(e_2)\}$.

So, there exist $e_1 \in f^{-1}(w_1)$ and $e_2 \in f^{-1}(w_2)$ such that $q \in F_A(e_1) \cap G_B(e_2)$, where

$$w = w_1 + w_2 = f(e_1) + f(e_2) = f(e_1 + e_2).$$

Now $P = f(F_A + G_B)(w) = \cup_{e\in f^{-1}(w)}(F_A + G_B)(e)$. Since $w = f(e_1 + e_2)$, $(F_A + G_B)(e_1 + e_2) \subseteq P$, that is, $\cup_{(e_1+e_2)=e_1'+e_2'}\{F_A(e_1') \cap G_B(e_2')\} \subseteq P$, that is, $F_A(e_1) \cap G_B(e_2) \subseteq P$. Then $q \in P$. Thus $Q \subseteq P$. So $P = Q$. This completes the proof of (1).

(2) Let the range of f be M and $w \in E_2$. Put $P = tf(F_A)(w)$ and $Q = f(tF_A)(w)$.

Case(i): If $w \in E_2 \setminus M$, then $f^{-1}(w) = \varnothing$. Thus for $t \neq 0$,

$$\begin{aligned} P &= tf(F_A)(w) = f(F_A)(t^{-1}w) \\ &= \cup_{e \in f^{-1}(t^{-1}w)} F_A(e) = \cup_{te \in f^{-1}(w)} F_A(e) \\ &= \varnothing.[\text{ as } f^{-1}(w) = \varnothing] \end{aligned}$$

Also $P = \varnothing$, when $t = 0$. So, if $w \in E_2 \setminus M$, then $P = \varnothing$.
Now

$$\begin{aligned} Q &= f(tF_A)(w) = \cup_{e \in f^{-1}(w)} tF_A(e) \\ &= \varnothing.[\text{ as } f^{-1}(w) = \varnothing] \end{aligned}$$

Hence $P = Q$, for $w \in E_2 \setminus M$.

Case(ii): If $w \in M$ and $t \neq 0$, then

$$\begin{aligned} P &= f(F_A)(t^{-1}w) = \cup_{e \in f^{-1}(t^{-1}w)} F_A(e) \\ &= \cup_{tf(e)=w} F_A(e) = \cup_{f(te)=w} F_A(e) \end{aligned}$$

and

$$\begin{aligned} Q &= f(tF_A)(w) = \cup_{e' \in f^{-1}(w)} tF_A(e') \\ &= \cup_{f(e')=w} F_A(t^{-1}e') \\ &= \cup_{f(te)=w} F_A(e).[\text{ taking } t^{-1}e' = e] \end{aligned}$$

Thus $P = Q$, for $w \in M$ and $t \neq 0$.

Case(iii): If $w(\neq 0) \in M$ and $t = 0$, then

$$P = 0f(F_A)(w) = \varnothing$$

and

$$\begin{aligned} Q &= f(0F_A)(w) = \cup_{f(e)=w} 0F_A(e) \\ &= \varnothing.[\text{ since } f(e) = w \neq 0,\ e \neq 0] \end{aligned}$$

Thus $P = Q$, for $w(\neq 0) \in M$ and $t = 0$.

Case(iv): If $w(= 0) \in M$ and $t = 0$, then

$$\begin{aligned} P &= 0f(F_A)(0) = \cup_{e_2 \in E_2} f(F_A)(e_2) \\ &= \cup_{e_2 \in E_2} \cup_{e_1 \in f^{-1}(e_2)} F_A(e_1) = \cup_{e_1 \in E_1} F_A(e_1) \end{aligned}$$

and

$$\begin{aligned} Q &= f(0F_A)(0) = \cup_{e \in f^{-1}(0)} 0F_A(e) \\ &= 0F_A(0)[\text{ as if } e(\neq 0) \in f^{-1}(0), \text{ then } 0F_A(e) = \varnothing] \\ &= \cup_{e_1 \in E_1} F_A(e_1). \end{aligned}$$

Thus $P = Q$ for $w(= 0) \in M$ and $t = 0$. This completes the proof of part (2). □

Proposition 3.9. *If S is an ordinary subset of a linear space E over $\mathbb{R}$ or $\mathbb{C}$, F_A is a soft set over (U, E) and $x \in E$, then*

(1) *$(x + F_A)(e) = F_A(e - x)$ for all $e \in E$ where $x + F_A$ means $1_x + F_A$ and*

$$1_x(e) = \begin{cases} U & \text{if } e = x \\ \varnothing & \text{if } e \neq x. \end{cases}$$

(2) *$x + F_A = T_x(F_A)$ for the translation mapping $T_x : E \to E$ defined by $T_x(e) = x + e$ for all $e \in E$.*

(3) *$S + F_A = \sqcup_{a\in S}(a + F_A)$.*

Proof. (1) $(x + F_A)(e) = \cup_{e=e_1+e_2}\{1_x(e_1) \cap F_A(e_2)\}$
$= \{1_x(x) \cap F_A(e - x)\} \cup_{e_1 \neq x} \{\varnothing \cap F_A(e - e_1)\}$
$= F_A(e - x) \cup \varnothing$
$= F_A(e - x)$.

(2) $T_x(F_A)(e) = \cup_{e' \in T_x^{-1}(e)} F_A(e') = F_A(e - x)$.
Thus, by (1), we get $x + F_A = T_x(F_A)$.

(3) $(S + F_A)(e) = \cup_{e=e_1+e_2}\{1_S(e_1) \cap F_A(e_2)\}$
$= \cup_{a\in S}\{1_S(a) \cap F_A(e - a)\} \cup_{a\in E\setminus S} \{1_S(a) \cap F_A(e - a)\}$
$= \cup_{a\in S} F_A(e - a) \cup \varnothing$
$= \cup_{a\in S} F_A(e - a)$
$= \cup_{a\in S}(a + F_A)(e)$. □

Proposition 3.10. *If $F_A, F_{A_1}, F_{A_2}, \cdots, F_{A_n}$ are soft sets over (U, E), where E is a linear space. Then for the scalars $t_1, t_2, \cdots, t_n$, the following are equivalent:*

(1) *$t_1F_{A_1} + t_2F_{A_2} + \cdots + t_nF_{A_n} \sqsubseteq F_A$.*

(2) *For all $e_1, e_2, \cdots, e_n \in E$, $F_A(t_1e_1 + t_2e_2 + \cdots + t_ne_n) \supseteq \cap_{i=1}^n F_{A_i}(e_i)$*

Proof. (1) ⇒ (2): Without lost of generality, we may assume that the first m $(0 \leq m \leq n)$ scalars $t_1, t_2, \cdots, t_m$ are non-zero and the remaining scalars are zero. Then
$F_A(t_1e_1 + t_2e_2 + \cdots + t_me_m + 0e_{m+1} + \cdots + 0e_n)$
$\supseteq (t_1F_{A_1} + t_2F_{A_2} + \cdots + t_mF_{A_m} + 0F_{A_{m+1}} + \cdots + 0F_{A_n})$
$(t_1e_1 + t_2e_2 + \cdots + t_me_m + 0e_{m+1} + \cdots + 0e_n)$
$= \cup_{t_1e_1+t_2e_2+\cdots+t_me_m+0e_{m+1}+\cdots+0e_n=r_1+r_2+\cdots+r_m+\cdots+r_n}\{t_1F_{A_1}(r_1)$
$\cap t_2F_{A_2}(r_2) \cap \cdots \cap t_mF_{A_m}(r_m) \cap 0F_{A_{m+1}}(r_{m+1}) \cap \cdots \cap 0F_{A_n}(r_n)\}$
$\supseteq t_1F_{A_1}(t_1e_1) \cap t_2F_{A_2}(t_2e_2) \cap \cdots \cap t_mF_{A_m}(t_me_m) \cap 0F_{A_{m+1}}(0) \cap \cdots \cap 0F_{A_n}(0)$
$\supseteq t_1F_{A_1}(t_1e_1) \cap t_2F_{A_2}(t_2e_2) \cap \cdots \cap t_mF_{A_m}(t_me_m) \cap F_{A_{m+1}}(e_{m+1})$
$\cap \cdots \cap F_{A_n}(e_n)$ [as $0F_{A_i}(0) = \cup_{e\in E} F_{A_i}(e)$ for $i = m + 1, m + 2, \cdots, n$]
$= F_{A_1}(e_1) \cap F_{A_2}(e_2) \cap \cdots \cap F_{A_m}(e_m) \cap F_{A_{m+1}}(e_{m+1}) \cap \cdots \cap F_{A_n}(e_n)$.

(2) ⇒ (1): Let $F_A(t_1e_1 + t_2e_2 + \cdots + t_ne_n) \supseteq F_{A_1}(e_1) \cap F_{A_2}(e_2) \cap \cdots \cap F_{A_n}(e_n)$ for all $e_1, e_2, \cdots, e_n \in E$. Rearranging the order, let $t_i \neq 0$ for $1 \leq i \leq k$ and $t_i = 0$ for $k < i \leq n$. Then, from the hypothesis, we get
$F_A(t_1e_1 + t_2e_2 + \cdots + t_ke_k)$
$\supseteq F_{A_1}(e_1) \cap F_{A_2}(e_2) \cap \cdots \cap F_{A_k}(e_k) \cap F_{A_{k+1}}(x_{k+1}) \cap \cdots \cap F_{A_n}(x_n)$,
for all $x_{k+1}, x_{k+2}, \cdots, x_n \in E$.
Thus,
$F_A(t_1e_1 + t_2e_2 + \cdots + t_ke_k)$
$\supseteq F_{A_1}(e_1) \cap F_{A_2}(e_2) \cap \cdots \cap F_{A_k}(e_k) \cap 0F_{A_{k+1}}(0) \cap \cdots \cap 0F_{A_n}(0)$,
for all $e_1, e_2, \cdots, e_k \in E$.

On one hand,
$(t_1F_{A_1} + t_2F_{A_2} + \cdots + t_nF_{A_n})(e)$
$= \cup_{e=e_1+e_2+\cdots+e_n}\{t_1F_{A_1}(e_1) \cap t_2F_{A_2}(e_2) \cap \cdots \cap t_nF_{A_n}(e_n)\}$
$= \cup_{e=e_1+e_2+\cdots+e_n}\{t_1F_{A_1}(e_1) \cap t_2F_{A_2}(e_2) \cap \cdots \cap t_kF_{A_k}(e_k)$

$$\cap 0F_{A_{k+1}}(e_{k+1}) \cap \cdots \cap 0F_{A_n}(e_n)\}$$
$$= \cup_{e=e_1+e_2+\cdots+e_k}\{t_1F_{A_1}(e_1) \cap t_2F_{A_2}(e_2) \cap \cdots \cap t_kF_{A_k}(e_k)$$
$$\cap 0F_{A_{k+1}}(0) \cap \cdots \cap 0F_{A_n}(0)\}$$
$$= \cup_{e=e_1+e_2+\cdots+e_k}\{F_{A_1}(t_1^{-1}e_1) \cap F_{A_2}(t_2^{-1}e_2) \cap \cdots \cap F_{A_k}(t_k^{-1}e_k)$$
$$\cap 0F_{A_{k+1}}(0) \cap \cdots \cap 0F_{A_n}(0)\}$$
$$\subseteq \cup_{e=e_1+e_2+\cdots+e_k}F_A(t_1t_1^{-1}e_1 + t_2t_2^{-1}e_2 + \cdots + t_kt_k^{-1}e_k)$$
$$= \cup_{e=e_1+e_2+\cdots+e_k}F_A(e_1 + e_2 + \cdots + e_k)$$
$$= F_A(e).$$
□

Proposition 3.11. *If F_A and G_B are two soft sets over (U, E), where E is a linear space, then*
(1) $1F_A + 0G_B \sqsubseteq F_A$.
(2) $1F_A + 0G_B = F_A$ *iff* $\cup_{e\in E}F_A(e) \subseteq \cup_{e\in E}G_B(e)$.

Proof. (1) Let $e_1, e_2 \in E$. Then $F_A(1e_1 + 0e_2) = F_A(e_1) \supseteq F_A(e_1) \cap G_B(e_2)$. Thus, by previous proposition, we have $1F_A + 0G_B \sqsubseteq F_A$.

(2) Let $\cup_{e\in E}F_A(e) \subseteq \cup_{e\in E}G_B(e)$ and $e \in E$. Then

$$(1F_A + 0G_B)(e) = \cup_{e=e_1+e_2}\{1F_A(e_1) \cap 0G_B(e_2)\}$$
$$= 1F_A(e) \cap 0G_B(0) = F_A(e) \cap \{\cup_{x\in E}G_B(x)\}$$
$$= F_A(e).[\text{ as by the hypothesis we have } F_A(e) \subseteq \cup_{x\in E}G_B(x)]$$

Thus $1F_A + 0G_B = F_A$.

Conversely, suppose that $1F_A + 0G_B = F_A$ and $x \in \cup_{e\in E}F_A(e)$.
Then $x \in F_A(e) = (1F_A + 0G_B)(e)$ for some $e \in E$.
Thus $x \in \cup_{e=e_1+e_2}\{1F_A(e_1) \cap 0G_B(e_2)\}$.
So $x \in 1F_A(e) \cap 0G_B(0)$, i.e., $x \in 0G_B(0) = \cup_{e\in E}G_B(e)$.
Hence $\cup_{e\in E}F_A(e) \subseteq \cup_{e\in E}G_B(e)$. □

4. Conclusions

Ismat Beg [3] has defined the vector sum and the scalar multiplication of fuzzy sets in 2001. In this paper, the vector sum and scalar multiplication are being defined on soft sets over a linear space. Then we have established some propositions which will be needed in future for construction of a soft balanced set, absorbing set, convex set etc.

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