Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 12, No. 3, (September 2016), pp. 361–371
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

@FMI

http://www.kyungmoon.com

On fuzzy supra-preopen sets

Hakeem A. Othman

Received 26 November 2015; Revised 15 February 2016; Accepted 17 March 2016

Abstract. In this paper, we define the fuzzy closure* and fuzzy interior * operators. Then, we extend the notion of pre*open (supra preopen) (pre*closed (supra-preclosed)) sets to fuzzy topological spaces and study some notion based on this new concept. In the light of this concept, we also define fuzzy supra-precontinuous and fuzzy supra-preopen (supra-preclosed) mappings. The relationships between these new concepts and others types are investigated. Counter-examples are given to show that the inverse of these relations are not true in general. In addition, some of special results and properties, which belong to fuzzy supra-preopen (supra-preclosed) set, fuzzy supra-precontinuous and fuzzy supra-preopen (supra-preclosed) mappings are studied and discussed. The notion of fuzzy supra-preconnected are introduced.

2010 AMS Classification: 54A40, 54C08, 54C10

Keywords: Fuzzy closure*, Fuzzy interior *, Fuzzy supra-preopen set, Fuzzy supra-preclosed set, Fuzzy supra-precontinuous mapping, Fuzzy supra-preopen mapping, Fuzzy supra-preclosed mapping, Fuzzy supra-preconnected.

Corresponding Author: Hakeem A. Othman (hakim _ albdoie@yahoo.com)

1. Introduction

Dealing with the uncertainties in our real life phenomena has been a critical matter in recent years. Classical set theory was not suitable in such cases. Since 1965, Zadeh [9] has introduced a more suitable set theory providing the concept "fuzzy set". A large community of Mathematicians has put their relentless efforts to the investigation of various concepts of general topology in fuzzy setting likely, [1], [2], [8] and [4].

In 2012, Selvi and Dharani, [7] introduced the concept of pre*open (supra-preopen) sets in general topology. In this paper, we extend the notions of supra-preopen sets and closure* and interior * operators to fuzzy topology space and study some notions based on these new concepts. We further study the relations between fuzzy supra-preopen sets and other kinds of fuzzy open sets. We also introduce the concepts of fuzzy supra-precontinuous, fuzzy supra-preopen and fuzzy supra-preclosed mapping and discuss their relations with other weaker forms of fuzzy continuous mappings. The concept of fuzzy supra-preconnected are introduced and some of interesting results about these new concepts are investigated.

2. Preliminaries

Throughout this paper by (X, τ) or simply by X we mean a fuzzy topological space (fts, shorty) and $f : X \to Y$ means a mapping f from a fuzzy topological space X to a fuzzy topological space Y. If u is a fuzzy set and p is a fuzzy singleton in X then $N(p)$, $Int\ u$, $cl\ u$ and u^c denote respectively, the neighborhood system of p, the interior of u, the closure of u and complement of u.

Now we recall some of the basic definitions and results in fuzzy topology.

Definition 2.1 ([3])**.** A fuzzy singleton p in X is a fuzzy set defined by:
$p(x) = t$, for $x = x_0$ and $p(x) = 0$ otherwise, where $0 < t \leq 1$. The point p is said to have supported x_0 and value t.

Definition 2.2. A fuzzy set u in a fts X is said to be fuzzy $\alpha-$open [2] (resp. Fuzzy preopen [8], Fuzzy semi open [1] set, if $u \leq$ Int cl Int u (resp. $u \leq$ Int cl u, $u \leq$ cl Int u).

The family of all fuzzy α-open (resp. fuzzy preopen, fuzzy semi open) sets of X is denoted by $F\alpha O(X)$ (resp. $FPO(X)$, $FSO(X)$).

Definition 2.3. A fuzzy set μ in a fts X is said to be:

(i) Fuzzy infra-semiopen [5], if $\eta \leq \mu \leq Cl^*(\eta)$ where η is fuzzy open or equivalently $\mu \leq cl^*(Int(\mu))$.

(ii) Fuzzy infra-semiclosed [5], if $Int^*(\eta) \leq \mu \leq \eta$ where where η is fuzzy closed or equivalently $Int^*(Cl(\mu)) \leq \mu$.

(iii) Fuzzy infra-α-closed [6], if $Cl(Int^*(Cl(\mu))) \leq \mu$ and fuzzy infra-α-open if $\mu \leq Int(Cl^*(Int(\mu)))$.

The family of all fuzzy infra-semiopen, fuzzy infra-semiclosed, fuzzy infra-α-open and fuzzy infra-α-closed sets in X will be denoted by IFSO(X), IFSC(X), IFαO(X) and IFαC(X), respectively.

Definition 2.4. A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be:

(i) Fuzzy α-continuous [2], if $f^{-1}(v)$ is fuzzy α-open set in X for each fuzzy open set v in Y.

(ii) Fuzzy semi continuous [1], if $f^{-1}(v)$ is fuzzy semi open set in X for each fuzzy open set v in Y.

(iii) Fuzzy precontinuous [2], if $f^{-1}(v)$ is fuzzy preopen set in X for each fuzzy open set v in Y.

Definition 2.5. A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be:

(i) Fuzzy infra-α-continuous [6], if $f^{-1}(v)$ is fuzzy infra-α-open (infra-α-closed) set in X for each fuzzy open (closed) set v in Y.

(ii) Fuzzy infra-semicontinuous [5], if $f^{-1}(v)$ is fuzzy infra-semiopen (infra-semiclosed) set in X for each fuzzy open (closed) set v in Y.

3. Fuzzy supra-preopen set

Definition 3.1. Let λ any fuzzy set. Then,

(i) $Cl^*\lambda = \wedge\{\mu : \mu \geq \lambda,\ \mu$ is a fuzzy generalized closed set of $X\}$ is called the fuzzy closure*.

(ii) $Int^*\lambda = \vee\{\mu : \mu \leq \lambda,\ \mu$ is a fuzzy generalized open set of $X\}$ is called fuzzy the interior*.

Since every fuzzy closed set is fuzzy generalized closed and fuzzy generalized open set is fuzzy open set, we have the following Lemma.

Lemma 3.2. *Let λ any fuzzy set. Then,*

(1) $\lambda \leq Cl^*\lambda \leq Cl\lambda$.

(2) $Int\lambda \leq Int^*\lambda \leq \lambda$.

Example 3.3. Let $X = \{a, b\}$ and v_1, v_2, v_3 and v_4 be fuzzy sets of X defined as:

$$v_1(a) = 0.6,\quad v_1(b) = 0.4,\qquad v_2(a) = 0.3,\quad v_2(b) = 0.5,$$
$$v_3(a) = 0.3,\quad v_3(b) = 0.4,\qquad v_4(a) = 0.6,\quad v_4(b) = 0.5.$$

Let $\tau = \{0_x, v_1, v_2, v_1 \vee v_2, v_1 \wedge v_2, 1_x\}$. Clearly τ is a fuzzy topology on X. Easy computation,

$Int^*(v_4^c) = \bigvee \begin{pmatrix} 0.3 \leq x < 4 \\ 0.5 \end{pmatrix}$ but $Int(v_4^c) = v_2$, then $Int(v_4^c) \leq Int^*(v_4^c) \leq v_4^c$ and

$Cl^*(v_2) = \bigwedge \begin{pmatrix} 0.3 < x \leq 4 \\ 0.5 \end{pmatrix}$ but $Cl(v_2) = v_4^c$, then $v_2 \leq Cl^*(v_2) \leq Cl(v_2)$.

Definition 3.4. A fuzzy subset λ of fuzzy space X is called fuzzy supra-preopen (supra-preclosed) set if $\lambda \leq Int^*\,Cl\lambda$ ($Cl^*\,Int\lambda \leq \lambda$). The class of all fuzzy supra-preopen (supra-preclosed)sets in X will as denoted be $SFPO(X)$ ($SFPC(X)$).

Theorem 3.5. *A fuzzy set $\lambda \in SFPO(X)$ if and only if there exists a fuzzy generalized open set μ such that $\lambda \leq \mu \leq Cl^*\lambda$.*

Proof. **Necessity**: If $\lambda \in SFPO(X)$, then $\lambda \leq Int^*Cl\lambda$. Put $\mu = Int^*Cl\lambda$. Then μ is fuzzy generalized open set and $\lambda \leq \mu \leq Cl^*\lambda$.
Sufficiency: Let μ be fuzzy generalized open set such that $\lambda \leq \mu \leq Cl^*\lambda$. Then $\lambda \leq \mu \leq Cl\lambda$ and $Int^*\mu = \mu$. Thus $\lambda \leq Int^*Cl\lambda$. □

Theorem 3.6. *For any fuzzy subset λ of a fuzzy space X, the following statements are equivalent:*

(1) $\lambda \in SFPC(X)$.

(2) $Cl^*Int\lambda \leq \lambda$.

(3) *There exists a fuzzy generalized closed set μ such that $Int^*\lambda \leq \mu \leq \lambda$.*

Proof. (1) $\Rightarrow$ (2) Let $\lambda \in SFPC(X)$, then $\lambda^c \in SFPO(X)$ such that $\lambda^c \leq Int^*Cl\lambda^c$. Then, $(\lambda^c)^c \leq (Int^*Cl\lambda^c)^c$. Thus, $Cl^*Int\lambda \leq \lambda$.

(2) $\Rightarrow$ (3) Suppose $Cl^*Int\lambda \leq \lambda$ and let $\mu = Cl^*Int\lambda$. Then μ is a fuzzy generalized closed set and $Int^*\lambda \leq \mu \leq \lambda$.

(3) $\Rightarrow$ (1) Let μ be a fuzzy generalized closed set such that $Int^*\lambda \leq \mu \leq \lambda$. Then $\lambda^c \leq \mu^c \leq Cl^*\lambda^c$. Thus λ^c is fuzzy supra preopen set. So λ is fuzzy supra-preclosed set. □

Theorem 3.7. *For any fuzzy subset λ of a fuzzy space X, the following statements are hold:*
(1) *If $\lambda \leq \mu \leq Cl^*\lambda$ and $\mu \in SFPO(X)$ if and only if $\lambda \in SFPO(X)$.*
(2) *If $Int^*\lambda \leq \mu \leq \lambda$ and $\mu \in SFPC(X)$ if and only if $\lambda \in SFPC(X)$.*

Proof. (1) **Necessity**: If μ is fuzzy supra-preopen set, then $\lambda \leq \mu \leq Cl^*\lambda$. Thus $\lambda \leq \mu \leq Int^*Cl\mu \leq Int^*Cl\lambda$. So $\lambda \in SFPO(X)$.
Sufficiency: Let $\lambda \in SFPO(X)$ and $\lambda \leq Int^*Cl\lambda$. Put $\mu = Int^*Cl\lambda$. Then μ is a fuzzy generalized open set. Thus, μ is fuzzy supra-preopen set such that $\lambda \leq \mu \leq Cl^*\lambda$.
(2) Easy to prove by using the same technique of proof (1). □

Theorem 3.8. (1) *The arbitrary union of fuzzy supra-preopen set is a fuzzy supra-preopen set.* (2) *The arbitrary intersection of fuzzy supra-preclosed set is a fuzzy supra-preclosed set.*

Proof. (1) Let $\{\lambda_\alpha\}$ be family of fuzzy supra-preopen set. Then, for each α, $\lambda_\alpha \leq Int^*Cl\lambda_\alpha$ and $\vee\lambda_\alpha \leq \vee(Int^*Cl\lambda_\alpha) \leq Int^*Cl(\vee\lambda_\alpha)$. Thus $\vee\lambda_\alpha$ is a fuzzy supra-preopen set.
(2) Obvious. □

Remark 3.9. The intersection (union) of fuzzy supra-preopen (supra-preclosed) sets need not be fuzzy supra-preopen (supra-preclosed) set. These are illustrated by the following example:

Example 3.10. Let $X = \{a, b\}$ and v_1, v_2, v_3 and v_4 be fuzzy sets of X defined as:

$$\begin{array}{llll} v_1(a) = 0.6, & v_1(b) = 0.4, & v_2(a) = 0.4, & v_2(b) = 0.3, \\ v_3(a) = 0.7, & v_3(b) = 0.6, & v_4(a) = 0.4, & v_4(b) = 0.8. \end{array}$$

Let $\tau = \{0_x, v_1, v_2, 1_x\}$. Clearly τ is a fuzzy topology on X.

- v_3 and v_4 are fuzzy supra-preopen sets. But $(v_3 \wedge v_4)$ is not a fuzzy supra-preopen set.
- v_3^c and v_4^c are fuzzy supra-preclosed sets. But $(v_2^c \vee v_3^c)$ is not a fuzzy supra-preclosed set.

Definition 3.11. Let λ any fuzzy set, then
(i) $SpCl\ \lambda = \wedge\{\mu : \mu \geq \lambda,\ \mu$ is a fuzzy supra-preclosed set of $X\}$ is called fuzzy supra-preclosure.
(ii) $SpInt\ \lambda = \vee\{\mu : \mu \leq \lambda, \mu$ is a fuzzy supra-preopen set of $X\}$ is called fuzzy supra-preInterior.

Theorem 3.12. *Let λ be a fuzzy set of a fts X. Then, the following properties are true:*
(1) *$(SpInt\lambda)^c = IpCl\lambda$.*
(2) *$(SpCl\lambda)^c = SpInt\lambda$.*
(3) *$SpInt\lambda \leq \lambda \wedge Int^*Cl\lambda$.*
(4) *$SpCl\lambda \geq \lambda \vee Cl^*Int\lambda$.*

Proof. We will prove only (1) and (4).

$$\begin{aligned}(1)\quad (SpInt\lambda)^c &= \big(\vee\{v : v \leq \lambda,\ v\ is\ a\ fuzzy\ supra-preopen\ set\ of\ X\}\big)^c \\ &= \wedge\{v^c : v^c \geq \lambda,\ v^c\ is\ a\ fuzzy\ supra-preclosed\ set\ of\ X\} \\ &= SpCl\lambda.\end{aligned}$$

(4) Since $\lambda \leq SpCl\lambda$ and $SpCl\lambda$ is a fuzzy supra-preclosed set, $Cl^*Int(SpCl\lambda) \leq SpCl\lambda$. Thus, $Spcl\lambda \geq \lambda \vee Cl^*Int\lambda$. □

Proposition 3.13. *Let λ and μ be the fuzzy sets in fts X and $\lambda \leq \mu$. Then, the following statements hold:*

1. *$SpInt(\lambda)$ is the largest fuzzy supra-preopen set contained in λ.*
2. *$SpInt\lambda \leq \lambda$.*
3. *$SpInt\lambda \leq SpInt\mu$.*
4. *$SpInt(SpInt\lambda) = SpInt\lambda$.*
5. *$\lambda \in FP^*0(X) \Leftrightarrow SpInt\lambda = \lambda$.*

Proposition 3.14. *Let λ and μ be the fuzzy sets in fts X and $\lambda \leq \mu$. Then, the following statements hold:*

1. *$SpCl(\lambda)$ is the smallest fuzzy supra-preclosed set containing λ.*
2. *$\lambda \leq SpCl(\lambda)$.*
3. *$SpCl\lambda \leq SpCl\mu$.*
4. *$SpCl(SpCl\lambda) = SpCl\lambda$.*
5. *$\lambda \in SFPC(X) \Leftrightarrow SpCl\lambda = \lambda$.*

Using Proposition 3.13 and 3.14, we can easily prove the next Theorem.

Theorem 3.15. *Let λ and μ be the fuzzy sets in fts X. Then the following statements hold:*

1. *$SpInt\lambda \vee SpInt\mu \leq SpInt(\lambda \vee \mu)$.*
2. *$SpInt\lambda \wedge SpInt\mu \geq SpInt(\lambda \wedge \mu)$.*
3. *$SpCl\lambda \vee SpCl\mu \leq SpCl(\lambda \vee \mu)$.*
4. *$SpCl\lambda \wedge SpCl\mu \geq SpCl(\lambda \wedge \mu)$.*

Theorem 3.16. *Let λ be a fuzzy set of a fts X. Then,*

$$Int^*\lambda \leq SpInt\lambda \leq \lambda \leq SpCl\lambda \leq Cl^*\lambda.$$

Proof. We know that $Int^*\lambda \leq \lambda$. This implies that $SpInt(Int^*\lambda) \leq SpInt\lambda$. Then,

$$SpInt(Int^*\lambda) = Int^*\lambda \quad and \quad Int^*\lambda \leq SpInt\lambda. \tag{3.1}$$

Also, we know that $\lambda \leq Cl^*\lambda$. This implies that $SpCl\lambda \leq SpCl(Cl^*\lambda)$. Then,

$$SpCl(Cl^*\lambda) = Cl^*\lambda \quad and \quad SpCl \leq Cl^*\lambda. \tag{3.2}$$

From 3.1 and 3.2, we obtain

$$Int^*\lambda \leq SpInt\lambda \leq \lambda \leq SpCl\lambda \leq Cl^*\lambda.$$

□

Theorem 3.17. *Let λ be a fuzzy set of a fts X. Then, the following statements hold:*
(1) *If λ is a fuzzy preopen (preclosed) set, then λ is a fuzzy supra-preopen (supra-preclosed) set.*
(2) *If λ is a fuzzy infra-α-open (supra-α-closed) set, then λ is a fuzzy supra-preopen (supra-preclosed) set.*
(3) *If λ is a fuzzy infra-α-open (supra-α-closed) set, then λ is a fuzzy infra-semiopen (supra-semiclosed) set.*

Proof. It is clear from Definitions 2.2, 2.3, 3.4 and basic relations. □

The following "Implication Figure 1" illustrates the relation of different classes of fuzzy open sets.

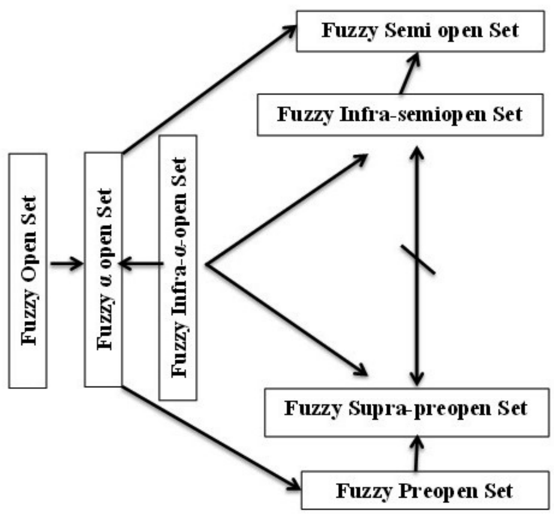

FIGURE 1.

Remark 3.18. The converse of these relations need not be true, in general as shown by the following examples.

Example 3.19. Let $X = \{a, b\}$ and v_1 and v_2 be fuzzy sets of X defined as:

$$v_1(a) = 0.3, \quad v_1(b) = 0.4,$$
$$v_2(a) = 0.3, \quad v_2(b) = 0.5.$$

Let $\tau = \{0_x, v_1, 1_x\}$. Clearly τ is a fuzzy topological space on X. By easy computation, we can see v_2 is a fuzzy supra-preopen set which is neither a fuzzy preopen set nor a fuzzy infra-α-open set.

Example 3.20. Let $X = \{a, b\}$ and v_1, v_2 and v_3 be fuzzy sets of X defined as:

$$v_1(a) = 0.3, \quad v_1(b) = 0.5,$$
$$v_2(a) = 0.6, \quad v_2(b) = 0.7,$$
$$v_3(a) = 0.7, \quad v_3(b) = 0.5.$$

Let $\tau = \{0_x, v_1, 1_x\}$. Clearly τ is a fuzzy topological space on X. By easy computation, we can see v_2 is a fuzzy supra-preopen set which is neither a fuzzy infra-semiopen set nor a fuzzy open set.

4. Fuzzy supra precontinuous mapping

Definition 4.1. A mapping $f : X \to Y$ is said to be:

(i) Fuzzy supra-precontinuous if $f^{-1}(\lambda)$ is a fuzzy supra-preopen (supra-preclosed) set in X for each fuzzy open (closed) set λ in Y.

(ii) Fuzzy supra-preirresolute if $f^{-1}(\lambda)$ is a fuzzy supra-preopen (supra-preclosed) set in X for each fuzzy supra-preopen (supra-preclosed) set λ in Y.

Theorem 4.2. *For a mapping $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

(1) *f is fuzzy supra-precontinuous.*

(2) *For every fuzzy singleton p in X and every fuzzy open set μ in Y such that $f(p) \leq \mu$, there exists a fuzzy supra preopen set λ in X such that $p \leq \lambda$ and $\lambda \leq f^{-1}(\mu)$.*

(3) *For every fuzzy singleton p in X and every fuzzy open set μ in Y such that $f(p) \leq \mu$, there exists a fuzzy supra-preopen set λ in X such that $p \leq \lambda$ and $f(\lambda) \leq \mu$.*

(4) *The inverse image of each fuzzy closed set in Y is fuzzy supra-preclosed.*

(5) *$Cl^*Int(f^{-1}(\mu)) \leq (f^{-1}(Cl\mu))$ for each μ in Y.*

(6) *$f(ClInt^*(\lambda)) \leq Cl\ f(\lambda)$ for each λ in X.*

Proof. (1)⇒(2): Let fuzzy singleton p be in X and every open set μ in Y such that $f(p) \subseteq \mu$, there exists a fuzzy open set m be in Y such that $f(p) \leq m \leq \mu$. Since f is supra-precontinuous, $\lambda = f^{-1}(m)$ is fuzzy supra-preopen. Then we have

$$p \leq f^{-1}(f(p)) \leq f^{-1}(m) \leq f^{-1}(\mu)$$

or

$$p \leq \lambda = f^{-1}(m) \leq f^{-1}(\mu).$$

(2)⇒(3): Let fuzzy singleton p be in X and every fuzzy open set μ be in Y such that $f(p) \subseteq \mu$, there exists a fuzzy supra-preopen λ such that $p \leq \lambda$ and $\lambda \leq f^{-1}(\mu)$. Then, we have $p \leq \lambda$ and $f(\lambda) \leq f^{-1}((f(\mu))) \leq \mu$.

(3)⇒(1): Let μ be a fuzzy open set in Y and let us take $p \leq f^{-1}(\mu)$. This shows that $f(p) \leq f(f^{-1}(\mu)) \leq \mu$. Since μ is a fuzzy open set, there exists a fuzzy supra-preopen set λ such that $p \leq \lambda$ and $f(\lambda) \leq \mu$. Thus $p \leq \lambda \leq f^{-1}(f(\lambda)) \leq f^{-1}(\mu)$. So $f^{-1}(\mu)$ is a fuzzy supra-preopen set in X. Hence f is fuzzy supra-precontinuous.

(1)⇒(4): Let μ be fuzzy closed in Y. This implies that $I_Y - \mu$ is a fuzzy open set. Thus $f^{-1}(I_Y - \mu)$ is fuzzy supra-preopen set in X, i.e., $(I_X - f^{-1}(\mu))$ is fuzzy supra-preopen set in X. So $f^{-1}(\mu)$ is a fuzzy supra-preclosed set in X.

(4)⇒(5): Let μ in Y. Then $f^{-1}(Cl\ \mu)$ is fuzzy supra-preclosed in X, i.e.,

$$Cl^*Int(f^{-1}(\mu)) \leq f^{-1}(Cl\ \mu).$$

(5)⇒(6): Let λ in X and let $\mu = f(\lambda)$ in Y. Then $Cl^*Int(f^{-1}(f(\lambda))) \leq f^{-1}(Cl(f(\lambda)))$. Thus $Cl^*Int\ (\lambda) \leq f^{-1}(Cl(f(\lambda)))$. So $f(Cl^*Int\ (\lambda)) \leq Cl\ f(\lambda)$.

(6)⇒(1): Let μ be a fuzzy open set in Y and let $\lambda = f^{-1}(\mu^c)$ in (6). Then

$$f(Cl^*Int(f^{-1}(\mu^c))) \leq Clf(f^{-1}(\mu^c)) \leq Cl(\mu^c) = \mu^c.$$

That is $f^{-1}(\mu^c)$ is a fuzzy supra-preclosed set in X. Thus f is a supra-precontinuous mapping. □

Using the same arguments as Theorem 4.2 and Propositions 3.13 and 3.14, one can prove the following theorem.

Theorem 4.3. *For a mapping $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

(1) *f is fuzzy supra-precontinuous.*
(2) *The inverse image of each fuzzy closed set in Y is fuzzy supra-preclosed.*
(3) *$f(SpCl(\lambda)) \leq Cl(f(\lambda))$, for each fuzzy set λ in X.*
(4) *$SpCl(f^{-1}(\mu)) \leq f^{-1}(Cl(\mu))$, for each fuzzy set μ in Y.*
(5) *$f^{-1}(Int(\mu)) \leq SpCl(f^{-1}(\mu))$, for each fuzzy set μ in Y.*

Remark 4.4. If $f : X \to Y$ is a fuzzy supra-precontinuous mapping and $g : Y \to Z$ is a fuzzy supra-precontinuous mapping, then $gof : X \to Z$ may not be a fuzzy supra-precontinuous mapping, this can be shown by the following example.

Example 4.5. Let $X = \{a, b, c\}$ and v_1, v_2, v_3, v_4 and v_5 be fuzzy sets of X defined as:

$$\begin{array}{lll} v_1(a) = 0.8, & v_1(b) = 0.7, & v_1(c) = 0.8, \\ v_2(a) = 0.7, & v_2(b) = 0.7, & v_2(c) = 0.7, \\ v_3(a) = 0.8, & v_3(b) = 0.3, & v_3(c) = 0.2, \\ v_4(a) = 0.2, & v_4(b) = 0.7, & v_4(c) = 0.3, \\ v_5(a) = 0.2, & v_5(b) = 0.3, & v_5(c) = 0.3. \end{array}$$

Consider fts τ_1, τ_2 and τ_3 where $\tau_1 = \{0_x, v_1, v_2, 1_x\}$, $\tau_2 = \{0_x, v_3, 1_x\}$ and $\tau_3 = \{0_x, v_4, 1_x\}$ and the mapping $f : (X, \tau_1) \to (X, \tau_2)$ and $g : (X, \tau_2) \to (X, \tau_3)$ defined as : $f(a) = c$, $f(b) = a$, $f(c) = a$ and $g(a) = c$, $g(b) = b$, $g(c) = a$. It is clear that f and g are fuzzy supra-precontinuous mapping. But $gof : (X, \tau_1) \to (X, \tau_3)$ is not a fuzzy supra-precontinuous mapping. This is because $(gof)^{-1}(v_3) = v_5$ and v_5 is not a fuzzy supra-preopen set and hence gof is not a fuzzy supra-precontinuous mapping.

Theorem 4.6. *If $f : X \to Y$ is a fuzzy supra-precontinuous mapping and $g : Y \to Z$ is a fuzzy continuous mapping, then $gof : X \to Z$ is a fuzzy supra-precontinuous mapping.*

Proof. Let μ be a fuzzy open set of Z. Then $(gof)^{-1}(\mu) = f^{-1}(g^{-1}(\mu))$. Since g is fuzzy continuous, $g^{-1}(\mu)$ is a fuzzy open set of Y. Thus $f^{-1}(g^{-1}(v))$ is a fuzzy supra-preopen set in X. So, gof is a fuzzy supra-precontinuous mapping. □

The following "Implication Figure 2" illustrates the relation of different classes of fuzzy continuous mappings.

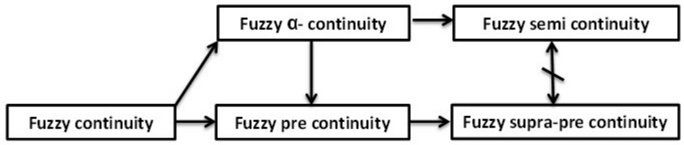

FIGURE 2.

Remark 4.7. We can see the converse of these relations need not be true in general as shown by the following example:

Example 4.8. Let $X = \{a, b\}$ and v_1, v_2 and v_3 be fuzzy sets of X defined as:

$$v_1(a) = 0.3, \quad v_1(b) = 0.4,$$
$$v_2(a) = 0.3, \quad v_2(b) = 0.5,$$
$$v_3(a) = 0.3, \quad v_3(b) = 0.3.$$

Consider fts τ_1, τ_2 and τ_3 where $\tau_1 = \{0_x, v_1, 1_x\}$, $\tau_2 = \{0_x, v_2, 1_x\}$ and $\tau_3 = \{0_x, v_3, 1_x\}$ and the mapping $f : (X, \tau_1) \to (X, \tau_2)$ and $g : (X, \tau_1) \to (Y, \tau_3)$ defined as : $f(a) = a$, $f(b) = b$ and $g(a) = a$, $g(b) = b$. It is clear that :

(1) f is a fuzzy supra-precontinuous mapping which is neither a fuzzy infra-α-continuous (fuzzy infra-semicontinuous) mapping nor a fuzzy precontinuous mapping.

(2) g is a fuzzy supra-precontinuous mapping which is not a fuzzy semicontinuous mapping.

(3) f is a fuzzy supra-precontinuous mapping which is neither a fuzzy α continuous mapping nor a fuzzy continuous mapping.

Definition 4.9. A mapping $f : X \to Y$ is said to be fuzzy supra-preopen (Fuzzy supra-preclosed) if $f(\lambda)$ is a fuzzy supra-preopen (fuzzy supra-preclosed) set in Y for each fuzzy open (fuzzy closed) set λ in X.

Theorem 4.10. *For a mapping $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

(1) *f is a fuzzy supra-preopen.*
(2) *$f(Int\lambda) \leq SpInt(f(\lambda))$, for each fuzzy set λ in X.*
(3) *$Int(f^{-1}(\mu)) \leq f^{-1}(SpInt(\mu))$, for each fuzzy set μ in Y.*
(4) *$f^{-1}(SpCl(\mu)) \leq Cl(f^{-1}(\mu))$, for each fuzzy set μ in Y.*
(5) *$f(Int\lambda) \leq Int^{*}Cl(f(\lambda))$, for each fuzzy set λ in X.*

Proof. (1)⇒(2). Let f be a fuzzy supra-preopen mapping and λ be a fuzzy set in X, $f(Int(\lambda)) \leq f(\lambda)$. Then $SpInt(f(Int(\lambda))) \leq SpIntf(\lambda)$. Thus, $f(Int(\lambda)) \leq SpIntf(\lambda)$.

(2)⇒(3). Let μ be a fuzzy set in Y. Then $f^{-1}(\mu)$ be a fuzzy set in X. We put $f^{-1}(\mu) = \lambda$ in (2). Then we get

$$f(Int(f^{-1}(\mu))) \leq SpInt(f(f^{-1}(\mu))) \leq SpInt(\mu).$$

Thus, $Int(f^{-1}(\mu)) \leq f^{-1}(SpInt(\mu))$.

(3)⇒(4). Let μ be a fuzzy set in Y. Then μ^c is a fuzzy set in Y. In (3), we put $\mu^c = \mu$. Then we get $Int(f^{-1}(\mu^c)) \leq f^{-1}(SpInt(\mu^c))$. Thus, $(Cl(f^{-1}(\mu)))^c \leq (f^{-1}(SpCl(\mu)))^c$. So, $f^{-1}(SpCl(\mu)) \leq Cl(f^{-1}(\mu))$.

(4)⇒(5). Let λ be a fuzzy set in X. Then $(f(\lambda))^c$ is a fuzzy set in Y. Using (4), we get $f^{-1}(SpCl((f(\lambda))^c)) \leq Cl(f^{-1}((f(\lambda))^c))$. Thus $(f^{-1}(SpInt(f(\lambda))))^c \leq Int(f^{-1}(f(\lambda)))^c$. So

$$Int(\lambda) \leq f^{-1}(SpInt(f(\lambda)))$$

and

$$f(Int(\lambda)) \leq SpInt(f(\lambda)) \leq Int^{*}Cl(SpInt(f(\lambda))).$$

Hence, $f(Int\lambda) \leq Int^{*}Cl(f(\lambda))$.

(5)⇒(1). Let λ be a fuzzy open set in X. By using (5), we have $f(\lambda) \leq Int^{*}Cl(f(\lambda))$. Then f is a fuzzy supra-preopen mapping. □

Corollary 4.11. *For a mapping $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

(1) *f is fuzzy supra-preclosed.*
(2) *$f(SpCl\mu) \leq Cl(f(\mu))$, for each fuzzy set μ in Y.*
(3) *$Int(f^{-1}(\mu)) \leq f^{-1}(SpInt(\mu))$, for each fuzzy set μ in Y.*
(4) *$Int(f^{-1}(\mu)) \leq f^{-1}(Int^{*}Cl(\mu))$, for each fuzzy set μ in Y.*

Now, we can generalize the definition of fuzzy connected in order to define fuzzy supra-preconnected as follows:

Definition 4.12. A fuzzy set λ in a fts (X, τ) is said to be fuzzy supra-preconnected if and only if λ cannot be expressed as the union of two fuzzy supra-preseparated sets.

Theorem 4.13. *Let $f : X \to Y$ be a fuzzy supra-precontinuous surjective mapping. If η is a fuzzy supra-preconnected subset in X, then $f(\eta)$ is fuzzy connected in Y.*

Proof. suppose that $f(\eta)$ is not supra-preconnected in Y. Then there, exists fuzzy supra-preseparated subsets λ and μ in Y such that $f(\eta) = \lambda \cup \mu$.
Since f is a fuzzy supra-precontinuous surjective mapping, $f^{-1}(\lambda)$ and $f^{-1}(\mu)$ are fuzzy supra-preopen sets in X and $\eta = f^{-1}(f(\eta)) = f^{-1}(\lambda \cup \mu) = f^{-1}(\lambda) \cup f^{-1}(\mu)$.
It is clear that $f^{-1}(\lambda)$ and $f^{-1}(\mu)$ are fuzzy supra pre separated in X. Therefore, η is not fuzzy supra-preconnected in X, which is a contradiction!!
Hence $f(\eta)$ is fuzzy supra-preconnected. □

5. Conclusion

In this work, we have presented some applications in fuzzy topological space. New class of operators and sets called fuzzy closure*, fuzzy interior * and fuzzy supra-preopen set (supra-preclosed) were introduced and its properties are studied. As an application of these new concepts, we introduced fuzzy supra precontinuous and fuzzy supra-preopen (supra-preclosed) mappings, their basic properties and their relationship with other types of them were investigated. We also defined Fuzzy supra-preconnected.

The notions of the sets and mappings in fuzzy topology were highly developed extensively in many practical engineering problems, computational fuzzy topology for fuzzy geometric design, computer-aided geometric design and mathematics science. Moreover, This study can be useful and open new windows in the study of supra preopen sets, supra-precontinuities, supra-preseparation axioms and supra-precompactness in fuzzy topological spaces.

References

[1] K. K. Azad, On fuzzy semi continuity, fuzzy almost continuity and weakly continuity, J. Math. Anal. Appl. 82 (1981) 14–32.

[2] A. S. Bin Shahna, On fuzzy strongly semi continuity and fuzzy precontinuity, Fuzzy Sets and Systems 44 (1991) 330–308.

[3] M. H. Ghanim, E. E. Kerre and A. S. Mashhour, Separation Axioms, Subspace and Sums in Fuzzy Topology, J. Math. Anal. Appl. 102 (1984) 189–202.

[4] Hakeem A. Othman, On fuzzy *sp*-open sets, Hindawi Publishing Corporation, Advances in Fuzzy Systems, 2011 (2011) 1–5.

[5] Hakeem A. Othman, On Fuzzy Infra-Semiopen sets, Punjab University Journal of Mathematics 48 (2) (2016) 1–10.

[6] Hakeem A. Othman, Strong and Weak Types of Fuzzy α-open Sets, Proceedings of the Jangjeon Mathematical Society, Accepted.

[7] T. Selvi and A. Punitha Dharani, Some new class of nearly closed and open sets, Asian Journal of Current Engineering and Maths. 1 (5) (2012) 305–307.

[8] M. K. Singal and N. Prakash, Fuzzy preopen sets and fuzzy preseparation axioms, Bull. Call. Math. Soc. 78 (1986) 57–69.

[9] L.A.Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.

Hakeem A. Othman (hakim _ albdoie@yahoo.com)
Department of Mathematics, Al-Qunfudah Center for Scientific Research,
AL-Qunfudhah University college, Umm Al-Qura University, KSA
Department of Mathematics, Rada'a College of Education and Science,
Albida University, Albida, Yemen