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New generalized difference sequence spaces of fuzzy numbers

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Abstract. The main aim of the present paper is to introduce the spaces $c_0^u(F, \Lambda, \triangle_n^m, p)$, $c^u(F, \Lambda, \triangle_n^m, p)$ and $l_\infty^u(F, \Lambda, \triangle_n^m, p)$. We examine some topological properties of these new difference sequence spaces of fuzzy numbers by using a sequence of modulus functions.

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1. Introduction

A sequence space is defined to be a linear space of real or complex sequences. Throughout the paper $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{C}$ denotes the set of non-negative integers, the set of real numbers and the set of complex numbers respectively. Let ω denote the space of all sequences (real or complex); l_∞ and c respectively, denotes the space of all bounded sequences and the space of convergent sequences.

Throughout the paper $p = (p_k)$ is a sequence of positive real numbers. The notion of paranormed sequences was studied at the initial stage by Simons [31]. It was further investigated by Ganie and Sheikh [14], Maddox [19], Tripathy and Sen [36] and many others.

Following Ruckle [26] and Maddox [19], a modulus function f is a function from $[0, \infty)$ to $[0, \infty)$ such that

(i) $f(x) = 0$ if and only if $x = 0$,
(ii) $f(x + y) \leq f(x) + f(y)$ $\forall x, y \geq 0$,
(iii) f is increasing,
(iv) f is continuous from right at $x = 0$.

The concepts of fuzzy sets and fuzzy set operations were first introduced by Zadeh [37] and subsequently several authors have discussed various aspects of the theory and applications of fuzzy sets such as fuzzy topological spaces, similarity relations and fuzzy orderings, fuzzy measures of fuzzy events, fuzzy mathematical programming. Matloka [20] introduced bounded and convergent sequences of fuzzy numbers and studied their some properties. Matloka [20] also has shown that every convergent sequence of fuzzy numbers is bounded. Later on sequences of fuzzy numbers have been discussed by Altin [2], Altinok [3], Başarir and Mursaleen [4], Bilgin [5], Chaudhury and Das [6], Çolak [7, 8, 9], Diamond and Kloeden [10], Esi [11, 12], Fang [13], Ganie and Sheikh [15, 30], Hazarika [16], Kelava [17], Nanda [22], Savaş [27, 28], Tripathy et al [32, 33, 34, 35] etc.

Let D denote the set of all closed and bounded intervals $X = [a_1, a_2]$ on the real line $\mathbb{R}$. For $X, Y \in D$ we define

$d(X,Y) = \max(|a_1 - b_1|, |a_2 - b_2|)$, where $X = [a_1, a_2]$, $Y = [b_1, b_2]$.

It is known that (D, d) is a complete metric space.

Let $I = [0, 1]$. A fuzzy real number X is a fuzzy set on $\mathbb{R}$ and is a mapping $X : \mathbb{R} \to I$ associating each real number t with its grade membership $X(t)$.

A fuzzy real number X is called convex if

$$X(t) \geq X(s) \wedge X(r) = \min(X(s), X(r)), \text{ where } s < t < r.$$

A fuzzy real number X is called normal if there exists $t_0 \in \mathbb{R}$ such that $X(t_0) = 1$.

A fuzzy real number X is called upper semi-continuous if for each $\varepsilon > 0$, $X^{-1}([0, a+\varepsilon))$ for all $a \in I$ and given $\varepsilon > 0$, $X^{-1}([0, a+\varepsilon))$ is open in the usual topology of $\mathbb{R}$. The set of all upper-semi continuous, normal, convex fuzzy numbers is denoted by $R(I)$. The α-level set of a fuzzy real number X for $0 < \alpha \leq 1$ denoted by X^α is defined by $X^\alpha = \{t \in \mathbb{R} : X(t) \geq \alpha\}$. The 0-level set is the closure of strong 0-cut.

For each $r \in \mathbb{R}$, $\bar{r} \in \mathbb{R}(I)$ is defined by

$$\bar{r} = \begin{cases} \bar{r}, & \text{if } t = r, \\ 0, & \text{if } t \neq r. \end{cases}$$

The absolute value of $|X|$ of $X \in \mathbb{R}(I)$ is defined by (see for instance Kaleva and Seikkla [17])

$$|X|(t) = \begin{cases} \max\{X(t), X(-t)\}, & \text{if } t \geq 0, \\ 0, & \text{if } t < 0. \end{cases}$$

Let $\bar{d} : \mathbb{R}(I) \times \mathbb{R}(I) \to \mathbb{R}$ be defined by

$$\bar{d}(X, Y) = \sup_{0 \leq \alpha \leq 1} d(X^\alpha, Y^\alpha).$$

Then $\bar{d}$ defines a metric on $\mathbb{R}(I)$ (Matloka [20]). The additive identity and multiplicative identity in $\mathbb{R}(I)$ are denoted by $\bar{0}$ and $\bar{1}$ respectively.

Throughout the article ω^F, c^F, c_0^F and l_∞^F denote the classes of all, convergent, null, bounded sequence spaces of fuzzy real numbers.

A fuzzy real valued sequence $\{X_n\}$ is said to be convergent to fuzzy real number X, if for $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that $\bar{d}(X_n, X) < \varepsilon$ for all $k \geq n_0$.

A fuzzy real valued sequence $\{X_n\}$ is said to be solid (normal) if $(X_k) \in E^F$ implies that $(\alpha_k X_k) \in E^F$ for all sequences of scalars (α_k) with $|\alpha_k| \leq 1$, for all $k \in \mathbb{N}$.

Let $K = \{k_1 < k_2 < ...\} \subseteq \mathbb{N}$ and E^F be a sequence space. A k-step space of E^F is a sequence space $\lambda_K^{E^F} = \{(X_{k_n}) \in \omega^F : (X_n) \in E^F\}$.

A canonical preimage of a sequence $\{X_k\} \in \lambda_K^{E^F}$ is a sequence $\{Y_n\} \in \omega^F$ defined as

$$Y_n = \begin{cases} X_n, & \text{if} \quad k \in K, \\ \bar{0}, & \quad \text{otherwise}. \end{cases}$$

A canonical preimage of a step space $\lambda_K^{E^F}$ is a set of all elements in $\lambda_K^{E^F}$, i.e., Y is in canonical preimage of $\lambda_K^{E^F}$ if and only if Y is canonical preimage of some $X \in \lambda_K^{E^F}$.

A sequence space E^F is said to be monotone if it contains the canonical preimages of its step spaces.

A sequence space E^F is said convergence free if $(Y_k) \in E^F$ whenever $(X_k) \in E^F$ and $Y_k = \bar{0}$ whenever $X_k = \bar{0}$.

The difference sequence spaces, $Z(\Delta) = \{ x = (x_k) : \Delta x \in Z\}$,where $Z = l_\infty , c$ and c_0, were studied by Kizmaz [18].

It was further generalized by Tripathy and Esi [33], as follows. Let $m \geq 0$ be an integer then $H(\Delta^m) = \{ x = (x_k) : \Delta^m x \in Z\}$, for $Z = l_\infty , c$ and c_0, where $\Delta^m x_k = x_k - x_{k+m}$. Further, in [32] Tripathy et al generalized the above notions and unified these as follows:

$$\Delta_n^m x_k = \{ x \in \omega : (\Delta_n^m x_k) \in Z\},$$

where

$$\Delta_n^m x_k = \sum_{\mu=0}^{n} (-1)^\mu \begin{pmatrix} n \\ r \end{pmatrix} x_{k+m\mu},$$

and

$$\Delta_n^0 x_k = x_k \forall\ k \in \mathbb{N}.$$

The idea of Kizmaz [18] was applied by Savaş [27, 28] for introducing the notion of difference sequences for fuzzy real numbers and study their different properties. The difference sequence space were further studies by Çolak [8, 9], Ganie et al [14, 15], Mursaleen [1, 21], Raj et al [23, 24, 25], Sharma [29] and many others.

For (a_k) and (b_k) be two sequence with complex terms and $p = (p_k) \in l_\infty$, we have the following known inequality:

$$|a_k + b_k|^{p_k} \leq K\left(|a_k|^{p_k} + |b_k|^{p_k}\right), \tag{1.1}$$

where $K = \max\{1, 2^{M-1}\}$ and $M = \sup_k p_k$.

2. Major section

Let $X = (X_k)$ be a sequence of fuzzy numbers and $\Lambda = (f_k)$ be a sequence of moduli. Let $u = (u_k)$ be a sequence such that $u_k \neq 0$ for all $k \in \mathbb{N}$. We define the following classes of difference sequences of fuzzy numbers:

$$c_0^u(F, \Lambda, \triangle_n^m, p) = \left\{ X = (X_k) : \lim_k [f_k(\bar{d}(u_k \triangle_n^m X_k, \bar{0}))]^{p_k} = 0 \right\},$$

$$c^u(F, \Lambda, \triangle_n^m, p) = \left\{ X = (X_k) : \lim_k [f_k(\bar{d}(u_k \triangle_n^m X_k, X_0))]^{p_k} = 0 \right\},$$

$$l_\infty^u(F, \Lambda, \triangle_n^m, p) = \left\{ X = (X_k) : \sup_k [f_k(\bar{d}(u_k \triangle_n^m X_k, \bar{0}))]^{p_k} < \infty \right\},$$

for some X_0 and $p = (p_k)$ is a sequence of real numbers such that $p_k > 0$ for all k and $\sup_k p_k = M < \infty$.

Note that for $m = 1 = n$, $f_k(x) = x$ and $u_k = p_k = 1$ for all $k \in \mathbb{N}$, then these spaces are reduced to $c_0(F, \triangle)$, $c(F, \triangle)$ and $l_\infty(F, \triangle)$, introduced by Mursaleen and Başarir [21]. Again if we take $m = 0$, $n = 1$, $f_k(x) = x$ and $p_k = 1$ for all $k \in \mathbb{N}$, then these spaces are respectively reduced to $c_0(F)$, $c(F)$ and $l_\infty(F)$ introduced by Nanda [22].

Theorem 2.1. *If $\bar{d}$ is a translation invariant metric, then $c_0^u(F, \Lambda, \triangle_n^m, p)$, $c^u(F, \Lambda, \triangle_n^m, p)$ and $l_\infty^u(F, \Lambda, \triangle_n^m, p)$ are closed under the operation of addition and scalar multiplication.*

Proof. As $\bar{d}$ is translation invariant metric, it implies that

$$\bar{d}(\triangle_n^m X_k + \triangle_n^m Y_k, X_0 + Y_0) \leq \bar{d}(u_k \triangle_n^m X_k, X_0) + \bar{d}(u_k \triangle_n^m Y_k, Y_0) \tag{2.1}$$

and

$$\bar{d}(u_k \triangle_n^m \lambda X_k, \lambda X_0) \leq |\lambda| \bar{d}(u_k \triangle_n^m X_k, X_0) \tag{2.2}$$

where λ is a scalar and $|\lambda| > 1$. We shall prove only for $c^u(F, \Lambda, \triangle_n^m, p)$. The others can be treated similarly. Suppose that $X = (X_k)$, $Y = (Y_k) \in c^u(F, \Lambda, \triangle_n^m, p)$. Then

$$\begin{aligned}
&[f_k(\bar{d}(u_k \triangle_n^m X_k + u_k \triangle_n^m Y_k, X_0 + Y_0))]^{p_k} \\
&\leq [f_k(\bar{d}(u_k \triangle_n^m X_k, X_0) + \bar{d}(u_k \triangle_n^m Y_k, Y_0))]^{p_k} \text{ [By (2.1)]} \\
&\leq [f_k(\bar{d}(u_k \triangle_n^m X_k, X_0)) + f_k(\bar{d}(u_k \triangle_n^m Y_k, Y_0))]^{p_k} \\
&\leq K^M [f_k(\bar{d}(u_k \triangle_n^m X_k, X_0))]^{p_k} + K^M [f_k(\bar{d}(u_k \triangle_n^m Y_k, Y_0))]^{p_k} \text{ [By (1.1)]}.
\end{aligned}$$

Thus $X+Y \in c^u(F,\Lambda,\triangle_n^m,p)$. Let $X=(X_k)\in c^u(F,\Lambda,\triangle_n^m,p)$. For $\lambda\in\mathbb{R}$, there exists an integer K such that $|\lambda|\le K$. So, by taking into account the property 2.2 and the modulus functions f_k for all $k\in\mathbb{N}$, we have

$$\begin{aligned}[f_k(\bar{d}(\lambda u_k\triangle_n^m X_k,\lambda X_0))]^{p_k} &\le [f_k|\lambda|(\bar{d}(u_k\triangle_n^m X_k,X_0))]^{p_k}\\ &\le K^M[f_k(\bar{d}(u_k\triangle_n^m X_k,X_0))]^{p_k}.\end{aligned}$$

This implies that $\lambda X\in c^u(F,\Lambda,\triangle_n^m,p)$. □

Theorem 2.2. *Let $p=(p_k)\in l_\infty$. Then the classes of sequences $c_0^u(F,\Lambda,\triangle_n^m,p)$, $c^u(F,\Lambda,\triangle_n^m,p)$ and $l_\infty^u(F,\Lambda,\triangle_n^m,p)$, are paranormed spaces, paranormed by g defined*

$$g(X)=\sup_k\left[f(\bar{d}(u_k\triangle_n^m(\alpha_k X_k),\bar{0}))\right]^{\frac{p_k}{M}},$$

where $M=\max(1,\sup_k p_k)$.

Proof. Clearly, $g(X)=g(-X)$ for all $X\in c_0^u(F,\Lambda,\triangle_n^m,p)$. Since, $\frac{p_k}{M}\le 1$ with $M\ge 1$, by Minkowski's inequality, we have

$$\begin{aligned}&[f_k(\bar{d}(u_k\triangle_n^m X_k+u_k\triangle_n^m Y_k,\bar{0}))]^{\frac{p_k}{M}}\\ &\le\left[f_k\left(\bar{d}(u_k\triangle_n^m X_k,\bar{0})+\bar{d}(u_k\triangle_n^m Y_k,\bar{0})\right)\right]^{\frac{p_k}{M}}\\ &\le\left[f_k\left(\bar{d}(u_k\triangle_n^m X_k,\bar{0})\right)\right]^{\frac{p_k}{M}}+\left[f_k\left(\bar{d}(u_k\triangle_n^m Y_k,\bar{0})\right)\right]^{\frac{p_k}{M}},\end{aligned}$$

which shows that $g(X+Y)\le g(X)+g(Y)$.

It remains to show that the scalar multiplication is continuous. For that, let β be any scalar, then by definition by g, we have

$$g(\beta X)=\sup_k(f(\bar{d}(u_k\triangle_n^m(\alpha_k X_k),\bar{0})))^{\frac{p_k}{M}}\le K_\beta^{\frac{H}{M}}g(X),$$

where K_β is an integer with $|\beta|<K_\beta$.

Taking $\beta\to 0$ for fixed X with $g(X)\ne 0$, we have by property of f and for $|\beta|<1$ that

$$[f_k\left(\bar{d}(u_k\triangle_n^m X_k,\bar{0})\right)]^{p_k}<\epsilon.$$

Since f is continuous and by taking β enough small, it follows that $g(\beta X)\to 0$ as $\beta\to 0$, which shows that the scalar multiplication is continuous and the result follows. □

Theorem 2.3. *Let $\Lambda=(f_k)$ be a sequence of moduli. Then,*

$$c_0^u(F,\Lambda,\triangle_n^m,p)\subset c^u(F,\Lambda,\triangle_n^m,p)\subset l_\infty^u(F,\Lambda,\triangle_n^m,p).$$

Proof. $c_0^u(F,\Lambda,\triangle_n^m,p)\subset c^u(F,\Lambda,\triangle_n^m,p)$ is trivial. So, let $X=(X_K)\in c^u(F,\Lambda,\triangle_n^m,p)$. Then, there is some fuzzy number X_0 such that

$$\lim_k[f_k(\bar{d}(u_k\triangle_n^m X_k,\bar{0}))]^{p_k}=0.$$

Now, from (1.1), we have

$$[f_k(\bar{d}(u_k\triangle_n^m X_k,\bar{0}))]^{p_k}\le K[f_k(\bar{d}(u_k\triangle_n^m X_k,X_0))]^{p_k}+K[f_k(\bar{d}(u_k\triangle_n^m X_k,\bar{0}))]^{p_k}.$$

As $X=(X_k)\in c^u(F,\Lambda,\triangle_n^m,p)$, we obtain $X=(X_k)\in l_\infty^u(F,\Lambda,\triangle_n^m,p)$ and this proves the result. □

Theorem 2.4. *The classes* $c^u(F, \Lambda, \triangle_n^m, p)$ *and* $l_\infty^u(F, \Lambda, \triangle_n^m, p)$ *are neither solid nor monotone (in general).*

Proof. Let $f(x) = x$, for all $x \in [0, \infty)$, $m = 2,\ n = 1$, and $u_k = 1 = p_k$ for all $k \in \mathbb{N}$ and consider the sequence Fuzzy numbers (X_k) defined by

$$X_k(t) = \begin{cases} t+1, & \text{if } -1 \le t \le 0, \\ 1-t, & \text{if } \ 0 \le t \le 1. \\ 0, & \quad \text{otherwise.} \end{cases}$$

Then clearly $(X_k) \in c^u(F, \Lambda, \triangle_n^m, p)$. For, N, a class of sequences, consider its J-step space N_j defined as follows:

If $(X_k) \in N_j$, then its canonical pre-image $(Y_k) \in N_j$ is given by

$$Y_k = \begin{cases} X_k, & \text{if } k = \text{even}, \\ \bar{0}, & \text{if } k = \text{odd}. \end{cases}$$

Then $(Y_k) \notin c^u\left(F, \triangle_1^2, p\right)$. Thus, the class of sequences $c^u\left(F, \triangle_1^2, p\right)$ is not monotone. So, it is not solid. Hence, the class of sequences $c^u(F, \triangle_n^m, p)$ is not monotone in general.

We may consider the following example:

Let $p_k = 1$, $f_k(x) = |x|$, $u_k = 1$, for all $k \in \mathbb{N}$, $m = n = 1$. Consider the sequence of fuzzy numbers $X_k = \bar{1}$ and the sequence of scalars (α_k), defined by $\alpha_k = (-1)^k$ for all $k \in \mathbb{N}$. Then, (X_k) belongs $c^u(F, \Lambda, \triangle_n^m, p)$ but $(\alpha_k Xk)$ does not belong to $c^u(F, \Lambda, \triangle_n^m, p)$. □

Theorem 2.5. *The spaces* $c_0^u(F, \Lambda, \triangle_n^m, p)$, $c^u(F, \Lambda, \triangle_n^m, p)$ *and* $l_\infty^u(F, \Lambda, \triangle_n^m, p)$ *are not symmetric in general.*

Proof. We only consider the case $c^u(F, \Lambda, \triangle_n^m, p)$. To prove the result we consider the following example:

Let $f(x) = x$, for all $x \in [0, \infty)$, $m = n = 1,\ u_k = 1 = p_k$ for all $k \in \mathbb{N}$ and consider the sequence $(X_k) = (H, N, H, N, ...) = (X_1, X_2, X_3, ...)$, where

$$X_k = \begin{cases} H, & \text{if } k = odd, \\ N, & \text{if } k = even, \end{cases}$$

and the fuzzy number H and N are defined as follows:

$$H(t) = \begin{cases} \frac{t+4}{4}, & \text{if } -4 \le t \le 0, \\ \frac{4-t}{4}, & \text{if } \ 0 \le t \le 4. \\ 0, & \quad \text{otherwise.} \end{cases}$$

and the fuzzy number N is defined by

$$N(t) = \begin{cases} \frac{t+5}{5}, & \text{if } -5 \le t \le 0, \\ \frac{5-t}{5}, & \text{if } \ 0 \le t \le 5. \\ 0, & \quad \text{otherwise.} \end{cases}$$

Thus we have

$$[H]^{\alpha} = [4\alpha - 4, 4 - 4\alpha] \quad and \quad [N]^{\alpha} = [5\alpha - 5, 5 - 5\alpha].$$

So

$$[H - N]^{\alpha} = [9\alpha - 9, 9 - 9\alpha] = [\triangle X_1]^{\alpha},$$
$$[N - H]^{\alpha} = [9\alpha - 9, 9 - 9\alpha] = [\triangle X_2]^{\alpha} \quad etc,$$

from which we conclude that $(X_k) \in c^u\left(F, \triangle_1^1, p\right)$.

We now consider the rearrangement (Y_k) of (X_k) which is defined by $(Y_k) = (H, H, N, N, H, H, N, N, ...) = (Y_1, Y_2, Y_3, ...)$. Then, as above,

$$[H - H]^{\alpha} = [8\alpha - 8, 8 - 8\alpha] = [\triangle Y_1]^{\alpha},$$
$$[H - N]^{\alpha} = [9\alpha - 9, 9 - 9\alpha] = [\triangle Y_2]^{\alpha},$$
$$[N - N]^{\alpha} = [10\alpha - 10, 10 - 10\alpha] = [\triangle Y_3]^{\alpha} \quad etc.$$

Thus it follows that $(Y_k) \notin c^u\left(F, \triangle_1^1, p\right)$. So, the class of sequences $c^u\left(F, \Lambda, \triangle_n^m, p\right)$ is not symmetric, and the result follows.

Alternatevily, we may consider the following example:

Let $p_k = 1, f_k(x) = |x|,\ u_k = 1$, for all $k \in \mathbb{N},\ m = n = 1$. consider the sequence of fuzzy numbers $X_k = \bar{k}$, for all $k \in \mathbb{N}$. Consider the rearranged sequence (Y_k) of (X_k), defined by $(Y_k) = (X_1, X_2, X_4, X_3, X_9, X_5, X_{16}, ...)$. Then the sequence (X_k) belongs to $c^u\left(F, \Lambda, \triangle_n^m, p\right)$ but the rearranged sequence (Y_k) does not.

□

3. Conclusions

We have introduced the spaces $c_0^u\left(F, \Lambda, \triangle_n^m, p\right), c^u\left(F, \Lambda, \triangle_n^m, p\right)$ and $l_\infty^u\left(F, \Lambda, \triangle_n^m, p\right)$ and have shown them to be paranormed spaces . Also, we have given some topological properties of these new difference sequence spaces of fuzzy numbers by using a sequence of modulus functions. Moreover, we have shown them that they are not monotone and symmetric in general.

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