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On fuzzy H-closedness of fuzzy topological spaces (Part-2)

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Abstract. Afsan [6] has introduced the fuzzy covering axioms weakly fuzzy H-closedness and fuzzy $H^\star$-closed which are "good extension" of H-closed spaces. The main purpose of this paper is to study some other types of fuzzy covering axioms which are also good extension of H-closed spaces. In fact, we have introduced and investigated several properties of strongly fuzzy H-closed and fuzzy ultra-H-closed spaces. We have achieved several characterizations of these notions via. α-net [11], α-filter [11] and fuzzy upper weak-θ-limits [6], fuzzy upper $\mathcal{I}$-θ-limits [6] of fuzzy nets of fuzzy closed sets. We have also shown that strongly fuzzy H-closedness is preserved under fuzzy θ-continuous closed surjection. Besides these, we have established the mutual relationship among these fuzzy covering axioms and weakly fuzzy H-closed spaces [6]. We have shown that every fuzzy regular strongly fuzzy H-closed (resp. fuzzy ultra-H-closed) is strongly fuzzy compact [14] (resp. fuzzy ultra-compact [14]).

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1. Introduction

The fuzzy mathematics started its journey by Zadeh [20], when he published his famous paper "Fuzzy sets" in 1965 and now it becomes a vast in collaborate with its different branches. On of its important branch is fuzzy topology which was first introduced by Chang just after three years later of Zadeh. Among several mathematician Lowen was taken most important role in developing this field of fuzzy mathematics and today it is a matured field in mathematical activities. Fuzzy compactness was introduced by Chang which is not a "good extension" of compactness of ordinary topological space and it fails to satisfy most important properties of compactness of ordinary topology. Then Lowen introduced the notion of a new fuzzy covering axiom, namely weak fuzzy compactness which is a "good extension" of compactness of ordinary topology. Also, strong fuzzy compactness and ultra-fuzzy compactness were introduced by Lowen which are also "good extension" of compactness of ordinary topology. Some recent research works related to fuzzy compactness are found in the study of Afsan [1, 2, 3, 4, 5, 6] and Tok [19]. H-closedness is an important covering property of ordinary topology introduced by by closure operators.

To fuzzyfy The celebrated notion H-closedness of ordinary topology the notions almost compact fuzzy sets [15], fuzzy almost compact space[9] and θ-compact [7] were invented, but none of them is a "good extension" of H-closed topological space. Observing this serious drawback, Afsan [6] introduced the fuzzy covering axioms weak fuzzy H-closedness and fuzzy $H^\star$-closed which are "good extension" of H-closed spaces.

The purpose of the present paper is to continue the study of some other types of fuzzy covering axioms which are also "good extension" of H-closed spaces. In section 3, we have introduced the fuzzy covering property strong fuzzy H-closedness along with its several characterizations in terms of θ-cluster point [6] of α-net [11], α-filter [11] and fuzzy upper weak-θ-limits [6], fuzzy upper $\mathcal{I}$-θ-limits [6] of fuzzy net of fuzzy closed sets. We have also shown that strong fuzzy H-closedness is preserved under fuzzy θ-continuous closed surjection. We have established the examples which show that the class of strongly fuzzy H-closed spaces are properly contained in the class of weakly fuzzy H-closed spaces [6] and the class of strongly fuzzy compact spaces [14] are properly contained in the class of strongly fuzzy H-closed spaces. It has been shown that fuzzy regular strongly fuzzy H-closed spaces are strongly fuzzy compact [14]. In section 4, we have initiated another fuzzy covering axiom fuzzy ultra-H-closedness along with its characterizations in terms of α-net [11], α-filter [11] and the fuzzy net of fuzzy closed sets. We have established that every fuzzy ultra-H-closed space is strongly fuzzy H-closed space, but converse is not true in general. We have also shown that fuzzy ultra-compact spaces [14] are fuzzy ultra-H-closed, but in presence of fuzzy regularity, these two notions are equivalent.

2. Preliminaries

Throughout this paper, spaces (X, σ) and (Y, δ) (or simply X and Y) represent non-empty fuzzy topological spaces due to Chang [8] and the symbols I and I^X have been used for the unit closed interval $[0, 1]$ and the set of all functions with domain X and codomain I respectively. The support of a fuzzy set A is the set $\{x \in X : A(x) > 0\}$ and is denoted by $supp(A)$. A fuzzy set with only non-zero value $\lambda \in (0, 1]$ at only one element $x \in X$ is called a fuzzy point and is denoted by x_λ and the set of all fuzzy points of a fuzzy topological space is denoted by $FP(X)$. For any two fuzzy sets A, B of X, $A \leq B$ if and only if $A(x) \leq B(x)$ for all $x \in X$. A fuzzy point x_λ is said to be in a fuzzy set A (denoted by $x_\lambda \in A$) if $x_\lambda \leq A$, that is, if $\lambda \leq A(x)$. The constant fuzzy set of X with value $\alpha \in I$ is denoted by $\underline{\alpha}$. A fuzzy set A is said to be quasi-coincident with B (written as $A\hat{q}B$) [17] if $A(x) + B(x) > 1$ for some $x \in X$. A fuzzy open set A of X is called fuzzy quasi-neighborhood of a fuzzy point x_λ if $x_\lambda \hat{q} A$.

It is well-known that a function $\psi : X \to Y$ is fuzzy θ-continuous [16] if for every fuzzy point x_λ and every fuzzy quasi-neighborhood V of a fuzzy point $\psi(x_\lambda)$, there exists a fuzzy quasi-neighborhood U of a fuzzy point x_λ such that $\psi(cl(U)) \leq cl(V)$.

Throughout the paper, $\mathcal{D}$ stands for a directed set. An ideal on a non-empty set S is defined as a non-empty family $\mathcal{I}$ of subsets of S satisfying:

(i) $\varnothing \in \mathcal{I}$,

(ii) $A \in \mathcal{I}$ and $B \subset A \Rightarrow B \in \mathcal{I}$ and

(iii) $A \in \mathcal{I}$ and $B \in \mathcal{I} \Rightarrow A \cup B \in \mathcal{I}$.

An ideal $\mathcal{I}$ on $\mathcal{D}$ is called non-trivial if $\mathcal{D} \notin \mathcal{I}$. A non-trivial ideal $\mathcal{I}$ on $\mathcal{D}$ is called admissible [10] if $\mathcal{D} - M_\lambda \in \mathcal{I}$, where $M_\lambda = \{n \in \mathcal{D} : n \geq \lambda\}$ for all $\lambda \in \mathcal{D}$. Throughout this paper, $\mathcal{I}$ stands for an admissible ideal on $\mathcal{D}$.

Lowen [13] defined the notion of prefilter. A non-empty collection $\mathcal{F}$ of non-zero fuzzy subsets of X is called a prefilter(=filter) if

(i) $\mu \wedge \nu \in \mathcal{F}$ for all $\mu, \nu \in \mathcal{F}$ and

(ii) $\mu \leq \nu$ and $\mu \in \mathcal{F}$ implies $\nu \in \mathcal{F}$.

A non-empty collection $\mathcal{B}$ of non-zero fuzzy subsets of X is called a filterbase if for all $\mu, \nu \in \mathcal{F}$, there exists an $\eta \in \mathcal{F}$ such that $\eta \leq \mu \wedge \nu$. Clearly, the family $< \mathcal{B} >= \{\nu \in I^X : \mu \leq \nu, \mu \in \mathcal{B}\}$ is a filter.

A prefilter $\mathcal{F}$ is called prime if $\mu \vee \nu \in \mathcal{F}$ implies either $\mu \in \mathcal{F}$ or $\nu \in \mathcal{F}$. The collection all prime prefilters finer than a given prefilter $\mathcal{F}$ is denoted by $\mathcal{P}(\mathcal{F})$. The set of minimal elements of $\mathcal{P}(\mathcal{F})$ is denoted by $\mathcal{P}_m(\mathcal{F})$.

A filter $\mathcal{G}$ on X and a prefilter $\mathcal{F}$ on X are said to be compatible if $\mu(x) \neq 0$ for some $x \in F$ for all $\mu \in \mathcal{F}$ and $F \in \mathcal{G}$. Suppose μ_F be the fuzzy set on X defined by $\mu_F(x) = \mu(x)$ for all $x \in F$ and $\mu_F(x) = 0$ for all $x \in X - F$. We use the symbol $(\mathcal{F}, \mathcal{G}) =< \{\mu_F : \mu \in \mathcal{F}, F \in \mathcal{G}\} >$. Lowen [13] shaw that $\mathcal{P}_m(\mathcal{F}) = \{(\mathcal{F}, \mathcal{U}) : \mathcal{U}$ is an ultrafilter on X and is compatible with $\mathcal{F}\}$. The adherence of a prefilter $\mathcal{F}$ on X is the fuzzy set $ad\mathcal{F}$ defined by $ad\mathcal{F}(x) = \inf\{cl(\mu)(x) : \mu \in \mathcal{F}\}$ for all $x \in X$ and the limit of $\mathcal{F}$ is the fuzzy set $\lim \mathcal{F}$ defined by $\lim \mathcal{F}(x) = \inf\{ad\mathcal{B}(x) : \mathcal{B} \in \mathcal{P}_m(\mathcal{F})\}$ for all $x \in X$.

Let $\mathcal{F}$ be a prefilter on X and $\mu \in I^X$. Then the characteristic set of $\mathcal{F}$ with respect to μ is the set $\mathcal{C}^\mu(\mathcal{F}) = \{a \in I : \forall \nu \in \mathcal{F}, \exists x \in X$ such that $\nu(x) > \mu(x) + a\}$ and the spermium of this set is called the characteristic value of $\mathcal{F}$ with respect to μ is the set $c^\mu(\mathcal{F})$. A characteristic set is one of the form $\varnothing$, $\{0\}$, $[0, c]$ for some $c \in I - \{1\}$ or $[0, c[$ for some $c \in I$. We use the following notations:

(i) $\mathcal{W}(X)$, the set of all pefilters on X,

(ii) $\mathcal{W}^\mu(X) = \{\mathcal{F} \in \mathcal{W}(X) : \mathcal{C}^\mu(\mathcal{F}) \neq \varnothing\}$,

(iii) $\mathcal{W}^\mu_+(X) = \{\mathcal{F} \in \mathcal{W}(X) : c^\mu(\mathcal{F}) > 0\}$ and

(iv) $\mathcal{W}^\mu_K(X) = \{\mathcal{F} \in \mathcal{W}(X) : c^\mu(\mathcal{F}) = K\}$, where K is some nonempty characteristic set.

A member of $\mathcal{W}^\mu_K(X)$ is called K-filter. A prefilter $\mathcal{F}$ is called ϵ-prefilter if $\mu \not\lesssim \epsilon$ for each $\mu \in \mathcal{F}$.

Definition 2.1 ([6])**.** Let $\{A_n : n \in \mathcal{D}\}$ be a net of fuzzy sets of a fuzzy topological space X. The fuzzy upper weak-θ-limit of $\{A_n : n \in \mathcal{D}\}$ is defined and denoted by $FUWL_\theta(A_n) = \vee\{x_\lambda \in FP(X) :$ for every fuzzy quasi-neighborhood U of x_λ and for every $n_0 \in \mathcal{D}$, there exists an $n(\geq n_0) \in \mathcal{D}$ such that $int(A_n)\hat{q}cl(U)\}$.

Definition 2.2 ([6]). Let $\{A_n : n \in \mathcal{D}\}$ be a fuzzy net of fuzzy sets of a fuzzy topological space X. Then the fuzzy upper $\mathcal{I}$-θ-limit of $\{A_n : n \in \mathcal{D}\}$ is defined and denoted by $FIUL_\theta(A_n) = \vee\{x_\lambda \in FP(X)$: for every fuzzy quasi-neighborhood U of x_λ, $\{n \in \mathcal{D} : A_n \hat{q} cl(U)\} \notin \mathcal{I}\}$.

Definition 2.3 ([6]). A fuzzy subset $\eta \in I^X$ is called weakly fuzzy H-set if for each family $\{\mu_\alpha \in \delta : \alpha \in \Delta\}$ satisfying $\bigvee\{\mu_\alpha \in \delta : \alpha \in \Delta\} \geq \eta$ and for each $\epsilon > 0$, there exist finite number of indices $\alpha_1, \alpha_2, ..., \alpha_p \in \Delta$ such that $\bigvee_{i=1}^{p} cl(\mu_{\alpha_i}) \geq \underline{\eta - \epsilon}$. If $\eta = \underline{1}$ and η is a weakly fuzzy H-set, then X is called a weakly fuzzy H-closed space.

Every weakly fuzzy compact space due to Lowen [12] is weakly fuzzy H-closed.

Definition 2.4 ([6]). [6] A fuzzy topological space X is called fuzzy $H^\star$-closed if for each $\alpha \in (0, 1]$, for each $\beta \in (0, \alpha)$ and for each family $\mathcal{U}_{\alpha\beta}$ of fuzzy open sets with the property that $\bigvee \mathcal{U}_{\alpha\beta} \geq \underline{\alpha}$, there exists finite subfamily $\mathcal{U}^0_{\alpha\beta}$ satisfying $\bigvee cl(\mathcal{U}^0_{\alpha\beta}) \geq \underline{\alpha - \beta}$.

Clearly, every fuzzy $H^\star$-closed space is weakly fuzzy H-closed.

3. Strongly fuzzy H-closed fuzzy topological spaces

Definition 3.1. A fuzzy topological space X is called strongly fuzzy H-closed if for each $\alpha \in [0, 1)$ and each family $\mathcal{U}_\alpha$ with the property that for each $x \in X$, there exists an $U \in \mathcal{U}_\alpha$ satisfying $U(x) > \alpha$, has finite subfamily $\mathcal{U}^0_\alpha$ with the property that for each $x \in X$, there exists an $U \in \mathcal{U}^0_\alpha$ satisfying $cl(U)(x) > \alpha$.

Theorem 3.2. *Every strongly fuzzy H-closed space is weakly fuzzy H-closed.*

Proof. Let X be a strongly fuzzy H-closed space. Let $\epsilon \in (0, 1)$ and $\Sigma = \{U_j : j \in \Delta\}$ be a fuzzy open cover of X. Let $\alpha \in (1 - \epsilon, 1)$. Since for each $x \in X$, $\sup\{U_j(x) : j \in \Delta\} = 1 > \alpha$, for each $x \in X$, there exists a $j \in \Delta$ such that $U_j(x) > \alpha$. Since X is strongly fuzzy H-closed, there exists finite number of indices $j_1, j_2, ..., j_n \in \Delta$ such that for each $x \in X$, there exists an $i \in \{1, 2, ..., n\}$ such that $cl(U_{j_i})(x) > \alpha > 1 - \epsilon$ and so $\bigvee_{i=1}^{n} cl(U_{j_i}) > 1 - \epsilon$. Then X is weakly fuzzy H-closed. □

Let (X, δ) be a fuzzy topological space. Then for each $\alpha \in [0, 1)$, $\iota_\alpha(\delta) = \{\mu^{-1}(\alpha, 1] : \mu \in \delta\}$ is a topology on X [13]. The topology generated by the subbase $\{\mu^{-1}(\alpha, 1] : \mu \in \delta, \alpha \in [0, 1]\}$ is denoted by $\iota(\delta)$. It is obvious that $\iota(\delta) = \sup\{\iota_\alpha(\delta) : \alpha \in [0, 1)\}$.

Following example shows that weakly fuzzy H-closed spaces may not be strongly fuzzy H-closed.

Example 3.3. Let $X = I$, $\sigma_1 = \{\chi_x : x \in X - \mathbb{Q}\}$ and $\sigma_2 = \{\nu^s_x = \frac{s}{q} + \frac{1}{q}\chi_x : x = \frac{p}{q} \in X \cap \mathbb{Q}, s \in \mathbb{N}, 0 \leq s \leq q\}$ where χ_x, is characteristic function of $\{x\}$. Consider the fuzzy topology δ generated by $\sigma_1 \cup \sigma_2 \cup \{\alpha : \alpha$ is constant$\}$. Then Lowen [14] shaw that (X, δ) is a fuzzy compact and thus weakly fuzzy compact. So, (X, δ) is weakly fuzzy H-closed. Also $(X, \iota_\alpha(\delta))$ is a discrete space for each $\alpha \in [0, 1)$. If possible, let (X, δ) be strongly fuzzy H-closed. Suppose an $\alpha \in [0, 1)$ and $\mathcal{U}_\alpha$ with the property that for each $x \in X$, there exists an $U \in \mathcal{U}_\alpha$ satisfying $U(x) > \alpha$. Then there exists a finite subfamily $\mathcal{U}_\alpha^0$ with the property that for each $x \in X$, there exists an $U \in \mathcal{U}_\alpha^0$ satisfying $cl(U)(x) > \alpha$. We observe that $\{U^{-1}(\alpha, 1] : U \in \mathcal{U}_\alpha\} = X$. And since (X, δ) is strongly fuzzy H-closed, $\{U^{-1}(\alpha, 1] : U \in \mathcal{U}_\alpha^0\} \subset \{(cl(U))^{-1}(\alpha, 1] : U \in \mathcal{U}_\alpha^0\} = X$. Thus $(X, \iota_\alpha(\delta))$ is compact which is not possible.

Theorem 3.4. *A fuzzy topological space (X, τ) is H-closed if and only if $(X, \omega(\tau))$ is strongly fuzzy H-closed.*

Proof. Let $(X, \omega(\tau))$ be a strongly fuzzy H-closed space. Since every strongly fuzzy H-closed space is weakly fuzzy H-closed, by Theorem 3.2, (X, τ) is H-closed.

Conversely, let (X, τ) be a H-closed space, $\alpha \in [0, 1)$ and $\mathcal{U}_\alpha$ be a family of fuzzy open sets of $(X, \omega(\tau))$ with the property that for each $x \in X$, there exists an $U \in \mathcal{U}_\alpha$ satisfying $U(x) > \alpha$. Define $\Omega(U) = \{x \in X : U(x) > \alpha\}$ for each $U \in \mathcal{U}_\alpha$. Then $\{\Omega(U) : U \in \mathcal{U}_\alpha\}$ is an open cover of X. Since X is H-closed, there exist finite number of sets $U_1, U_2, ..., U_p \in \mathcal{U}_\alpha$ such that $\bigcup_{i=1}^{p} cl(\Omega(U_i)) \supset X$. Now consider the family $\mathcal{U}_\alpha^0 = \{U_1, U_2, ..., U_p\}$. If possible, let there exists an $x \in X$ such that $cl(U_i)(x) \leq \alpha$ for each $i = 1, 2, ..., p$. We shall show that $x \notin cl(\Omega(U_i))$ for each $i = 1, 2, ..., p$. We fix an $i \in \{1, 2, ..., p\}$. we can select a real number λ such that $cl(U_i)(x) < \lambda \leq \alpha$. Since $\underline{1} - cl(U_i) \in (X, \omega(\tau))$,

$$U = \{x \in X : \underline{1} - cl(U_i)(x) > 1 - \lambda\} = \{x \in X : cl(U_i)(x) < \lambda\}$$

is an open set in (X, τ) containing x and $U \cap \Omega(U_i) = \varnothing$. Thus $x \notin cl(\Omega(U_i))$. □

Remark 3.5. (1) Every fuzzy almost compact space due to A. Di Concilio and G. Gerla [9] is strongly fuzzy H-closed.

(2) Every strongly fuzzy compact space due to Lowen [12] is strongly fuzzy H-closed.

Example 3.6. (1) Let $X = [0, 1]$ be the topological space equipped with the subspace usual topology τ inherited from the usual Euclidean space $\mathbf{R}$. Then (X, τ) is a H-closed space. Then by Theorem 3.4, $(X, \omega(\tau))$ is strongly fuzzy H-closed. But the space $(X, \omega(\tau))$ is not fuzzy almost compact as the fuzzy clopen cover $\{\mu_\alpha : \alpha \in [0, 1]\}$, $\mu_\alpha(x) = \alpha$ for all $x \in X$, has no finite fuzzy subcover for X.

(2) Let the topological space $Z = \{(\frac{1}{p}, \frac{1}{q}) : p \in \mathbf{N}, \mid q \mid \in \mathbf{N}\} \cup \{(\frac{1}{p}, 0) : p \in \mathbf{N}\}$ equipped with the subspace usual topology inherited from the usual Euclidean space $\mathbf{R}^2$. Let the space $X = Z \cup \{s, t\}$ with the topology τ in which a subset $U \subset X$ is declare open if $U \cap Z$ is open in Z and $\{(\frac{1}{p}, \frac{1}{q}) : p \geq r, q \in \mathbf{N}\} \subset U$ for some $r \in \mathbf{N}$ when $s \in U$ and $\{(\frac{1}{p}, \frac{1}{q}) : p \geq r, -q \in \mathbf{N}\} \subset U$ for some $r \in \mathbf{N}$ when $t \in U$. Then (X, τ) is non-compact H-closed space [18]. Thus $(X, \omega(\tau))$ is strongly fuzzy H-closed, but not strongly fuzzy compact.

Theorem 3.7. *Let X be fuzzy regular. Then X is strongly fuzzy H-closed if and only if X is strongly fuzzy compact.*

Proof. The proof is analogous to the proof of Theorem 3.7 [6]. □

Theorem 3.8. *A fuzzy topological space X is strongly fuzzy H-closed if and only if for each $\alpha \in [0, 1)$ and for every net $\{F_n : n \in \mathcal{D}\}$ of fuzzy closed sets with $FUWL_\theta(F_n) < \underline{1 - \alpha}$, there exists an $n_0 \in \mathcal{D}$ such that $int(F_n) < \underline{1 - \alpha}$ for all $n \geq n_0$.*

Proof. Let X be a strongly fuzzy H-closed space, $\alpha \in [0,1)$ and $\{F_n : n \in \mathcal{D}\}$ be a net of fuzzy closed sets with $FUWL_\theta(F_n) < \underline{1-\alpha}$. Then for each fuzzy point x_λ of X, there exist $U_{x_\lambda} \in \mathcal{Q}(X, x_\lambda)$ for which $FUWL_\theta(F_n) < x_\lambda \leq \underline{1-\alpha}$ and an $n(x_\lambda)$ such that $int(F_n)\bar{q}cl(U_{x_\lambda})$, i.e. $cl(U_{x_\lambda}) \leq \underline{1} - int(F_n)$ for all $n \geq n(x_\lambda)$. Since $U_{x_\lambda} \in \mathcal{Q}(X, x_\lambda)$, $U_{x_\lambda}(x) > 1 - \lambda \geq \alpha$. Consider the family $\mathcal{U}_\alpha = \{U_{x_\lambda} : x_\lambda \in Pt(X), FUWL_\theta(F_n) < x_\lambda \leq \underline{1-\alpha}\}$. Now since X is strongly fuzzy H-closed, there exists a finite number of points $x^1_{\lambda_1}, x^2_{\lambda_2}, ..., x^n_{\lambda_n} \in Pt(X)$ such that for each $x \in X$, there exists $i \in \{1, 2, ...n\}$ such that $cl(U_{x^i_{\lambda_i}})(x) > \alpha$ and thus $\bigvee\{cl(U_{x^i_{\lambda_i}}) : i = 1, 2, ..., n\} > \underline{\alpha}$. Consider $n_0 \in \mathcal{D}$ such that $n_0 \geq n(x^i_{\lambda_i})$ for each $i \in \{1, 2, ...n\}$. So, $\underline{1} - int(F_n) \geq \bigvee\{cl(U_{x^i_{\lambda_i}}) : i = 1, 2, ..., n\} > \underline{\alpha}$ for all $n \geq n_0$. Hence $int(F_n) < \underline{1-\alpha}$ for all $n \geq n_0$.

Conversely, let the condition of the theorem holds. Suppose $\alpha \in [0,1)$ and $\mathcal{U}_\alpha \subset \delta$ be a family with the property that for each $x \in X$, there exists an $U \in \mathcal{U}_\alpha$ satisfying $U(x) > \alpha$. Let $\mathcal{D} = \{n \subset \mathcal{U}_\alpha :| n |< \aleph_0\}$ and the relation "$\leq$" defined by $n_1 \leq n_2$ if and only if $n_2 \subset n_1$. Then $(\mathcal{D}, \leq)$ is a directed set. Consider the fuzzy net $\{F_n : n \in \mathcal{D}\}, F_n = \bigwedge_{U \in n} U'$. We claim that $FUWL_\theta(F_n) < \underline{1-\alpha}$. Here we note that $F_{n_1} \leq F_{n_2}$ if $n_2 \leq n_1$. Let $x_\lambda \notin \bigwedge\{F_n : n \in \mathcal{D}\}$. Then there exists an $n_0 \in \mathcal{D}$ such that $x_\lambda \notin F_{n_0}$. Thus there exists $U \in \mathcal{Q}(X, x_\lambda)$ such that $U\bar{q}F_{n_0}$, i.e., $U \leq \underline{1 - F_{n_0}}$, i.e., $cl(U)\bar{q}int(F_{n_0})$. So, $cl(U)\bar{q}int(F_n)$ for all $n \geq n_0$ and thus $x_\lambda \notin \underline{FUWL_\theta(F_n)}$. Hence $FUWL_\theta(F_n) \leq \bigwedge\{F_n : n \in \mathcal{D}\} = \bigwedge\{U' : U \in \mathcal{U}_\alpha\} < \underline{1-\alpha}$. By the condition, there exists an $n_0 \in \mathcal{D}$ such that $int(F_n) < \underline{1-\alpha}$ for all $n \geq n_0$. Then, $\underline{\alpha} < 1 - int(F_{n_0}) = 1 - \bigwedge_{U \in n_0} int(U') = \bigvee_{U \in n_0} cl(U)$. Thus the family $\{U : U \in n_0\}$ fulfil the requirement for being X strongly fuzzy H-closed. □

Theorem 3.9. *Let X be a strongly fuzzy H-closed topological space. Then for each $\alpha \in [0,1)$, for each fuzzy net $\{F_n : n \in \mathcal{D}\}$ of fuzzy closed sets and for every ideal $\mathcal{I}$ on $\mathcal{D}$ with $FIUL_\theta(F_n) \leq \underline{1-\alpha}$, $\{n \in \mathcal{D} : F_n \not\leq \underline{1-\alpha}\} \in \mathcal{I}$.*

Proof. Let X be a strongly fuzzy H-closed space and $\{F_n : n \in \mathcal{D}\}$ be a fuzzy net of fuzzy closed sets, $\mathcal{I}$ be an ideal on $\mathcal{D}$ with $FIUL_\theta(F_n) \leq \underline{1-\alpha}$ and an $\alpha \in [0,1)$. Then for each fuzzy point x_λ of X satisfying $FIUL_\theta(F_n) < x_\lambda \leq \underline{1-\alpha}$, there exists a fuzzy quasi-neighborhood U_{x_λ} of x_λ such that $\{n \in \mathcal{D} : F_n\hat{q}cl(U_{x_\lambda})\} \in \mathcal{I}$. Since X is strongly fuzzy H-closed and $\{U_{x_\lambda} : x_\lambda \in FP(X), FIUL_\theta(F_n) < x_\lambda \leq \underline{1-\alpha}\}$ satisfies the property that for each $x \in X$, $U_{x_\lambda}(x) > 1 - \lambda \geq \alpha$, there exist finite number of fuzzy points $e_1, e_2, ..e_p \in FP(X)$ such that for each $x \in X$, $cl(U_{e_i})(x) > \alpha$ for some $i \in \{1, 2, ..p\}$. Here $\{n \in \mathcal{D} : F_n\hat{q} \vee_{i=1}^p cl(U_{e_i})\} = \bigcup_{i=1}^p \{n \in \mathcal{D} : F_n\hat{q}cl(U_{e_i})\} \in \mathcal{I}$. Since $\{n \in \mathcal{D} : F_n \not\leq \underline{1-\alpha}\} \subset \{n \in \mathcal{D} : F_n\hat{q} \vee_{i=1}^p cl(U_{e_i})\}$, $\{n \in \mathcal{D} : F_n \not\leq \underline{1-\alpha}\} \in \mathcal{I}$. □

Definition 3.10 ([6])**.** Let $\mathcal{S} = \{s_k : k \in D\}$ be a fuzzy net in a fuzzy topological space X and $x_\lambda \in Pt(X)$.

(i) x_λ is called a θ-cluster point of a fuzzy net $\mathcal{S}$ if for each $F \in \mathcal{R}(X, x_\lambda)$ and for each $k \in D$, there exists $k_0 \in D$ such that $k_0 \geq k$ and $s_{k_0} \notin int(F)$. We write $\Theta(\mathcal{S}) = \vee\{x_\lambda \in Pt(X) : x_\lambda$ is a θ-cluster point of a fuzzy net $\mathcal{S}\}$.

(ii) x_λ is called a θ-limit point of a fuzzy net $\mathcal{S}$ if for each $F \in \mathcal{R}(X, x_\lambda)$, there exists $k_0 \in D$ such that $s_k \notin int(F)$ for all $k \geq k_0$.

Recall that a fuzzy net $\mathcal{S} = \{s_k : k \in D\}$ is called an α-net [11], $\alpha \in (0, 1]$ if the net (called value net) $\hat{\mathcal{S}} = \{\lambda_k : k \in D\}$, where λ_k is the value of the fuzzy point x_k converges to α.

Theorem 3.11. *For a fuzzy topological space X, following conditions are equivalent:*
(1) *X is strongly fuzzy H-closed.*
(2) *Every constant α-net has a fuzzy θ-cluster point with value $\alpha \in (0, 1]$.*

Proof. (1)⇒(2): Let X be strongly fuzzy H-closed space. If possible, let $\mathcal{S} = \{s_k : k \in D\}$ be a constant α-net which has no fuzzy θ-cluster point with value $\alpha \in (0, 1]$. Then for each $x \in X$, there exist $k_x \in D$ and $V_x \in \mathcal{R}(X, x_\alpha)$ such that $s_k \in int(V_x)$ for all $k \geq k_x$. Consider the family $\Sigma = \{V'_x : x \in X\}$. Then for each $x \in X$, there exists $V'_x \in \Sigma$ such that $V_x(x) < \alpha$, i.e., $V'_x(x) > 1 - \alpha$. Since X is strongly fuzzy H-closed space, there exists finite number of points $x^1, x^2, ..., x^n \in X$ such that for each $x \in X$, there exists $i \in \{1, 2, ..., n\}$ such that $cl(V_{x'_i})(x) > 1-\alpha$, i.e., $int(V_{x'_i})(x) < \alpha$. Consider a $k_0 \in D$ with the property $k_0 \geq k_{x_i}$ for each $i \in \{1, 2, ..., n\}$. Then for each $i \in \{1, 2, ..., n\}$, $s_k \in int(V_{x_i})$ for all $k \geq k_0$. Suppose $supp(s_k) = x^k$ for each $k \in D$. Then we get $int(V_{x_i})(x^k) > \alpha$ for all $k \geq k_0$ and for all $i \in \{1, 2, ..., n\}$, which is a contradiction.

(2)⇒(1): Let X be not a strongly fuzzy H-closed space. Then there exists an $\varepsilon(= 1 - \alpha) \in [0, 1)$ and a family $\mathcal{U}$ of fuzzy open sets of X with the property that for each $x \in X$, there exists an $U \in \mathcal{U}$ satisfying $U(x) > \varepsilon$ such that for each finite subfamily $\mathcal{U}^0$, there exists an $x \in X$ with the property that $cl(U)(x) \leq \varepsilon$, i.e., $int(U')(x) \geq \alpha$ for each $U \in \mathcal{U}^0$. For each finite subfamily $\mathcal{U}^0$ of $\mathcal{U}$, let $int(\mathcal{U}'^0) = \wedge\{int(U') : U \in \mathcal{U}\}$. Now consider the set $D = \{\mathcal{U}'^0 : \mathcal{U}^0$ is a finite subfamily of $\mathcal{U}\}$. Then D becomes to a directed by the order " $\geq$ " defined by $\mathcal{U}'^0 \geq \mathcal{U}'^1$ if and only if $\mathcal{U}'^0 \supset \mathcal{U}'^1$. Now consider $\mathcal{U}'^0 \in D$. Then there exists an $x \in X$ such that $int(U')(x) \geq \alpha$ for each $U \in \mathcal{U}^0$. Thus for each $\mathcal{U}'^0 \in D$, we can select a fuzzy point $s(\mathcal{U}'^0) = x_\alpha$ such that $x_\alpha \in int(U')$ for each $U \in \mathcal{U}^0$. So we get a constant fuzzy α-net $\mathcal{S} = \{s(\mathcal{U}'^0) : \mathcal{U}'^0 \in D\}$.

We claim that $\mathcal{S}$ has no fuzzy θ-cluster point with value α. Let y_α be any fuzzy point with value α. Then there exists an $U_0 \in \mathcal{U}$ satisfying $U_0(x) > \varepsilon$, i.e., $U'(y) < \alpha$. Thus $U'_0 \in \mathcal{R}(X, y_\alpha)$. Suppose $\{U'_0\} \in D$. Then for all $\mathcal{U}'^0 \in D$ with $\mathcal{U}'^0 \geq \{U'_0\}$, we get $U'_0 \in \mathcal{U}'^0$ and so $s(\mathcal{U}'^0) \in int(U'_0)$. Hence y_α is not fuzzy θ-cluster point. □

For a fuzzy set F of a fuzzy topological space X, $h(F) = \sup\{F(x) : x \in X\}$ is called the hight of the fuzzy set F. Let $\mathcal{F}$ be a fuzzy filter on X and $\inf\{h(F) : F \in \mathcal{F}\} = \alpha$. Then the fuzzy filter $\mathcal{F}$ is called an α-filter. If for each $F \in \mathcal{F}$, there exists an $x \in X$ such that $F(x) \geq \alpha$, then $\mathcal{F}$ is called a constant α-filter. A fuzzy point x_λ is called its θ-adherent point if for each $U \in \mathcal{R}(X, x_\lambda)$ and each $F \in \mathcal{F}$, $F \not\leq int(U)$.

Theorem 3.12. *For a fuzzy topological space X, then following conditions are equivalent:*
(1) *X is fuzzy strongly H-closed.*
(2) *Every constant α-filter $\mathcal{F}$ has a fuzzy θ-adherent point with value α.*

Proof. (1) ⇒(2): Let X be a strongly fuzzy H-closed space. Suppose $\mathcal{F}$ be a constant α-filter in X. Consider the directed set $D = \mathcal{F}$ ordered by " $\geq$ " that is defined by $F_1 \geq F_2$ if and only if $F_1 \leq F_2$(i.e. $F_1(x) \geq F_2(x)$ for all $x \in X$). For each $F \in D$, we can find an $x^F \in X$ such that $\alpha \leq F(x)$. Here $\mathcal{S} = \{x^F_\alpha : F \in D\}$ is a constant α-net in X [11]. Since X is fuzzy strong H-closed, there exists a fuzzy point $x_\alpha \in \Theta(\mathcal{S})$. We claim that x_α is a fuzzy θ-adherent point of $\mathcal{F}$. Let $U \in \mathcal{R}(X, x_\lambda)$ and $F \in \mathcal{F}$. Since $F \in D$, there exists $E \in D$ such that $E \geq F$ and $x^E_\alpha \notin int(U)$. Since $x^E_\alpha \in E$, $x^E_\alpha \in F$. So $F \nleq int(U)$. Thus x_α is a fuzzy θ-adherent point of $\mathcal{F}$.

(2) $\Rightarrow$(1): Let $\mathcal{S} = \{s_k : k \in D\}$ be a constant α-net in X. For each $n \in D$, let $F_n = \vee\{s_k : k \geq n\}$. Then $\mathcal{F} = \{F \in I^X : F \geq F_n$ for some $n \in D\}$ is a fuzzy constant α-filter in X. Thus there exists an $x_\alpha \in Pt(X)$ which is a fuzzy θ-adherent point of $\mathcal{F}$. Now we shall show that $x_\alpha \in \Theta(\mathcal{S})$. Let $U \in \mathcal{R}(X, x_\alpha)$ and $n \in D$. Since $F_n \nleq int(U)$, we get a $k \in D$ such that $k \geq n$ and $s_k \notin int(U)$. □

Theorem 3.13. *Let X be a strongly fuzzy H-closed topological space, Y be a fuzzy Hausdorff space and $\psi : X \to Y$ be a fuzzy θ-continuous surjection. Then Y is strongly fuzzy H-closed.*

Proof. Suppose $\alpha \in [0, 1)$ and $\mathcal{V}_\alpha \subset \sigma$ be a family with the property that for each $y \in Y$, there exists an $V_y \in \mathcal{U}_\alpha$ satisfying $V(y) > \alpha$. Suppose $x \in X$ such that $\psi(x) = y$. Since $V \in \mathcal{Q}(Y, y_{1-\alpha}$ and ψ is fuzzy θ-continuous, there exists an $U_V \in \mathcal{Q}(X, x_{1-\alpha})$ such that $\psi(cl(U_V)) \leq cl(V)$. Consider the family $\mathcal{U}_\alpha = \{U_V : V \in \mathcal{V}_\alpha\}$. We note that $U_V(x) > \alpha$. Since X is strongly fuzzy H-closed, there exist finite number of fuzzy set $V_1, V_2, ..., V_n \in \sigma$ such that for each $x \in X$, there exists $i \in \{1, 2, ..., n\}$ such that $cl(U_{V_i})(x) > \alpha$ i.e., $\psi(cl(U_{V_i}))(y) > \alpha$. Thus Y is strongly fuzzy H-closed. □

Theorem 3.14. *A fuzzy topological space X is strongly fuzzy H-closed if and only if for each $\alpha > 0$ and fuzzy open α-prefilter $\mathcal{F}$, $ad\mathcal{F} \nleq \alpha$.*

Proof. Let X be strongly fuzzy H-closed and $\mathcal{F} \in \mathcal{W}^\alpha_+(X)$ be a fuzzy open α-prefilter such that $ad\mathcal{F} \leq \alpha$. Consider the family $\mathcal{U}_\alpha = \{\underline{1} - cl(\mu) : \mu \in \mathcal{F}\}$. Let $x \in X$. Then $1 - \alpha < \underline{1} - ad\mathcal{F}(x) = \bigvee\{\underline{1} - cl(\mu)(x) : \mu \in \mathcal{F}\}$. So there exists an $\underline{1} - cl(\mu) \in \mathcal{U}_\alpha$ such that $\underline{1} - cl(\mu)(x) > 1 - \alpha$. Since X is strongly fuzzy H-closed, there exists finite subfamily $\mathcal{F}^0_\alpha$ of $\mathcal{F}_\alpha$ with the property that for each $x \in X$, there exists an $\mu \in \mathcal{F}^0_\alpha$ satisfying $cl(\underline{1} - cl(\mu))(x) > 1 - \alpha$, i.e., $1 - int(cl(\mu))(x) > 1 - \alpha$, i.e, $\mu(x) < \alpha$.

Conversely, let X be not strongly fuzzy H-closed. Then there exists an $\alpha > 0$ and family $\mathcal{U}_\alpha$ with the property that for each $x \in X$, there exists an $U \in \mathcal{U}_\alpha$ satisfying $U(x) > 1 - \alpha$ such that for each finite set $\mathcal{U} \subset \mathcal{U}_\alpha$ with the property that there exists an $x \in X$ such that $cl(U)(x) \leq 1 - \alpha$ for all $U \in \mathcal{U}$. Consider the family $\mathcal{F} = \{\mu \in \delta : \mu \geq \underline{1} - \bigvee\{cl(U) : U \in \mathcal{U}\}\}$. Then $\mathcal{F}$ is a fuzzy open α-prefilter on X. Now

$$\begin{aligned} ad\mathcal{F}(x) &= \bigwedge\{cl(\mu)(x) : \mu \in \mathcal{F}\} \\ &\leq \bigwedge\{cl(\underline{1} - \bigvee\{cl(U)(x) : U \in \mathcal{U}\}) : \mathcal{U} \subset \mathcal{U}_\alpha \text{ is finite }\} \\ &= \underline{1} - \bigvee\{\{U(x) : U \in \mathcal{U}\} : \mathcal{U} \subset \mathcal{U}_\alpha \text{ is finite }\} \\ &= \underline{1} - \bigvee\{U(x) : U \in \mathcal{U}_\alpha\} \leq \alpha. \text{ Thus } ad\mathcal{F} \leq \alpha. \end{aligned}$$ □

4. Fuzzy ultra-H-closed topological spaces

Let (X, δ) be a fuzzy topological space. Then for each $\alpha \in [0, 1)$, $\iota_\alpha(\delta) = \{\mu^{-1}(\alpha, 1] : \mu \in \delta\}$ is a topology on X [13]. The topology generated by the subbase $\{\mu^{-1}(\alpha, 1] : \mu \in \delta, \alpha \in [0,1]\}$ is denoted by $\iota(\delta)$. It is obvious that $\iota(\delta) = \sup\{\iota_\alpha(\delta) : \alpha \in [0,1)\}$ [14].

Definition 4.1. A fuzzy topological space (X, δ) is said to be fuzzy ultra-H-closed if and only if $(X, \iota(\delta))$ is a H-closed space.

Theorem 4.2. *A fuzzy topological space (X, τ) is H-closed if and only if $(X, \omega(\tau))$ is fuzzy ultra-H-closed.*

Proof. Since $\iota(\omega(\tau)) = \tau$, $(X, \omega(\tau))$ is fuzzy ultra-H-closed if and only if (X, τ) is H-closed. □

Remark 4.3. Every fuzzy ultra-compact space due to Lowen [14] is fuzzy ultra-H-closed, but the converse need not be true. In fact, the space $(X, \omega(\tau))$ of Example 3.6 (2) is fuzzy ultra-H-closed, but not fuzzy ultra-compact.

Theorem 4.4. *Let (X, δ) be fuzzy regular. Then X is fuzzy ultra-H-closed if and only if X is fuzzy ultra-compact.*

Proof. Since (X, δ) is a fuzzy regular space and fuzzy ultra-H-closed, $(X, \iota(\delta))$ is regular and H-closed. So $(X, \iota(\delta))$ is compact. Thus (X, δ) is fuzzy ultra-compact. Converse part follows from the Remark 4.3. □

Two nets $\mathcal{S} = \{s_k : k \in D\}$ and $\mathcal{S}' = \{s'_k : k \in D\}$ in a fuzzy topological space X are said to be similar [11] if for each $k \in D$, $supp(s_k) = supp(s'_k)$.

Definition 4.5. Let $\mathcal{S} = \{s_k : k \in D\}$ be an α-net fuzzy θ-converges to a fuzzy point x_λ. Also let each constant a-net (where $a \in (0,1]$) similar to $\mathcal{S}$ fuzzy θ-converges to a fuzzy point x_a. Then x_λ is called transitive θ-limit point of the net $\mathcal{S}$.

A fuzzy topological topological space X is called said to satisfy the property $\mathcal{P}$ if $cl(A \cap B) = cl(A) \cap cl(B)$ for all $A, B \in \iota(\delta)$.

Lemma 4.6. *Let (X, δ) be a fuzzy topological space and $\alpha \in (0,1]$. If x_λ is a fuzzy transitive θ-limit of the constant α-net $\mathcal{S} = \{s_k : k \in D\}$, then $\widehat{\mathcal{S}} = \{supp(s_k) : k \in D\}$ θ-converges to x in the topological space $(X, \iota(\delta))$.*

Proof. Let $x^k = supp(s_k)$ for each $k \in D$. Let U be an open set of the topological space $(X, \iota(\delta))$ containing x. Then there exist fuzzy open subsets $A_1, A_2, ..., A_n$ of (X, δ) and $a_1, a_2, ..., a_n \in (0,1]$ such that $x \in \bigcap_{i=1}^n A_i^{-1}(a_i, 1] \subset U$. Here $a_i < A_i(x) \leq 1$, i.e. $1 - a_i > A'_i(x) \geq 0$, i.e. so $A'_i \in \mathcal{R}(X, x_{1-a_i})$ for each $i \in \{1, 2, ..., n\}$. Clearly, $\mathcal{S}_i = \{x^k_{1-a_i} : k \in D\}$, $i = 1, 2, ..., n$ is a constant $(1 - a_i)$-net similar to $\mathcal{S}$. Since x_λ is a fuzzy transitive θ-limit of the constant α-net $\mathcal{S} = \{s_k : k \in D\}$, $\mathcal{S}_i$ fuzzy θ-converges to a fuzzy point x_{1-a_i}, $i = 1, 2, ..., n$. Thus there exist $p_1, p_2, ..., p_n \in D$ such that $x^k_{1-a_i} \notin int(A'_i)$ for all $k \geq p_i$, $i \in \{1, 2, ..., n\}$. Consider a $p \in D$ with the property $p \geq p_i$ for each $i \in \{1, 2, ..., n\}$. We claim that $x^k \in cl(U)$ for all $k \geq p$. Now fix a $k \geq p$. Then $k \geq p_i$ for each $i \in \{1, 2, ..., n\}$. So $x^k_{1-a_i} \notin int(A'_i)$, i.e. $1 - cl(A_i)(x^k) < 1 - a_i$, i.e. $cl(A_i)(x^k) > a_i$. So, by the property $\mathcal{P}$ of $(X, \iota(\delta))$, $x^k \in \bigcap_{i=1}^n (cl(A_i))^{-1}(a_i, 1] = \bigcap_{i=1}^n cl((A_i)^{-1}(a_i, 1]) = cl(\bigcap_{i=1}^n (A_i)^{-1}(a_i, 1]) \subset cl(U)$. Hence $\widehat{\mathcal{S}} = \{x^k : k \in D\}$ θ-converges to x in the topological space $(X, \iota(\delta))$. □

Theorem 4.7. *If a fuzzy topological space (X, δ) satisfies the property $\mathcal{P}$ and every α-net has a subnet having fuzzy transitive θ-limit point with value α, then (X, δ) is fuzzy ultra-H-closed space.*

Proof. We shall show that $(X, \iota(\delta))$ is H-closed. Let $\widehat{\mathcal{S}} = \{x^k : k \in D\}$ be a net in $(X, \iota(\delta))$. Let $\alpha(0, 1]$ and $\mathcal{S}_\alpha = \{x^k_\alpha : k \in D\}$ be a constant α-net. Then there exists a subnet $\mathcal{T}_\alpha = \phi o \mathcal{S}_\alpha = \{x^m_\alpha : m \in M\}$ of $\mathcal{S}_\alpha$ defined by the mapping $\phi : M \to D$ which has fuzzy transitive θ-limit point with value α. Then by Lemma 4.6, x is a θ-limit point of the net $\widehat{\mathcal{T}_\alpha} = \{x^m : m \in M\}$ in $(X, \iota(\delta))$. We claim that x is θ-cluster point of the net $\widehat{\mathcal{S}_\alpha}$ in $(X, \iota(\delta))$. Let U be an open set containing x in $(X, \iota(\delta))$ and $k_0 \in D$. Then there exists an $m_1 \in M$ such that $m \geq m_1$ implies $\phi(m) \geq k_0$. Since x is θ-limit point of the net $\widehat{\mathcal{T}_\alpha}$, there exists an $m_2 \in M$ such that $x^m \in cl(U)$ for all $m \geq m_2$. Suppose an $m_0 \in M$ such that $m_0 \geq m_1$ and $m_0 \geq m_2$. Then $x^{m_0} \in cl(U)$ and $\phi(m_0) \geq k_0$. □

Theorem 4.8. *Every fuzzy ultra-H-closed spaces is strongly fuzzy H-closed.*

Proof. Let (X, δ) be a fuzzy ultra-H-closed space. Then $(X, \iota(\delta))$ is H-closed. Let $\alpha \in [0, 1)$ and $\mathcal{U}_\alpha$ be a family with the property that for each $x \in X$, there exists an $U \in \mathcal{U}_\alpha$ satisfying $U(x) > \alpha$. Then $\cup\{U^{-1}((\alpha, 1] : U \in \mathcal{U}_\alpha\} = X$. Since $(X, \iota(\delta))$ is H-closed, there exists finite subfamily $\mathcal{U}^0_\alpha$ of $\mathcal{U}_\alpha$ such that $\cup\{cl(U^{-1}((\alpha, 1]) : U \in \mathcal{U}^0_\alpha\} = X$. Suppose an $x \in X$. Then there exists an $U \in \mathcal{U}^0_\alpha$ such that $x \in cl(U^{-1}((\alpha, 1]) \subset (cl(U))^{-1}((\alpha, 1])$. Thus $cl(U)(x) > \alpha$. □

Remark 4.9. Strongly fuzzy H-closed spaces need not be fuzzy ultra-H-closed in general.

Example 4.10. Let $X = \mathbb{N}$. Then for each each $\alpha \in (0, 1)$, there exists an $n \in \mathbb{N}$ such that $\frac{n-1}{n} < \alpha \leq \frac{n+1}{n}$. For each $a_i \in [\frac{n-1}{n}, \alpha]$, $i, 2, .., n$, define $U(\alpha, a_1, a_2, ..., a_n)(x) = \alpha$ when $x > n$ and $= \alpha_i$ when $x = \alpha_i$, $i = 1, 2, ..., n$. Then W. Guojun [11] shaw that $\delta = \{\underline{0}, \underline{1}\} \cup \{U'(\alpha, a_1, a_2, ..., a_n) : a_i \in [\frac{n-1}{n}, \alpha], \alpha \in (0, 1)\}$ is a fuzzy topology on X such that (X, δ) is which is fuzzy strongly compact and $(X, \iota(\delta))$ is discrete space. So by Remark 3.5 fuzzy strongly H-closed. Also $(X, \iota(\delta))$ is not H-closed and so (X, δ) is not fuzzy ultra-H-closed.

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