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Direct product of fuzzy ideals of near-rings

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Abstract. In this paper, we define the new notions of direct product of finite number "n" of fuzzy subset of $R_1 \times R_2 \times \cdots \times R_n$, where $R_1, R_2, \cdots, R_n$ are near-rings and study the direct product of n fuzzy ideal (resp. subnear-ring,R-subgroup) of $R_1 \times R_2 \times \cdots \times R_n$. We also discuss some of its basic properties with examples.

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1. Introduction

The concept of fuzzy set was first proposed by Zadeh[16] in 1965. The notion of fuzzy subnear-rings and fuzzy ideals of near-rings was introduced by Abou Said[1]. In [9], Rosenfeld defined fuzzy subgroup and gave some of its properties. Further, Kim[5] and Dutta[4] discussed some characterizations of fuzzy ideals of near-rings. Davvaz[3] and Narayanan et al[8] defined generalization of fuzzy ideals of near-rings, that is, $(\in, \in \vee q)$-fuzzy ideals of near-rings. Saikia et al.[13], defined the concept of fuzzy R-subgroups and fuzzy ideals of near-rings and near-ring groups. Then [14], he also discussed characterizations of fuzzy substructures of a near-rings and near-ring groups. Sherwood[15], presented the notion of product of fuzzy subgroups. After sixteen years, Ray[10] discussed some properties of the direct product of fuzzy subgroups. Further, Mushtaq et al.[7] initiated the notion of direct product of different types of ideals in Abel Grassmann's groupoids such as ideals, prime ideals, minimal ideals and some of its properties. Recently, Abdullah et al.[11], introduced the idea of direct product of different fuzzy ideals of LA-semigroups such as ideals and bi-ideals. In [12], Abdullah et al. initiated the notion of direct product of intuitionistic fuzzy H-ideals of BCK-algebras. Aktas et al.[2] introduced the notion of generalized product of fuzzy subgroups and t-level subgroups. In this paper, we introduce the new notions of direct product of $n(n = 1, 2, \cdots, n)$ fuzzy subnear-ring, fuzzy ideal and fuzzy R-subgroup of $R_1 \times R_2 \times \cdots \times R_n$ and related properties are investigated with examples.

2. Preliminaries

In this section, we have listed some basic definitions of near-rings. Throughout this paper, $R, R_1, R_2, \cdots, R_n$ denote left near-rings.

Definition 2.1 ([3, 6]). A near-ring is an algebraic system $(R, +, \cdot)$ consisting of a non empty set R together with two binary operations called $+$ and $\cdot$ such that $(R, +)$ is a group not necessarily abelian and $(R, \cdot)$ is a semigroup connected by the following distributive law: $x \cdot (y + z) = x \cdot y + x \cdot z$ valid for all $x, y, z \in R$. We will use the word 'near-ring'to mean 'left near-ring'. We denote xy instead of $x \cdot y$.
An ideal I of a near-ring R is a subset of R such that

(i) $(I, +)$ is a normal subgroup of $(R, +)$,
(ii) $RI \subseteq I$,
(iii) $(x + a)y - xy \in I$, for any $a \in I$ and $x, y \in R$.

Note that I is a left ideal of R if I satisfies (i) and (ii) and right ideal of R if I satisfies (i) and (iii).

A two sided R-subgroup of a near-ring R is a subset H of R such that

(i) $(H, +)$ is a subgroup of $(R, +)$,
(ii) $RH \subset H$,
(iii) $HR \subset H$.

If H satisfies (i) and (ii), then it is called a left R-subgroup of R. If H satisfies (i) and (iii) then it is called a right R-subgroup of R.

Definition 2.2 ([1]). Let R be a near-ring and μ be a fuzzy subset of R. Then μ is a fuzzy subnear-ring of R if

(i) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in R$,
(ii) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in R$.

Definition 2.3 ([1]). Let R be a near-ring and μ be a fuzzy subset of R. Then μ is a fuzzy ideal of R if

(i) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in R$,
(ii) $\mu(y + x - y) \geq \mu(x)$, for all $x, y \in R$,
(iii) $\mu(xy) \geq \mu(y)$, for all $x, y \in R$,
(iv) $\mu((x + z)y - xy) \geq \mu(z)$, for all $x, y, z \in R$.

A fuzzy subset with (i)-(iii) is called a fuzzy left ideal of R whereas a fuzzy subset with (i), (ii) and (iv) is called a fuzzy right ideal of R.

Definition 2.4 ([6]). A fuzzy subset μ of a near-ring R is called a fuzzy R-subgroup of R if

(i) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$,
(ii) $\mu(xy) \geq \mu(y)$,
(iii) $\mu(xy) \geq \mu(x)$, for all $x, y \in R$.

Note that μ is a fuzzy left ideal of R if it satisfies (i) and (ii) and μ is a fuzzy right ideal of R if it satisfies (i) and (iii).

Definition 2.5. Let μ_1 and μ_2 be any two fuzzy subsets of R_1 and R_2 respectively. Then the direct product of fuzzy subsets is defined by $\mu_1 \times \mu_2 : R_1 \times R_2 \to [0, 1]$

$$(\mu_1 \times \mu_2)(x_1, x_2) = \min\{\mu_1(x_1), \mu_2(x_2)\}$$

for all $x_1 \in R_1$ and $x_2 \in R_2$.

Definition 2.6. Let $\mu_1, \mu_2, \cdots, \mu_n$ be n(finite) fuzzy subsets of $R_1, R_2, \cdots, R_n$ respectively. Then the direct product of fuzzy subsets of near-rings is defined by $\mu_1 \times \mu_2 \times \cdots \times \mu_n : R_1 \times R_2 \times \cdots \times R_n \to [0, 1]$ as

$$(\mu_1 \times \mu_2 \times \cdots \times \mu_n)(x_1, x_2, \cdots, x_n) = \min\{\mu_1(x_1), \mu_2(x_2), \cdots, \mu_n(x_n)\}$$

for all $x_1 \in R_1, x_2 \in R_2, \cdots, x_n \in R_n$.

3. Direct product of n-fuzzy ideals of near-rings

Definition 3.1. Let $\mu_1, \mu_2, \cdots, \mu_n$ be n(finite)-fuzzy subsets of near-rings $R_1, R_2, \cdots, R_n$. Then $\mu_1 \times \mu_2 \times \cdots \times \mu_n$ is said to be a fuzzy subnear-ring of $R_1 \times R_2 \times \cdots \times R_n$ if it satisfies the following conditions:

(i) $$(\mu_1 \times \mu_2 \times \cdots \times \mu_n)((x_1, x_2, \cdots, x_n) - (y_1, y_2, \cdots, y_n)) \geq \min\{(\mu_1 \times \mu_2 \times \cdots \times \mu_n)(x_1, x_2, \cdots, x_n), (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(y_1, y_2, \cdots, y_n)\},$$

(ii) $$(\mu_1 \times \mu_2 \times \cdots \times \mu_n)((x_1, x_2, \cdots, x_n)(y_1, y_2, \cdots, y_n)) \geq \min\{(\mu_1 \times \mu_2 \times \cdots \times \mu_n)(x_1, x_2, \cdots, x_n), (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(y_1, y_2, \cdots, y_n)\},$$

for all $(x_1, x_2, \cdots, x_n), (y_1, y_2, \cdots, y_n) \in R_1 \times R_2 \times \cdots \times R_n$.

Definition 3.2. Let $\mu_1, \mu_2, \cdots, \mu_n$ be n(finite)-fuzzy subsets of $R_1, R_2, \cdots, R_n$ respectively. Then $\mu_1 \times \mu_2 \times \cdots \times \mu_n$ is a fuzzy ideal of $R_1 \times R_2 \times \cdots \times R_n$ if it satisfies the following:

(i) $$(\mu_1 \times \mu_2 \times \cdots \times \mu_n)((x_1, x_2, \cdots, x_n) - (y_1, y_2, \cdots, y_n)) \geq \min\{(\mu_1 \times \mu_2 \times \cdots \times \mu_n)(x_1, x_2, \cdots, x_n), (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(y_1, y_2, \cdots, y_n)\},$$

(ii) $$(\mu_1 \times \mu_2 \times \cdots \times \mu_n)((y_1, y_2, \cdots, y_n) + (x_1, x_2, \cdots, x_n) - (y_1, y_2, \cdots, y_n)) \geq (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(x_1, x_2, \cdots, x_n),$$

(iii) $$(\mu_1 \times \mu_2 \times \cdots \times \mu_n)((x_1, x_2, \cdots, x_n)(y_1, y_2, \cdots, y_n)) \geq (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(y_1, y_2, \cdots, y_n),$$

(iv) $$(\mu_1 \times \mu_2 \times \cdots \times \mu_n)(((x_1, x_2, \cdots, x_n) + (z_1, z_2, \cdots, z_n))(y_1, y_2, \cdots, y_n) - (x_1, x_2, \cdots, x_n)(y_1, y_2, \cdots, y_n)) \geq (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(z_1, z_2, \cdots, z_n),$$

for all $(x_1, x_2, \cdots, x_n), (y_1, y_2, \cdots, y_n), (z_1, z_2, \cdots, z_n) \in R_1 \times R_2 \times \cdots \times R_n$.

A fuzzy subset $\mu_1 \times \mu_2 \times \cdots \times \mu_n$ of $R_1 \times R_2 \times \cdots \times R_n$ is a fuzzy left ideal of $R_1 \times R_2 \times \cdots \times R_n$ if it satisfies (i),(ii) and (iii). A fuzzy subset $\mu_1 \times \mu_2 \times \cdots \times \mu_n$ of $R_1 \times R_2 \times \cdots \times R_n$ is a fuzzy right ideal of $R_1 \times R_2 \times \cdots \times R_n$ if it satisfies (i),(ii) and (iv).

Definition 3.3. Let $\mu_1, \mu_2, \cdots, \mu_n$ be n(finite)-fuzzy subsets of $R_1, R_2, \cdots, R_n$ respectively. Then $\mu_1 \times \mu_2 \times \cdots \times \mu_n$ is a fuzzy R-subgroup of $R_1 \times R_2 \times \cdots \times R_n$ if it satisfies the following:

(i) $(\mu_1 \times \mu_2 \times \cdots \times \mu_n)((x_1, x_2, \cdots, x_n) - (y_1, y_2, \cdots, y_n))$
$\geq \min\{(\mu_1 \times \mu_2 \times \cdots \times \mu_n)(x_1, x_2, \cdots, x_n), (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(y_1, y_2, \cdots, y_n)\}$,

(ii) $(\mu_1 \times \mu_2 \times \cdots \times \mu_n)((x_1, x_2, \cdots, x_n)(y_1, y_2, \cdots, y_n))$
$\geq (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(y_1, y_2, \cdots, y_n)$,

(iii) $(\mu_1 \times \mu_2 \times \cdots \times \mu_n)((x_1, x_2, \cdots, x_n)(y_1, y_2, \cdots, y_n))$
$\geq (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(x_1, x_2, \cdots, x_n)$,

for all $(x_1, x_2, \cdots, x_n), (y_1, y_2, \cdots, y_n) \in R_1 \times R_2 \times \cdots \times R_n$.

A fuzzy subset $\mu_1 \times \mu_2 \times \cdots \times \mu_n$ of $R_1 \times R_2 \times \cdots \times R_n$ is a fuzzy left R-subgroup of $R_1 \times R_2 \times \cdots \times R_n$ if it satisfies (i) and (ii). A fuzzy subset $\mu_1 \times \mu_2 \times \cdots \times \mu_n$ of $R_1 \times R_2 \times \cdots \times R_n$ is a fuzzy right R-subgroup of $R_1 \times R_2 \times \cdots \times R_n$ if it satisfies (i) and (iii).

Example 3.4. Let $R_1 = \{0, 1, 2\}$ be a near-ring with two binary operations $+_{R_1}$ and $\cdot_{R_1}$, $R_2 = \{0, 1\}$ be a near-ring with two binary operations $+_{R_2}$ and $\cdot_{R_2}$ and $R_3 = \{0, a, b, c\}$ be a near-ring with two binary operations $+_{R_3}$ and $\cdot_{R_3}$ are defined by

$+_{R_1}$	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

$\cdot_{R_1}$	0	1	2
0	0	1	2
1	0	1	2
2	0	1	2

$+_{R_2}$	0	1
0	0	1
1	1	0

$\cdot_{R_2}$	0	1
0	0	1
1	0	1

$+_{R_3}$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

$\cdot_{R_3}$	0	a	b	c
0	0	a	0	a
a	0	a	0	a
b	0	a	b	c
c	0	a	b	c

Let $R_1 \times R_2 \times R_3 = \{(0,0,0), (0,0,a), (0,0,b), (0,0,c), (0,1,0), (0,1,a), (0,1,b), (0,1,c), (1,0,0), (1,0,a), (1,0,b), (1,0,c), (1,1,0), (1,1,a), (1,1,b), (1,1,c), (2,0,0), (2,0,a), (2,0,b), (2,0,c), (2,1,0), (2,1,a), (2,1,b), (2,1,c)\}$. Define fuzzy subset $\mu_1 : R_1 \to [0,1]$ by $\mu_1(0) = 0.4$ and $\mu_1(1) = 0.3 = \mu_1(2)$. Let us defined fuzzy subsets $\mu_2 : R_2 \to [0,1]$ and $\mu_3 : R_3 \to [0,1]$ by $\mu_2(0) = 0.7, \mu_2(1) = 0.8, \mu_3(0) = 0.9, \mu_3(a) = 0.5, \mu_3(b) = 0.6$ and $\mu_3(c) = 0.4$. Then

$(\mu_1 \times \mu_2 \times \mu_3)(0,0,0) = (\mu_1 \times \mu_2 \times \mu_3)(0,0,a) = (\mu_1 \times \mu_2 \times \mu_3)(0,0,b) = 0.4$,
$(\mu_1 \times \mu_2 \times \mu_3)(0,0,c) = (\mu_1 \times \mu_2 \times \mu_3)(0,1,0) = (\mu_1 \times \mu_2 \times \mu_3)(0,1,a) = 0.4$,
$(\mu_1 \times \mu_2 \times \mu_3)(0,1,b) = (\mu_1 \times \mu_2 \times \mu_3)(0,1,c) = 0.4$

and

$(\mu_1 \times \mu_2 \times \mu_3)(1,0,0) = (\mu_1 \times \mu_2 \times \mu_3)(1,0,a) = (\mu_1 \times \mu_2 \times \mu_3)(1,0,b) = 0.3$,
$(\mu_1 \times \mu_2 \times \mu_3)(1,0,c) = (\mu_1 \times \mu_2 \times \mu_3)(1,1,0) = (\mu_1 \times \mu_2 \times \mu_3)(1,1,a) = 0.3$,
$(\mu_1 \times \mu_2 \times \mu_3)(1,1,b) = (\mu_1 \times \mu_2 \times \mu_3)(1,1,c) = (\mu_1 \times \mu_2 \times \mu_3)(2,0,0) = 0.3$,
$(\mu_1 \times \mu_2 \times \mu_3)(2,0,a) = (\mu_1 \times \mu_2 \times \mu_3)(2,0,b) = (\mu_1 \times \mu_2 \times \mu_3)(2,0,c) = 0.3$,
$(\mu_1 \times \mu_2 \times \mu_3)(2,1,0) = (\mu_1 \times \mu_2 \times \mu_3)(2,1,a) = (\mu_1 \times \mu_2 \times \mu_3)(2,1,b) = 0.3$,
$(\mu_1 \times \mu_2 \times \mu_3)(2,1,c) = 0.3$.

Clearly, $\mu_1 \times \mu_2 \times \mu_3$ is a fuzzy ideal of $R_1 \times R_2 \times R_3$.

Theorem 3.5. *Let $\mu_1, \mu_2, \cdots, \mu_n$ be n fuzzy ideals (subnear-rings, R-subgroups) of $R_1, R_2, \cdots, R_n$ respectively. Then $\mu_1 \times \mu_2 \times \cdots \times \mu_n$ is a fuzzy ideal (subnear-ring,R-subgroup) of $R_1 \times R_2 \times \cdots \times R_n$.*

Proof. Let $\mu_1, \mu_2, \cdots, \mu_n$ be fuzzy ideals of $R_1, R_2, \cdots, R_n$.
Let $(x_1, x_2, \cdots, x_n), (y_1, y_2, \cdots, y_n), (z_1, z_2, \cdots, z_n) \in R_1 \times R_2 \times \cdots \times R_n$. Then

$$\begin{aligned}
&(\mu_1 \times \mu_2 \times \cdots \times \mu_n)((x_1, x_2, \cdots, x_n) - (y_1, y_2, \cdots, y_n))\\
&= (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(x_1 - y_1, x_2 - y_2, \cdots, x_n - y_n)\\
&= \min\{\mu_1(x_1 - y_1), \mu_2(x_2 - y_2), \cdots, \mu_n(x_n - y_n)\}\\
&\geq \min\{\min\{\mu_1(x_1), \mu_1(y_1)\}, \min\{\mu_2(x_2), \mu_2(y_2\}, \cdots, \min\{\mu_n(x_n), \mu_n(y_n)\}\}\\
&= \min\{\min\{\mu_1(x_1), \mu_2(x_2), \cdots, \mu_n(x_n)\}, \min\{\mu_1(y_1), \mu_2(y_2), \cdots, \mu_n(y_n)\}\}\\
&= \min\{(\mu_1 \times \mu_2 \times \cdots \times \mu_n)(x_1, x_2, \cdots, x_n), (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(y_1, y_2, \cdots, y_n)\}.
\end{aligned}$$

And

$$\begin{aligned}
&(\mu_1 \times \mu_2 \times \cdots \times \mu_n)((y_1, y_2, \cdots, y_n) + (x_1, x_2, \cdots, x_n) - (y_1, y_2, \cdots, y_n))\\
&= (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(y_1 + x_1 - y_1, y_2 + x_2 - y_2, \cdots, y_n + x_n - y_n)\\
&= \min\{\mu_1(y_1 + x_1 - y_1), \mu_2(y_2 + x_2 - y_2), \cdots, \mu_n(y_n + x_n - y_n)\}\\
&\geq \min\{\mu_1(x_1), \mu_2(x_2), \cdots, \mu_n(x_n)\}\\
&= (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(x_1, x_2, \cdots, x_n).
\end{aligned}$$

Next,

$$\begin{aligned}
&(\mu_1 \times \mu_2 \times \cdots \times \mu_n)((x_1, x_2, \cdots, x_n)(y_1, y_2, \cdots, y_n))\\
&= (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(x_1y_1, x_2y_2, \cdots, x_ny_n)\\
&= \min\{\mu_1(x_1y_1), \mu_2(x_2y_2), \cdots, \mu_n(x_ny_n)\}\\
&\geq \min\{\mu_1(y_1), \mu_2(y_2), \cdots, \mu_n(y_n)\}\\
&= (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(y_1, y_2, \cdots, y_n).
\end{aligned}$$

Further,

$$\begin{aligned}
&(\mu_1 \times \mu_2 \times \cdots \times \mu_n)(((x_1, x_2, \cdots, x_n) + (z_1, z_2, \cdots, z_n))(y_1, y_2, \cdots, y_n)\\
&\qquad\qquad - (x_1, x_2, \cdots, x_n)(y_1, y_2, \cdots, y_n))\\
&= \mu_1 \times \mu_2 \times \cdots \times \mu_n((x_1 + z_1)y_1 - x_1y_1, (x_2 + z_2)y_2 - x_2y_2\\
&\qquad\qquad , \cdots, (x_n + z_n)y_n - x_ny_n)\\
&= \min\{\mu_1((x_1 + z_1)y_1 - x_1y_1), \mu_2((x_2 + z_2)y_2 - x_2y_2), \cdots,\\
&\qquad\qquad \mu_n((x_n + z_n)y_n - x_ny_n)\}\\
&\geq \min\{\mu_1(z_1), \mu_2(z_2), \cdots, \mu_n(z_n)\}\\
&= (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(z_1, z_2, \cdots, z_n).
\end{aligned}$$

Therefore, $\mu_1 \times \mu_2 \times \cdots \times \mu_n$ is a fuzzy ideal of $R_1 \times R_2 \times \cdots \times R_n$. □

Example 3.6. In Example 3.4 R_1, R_2 and R_3 are near-rings. Let $R_1 \times R_2 \times R_3 = \{(0,0,0), (0,0,a), (0,0,b), (0,0,c), (0,1,0), (0,1,a), (0,1,b), (0,1,c), (1,0,0), (1,0,a), (1,0,b), (1,0,c), (1,1,0), (1,1,a), (1,1,b), (1,1,c), (2,0,0), (2,0,a), (2,0,b), (2,0,c), (2,1,0), (2,1,a), (2,1,b), (2,1,c)\}$. Let us defined fuzzy subsets $\mu_1 : R_1 \to [0,1], \mu_2 :$

$R_2 \to [0,1]$ and $\mu_3 : R_3 \to [0,1]$ by $\mu_1(0) = 0.7, \mu_1(1) = 0.5 = \mu_1(2), \mu_2(0) = 0.6, \mu_2(1) = 0.4, \mu_3(0) = 0.9, \mu_3(a) = 0.7$ and $\mu_3(b) = 0.6 = \mu_3(c)$. Then

$$\begin{aligned}(\mu_1 \times \mu_2 \times \mu_3)(0,0,0) &= (\mu_1 \times \mu_2 \times \mu_3)(0,0,a) = (\mu_1 \times \mu_2 \times \mu_3)(0,0,b)\\ &= (\mu_1 \times \mu_2 \times \mu_3)(0,0,c) = 0.6,\\ (\mu_1 \times \mu_2 \times \mu_3)(1,0,0) &= (\mu_1 \times \mu_2 \times \mu_3)(1,0,a) = (\mu_1 \times \mu_2 \times \mu_3)(1,0,b) = 0.5,\\ (\mu_1 \times \mu_2 \times \mu_3)(1,0,c) &= (\mu_1 \times \mu_2 \times \mu_3)(2,0,0) = (\mu_1 \times \mu_2 \times \mu_3)(2,0,a) = 0.5,\\ (\mu_1 \times \mu_2 \times \mu_3)(2,0,b) &= (\mu_1 \times \mu_2 \times \mu_3)(2,0,c) = 0.5\end{aligned}$$

and

$$\begin{aligned}(\mu_1 \times \mu_2 \times \mu_3)(0,1,0) &= (\mu_1 \times \mu_2 \times \mu_3)(0,1,a) = 0.4(\mu_1 \times \mu_2 \times \mu_3)(0,1,b) = 0.4,\\ (\mu_1 \times \mu_2 \times \mu_3)(0,1,c) &= (\mu_1 \times \mu_2 \times \mu_3)(1,1,0) = (\mu_1 \times \mu_2 \times \mu_3)(1,1,a) = 0.4,\\ (\mu_1 \times \mu_2 \times \mu_3)(1,1,b) &= (\mu_1 \times \mu_2 \times \mu_3)(1,1,c) = (\mu_1 \times \mu_2 \times \mu_3)(2,1,0) = 0.4,\\ (\mu_1 \times \mu_2 \times \mu_3)(2,1,a) &= (\mu_1 \times \mu_2 \times \mu_3)(2,1,b) = (\mu_1 \times \mu_2 \times \mu_3)(2,1,c) = 0.4.\end{aligned}$$

Thus, $\mu_1 \times \mu_2 \times \mu_3$ is a fuzzy ideal of $R_1 \times R_2 \times R_3$.

The converse part of Theorem 3.5 is not true in general as shown in the following Example.

Example 3.7. Consider Example 3.4, $\mu_1 \times \mu_2 \times \mu_3$ is a fuzzy ideal of $R_1 \times R_2 \times R_3$, but μ_2 and μ_3 are not fuzzy ideals of R_2 and R_3 respectively.

Theorem 3.8. *Let μ be a fuzzy subset of R, Then μ is a fuzzy ideal (subnear-ring,R-subgroup) of R if and only if $\mu \times \mu \times \cdots \times \mu$(n-times) is a fuzzy ideal (subnear-ring,R-subgroup) of $R \times R \times \cdots \times R$(n-times).*

Proof. Assume that μ is a fuzzy ideal of R. By Theorem 3.5, we have $\mu \times \mu \times \cdots \mu$ is a fuzzy ideal of $R \times R \times \cdots \times R$.

Conversely, assume that $\mu \times \mu \times \cdots \times \mu$ is a fuzzy ideal of $R \times R \times \cdots \times R$. For $(x_1, x_2, \cdots, x_n), (y_1, y_2, \cdots, y_n), (z_1, z_2, \cdots, z_n) \in R \times R \times \cdots \times R$. Then,

$$\begin{aligned}&\min\{\mu(x_1 - y_1), \mu(x_2 - y_2), \cdots, \mu(x_n - y_n)\}\\ &= (\mu \times \mu \times \cdots \times \mu)\,(x_1 - y_1, x_2 - y_2, \cdots, x_n - y_n)\\ &= (\mu \times \mu \times \cdots \times \mu)\,((x_1, x_2, \cdots, x_n) - (y_1, y_2, \cdots, y_n))\\ &\geq \min\{(\mu \times \mu \times \cdots \times \mu)\,(x_1, x_2, \cdots, x_n), (\mu \times \mu \times \cdots \times \mu)\,(y_1, y_2, \cdots, y_n)\}\\ &= \min\{\min\{\mu(x_1), \mu(x_2), \cdots, \mu(x_n)\}, \min\{\mu(y_1), \mu(y_2), \cdots, \mu(y_n)\}\}.\end{aligned}$$

If $\mu(x_1 - y_1) < \mu(x_2 - y_2), \cdots, \mu(x_n - y_n)$, then $\mu(x_1) < \mu(x_2), \mu(x_3), \cdots, \mu(x_n)$ and $\mu(y_1) < \mu(y_2), \mu(y_3), \cdots, \mu(y_n)$, we have $\mu(x_1 - y_1) \geq \min\{\mu(x_1), \mu(y_1)\}$.

$$\begin{aligned}&\min\{\mu(y_1 + x_1 - y_1), \mu(y_2 + x_2 - y_2), \cdots, \mu\,(y_n + x_n - y_n)\}\\ &= (\mu \times \mu \times \cdots \times \mu)\,((y_1, y_2, \cdots, y_n) + (x_1, x_2, \cdots, x_n) - (y_1, y_2, \cdots, y_n))\\ &\geq (\mu \times \mu \times \cdots \times \mu)(x_1, x_2, \cdots, x_n)\\ &= \min\{\mu(x_1), \mu(x_2), \cdots, \mu(x_n)\}.\end{aligned}$$

If $\mu(y_1 + x_1 - y_1) < \mu(y_2 + x_2 - y_2), \mu(y_3 + x_3 - y_3), \cdots, \mu(y_n + x_n - y_n)$, then $\mu(x_1) < \mu(x_2), \mu(x_3), \cdots, \mu(x_n)$, we have, $\mu(y_1 + x_1 - y_1) \geq \mu(x_1)$. Now,

$$\begin{aligned} &\min\{\mu(x_1 y_1), \mu(x_2, y_2), \cdots, \mu(x_n, y_n)\} \\ &= (\mu \times \mu \times \cdots \times \mu)((x_1, x_2, \cdots, x_n)(y_1, y_2, \cdots, y_n)) \\ &\geq (\mu \times \mu \times \cdots \times \mu)(y_1, y_2, \cdots, y_n) \\ &= \min\{\mu(y_1), \mu(y_2), \mu(y_3), \cdots, \mu(y_n)\}. \end{aligned}$$

If $\mu(x_1 y_1) < \mu(x_2, y_2), \cdots, \mu(x_n, y_n)$, Then $\mu(y_1) < \mu(y_2), \mu(y_3), \cdots, \mu(y_n)$. Thus we have $\mu(x_1 y_1) \geq \mu(y_1)$. Similarly, $\mu((x_1 + z_1)y_1 - x_1 y_1) \geq \mu(z_1)$. So, μ is a fuzzy ideal of R. □

Theorem 3.9. *Let μ_1 and μ_2 be two fuzzy ideals (subnear-rings,R-subgroups) of R. then $\mu_1 \times \mu_2$ is a fuzzy ideal (subnear-ring,R-subgroup) of $R \times R$.*

Proof. The proof is straightforward from Theorem 3.5. □

The converse of Theorem 3.9 is not true. The given example state that if $\mu_1 \times \mu_2$ fuzzy ideal of $R \times R$, then μ_1 and μ_2 need not be fuzzy ideals of R.

Example 3.10. Consider Example 3.4, R_1 is a near-ring. Let $R_1 \times R_1 = \{(0,0), (0,1), (0,2), (1,0), (1,1), (1,2), (2,0), (2,1), (2,2)\}$.

$+_{R_1 \times R_1}$	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$	$(2,0)$	$(2,1)$	$(2,2)$
$(0,0)$	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$	$(2,0)$	$(2,1)$	$(2,2)$
$(0,1)$	$(0,1)$	$(0,2)$	$(0,0)$	$(1,1)$	$(1,2)$	$(1,0)$	$(2,1)$	$(2,2)$	$(2,0)$
$(0,2)$	$(0,2)$	$(0,0)$	$(0,1)$	$(1,2)$	$(1,0)$	$(1,1)$	$(2,2)$	$(2,0)$	$(2,1)$
$(1,0)$	$(1,0)$	$(1,1)$	$(1,2)$	$(2,0)$	$(2,1)$	$(2,2)$	$(0,0)$	$(0,1)$	$(0,2)$
$(1,1)$	$(1,1)$	$(1,2)$	$(1,0)$	$(2,1)$	$(2,2)$	$(2,0)$	$(0,1)$	$(0,2)$	$(0,0)$
$(1,2)$	$(1,2)$	$(1,0)$	$(1,1)$	$(2,2)$	$(2,0)$	$(2,1)$	$(0,2)$	$(0,0)$	$(0,1)$
$(2,0)$	$(2,0)$	$(2,1)$	$(2,2)$	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$
$(2,1)$	$(2,1)$	$(2,2)$	$(2,0)$	$(0,1)$	$(0,2)$	$(0,0)$	$(1,1)$	$(1,2)$	$(1,0)$
$(2,2)$	$(2,2)$	$(2,0)$	$(2,1)$	$(0,2)$	$(0,0)$	$(0,1)$	$(1,2)$	$(1,0)$	$(1,1)$

$\cdot_{R_1 \times R_1}$	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$	$(2,0)$	$(2,1)$	$(2,2)$
$(0,0)$	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$	$(2,0)$	$(2,1)$	$(2,2)$
$(0,1)$	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$	$(2,0)$	$(2,1)$	$(2,2)$
$(0,2)$	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$	$(2,0)$	$(2,1)$	$(2,2)$
$(1,0)$	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$	$(2,0)$	$(2,1)$	$(2,2)$
$(1,1)$	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$	$(2,0)$	$(2,1)$	$(2,2)$
$(1,2)$	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$	$(2,0)$	$(2,1)$	$(2,2)$
$(2,0)$	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$	$(2,0)$	$(2,1)$	$(2,2)$
$(2,1)$	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$	$(2,0)$	$(2,1)$	$(2,2)$
$(2,2)$	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$	$(2,0)$	$(2,1)$	$(2,2)$

Let us define fuzzy sets $\mu_1 : R_1 \to [0,1], \mu_2 : R_1 \to [0,1]$ by $\mu_1(0) = 0.5, \mu_1(1) = 0.4 = \mu_1(2)$ and $\mu_2(0) = 0.6, \mu_2(1) = 0.4, \mu_2(2) = 0.5$. Then
$(\mu_1 \times \mu_2)(0,0) = 0.5 = (\mu_1 \times \mu_2)(0,2)$ and $(\mu_1 \times \mu_2)(0,1) = (\mu_1 \times \mu_2)(1,0) = (\mu_1 \times \mu_2)(1,1) = (\mu_1 \times \mu_2)(1,2) = (\mu_1 \times \mu_2)(2,0) = (\mu_1 \times \mu_2)(2,1) = (\mu_1 \times \mu_2)(2,2) = 0.4$.
Hence $\mu_1 \times \mu_2$ is a fuzzy ideal of $R_1 \times R_1$ but μ_2 is not a fuzzy ideal of R_1.

Theorem 3.11. *Let μ_1 and μ_2 be two fuzzy subsets of R such that $\mu_1 \times \mu_2$ is a fuzzy ideal of $R \times R$. Then*

(1) *If $\mu_2(x) \leq \mu_1(0)$ for any $x \in R$, then μ_2 is a fuzzy ideal of R.*

(2) *If $\mu_1(x) \leq \mu_1(0)$ for all $x \in R$ and $\mu_2(y) > \mu_1(0)$ for some $y \in R$, then μ_1 is a fuzzy ideal of R.*

Proof. (1) Suppose $\mu_2(x) \leq \mu_1(0)$ for all $x \in R$. Then

$$\begin{aligned}\mu_2(x-y) &= \min\{\mu_1(0), \mu_2(x-y)\}\\ &= (\mu_1 \times \mu_2)(0, x-y)\\ &= (\mu_1 \times \mu_2)((0,x)-(0,y))\\ &\geq \min\{(\mu_1 \times \mu_2)(0,x), (\mu_1 \times \mu_2)(0,y)\}\\ &= \min\{\min\{\mu_1(0), \mu_2(x)\}, \min\{\mu_1(0), \mu_2(y)\}\}\\ &= \min\{\mu_2(x), \mu_2(y)\},\end{aligned}$$

$$\begin{aligned}\mu_2(y+x-y) &= \min\{\mu_1(0), \mu_2(y+x-y)\}\\ &= (\mu_1 \times \mu_2)(0, y+x-y)\\ &= (\mu_1 \times \mu_2)((0,y)+(0,x)-(0,y))\\ &\geq (\mu_1 \times \mu_2)(0,x)\\ &= \min\{\mu_1(0), \mu_2(x)\}\\ &= \mu_2(x),\end{aligned}$$

and

$$\begin{aligned}\mu_2(xy) &= \min\{\mu_1(0), \mu_2(xy)\}\\ &= (\mu_1 \times \mu_2)(0, xy)\\ &= (\mu_1 \times \mu_2)((0,x)(0,y))\\ &\geq (\mu_1 \times \mu_2)(0,y)\\ &= \min\{\mu_1(0), \mu_2(y)\}\\ &= \mu_2(y).\end{aligned}$$

Similarly, $\mu_2((x+z)y - xy) \geq \mu_2(z)$. Thus μ_2 is a fuzzy ideal of R.

(2) Assume that $\mu_1(x) \leq \mu_1(0)$ for all $x \in R$ and $\mu_2(y) > \mu_1(0)$ for some $y \in R$. It follows that $\mu_2(0) \geq \mu_2(y) > \mu_1(x)$ for all $x \in R$. Let $x, y, z \in R$. Then

$$\begin{aligned}\mu_1(x-y) &= \min\{\mu_1(x-y), \mu_2(0)\}\\ &= (\mu_1 \times \mu_2)((x-y), 0)\\ &= (\mu_1 \times \mu_2)((x,0)-(y,0))\\ &\geq \min\{(\mu_1 \times \mu_2)(x,0) - (\mu_1 \times \mu_2)(y,0)\}\\ &= \min\{\min\{\mu_1(x), \mu_2(0)\}, \min\{\mu_1(y), \mu_2(0)\}\}\\ &= \min\{\mu_1(x), \mu_1(y)\}\end{aligned}$$

and

$$\begin{aligned}\mu_1(y+x-y) &= \min\{\mu_1(y+x-y),\mu_2(0)\}\\ &= (\mu_1\times\mu_2)(y+x-y,0)\\ &= (\mu_1\times\mu_2)((y,0)+(x,0)-(y,0))\\ &\geq (\mu_1\times\mu_2)(x,0)\\ &= \min\{\mu_1(x),\mu_2(0)\}\\ &= \mu_1(x).\end{aligned}$$

Similarly, $\mu_1(xy)\geq\mu(y)$ and $\mu_1((x+z)y-xy)\geq\mu_1(z)$. Thus μ_1 is a fuzzy ideal of R. □

Definition 3.12. Let $\mu_1\times\mu_2\times\cdots\times\mu_n$ be a fuzzy subset of $R_1\times R_2\times\cdots\times R_n$ and $t\in[0,1]$. Then the subset $(\mu_1\times\mu_2\times\cdots\times\mu_n)_t=\{(x_1,x_2,\cdots,x_n)\in R_1\times R_2\times\cdots\times R_n|(\mu_1\times\mu_2\times\cdots\times\mu_n)(x_1,x_2,\cdots,x_n)\geq t\}$ is called a level subset of $R_1\times R_2\times\cdots\times R_n$.

Theorem 3.13. *Let $\mu_1,\mu_2,\cdots,\mu_n$ be fuzzy subsets of $R_1,R_2,\cdots,R_n$ respectively. Then $\mu_1\times\mu_2\times\cdots\times\mu_n$ is a fuzzy ideal (subnear-ring,R-subgroup) of $R_1\times R_2\times\cdots\times R_n$ if and only if $t\in[0,1]$ $(\mu_1\times\mu_2\times\cdots\times\mu_n)_t$ is an ideal (subnear-ring,R-subgroup) of $R_1\times R_2\times\cdots\times R_n$.*

Proof. Assume that $\mu_1\times\mu_2\times\cdots\times\mu_n$ be fuzzy ideal of $R_1\times R_2\times\cdots\times R_n$. Let $(x_1,x_2,\cdots,x_n),(y_1,y_2,\cdots,y_n)\in(\mu_1\times\mu_2\times\cdots\times\mu_n)_t$. Then

$$(\mu_1\times\mu_2\times\cdots\times\mu_n)(x_1,x_2,\cdots,x_n)\geq t$$

and

$$(\mu_1\times\mu_2\times\cdots\times\mu_n)(y_1,y_2,\cdots,y_n)\geq t.$$

Thus

$$\begin{aligned}&(\mu_1\times\mu_2\times\cdots\times\mu_n)((x_1,x_2,\cdots,x_n)-(y_1,y_2,\cdots,y_n))\\ &\geq\min\{(\mu_1\times\mu_2\times\cdots\times\mu_n)(x_1,x_2,\cdots,x_n),(\mu_1\times\mu_2\times\cdots\mu_n)(y_1,y_2,\cdots,y_n)\}\\ &\geq t.\end{aligned}$$

Furthermore

$$(x_1,x_2,\cdots,x_n)\in(\mu_1\times\mu_2\times\cdots\times\mu_n)_t$$

and

$$(y_1,y_2,\cdots,y_n)\in R_1\times R_2\times\cdots\times R_n.$$

Then

$$\begin{aligned}&(\mu_1\times\mu_2\times\cdots\times\mu_n)((y_1,y_2,\cdots,y_n)+(x_1,x_2,\cdots,x_n)-(y_1,y_2,\cdots,y_n))\\ &\geq(\mu_1\times\mu_2\times\cdots\times\mu_n)(x_1,x_2,\cdots,x_n)\\ &\geq t.\end{aligned}$$

This implies that

$$(y_1,y_2,\cdots,y_n)+(x_1,x_2,\cdots,x_n)-(y_1,y_2,\cdots,y_n)\in(\mu_1\times\mu_2\times\cdots\times\mu_n)_t.$$

On the other hand, for $(x_1, x_2, \cdots, x_n) \in R_1 \times R_2 \times \cdots \times R_n$ and $(y_1, y_2, \cdots, y_n) \in (\mu_1 \times \mu_2 \times \cdots \times \mu_n)_t$, $(\mu_1 \times \mu_2 \times \cdots \times \mu_n)(y_1, y_2, \cdots, y_n) \geq t$. Since $\mu_1 \times \mu_2 \times \cdots \times \mu_n$ is a fuzzy ideal of $R_1 \times R_2 \times \cdots \times R_n$,

$$\begin{aligned} &(\mu_1 \times \mu_2 \times \cdots \times \mu_n)((x_1, x_2, \cdots, x_n)(y_1, y_2, \cdots, y_n)) \\ &\geq (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(y_1, y_2, \cdots, y_n) \\ &\geq t. \end{aligned}$$

Thus $(x_1, x_2, \cdots, x_n)(y_1, y_2, \cdots, y_n) \in (\mu_1 \times \mu_2 \times \cdots \times \mu_n)_t$.
Similarly, $(((x_1, x_2, \cdots, x_n) + (z_1, z_2, \cdots, z_n))(y_1, y_2, \cdots, y_n) - (x_1, x_2, \cdots, x_n)(y_1, y_2, \cdots, y_n)) \in (\mu_1 \times \mu_2 \times \cdots \times \mu_n)_t$. So $(\mu_1 \times \mu_2 \times \cdots \times \mu_n)_t$ is an ideal of $R_1 \times R_2 \times \cdots \times R_n$.

Conversely, assume that $(\mu_1 \times \mu_2 \times \cdots \times \mu_n)_t$ is an ideal of $R_1 \times R_2 \times \cdots \times R_n$. Suppose that

$$\begin{aligned} &(\mu_1 \times \mu_2 \times \cdots \times \mu_n)((x_1, x_2, \cdots, x_n) - (y_1, y_2, \cdots, y_n)) \\ &< \min\{(\mu_1 \times \mu_2 \times \cdots \times \mu_n)(x_1, x_2, \cdots, x_n), (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(y_1, y_2, \cdots, y_n)\}. \end{aligned}$$

Choose $t \in (0, 1]$ such that

$$\begin{aligned} &(\mu_1 \times \mu_2 \times \cdots \times \mu_n)((x_1, x_2, \cdots, x_n) - (y_1, y_2, \cdots, y_n)) \\ &< t \\ &< \min\{(\mu_1 \times \mu_2 \times \cdots \times \mu_n)(x_1, x_2, \cdots, x_n), (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(y_1, y_2, \cdots, y_n)\}. \end{aligned}$$

Then $(x_1, x_2, \cdots, x_n), (y_1, y_2, \cdots, y_n) \in (\mu_1 \times \mu_2 \times \cdots \times \mu_n)_t$ but $(x_1, x_2, \cdots, x_n) - (y_1, y_2, \cdots, y_n) \notin (\mu_1 \times \mu_2 \times \cdots \times \mu_n)_t$, which is a contradiction to our assumption that $(\mu_1 \times \mu_2 \times \cdots \times \mu_n)_t$ is an ideal of $R_1 \times R_2 \times \cdots \times R_n$. So

$$\begin{aligned} &(\mu_1 \times \mu_2 \times \cdots \times \mu_n)((x_1, x_2, \cdots, x_n) - (y_1, y_2, \cdots, y_n)) \\ &\geq \min\{(\mu_1 \times \mu_2 \times \cdots \times \mu_n)(x_1, x_2, \cdots, x_n), (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(y_1, y_2, \cdots, y_n)\}. \end{aligned}$$

If there exists $(x_1, x_2, \cdots, x_n), (y_1, y_2, \cdots, y_n) \in R_1 \times R_2 \times \cdots \times R_n$ such that

$$\begin{aligned} &(\mu_1 \times \mu_2 \times \cdots \times \mu_n)((y_1, y_2, \cdots, y_n) + (x_1, x_2, \cdots, x_n) - (y_1, y_2, \cdots, y_n)) \\ &< (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(x_1, x_2, \cdots, x_n), \end{aligned}$$

let us select $t \in (0, 1]$ such that

$$\begin{aligned} &(\mu_1 \times \mu_2 \times \cdots \times \mu_n)((y_1, y_2, \cdots, y_n) + (x_1, x_2, \cdots, x_n) - (y_1, y_2, \cdots, y_n)) \\ &< t \\ &< (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(x_1, x_2, \cdots, x_n). \end{aligned}$$

Then $(x_1, x_2, \cdots, x_n) \in (\mu_1 \times \mu_2 \times \cdots \times \mu_n)_t$ but $((y_1, y_2, \cdots, y_n) + (x_1, x_2, \cdots, x_n) - (y_1, y_2, \cdots, y_n)) \notin (\mu_1 \times \mu_2 \times \cdots \times \mu_n)_t$ which is a contradiction. Thus

$$\begin{aligned} &(\mu_1 \times \mu_2 \times \cdots \times \mu_n)((y_1, y_2, \cdots, y_n) + (x_1, x_2, \cdots, x_n) - (y_1, y_2, \cdots, y_n)) \\ &\geq (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(x_1, x_2, \cdots, x_n). \end{aligned}$$

Similarly,

$$\begin{aligned} &(\mu_1 \times \mu_2 \times \cdots \times \mu_n)((x_1, x_2, \cdots, x_n)(y_1, y_2, \cdots, y_n)) \\ &\geq (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(y_1, y_2, \cdots, y_n) \end{aligned}$$

and

$$\begin{aligned}(\mu_1 \times \mu_2 \times \cdots \times \mu_n)(((x_1, x_2, \cdots, x_n) + (z_1, z_2, \cdots, z_n))(y_1, y_2, \cdots, y_n) \\ - (x_1, x_2, \cdots, x_n)(y_1, y_2, \cdots, y_n)) \\ \geq (\mu_1 \times \mu_2 \times \cdots \times \mu_n)(z_1, z_2, \cdots, z_n).\end{aligned}$$

Therefore $\mu_1 \times \mu_2 \times \cdots \times \mu_n$ is a fuzzy ideal of $R_1 \times R_2 \times \cdots \times R_n$. □

4. Conclusion

In this paper, we have defined direct product of n-fuzzy subnear-rings, n-fuzzy R-subgroups, n-fuzzy ideals. Also discussed some important properties. This work can be extended to other types of ideals of near-rings and other algebraic structures.

5. aknowledgement

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