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Somewhat fuzzy continuity and fuzzy Baire spaces

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Abstract. In this paper, several characterizations of fuzzy Baire spaces are obtained. The conditions underwhich somewhat fuzzy continuous and somewhat fuzzy open functions become fuzzy continuous and fuzzy open functions respectively on fuzzy topological spaces, are also investigated.

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1. Introduction

In order to deal with uncertainties, the idea of fuzzy sets, fuzzy set operations was introduced by L.A.Zadeh [12] in his classical paper in the year 1965, describing fuzziness mathematically for the first time. Since then, the notion of fuzziness has been applied for the study in all branches of Mathematics. Among the first field of Mathematics to be considered in the context of fuzzy sets was general topology. The concept of fuzzy topology was defined by C.L.Chang [5] in the year 1968. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then, much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed.

In classical topology, Baire space named in honor of Rene Louis Baire, was first introduced in Bourbaki's [4] Topologie Generale Chapter IX. The concepts of Baire spaces have been studied extensively in classical topology in [6] and [7]. The concept of Baire spaces in fuzzy setting was introduced and studied by G.Thangaraj and S.Anjalmose in [11]. The class of somewhat continuous functions was introduced by Karl R.Gentry and Hughes B.Hoyle III in [8]. Later, the concept of "somewhat" in classical topology has been extended to fuzzy topological spaces. Somewhat fuzzy continuous and somewhat fuzzy open functions on fuzzy topological spaces were introduced and studied by G.Thangaraj and G.Balasubramanian in [10]. The purpose of this paper is to study several characterizations of fuzzy Baire spaces. The conditions underwhich somewhat fuzzy continuous and somewhat fuzzy open functions become fuzzy continuous and fuzzy open functions on fuzzy topological spaces, are also investigated in this paper.

2. Preliminaries

By a fuzzy topological space we shall mean a non-empty set X together with a fuzzy topology T (in the sense of Chang) and denote it by (X, T).

Definition 2.1 ([5])**.** Let λ and μ be any two fuzzy sets in a fuzzy topological space (X, T). Then we define

(i) $\lambda \vee \mu : X \to [0, 1]$ as follows : $(\lambda \vee \mu)(x) = Max\{\lambda(x), \mu(x)\}$ where $x \in X$,
(ii) $\lambda \wedge \mu : X \to [0, 1]$ as follows : $(\lambda \wedge \mu)(x) = Min\{\lambda(x), \mu(x)\}$ where $x \in X$,
(iii) $\mu = \lambda^c \Leftrightarrow \mu(x) = 1 - \lambda(x)$ where $x \in X$.

More generally, for a family $\{\lambda_i / i \in I\}$ of fuzzy sets in (X, T), the union $\psi = \vee_i \lambda_i$ and intersection $\delta = \wedge_i \lambda_i$ are defined respectively as $\psi(x) = sup_i\{\lambda_i(x), x \in X\}$ and $\delta(x) = inf_i\{\lambda_i(x), x \in X\}$.

Definition 2.2 ([1])**.** Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). We define the interior and the closure of λ as follows:

(i) $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$,
(ii) $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$.

Lemma 2.3 ([1])**.** *Let λ be any fuzzy set in a fuzzy topological space (X, T). Then $1 - cl(\lambda) = int(1 - \lambda)$ and $1 - int(\lambda) = cl(1 - \lambda)$.*

Definition 2.4 ([10])**.** A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy dense if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$. That is., $cl(\lambda) = 1$ in (X, T).

Definition 2.5 ([10])**.** A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X, T) such that $\mu < cl(\lambda)$. That is., $intcl(\lambda) = 0$, in (X, T).

Definition 2.6 ([3])**.** Let (X, T) be a fuzzy topological space and λ be a fuzzy set in X. Then λ is called a fuzzy G_δ-set if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, for each $\lambda_i \in T$.

Definition 2.7 ([3])**.** Let (X, T) be a fuzzy topological space and λ be a fuzzy set in X. Then λ is called a fuzzy F_σ-set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, for each $1 - \lambda_i \in T$.

Definition 2.8 ([10])**.** A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of fuzzy second category.

Definition 2.9 ([11])**.** Let λ be a fuzzy first category set in a fuzzy topological space (X, T). Then $1 - \lambda$ is called a fuzzy residual set in (X, T).

Lemma 2.10 ([1])**.** *For a family $\mathscr{A} = \{\lambda_\alpha\}$ of fuzzy sets of a fuzzy space X, $\vee\big(cl(\lambda_\alpha)\big) \leq cl\big(\vee(\lambda_\alpha)\big)$. In case $\mathscr{A}$ is a finite set, $\vee\big(cl(\lambda_\alpha)\big) = cl\big(\vee(\lambda_\alpha)\big)$. Also $\vee\big(int(\lambda_\alpha)\big) \leq int\big(\vee(\lambda_\alpha)\big)$.*

Definition 2.11 ([9])**.** Let λ be a fuzzy set in a fuzzy topological space (X, T). The fuzzy boundary of λ is defined as $Bd(\lambda) = cl(\lambda) \wedge cl(1 - \lambda)$. Obviously $Bd(\lambda)$ is a fuzzy closed set.

Definition 2.12 ([3])**.** A fuzzy topological space (X, T) is called a fuzzy submaximal space if $cl(\lambda) = 1$, for any non-zero fuzzy set λ in (X, T), then $\lambda \in T$.

Definition 2.13 ([2])**.** A fuzzy set λ in a topological space (X, T) is called a fuzzy simply open set, if $Bd(\lambda)$ is a fuzzy nowhere dense set in (X, T). That is., λ is a fuzzy simply open set in (X, T) if $cl(\lambda) \wedge cl(1 - \lambda)$, is a fuzzy nowhere dense set in (X, T).

Definition 2.14 ([11])**.** Let (X,T) be a fuzzy topological space. Then (X, T) is called a fuzzy Baire space if $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T).

Theorem 2.15 ([11])**.** *Let (X, T) be a fuzzy topological space. Then the following are equivalent :*
(1) *(X, T) is a fuzzy Baire space.*
(2) *$int(\lambda) = 0$ for every fuzzy first category set λ in (X, T).*
(3) *$cl(\mu) = 1$ for every fuzzy residual set μ in (X, T).*

Theorem 2.16 ([11])**.** *If a fuzzy first category set λ in a fuzzy Baire space (X, T) is a fuzzy closed set, then λ is a fuzzy nowhere dense set in (X, T).*

Theorem 2.17 ([11])**.** *If λ is a fuzzy first category set in (X, T), then there is a non-zero fuzzy F_σ set η in (X, T) such that $\lambda \leq \eta$.*

3. Fuzzy Baire spaces

Proposition 3.1. *If λ is a fuzzy first category set in a fuzzy Baire space (X, T), then λ is not a fuzzy nowhere dense set in (X, T).*

Proof. Let λ be a fuzzy first category set in (X, T). Since (X, T) is a fuzzy Baire space by Theorem 2.15, $int(\lambda) = 0$ in (X, T). Now $cl(1-\lambda) = 1-int(\lambda) = 1-0 = 1$ and $int[cl(\lambda)] = 1 - (1 - int[cl(\lambda)]) = 1 - cl[int(1 - \lambda)] > 1 - cl(1 - \lambda) > 1 - 1 = 0$. That is, $int[cl(\lambda) \neq 0]$ and hence λ is not a fuzzy nowhere dense set in (X, T). □

Remark 3.2. From the above proposition, one can see that if λ is a fuzzy first category set in a fuzzy Baire space, then λ is not a fuzzy open set, since $int(\lambda) = 0$ implies that $int(\lambda) \neq \lambda$ in (X, T).

Proposition 3.3. *If $int(\eta) = 0$, for each fuzzy F_σ set η in a fuzzy topological space (X, T), then (X, T) is a fuzzy Baire space.*

Proof. Let λ be a fuzzy first category set in (X, T). Then, by 2.17, there is a non-zero fuzzy F_σ set η in (X, T) such that $\lambda \leq \eta$. By hypothesis, $int(\eta) = 0$. Now $\lambda \leq \eta$ implies that $int(\lambda) \leq int(\eta)$ and hence $int(\lambda) \leq 0$. That is, $int(\lambda) = 0$. Thus, for a fuzzy first category set λ in (X, T), we have $int(\lambda) = 0$. So, by Theorem 2.15, (X, T) is a fuzzy Baire space. □

Proposition 3.4. *If $cl(\delta) = 1$, for each fuzzy G_δ set δ in a fuzzy topological space (X, T), then (X, T) is a fuzzy Baire space.*

Proof. Let λ be a fuzzy first category set in (X,T). Then, by Theorem 2.17, there is a non-zero fuzzy F_σ set η in (X,T) such that $\lambda \leq \eta$. Since η is a fuzzy F_σ set, $1-\eta$ is a fuzzy G_δ set in (X,T). Thus, by the hypothesis, $cl(1-\eta) = 1$ and $cl(1-\eta) = 1-int(\eta)$, implies that $1-int(\eta) = 1$. So $int(\eta) = 0$ in (X,T). Now $\lambda \leq \eta$ implies that $int(\lambda) \leq int(\eta)$. Hence $int(\lambda) \leq 0$. That is., $int(\lambda) = 0$. Therefore, by 2.15, (X,T) is a fuzzy Baire space. □

Proposition 3.5. *If λ is a fuzzy residual set in a fuzzy topological space (X,T), then there exists a fuzzy G_δ set μ in (X,T) such that $\mu \leq \lambda$.*

Proof. Let λ be a fuzzy residual set in (X,T). Then $1-\lambda$ is a fuzzy first category set in (X,T). Thus, by Theorem 2.17, there is a non-zero fuzzy F_σ set η in (X,T) such that $1-\lambda \leq \eta$. So $1-\eta \leq \lambda$. Since η is a fuzzy F_σ set in (X,T), $1-\eta$ is a fuzzy G_δ set in (X,T). Let $\mu = 1-\eta$. Then, we have $\mu \leq \lambda$, where μ is a fuzzy G_δ set in (X,T). □

Proposition 3.6. *If λ is a fuzzy set defined on a fuzzy Baire space (X,T) and if $\mu \leq \lambda$, where $\mu \in T$, then λ is not a fuzzy first category set in (X,T).*

Proof. Let λ be a fuzzy set defined on X such that $\mu \leq \lambda$, where μ is a non-zero fuzzy open set in (X,T). Now $\mu \leq \lambda$ implies that $int(\mu) \leq int(\lambda)$. Then $\mu \leq int(\lambda)$ (since $int(\mu) = \mu$). Suppose that λ is a fuzzy first category set in (X,T). Since (X,T) is a fuzzy Baire space, by Theorem 2.15, $int(\lambda) = 0$. Thus $\mu \leq 0$ in (X,T). That is., $\mu = 0$ in (X,T). But this is a contradiction to μ being a non-zero fuzzy open set in (X,T). So λ is not a fuzzy first category set in (X,T). □

Proposition 3.7. *A fuzzy topological space (X,T) is a fuzzy Baire space if and only if each first category and fuzzy simply open set in (X,T) is a fuzzy nowhere dense set in (X,T).*

Proof. Let λ be a fuzzy first category set in (X,T) such that $intcl[Bd(\lambda)] = 0$ in (X,T). Now $intcl[cl(\lambda) \wedge (1-\lambda)] \leq intcl[cl(\lambda) \wedge cl(1-\lambda)]$. Since λ is a fuzzy simply open set,

$$intcl[cl(\lambda) \wedge cl(1-\lambda)] = 0. \tag{3.1.1}$$

Since (X,T) is a fuzzy Baire space, by Theorem 2.15, $int(\lambda) = 0$ in (X,T). Then $cl(1-\lambda) = 1 - int(\lambda) = 1 - 0 = 1$. That is,

$$cl(1-\lambda) = 1. \tag{3.1.2}$$

From (3.1.1) and (3.1.2), $intcl[cl(\lambda) \wedge 1] = 0$. This implies that $intcl[cl(\lambda)] = 0$ in (X,T). Since $cl[cl(\lambda)] = cl(\lambda)$, $int[cl(\lambda)] = 0$. Thus λ is a fuzzy nowhere dense set in (X,T).

Conversely, let each fuzzy first category and fuzzy simply open set λ is a fuzzy nowhere dense set in (X,T). Then $int[cl(\lambda)] = 0$. But $int(\lambda) \leq int[cl(\lambda)]$ implies that $int(\lambda) \leq 0$. That is, $int(\lambda) = 0$. Thus, by Theorem 2.15, (X,T) is a fuzzy Baire space. □

4. Fuzzy Baire spaces and somewhat fuzzy continuous functions

Definition 4.1 ([10])**.** A function $f : (X, T) \to (Y, S)$ from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S) is called somewhat fuzzy continuous if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0$, implies that there exists a fuzzy open set δ in (X, T) such that $\delta \neq 0$ and $\delta \leq f^{-1}(\lambda)$. That is., $int[f^{-1}(\lambda)] \neq 0$.

It is clear that every fuzzy continuous function is a somewhat fuzzy continuous function. But the converse is not true in general [10].

Definition 4.2 ([10])**.** A function $f : (X, T) \to (Y, S)$ from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S) is called somewhat fuzzy open if for all $\lambda \in T$ and $(\lambda) \neq 0$, there exists a fuzzy open set η in (Y, S) such that $\eta \neq 0$ and $\eta \leq f(\lambda)$. That is., $int[f(\lambda)] \neq 0$.

It is to be noted that every fuzzy open function is a somewhat fuzzy open function but the converse is not true in general [10].

Proposition 4.3. *If $f : (X, T) \to (Y, S)$ is a somewhat fuzzy continuous function from a fuzzy Baire space (X, T) into a fuzzy topological space (Y, S) and if λ is a fuzzy open set in (Y, S) then $f^{-1}(\lambda)$ is a fuzzy second category set in (X, T).*

Proof. Let λ be a fuzzy open set in (Y, S). Since $f : (X, T) \to (Y, S)$, is a somewhat fuzzy continuous function, $int[f^{-1}(\lambda)] \neq 0$. We claim that $f^{-1}(\lambda)$ is a fuzzy second category set in (X, T). Assume the contrary. Suppose that $f^{-1}(\lambda)$ is a fuzzy first category set in (X, T). Then $f^{-1}(\lambda) = \vee_{i=1}^{\infty}(\lambda_i)$ where (λ_i)'s are fuzzy nowhere dense sets in (X, T). Now $int[f^{-1}(\lambda)] = int(\vee_{i=1}^{\infty}(\lambda_i))$. Since $int[f^{-1}(\lambda)] \neq 0$, $int(\vee_{i=1}^{\infty}(\lambda_i)) \neq 0$ where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X, T). This implies that (X, T) is not a fuzzy Baire space, a contradiction. Thus $f^{-1}(\lambda)$ must be a fuzzy second category set in (X, T). □

Proposition 4.4. *If $f : (X, T) \to (Y, S)$ is a somewhat fuzzy open function from a fuzzy topological space (X, T) into a fuzzy Baire space (Y, S) and if λ is a fuzzy open set in (X, T), then $f(\lambda)$ is a fuzzy second category set in (Y, S).*

Proof. Let λ be a fuzzy open set in (X, T). Since $f : (X, T) \to (Y, S)$ is a somewhat fuzzy open function, $int[f(\lambda)] \neq 0$ in (Y, S). We claim that $f(\lambda)$ is a fuzzy second category set in (X, T). Assume the contrary. Then $f(\lambda) = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (Y, S). Thus $int[f(\lambda)] = int(\vee_{i=1}^{\infty}(\lambda_i))$ and $int[f(\lambda)] \neq 0$ implies that $int(\vee_{i=1}^{\infty}(\lambda_i)) \neq 0$, a contradiction to (Y, S) being a fuzzy Baire space. So our assumption that $f(\lambda)$ is a fuzzy first category set, does not hold. Hence $f(\lambda)$ must be a fuzzy second category set in (Y, S). □

Proposition 4.5. *Let $f : (X, T) \to (Y, S)$ be a somewhat fuzzy continuous function from a fuzzy Baire space (X, T) into a fuzzy topological space (Y, S). If each fuzzy second category set is a fuzzy open set in (X, T), the f is a fuzzy continuous function from (X, T) into (Y, S).*

Proof. Let λ be a fuzzy open set in (Y, S). Since f is a somewhat fuzzy continuous function from the fuzzy Baire space (X, T) into a fuzzy topological space (Y, S), by Proposition 4.3, $f^{-1}(\lambda)$ is a fuzzy second category set in (X, T). By hypothesis $f^{-1}(\lambda)$ is a fuzzy open set in (X, T). Thus, for each fuzzy open set λ in (Y, S),

$f^{-1}(\lambda)$ is a fuzzy open set in (X, T). So f is a fuzzy continuous function from (X, T) into (Y, S). □

Proposition 4.6. *Let $f : (X, T) \to (Y, S)$ be a somewhat fuzzy open function from a fuzzy topological space (X, T) into a fuzzy Baire space (Y, S). If each fuzzy second category set is a fuzzy open set in (Y, S), then f is an fuzzy open function from (X, T) into (Y, S).*

Proof. Let λ be an fuzzy open set in (X, T). Since f is a somewhat fuzzy open function from a fuzzy topological space (X, T) into a fuzzy Baire space (Y, S), by Proposition 4.4, $f(\lambda)$ is a fuzzy second category set in (Y, S). By hypothesis, $f(\lambda)$ is an fuzzy open set in (Y, S). Thus, for each fuzzy open set λ in (X, T), $f(\lambda)$ is an fuzzy open set in (Y, S). So f is an fuzzy open function from (X, T) into (Y, S). □

Theorem 4.7 ([11])**.** *If a fuzzy topological space (X, T) is a fuzzy Baire space, then no non-zero fuzzy open set is a fuzzy first category set in (X, T).*

Proposition 4.8. *If $f : (X, T) \to (Y, S)$ is a somewhat fuzzy continuous and one - one function from the fuzzy Baire space (X, T) onto a fuzzy topological space (Y, S), then f is a fuzzy continuous function.*

Proof. Let λ be a fuzzy open set in (Y, S). Since $f : (X, T) \to (Y, S)$ is a somewhat fuzzy continuous function from the fuzzy Baire space (X, T) onto a space (Y, S), by Proposition 4.3, $f^{-1}(\lambda)$ is a fuzzy second category set in (X, T). Since (X, T) is a fuzzy Baire space, by Theorem 4.7, there exists a non-zero fuzzy open set μ in (X, T) such that $\mu = f^{-1}(\lambda)$. Then $f^{-1}(\lambda)$ is a fuzzy open set in (X, T). Thus, for a fuzzy open set λ in (Y, S), $f^{-1}(\lambda)$ is a fuzzy open set in (X, T), implies that f is a fuzzy continuous function from (X, T) onto (Y, S). □

Proposition 4.9. *If $f : (X, T) \to (Y, S)$ is a somewhat fuzzy open and one - one function from a fuzzy topological space (X, T) onto a fuzzy Baire space (Y, S), then f is an fuzzy open function from (X, T) onto (Y, S).*

Proof. Let λ be a fuzzy open set in (X, T). Since $f : (X, T) \to (Y, S)$ is a somewhat fuzzy open function from the fuzzy topological space (X, T) into a fuzzy Baire space (Y, S), by Proposition 4.4, $f(\lambda)$ is a fuzzy second category set in (Y, S). Since (X, T) is a fuzzy Baire space, by Theorem 4.7, there exists a fuzzy open set δ in (Y, S) such that $\delta = f(\lambda)$. Then $f(\lambda)$ is a fuzzy open set in (Y, S). Thus, for each fuzzy open set λ in (X, T), $f(\lambda)$ is a fuzzy open set in (Y, S), implies that f is an fuzzy open function from (X, T) onto (Y, S). □

Proposition 4.10. *If $f : (X, T) \to (Y, S)$ is a somewhat fuzzy continuous, one-one and somewhat fuzzy open function from a fuzzy Baire space (X, T) onto a fuzzy Baire space (Y, S) then f is a fuzzy continuous and fuzzy open function from (X, T) onto (Y, S).*

Proof. The proof follows from the Propositions 4.8 and 4.9. □

Theorem 4.11 ([11])**.** . *If $\lambda \leq \mu$ and μ is a fuzzy first category set in a fuzzy topological space (X, T), then λ is also a fuzzy first category set in (X, T).*

Proposition 4.12. *If $f : (X, T) \to (Y, S)$ is a somewhat fuzzy continuous function from a fuzzy Baire space (X, T) into a fuzzy topological spaces (Y, S) and if λ is a fuzzy set defined on Y and $\mu \leq \lambda$ where $\mu \in S$, then $f^{-1}(\lambda)$ is a fuzzy second category set in (X, T).*

Proof. Let λ be a fuzzy set defined on Y and $\mu \leq \lambda$, where μ is a non-zero fuzzy open set in (Y, S). Now $\mu \leq \lambda$ in (Y, S), implies that $f^{-1}(\mu) \leq f^{-1}(\lambda)$ in (X, T). Since f is a somewhat fuzzy continuous function and μ is a non-zero fuzzy open set in (Y, S), there exists a non-zero fuzzy open set δ in (X, T) such that $\delta \leq f^{-1}(\mu)$ in (X, T). Then $\delta \leq f^{-1}(\mu) \leq f^{-1}(\lambda)$ in (X, T). Thus $\delta \leq f^{-1}(\lambda)$ in (X, T). If $f^{-1}(\lambda)$ is a fuzzy first category set in (X, T), then, by Theorem 4.11, δ will be a fuzzy first category set in (X, T), a contradiction to Theorem 4.7. So $f^{-1}(\lambda)$ is not a fuzzy first category set in (X, T). Hence $f^{-1}(\lambda)$ is a fuzzy second category set in (X, T). □

Proposition 4.13. *If $f : (X, T) \to (Y, S)$ is a somewhat fuzzy open function from a fuzzy topological space (X, T) into a fuzzy Baire space (Y, S) and if λ is a fuzzy set defined on X and $\mu \leq \lambda$, where $\mu \in T$, then $f(\lambda)$ is a fuzzy second category set in (X, T).*

Proof. Let λ be a fuzzy set defined on X and $\mu \leq \lambda$, where μ is a non-zero fuzzy open set in (X, T). Now $\mu \leq \lambda$ in (X, T) implies that $f(\mu) \leq f(\lambda)$ in (Y, S). Since f is a somewhat fuzzy open function and μ is a non-zero fuzzy open set in (X, T) there exists a non-zero fuzzy open set η in (Y, S) such that $\eta \leq f(\mu)$ in (Y, S). Then $\eta \leq f(\mu) \leq f(\lambda)$ in (Y, S). Thus $\eta \leq f(\lambda)$ in (Y, S). If $f(\lambda)$ is a fuzzy first category set in (Y, S), then, by Theorem 4.11, η will be a fuzzy first category set in (Y, S), a contradiction to Theorem 4.7. So $f(\lambda)$ is not a fuzzy first category set in (Y, S). Hence $f(\lambda)$ is a fuzzy second category set in (Y, S). □

Proposition 4.14. *If $f : (X, T) \to (Y, S)$ is a somewhat fuzzy open function from a fuzzy Baire space (X, T) into a fuzzy submaximal space (Y, S) and if λ is a fuzzy dense set in (Y, S) then $f^{-1}(\lambda)$ is a fuzzy second category set in (X, T).*

Proof. Let λ be a fuzzy dense set in (Y, S). Since (Y, S) is a fuzzy submaximal space, then fuzzy dense set λ is a fuzzy open set in (Y, S). Since f is a somewhat fuzzy continuous function and $\lambda \in S$, there exists a non-zero fuzzy open set μ in (X, T) such that $\mu \leq f^{-1}(\lambda)$. Then, by Proposition 4.12 , $f^{-1}(\lambda)$ is a fuzzy second category set in (X, T). □

Proposition 4.15. *If $f : (X, T) \to (Y, S)$ is a somewhat fuzzy open function from a fuzzy submaximal space (X, T) into a fuzzy Baire space (Y, S) and if λ is a fuzzy dense set in (X, T), then $f(\lambda)$ is a fuzzy second category set in (Y, S).*

Proof. Let λ be a fuzzy dense set in (X, T). Since (X, T) is a fuzzy submaximal space, the fuzzy dense set λ is a fuzzy open set in (X, T). Since f is a somewhat fuzzy open function and $\lambda \in T$, there exists a non-zero fuzzy open set η in (Y, S) such that $\eta \leq f(\lambda)$. Then, by Proposition 4.13, $f(\lambda)$ is a fuzzy second category set in (Y, S). □

5. Conclusions

In this paper, characterization of fuzzy Baire spaces in terms of fuzzy G_δ and fuzzy F_σ sets are established. A necessary and sufficient condition for a fuzzy topological space to be a fuzzy Baire space, is also established by means of fuzzy simply open sets. A fuzzy continuous function between fuzzy topological spaces is always a fuzzy somewhat continuous but the converse is not true in general. Also a fuzzy open function between topological spaces is always a somewhat fuzzy open function but the converse is not true in general. The conditions under which somewhat fuzzy continuous and somewhat fuzzy open functions become fuzzy continuous and fuzzy open functions on fuzzy topological spaces, are also investigated in this paper.

References

[1] K. K. Azad, On fuzzy semi-continuity,fuzzy almost continuity and fuzzy weakly continuity, J. Math. Anal. Appl. 82 (1981) 14–32.

[2] M. Athar and B. Ahmad, Fuzzy boundary and fuzzy semi-boundary, Advances in Fuzzy Systems Vol.2008, 586893, 9 Pages.

[3] G. Balasubramanian, Maximal fuzzy topologies, Kybernetika. 31 (5) (1995) 459–464.

[4] N. Bourbaki, Topologie Generale, Chap.9 (Act. Sci. Ind. no. 1045) Paris 1948.

[5] C. L. Chang, Fuzzy Topological Spaces, J. Math. Anal. Appl. 24 (1968) 182–190.

[6] G. Gruenhage and D. Lutzer, Baire and Volterra spaces, Proc. Amer. Math. Soc. 128 (2000) 3115–3124.

[7] R. C. Haworth and R. A. McCoy, Baire spaces, Dissertationes. Math. 141 (1977) 1–77.

[8] Karl. R. Gentry and Hughes. B. Hoyle III, Somewhat continuous functions, Czech. Math. Jour. 21 (96) (1971) 5–12.

[9] P. M. Pu and Y. M. Liu, Fuzzy topology–I: neighborhood structure of a fuzzy point and Moore-Smith convergence, J. Math. Anal. Appl. 76 (2) (1980) 571–599.

[10] G. Thangaraj and G. Balasubramanian, On somewhat fuzzy continuous functions, J. Fuzzy Math. 11 (2) (2003) 725–736.

[11] G. Thangaraj and S. Anjalmose, On fuzzy Baire spaces, J.Fuzzy Math. 21 (3) (2013) 667–676.

[12] L.A.Zadeh, Fuzzy sets, Informatin and Control, 8 (1965) 338–353.

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