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# Some types of open soft multisets and some types of mappings in soft multi topological spaces

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Abstract. The main purpose of this paper is to introduce the notions of $\gamma$-operation, pre-open soft multisets, $\alpha$-open multisets, semi-open soft multisets, $\beta$-open soft multisets and $b$-open soft multisets in soft multi topological spaces. In addition, the relationships among these types are studied. Moreover, the concepts of pre-soft multi continuous, $\alpha$-soft multi continuous, semi-soft multi continuous, $\beta$-soft multi continuous and $b$-soft multi continuous functions are introduced and their properties are studied in detail. Also, the concepts of pre-soft (respectively semi-soft, $\alpha$-soft, $\beta$-soft, $b$-soft) multi irresolute functions are presented.

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Keywords: Soft multisets, Soft multi topological spaces, $\gamma$-operations, Pre-open soft multisets, $\alpha$-open soft multisets, Semi-open soft multisets, $\beta$-open soft multisets, $b$-open soft multisets.

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## 1. Introduction

In 2013, Tokat [6] was introduced the concept of soft multisets as a combining between soft sets and multisets. Moreover, in [6] the soft multi topology and its basic properties was given. In addition, the soft multi connectedness was studied in [7]. Furthermore, the soft multi compactness on soft multi topological spaces was presented in [8]. El-Sheikh et al. in [4] introduced the soft multi semi-compactness as a generalization of soft multi compactness. In 2015, Tokat et al. [9] presented the soft multi continuous functions.

El-Sheikh et al. [3] introduced some M-operations such as semi-open mset, pre-open mset, $\alpha$-open mset, $\beta$-open mset and $b$-open mset. Moreover, they investigated their properties and the relationships among these M-operations in detail. In addition, the concept of soft semi-open set was first studied by Chen [2]. He investigated soft semi-open sets in soft topological spaces and studied some properties of it. In

2014, Kandil et al. [5] introduced the notion of soft $\gamma$-operation and presented some forms of soft continuity in soft topological spaces. Later, Akdag and Ozkan [1] defined soft $b$-open sets and soft $b$-continuity.

In this paper, we firstly introduced the notions of $\gamma$-operation, pre-open soft msets, $\alpha$-open soft msets, semi open soft msets, $\beta$-open soft msets and $b$-open soft msets in soft multi topological spaces. Further, the relationships among these types of sub soft msets in soft multi topological spaces are obtained. Also, the concepts of pre-soft (respectively $\alpha$-soft, semi-soft, $\beta$-soft, $b$-soft) multi continuous functions are introduced. Also, we discuss some important properties in detail. Finally, the concepts of pre-soft (respectively semi-soft, $\alpha$-soft, $\beta$-soft, $b$-soft) multi irresolute functions are presented.

## 2. Preliminaries

In this section, we present the basic definitions and results of soft multiset theory which will be needed in the sequel.

**Definition 2.1** ([7])**.** Let $X$ be a universal multiset (for short, mset), $E$ be a set of parameters and $A \subseteq E$. Then, an ordered pair $(F, A)$ is called a soft mset, where $F$ is a mapping given by $F : A \to P^*(X)$, $P^*(X)$ is the power set of an mset $X$. For every $e \in A$, $F(e)$ mset represent by count function $C_{F(e)} : X^* \to N$, where $N$ represents the set of non-negative integers and $X^*$ represents the support set of $X$.

For example, let $X = \{2/x, 3/y, 1/z\}$ be an mset. Then the support set of $X$ is $X^* = \{x, y, z\}$.

**Definition 2.2** ([7])**.** Let $(F, A)$ and $(G, B)$ be two soft msets over $X$. Then, $(F, A)$ is said to be a sub soft mset of $(G, B)$, if

(i) $A \subseteq B$,

(ii) $C_{F(e)}(x) \leqslant C_{G(e)}(x)$, for all $x \in X^*$, $e \in A$. It is denoted by $(F, A)\widetilde{\subseteq}(G, B)$.

**Definition 2.3** ([7])**.** Two soft msets $(F, A)$ and $(G, B)$ over $X$ are equal if $(F, A)$ is a sub soft mset of $(G, B)$ and $(G, B)$ is a sub soft mset of $(F, A)$.

**Definition 2.4** ([7])**.** Let $(F, A)$ and $(G, B)$ be two soft msets over an mset $X$. Then

(i) The union of two soft msets $(F, A)$ and $(G, B)$ over $X$, denoted by $(F, A)\widetilde{\cup}(G, B)$, is the soft mset $(H, C)$, where

$C = A \cup B$ and $C_{H(e)}(x) = max\{C_{F(e)}(x), C_{G(e)}(x)\}$, for all $e \in A \cup B$, $x \in X^*$.

(ii) The intersection of two soft msets $(F, A)$ and $(G, B)$ over $X$, denoted by $(F, A)\widetilde{\cap}(G, B)$, is the soft mset $(H, C)$, where

$C = A \cap B$ and $C_{H(e)}(x) = min\{C_{F(e)}(x), C_{G(e)}(x)\}$, for all $e \in A \cap B$, $x \in X^*$.

(iii) The difference between two soft msets $(F, E)$ and $(G, E)$ over $X$ is the soft mset $(H, C) = (F, E)\backslash(G, E)$ and defined as $H(e) = F(e)\backslash G(e)$ for all e∈E where $C_{H(e)}(x) = max\{C_{F(e)}(x) - C_{G(e)}(x), 0\}$, for all $x \in X^*$.

**Definition 2.5** ([7])**.** A soft mset $(F, A)$ over $X$ is said to be:

(i) a null soft mset, denoted by $\widetilde{\phi}$, if for all $e \in A$, $F(e) = \phi$.

(ii) an absolute soft mset, denoted by $\widetilde{A}$, if for all $e \in A$, $F(e) = X$.

**Definition 2.6** ([7]). Let $V$ be a non-empty submset of $X$, then $\widetilde{V}$ denotes the soft mset $(H, E)$ over $X$ for which $H(e) = V$, for all $e \in E$. In particular, $(X, E)$ will be denoted by $\widetilde{X}$ or $X_E$.

**Remark 2.1** ([7]). Let $(F, E)$ be a soft mset over $X$. If for all $e \in E$ and $a \in X^*$, $C_{F(e)}(a) = n$ $(n \geqslant 1)$, then it is written $a \in F(e)$ instead of $a \in^n F(e)$.

**Definition 2.7** ([7]). Let $(F, E)$ be a soft mset over $X$ and a$\in X^*$. Then, $a \in (F, E)$ read as a belongs to the soft mset $(F, E)$ whenever $a \in F(e)$ for all $e \in E$.

**Definition 2.8.** [7] Let $a \in X^*$. Then, $(a, E)$ denoted to the soft mset over $X$ for which $a(e) = \{a\}$, for all $e \in E$.

**Definition 2.9.** [7] Let $(F, E)$ be a soft mset over $X$ and $V$ be a non-empty submset of $X$. Then, the sub soft mset of $(F, E)$ over $V$, denoted by $({}^V F, E)$, is defined as follows:
${}^V F(e) = V \cap F(e)$, for all $e \in E$ where $C_{{}^V F(e)}(x) = min\{C_V(x), C_{F(e)}(x)\}$, for all $x \in X^*$.
In other words $({}^V F, E) = \widetilde{V} \widetilde{\cap} (F, E)$.

**Definition 2.10** ([7]). The complement of a soft mset $(F, A)$ is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \to P^*(X)$ is a mapping given by $F^c(e) = X \backslash F(e)$ for all $e \in A$ where $C_{F^c(e)}(x) = C_X(x) - C_{F(e)}(x)$, for all $x \in X^*$.

**Definition 2.11** ([7]). Let $X$ be a universal mset and $E$ be a set of parameters. Then, the family of all soft msets over $X$ with parameters from $E$ is called a soft multi class and is denoted by $SMS(X)_E$.

**Definition 2.12** ([7]). Let $\tau \subseteq SMS(X)_E$. Then, $\tau$ is said to be a soft multi topology on $X$ if the following conditions hold:
(i) $\widetilde{\phi}, \widetilde{X}$ belong to $\tau$.
(ii) The union of any number of soft msets in $\tau$ belongs to $\tau$.
(iii) The intersection of any two soft msets in $\tau$ belongs to $\tau$.
$\tau$ is called a soft multi topology over $X$ and the triple $(X, \tau, E)$ is called a soft multi topological space over $X$. Also, The members of $\tau$ are called open soft msets. Furthermore, $OSM(X)_E$ is the set of all open sub soft msets of $\widetilde{X}$.

A soft mset $(F, E)$ in $SMS(X)_E$ is said to be a closed soft mset if its complement $(F, E)^c$ belongs to $\tau$.

**Definition 2.13** ([7]). Let $X$ be a universal mset and $E$ be the set of parameters. Then

(i) $\tau = \{\widetilde{\phi}, \widetilde{X}\}$ is called the indiscrete soft multi topology on $X$ and $(X, \tau, E)$ is said to be an indiscrete soft multi space over $X$.

(ii) $\tau = SMS(X)_E$ is called the discrete soft multi topology on $X$ and $(X, \tau, E)$ is said to be a discrete soft multi space over $X$.

**Definition 2.14** ([7]). Let $(X, \tau, E)$ be a soft multi topological space over $X$ and $Y$ be a non-empty submset of $X$. Then

$$\tau_Y = \{({}^Y F, E) : (F, E) \in \tau\}$$

is said to be the soft multi topology on $Y$ and $(Y, \tau_Y, E)$ is called a soft multi subspace of $(X, \tau, E)$.

**Definition 2.15** ([6]). Let $(X, \tau, E)$ be a soft multi topological space over $X$ and $(F, E)$ be a sub soft mset of $\widetilde{X}$. Then

(i) The soft multi closure of $(F, E)$, denoted by $cl(F, E)$ [or $\overline{(F, E)}$], is the intersection of all closed soft mset containing $(F, E)$.

(ii) The soft multi interior of $(F, E)$, denoted by $int(F, E)$ [or $(F, E)^o$], is the union of all open soft mset contained in $(F, E)$.

**Definition 2.16** ([8]). Let $(X, \tau_1, E)$ and $(Y, \tau_2, K)$ be two soft multi topological spaces. Let $\varphi : X^* \to Y^*$ and $\psi : E \to K$ be two functions. Then, the pair $(\varphi, \psi)$ is called a soft mset function and denoted by $f = (\varphi, \psi) : SMS(X)_E \to SMS(Y)_K$ is defined as follows:

Let $(F, E) \in SMS(X)_E$. Then, the image of $(F, E)$ under the soft mset function $f$ is the soft mset in $SMS(Y)_K$ defined by $f(F, E)$, where for $k \in \psi(E) \subseteq K$ and $y \in Y^*$,

$$C_{f(F,E)(k)}(y) = \begin{cases} sup_{e\in\psi^{-1}(k)\cap E, x\in\varphi^{-1}(y)} C_{F(e)}(x), & \text{if } \psi^{-1}(k) \neq \phi, \varphi^{-1}(y) \neq \phi; \\ 0, & \text{otherwise.} \end{cases}$$

Let $(G, K)$ be a soft mset in $SMS(Y)_K$. Then, the inverse image of $(G, K)$ under the soft mset function $f$ is the soft mset in $SMS(X)_E$ defined by $f^{-1}(G, K)$, where for $e \in \psi^{-1}(K) \subseteq E$ and $x \in X^*$,

$$C_{f^{-1}(G,K)(e)}(x) = C_{G(\psi(e))}(\varphi(x)).$$

**Theorem 2.1** ([8]). *Let $f : SMS(X)_E \to SMS(Y)_K$ be a soft mset function, $(F_i, A)$ be a soft mset in $SMS(X)_E$ and $(G_i, B)$ be a soft mset in $SMS(Y)_K$. Then*

(1) $f(\widetilde{\bigcup}_{i\in I}(F_i, A_i)) = \widetilde{\bigcup}_{i\in I} f(F_i, A_i)$.

(2) $f^{-1}(\widetilde{\bigcup}_{i\in I}(G_i, B)) = \widetilde{\bigcup}_{i\in I} f^{-1}(G_i, B)$.

**Theorem 2.2** ([9]). *Let $(X, \tau_1, E)$ and $(Y, \tau_2, K)$ be two soft multi topological spaces and $f : SMS(X)_E \to SMS(Y)_K$ be a soft multi function. Then $f$ is called a soft multi continuous function if and only if $f^{-1}(G, K)$ is an open soft mset over $X$, for each open soft mset $(G, K)$ over $Y$.*

**Definition 2.17** ([9]). Let $(X, \tau_1, E)$ and $(Y, \tau_2, K)$ be two soft multi topological spaces and $f : SMS(X)_E \to SMS(Y)_K$ be a soft multi function. If $f(F, E)$ is a closed soft mset over $Y$, for each closed soft mset $(F, E)$ over $X$, then $f$ is a closed soft multi function.

## 3. Some types of open soft multisets

In this section, some types of open soft msets are introduced and their properties are presented.

**Definition 3.1.** Let $(X, \tau, E)$ be a soft multi topological space. A mapping $\gamma : SMS(X)_E \to SMS(X)_E$ is said to be an operation on $OSM(X)_E$, if $N_E \widetilde{\subseteq} \gamma(N_E)$ *for all* $N_E \in OSM(X)_E$. The family of all $\gamma$-open soft msets is denoted by $OSM(\gamma) = \{N_E : N_E \widetilde{\subseteq} \gamma(N_E), N_E \in SMS(X)_E\}$. Also, the complement of $\gamma$-open soft mset is called a $\gamma$-closed soft mset and the set of all $\gamma$-closed soft msets denoted by CSM($\gamma$).

**Definition 3.2.** Let $(X, \tau, E)$ be a soft multi topological space. Different cases of $\gamma$-operations on $SMS(X)_E$ are as follows:

(i) If $\gamma = int(cl)$, then $\gamma$ is called a pre-open soft multi operator. The family of all pre-open soft msets is denoted by $POSM(X)_E$ and the family of all pre-closed soft msets is denoted by $PCSM(X)_E$.

(ii) If $\gamma = int(cl(int))$, then $\gamma$ is called an $\alpha$-open soft multi operator. The family of all $\alpha$-open soft msets is denoted by $\alpha OSM(X)_E$ and the family of all $\alpha$-closed soft msets is denoted by $\alpha CSM(X)_E$.

(iii) If $\gamma = cl(int)$, then $\gamma$ is called a semi open soft multi operator. The family of all semi open soft msets is denoted by $SOSM(X)_E$ and the family of all semi closed soft msets is denoted by $SCSM(X)_E$.

(iv) If $\gamma = cl(int(cl))$, then $\gamma$ is called a $\beta$-open soft multi operator. The family of all $\beta$-open soft msets is denoted by $\beta OSM(X)_E$ and the family of all $\beta$-closed soft msets is denoted by $\beta CSM(X)_E$.

(v) If $\gamma = cl(int) \cup int(cl)$, then $\gamma$ is called a $b$-open soft multi operator. The family of all $b$-open soft msets is denoted by $BOSM(X)_E$ and the family of all $b$-closed soft msets is denoted by $BCSM(X)_E$.

**Theorem 3.1.** *Let $(X, \tau, E)$ be a soft multi topological space and $\gamma : SMS(X)_E \to SMS(X)_E$ be an operation on $OSM(X)_E$.*
*Suppose $\gamma \in \{int(cl), int(cl(int)), cl(int), cl(int(cl)), cl(int) \cup int(cl)\}$. Then*

(1) *an arbitrary union of $\gamma$-open soft msets is a $\gamma$-open soft mset,*

(2) *an arbitrary intersection of $\gamma$-closed soft msets is a $\gamma$-closed soft mset.*

*Proof.* (1) The proof is given for the case of pre-open soft multi operator, i.e., $\gamma = int(cl)$ and the other cases are similar. Let $\{(N_i, E) : i \in I\} \subseteq POSM(X)_E$. Then $(N_i, E) \widetilde{\subseteq} int(cl((N_i, E)))$ for all $i \in I$. Thus

$$\widetilde{\bigcup}_i (N_i, E) \widetilde{\subseteq} \widetilde{\bigcup}_i int(cl((N_i, E))) \widetilde{\subseteq} int(\widetilde{\bigcup}_i cl((N_i, E))) = int(cl(\widetilde{\bigcup}_i (N_i, E))).$$

So $\widetilde{\bigcup}_i (N_i, E) \in POSM(X)_E$ for all $i \in I$.

(2) Immediate. □

**Remark 3.1.** A finite intersection of pre-open (respectively semi-open, $\beta$-open, $b$-open) soft msets need not to be a pre-open (respectively semi-open, $\beta$-open, $b$-open) soft mset, as shown in the following examples.

**Examples 3.1.** (1) Let $X = \{2/x, 2/y, 1/z\}$ be an mset, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{X}, \widetilde{\phi}, (F_1, E)\}$ be a soft multi topology on $X$, where

$$F_1(e_1) = \{1/x, 2/y\},\ F_1(e_2) = \{1/x, 2/y\}.$$

Let $(A, E), (B, E)$ be two sub soft msets of $\widetilde{X}$ such that

$$A(e_1) = \{2/x, 1/z\},\ A(e_2) = \{1/z\},\ B(e_1) = \{1/x, 1/y\},\ B(e_2) = \{1/y\}.$$

Then $(A, E)$ and $(B, E)$ are pre-open soft msets of $(X, \tau, E)$. Thus $(A \cap B)(e_1) = \{1/x\}$ and $(A \cap B)(e_2) = \phi$. So $(A \cap B, E)$ is not a pre-open soft mset.

(2) Let $X = \{2/x, 3/y, 1/z\}$ be an mset, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{X}, \widetilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ be a soft multi topology on $X$, where

$$F_1(e_1) = \{1/x\},\ F_1(e_2) = \{1/x\},$$
$$F_2(e_1) = \{1/y\},\ F_2(e_2) = \{1/y\},$$

$$F_3(e_1) = \{1/x, 1/y\},\ F_3(e_2) = \{1/x, 1/y\}.$$

Let $(A, E), (B, E)$ be two sub soft msets of $\widetilde{X}$ such that

$$A(e_1) = \{1/x, 1/z\},\ A(e_2) = \{1/x, 1/z\},$$
$$B(e_1) = \{1/y, 1/z\},\ B(e_2) = \{1/y, 1/z\}.$$

Then $(A, E)$, $(B, E)$ are semi-open soft msets of $(X, \tau, E)$. Thus $(A \cap B)(e_1) = \{1/z\}$ and $(A \cap B)(e_2) = \{1/z\}$. So $(A \cap B, E)$ is not a semi-open soft mset.

(3) Let $X = \{2/x, 2/y, 1/z\}$ be an mset, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{X}, \widetilde{\phi}, (F_1, E)\}$ be a soft multi topology on $X$, where $F_1(e_1) = \{1/x, 2/y\}$, $F_1(e_2) = \{1/x, 2/y\}$. Let $(A, E), (B, E)$ be two sub soft msets of $\widetilde{X}$ such that

$$A(e_1) = \{2/x, 1/z\},\ A(e_2) = \{1/z\},\ B(e_1) = \{1/x, 1/y\},\ B(e_2) = \{1/y\}.$$

Then $(A, E)$, $(B, E)$ are $\beta$-open soft msets of $(X, \tau, E)$. Thus $(A \cap B)(e_1) = \{1/x\}$ and $(A \cap B)(e_2) = \phi$. So $(A \cap B, E)$ is not a $\beta$-open soft mset.

(4) Let $X = \{2/x, 2/y, 1/z\}$ be an mset, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{X}, \widetilde{\phi}, (F_1, E)\}$ be a soft multi topology on $X$ where $F_1(e_1) = \{1/x, 2/y\}$, $F_1(e_2) = \{1/x, 2/y\}$. Let $(A, E), (B, E)$ be two sub soft msets of $\widetilde{X}$ such that

$$A(e_1) = \{2/x, 1/z\},\ A(e_2) = \{1/x, 1/z\},$$
$$B(e_1) = \{1/x, 1/y\},\ B(e_2) = \{1/x, 2/y\}.$$

Then $(A, E)$, $(B, E)$ are $b$-open soft msets of $(X, \tau, E)$. Thus $(A \cap B)(e_1) = \{1/x\}$ and $(A \cap B)(e_2) = \{1/x\}$. So $(A \cap B, E)$ is not a $b$-open soft mset.

**Definition 3.3.** Let $(X, \tau, E)$ be a soft multi topological space, $N_E \in SMS(X)_E$. Then

(i) The $\gamma$-interior of $N_E$ is denoted by $\gamma SM(int(N_E))$ and it is defined as

$$\gamma SM(int(N_E)) = \widetilde{\bigcup}\{G_E : G_E \widetilde{\subseteq} N_E,\ G_E \in OSM(\gamma)\}.$$

(ii) The $\gamma$-closure of $N_E$ is denoted by $\gamma SM(cl(N_E))$ and it is defined as

$$\gamma SM(cl(N_E)) = \widetilde{\bigcap}\{G_E : G_E \in CSM(\gamma),\ N_E \widetilde{\subseteq} G_E\}.$$

**Theorem 3.2.** *Let $(X, \tau, E)$ be a soft multi topological space, $\gamma : SMS(X)_E \to SMS(X)_E$ be one of the operations in Definition 3.2 and $F_E, G_E \in SMS(X)_E$. Then, the following properties are satisfied for the $\gamma SM$-interior operators which is denoted by $\gamma SM(int)$.*

(1) $\gamma SM(int(\widetilde{X})) = \widetilde{X}$ *and* $\gamma SM(int(\widetilde{\phi})) = \widetilde{\phi}$.
(2) $\gamma SM(int(F_E)) \widetilde{\subseteq} F_E$.
(3) $\gamma SM(int(F_E))$ *is the largest* $\gamma$*-open soft mset contained in* $F_E$.
(4) *If* $F_E \widetilde{\subseteq} G_E$, *then* $\gamma SM(int(F_E)) \widetilde{\subseteq} \gamma SM(int(G_E))$.
(5) $\gamma SM(int(\gamma SM(int(F_E)))) = \gamma SM(int(F_E))$.
(6) $\gamma SM(int(F_E)) \widetilde{\cup} \gamma SM(int(G_E)) \widetilde{\subseteq} \gamma SM(int(F_E \widetilde{\cup} G_E))$.
(7) $\gamma SM(int(F_E \widetilde{\cap} G_E)) \widetilde{\subseteq} \gamma SM(int(F_E)) \widetilde{\cap} \gamma SM(int(G_E))$.

*Proof.* Immediate. □

**Theorem 3.3.** *Let $(X, \tau, E)$ be a soft multi topological space, $\gamma : SMS(X)_E \to SMS(X)_E$ be one of the operations in Definition 3.2 and $F_E, G_E \in SMS(X)_E$. Then, the following properties are satisfied for the $\gamma SM$-closure operators which is denoted by $\gamma SM(cl)$.*

(1) $\gamma SM(cl(\widetilde{X})) = \widetilde{X}$ *and* $\gamma SM(cl(\widetilde{\phi})) = \widetilde{\phi}$.

(2) *$F_E \widetilde{\subseteq} \gamma SM(cl(F_E))$.*
(3) *$\gamma SM(cl(F_E))$ is the smallest $\gamma$-closed soft mset containing $F_E$.*
(4) *If $F_E \widetilde{\subseteq} G_E$, then $\gamma SM(cl(F_E)) \widetilde{\subseteq} \gamma SM(cl(G_E))$.*
(5) *$\gamma SM(cl(\gamma SM(cl(F_E)))) = \gamma SM(cl(F_E))$.*
(6) *$\gamma SM(cl(F_E)) \widetilde{\cup} \gamma SM(cl(G_E)) \widetilde{\subseteq} \gamma SM(cl(F_E \widetilde{\cup} G_E))$.*
(7) *$\gamma SM(cl(F_E \widetilde{\cap} G_E)) \widetilde{\subseteq} \gamma SM(cl(F_E)) \widetilde{\cap} \gamma SM(cl(G_E))$.*

*Proof.* Immediate. □

**Remark 3.2.** It should be noted that the family of all $\gamma$-open soft msets on a soft multi topological space $(X, \tau, E)$ forms a supra soft multi topology, i.e., the family all $\gamma$-open soft msets contains $\widetilde{X}, \widetilde{\phi}$ and closed under arbitrary soft multi union.

## 4. Relationships among soft multisets

In this section, the relationships are introduced among some special sub soft msets which are introduced in the above section. Moreover, some important results are obtained.

**Theorem 4.1.** *In a soft multi topological space $(X, \tau, E)$, the following statements hold:*
(1) *Every open soft mset is a $\gamma$-open soft mset.*
(2) *Every closed soft mset is a $\gamma$-closed soft mset.*

*Proof.* The proof is given for the case of pre-open soft multi operator, i.e., $\gamma = int(cl)$ and the other cases are similar.

(1) Let $N_E \in OSM(X)_E$. Then $int(N_E) = N_E$. Since $N_E \widetilde{\subseteq} cl(N_E)$, $N_E \widetilde{\subseteq} int(cl(N_E))$. Thus $N_E \in POSM(X)_E$.

(2) By a similar way. □

**Remark 4.1.** The converse of Theorem 4.1 is not true in general as shown in the following examples.

**Examples 4.1.** (1) Let $X = \{2/x, 2/y, 1/z\}$ be an mset, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{X}, \widetilde{\phi}, (F_1, E)\}$ be a soft multi topology on $X$, where $F_1(e_1) = \{1/x, 2/y\}$, $F_1(e_2) = \{1/y\}$. Let $(A, E)$ be a sub soft mset of $\widetilde{X}$ such that $A(e_1) = \{2/x, 1/z\}$, $A(e_2) = \{2/x\}$. Then $(A, E)$ is a pre-open soft mset of $(X, \tau, E)$. But it is not an open soft mset.

(2) Let $X = \{2/x, 3/y, 1/z\}$ be an mset, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{X}, \widetilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ be a soft multi topology on $X$, where

$$F_1(e_1) = \{1/x\},\ F_1(e_2) = \{1/x, 1/y\},$$
$$F_2(e_1) = \{1/x\},\ F_2(e_2) = \{1/x\},$$
$$F_3(e_1) = \{1/x, 1/y\},\ F_3(e_2) = \{1/x, 1/y\}.$$

Let $(A, E)$ be a sub soft mset of $\widetilde{X}$ such that $A(e_1) = \{1/x, 1/z\}$, $A(e_2) = \{2/y, 1/z\}$. Then $(A, E)$ is a semi-open soft mset of $(X, \tau, E)$. But it is not an open soft mset.

(3) Let $X = \{2/x, 2/y, 1/z\}$ be an mset, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{X}, \widetilde{\phi}, (F_1, E)\}$ be a soft multi topology on $X$, where $F_1(e_1) = \{2/x, 1/y\}$, $F_1(e_2) = \{1/x, 2/y\}$. Let $(A, E)$ be a sub soft mset of $\widetilde{X}$ such that $A(e_1) =$

$\{2/x, 2/y\}$, $A(e_2) = \{2/x, 2/y\}$. Then $(A, E)$ is an $\alpha$-open soft mset of $(X, \tau, E)$. But it is not an open soft mset.

(4) Let $X = \{2/x, 2/y, 1/z\}$ be an mset, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{X}, \widetilde{\phi}, (F_1, E)\}$ be a soft multi topology on $X$, where $F_1(e_1) = \{1/x, 2/y\}$, $F_1(e_2) = \{1/y\}$. Let $(A, E)$ be a sub soft mset of $\widetilde{X}$ such that $A(e_1) = \{2/x, 1/z\}$, $A(e_2) = \{2/x\}$. Then $(A, E)$ is a $\beta$-open soft mset of $(X, \tau, E)$. But it is not an open soft mset.

(5) Let $X = \{2/x, 2/y, 1/z\}$ be an mset, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{X}, \widetilde{\phi}, (F_1, E)\}$ be a soft multi topology on $X$, where $F_1(e_1) = \{2/x, 1/y\}$, $F_1(e_2) = \{1/x, 2/y\}$. Let $(A, E)$ be a sub soft mset of $\widetilde{X}$ such that $A(e_1) = \{2/x, 2/y\}$, $A(e_2) = \{2/x, 2/y\}$. Then $(A, E)$ is a $b$-open soft mset of $(X, \tau, E)$. But it is not an open soft mset.

**Theorem 4.2.** *Let $(X, \tau, E)$ be a soft multi topological space. Then, the following statements hold:*

(1) *Every $\alpha$-open (respectively $\alpha$-closed) soft mset is semi-open (respectively semi-closed).*

(2) *Every $\alpha$-open (respectively $\alpha$-closed) soft mset is pre-open (respectively pre-closed).*

(3) *Every pre-open (respectively pre-closed) soft mset is b-open (respectively b-closed).*

(4) *Every semi-open (respectively semi-closed) soft mset is b-open (respectively b-closed).*

(5) *Every b-open (respectively b-closed) soft mset is $\beta$-open (respectively $\beta$-closed).*

*Proof.* The assertion is proved in the case of open soft mset and the other case is similar.

(1) Let $N_E \in \alpha OSM(X)_E$. Then $N_E \widetilde{\subseteq} int(cl(int(N_E))) \widetilde{\subseteq} cl(int(N_E))$. Thus $N_E \in SOSM(X)_E$.

(2) Let $N_E \in \alpha OSM(X)_E$. Then $N_E \widetilde{\subseteq} int(cl(int(N_E)))$. Since $int(N_E) \widetilde{\subseteq} cl(N_E)$, $cl(int(N_E)) \widetilde{\subseteq} cl(N_E)$. Thus $N_E \widetilde{\subseteq} int(cl(int(N_E))) \widetilde{\subseteq} int(cl(N_E))$. So $N_E \widetilde{\subseteq} int(cl(N_E))$. It follows that $N_E \in POSM(X)_E$.

(3) Let $N_E \in POSM(X)_E$. Then $N_E \widetilde{\subseteq} int(cl(N_E))$. Thus $N_E \widetilde{\subseteq} int(cl(N_E)) \widetilde{\cup} cl(int(N_E))$. So $N_E \in BOSM(X)_E$.

(4) Let $N_E \in SOSM(X)_E$. Then $N_E \widetilde{\subseteq} cl(int(N_E))$. Thus $N_E \widetilde{\subseteq} int(cl(N_E)) \widetilde{\cup} cl(int(N_E))$. Hence $N_E \in BOSM(X)_E$.

(5) Let $N_E \in BOSM(X)_E$. Then

$$\begin{aligned} N_E &\widetilde{\subseteq} int(cl(N_E)) \widetilde{\cup} cl(int(N_E)) \\ &\widetilde{\subseteq} cl(int(N_E)) \widetilde{\cup} cl(int(cl(N_E))) \\ &\widetilde{\subseteq} cl[int(N_E) \widetilde{\cup} int(cl(N_E))] \\ &\widetilde{\subseteq} cl(int[N_E \widetilde{\cup} cl(N_E)]) \\ &\widetilde{\subseteq} cl(int(cl(N_E))). \end{aligned}$$

Thus $N_E \in \beta OSM(X)_E$. □

**Remark 4.2.** The converse of Theorem 4.2 is not true in general as shown in the following examples.

**Examples 4.2.** (1) Let $X = \{2/x, 3/y, 1/z\}$ be an mset, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{X}, \widetilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ be a soft multi topology on $X$, where

$$F_1(e_1) = \{1/x\},\ F_1(e_2) = \{1/x, 1/y\},$$
$$F_2(e_1) = \{1/x\},\ F_2(e_2) = \{1/x\},$$
$$F_3(e_1) = \{1/x, 1/y\},\ F_3(e_2) = \{1/x, 1/y\}.$$

Let $(A, E)$ be a sub soft mset of $\widetilde{X}$ such that

$$A(e_1) = \{1/x, 1/z\},\ A(e_2) = \{1/x, 2/y\}.$$

Then $(A, E)$ is a semi-open soft mset of $(X, \tau, E)$. But it is not an $\alpha$-open soft mset.

(2) Let $X = \{3/x, 2/y, 1/z\}$ be an mset, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{X}, \widetilde{\phi}, (F_1, E)\}$ be a soft multi topology on $X$, where $F_1(e_1) = \{2/x, 2/y\}$, $F_1(e_2) = \{2/x\}$. Let $(A, E)$ be a sub soft mset of $\widetilde{X}$ such that $A(e_1) = \{3/x, 1/z\}$, $A(e_2) = \{2/y, 1/z\}$. Then $(A, E)$ is a $\beta$-open soft mset of $(X, \tau, E)$. But it is not a semi-open soft mset.

(3) Let $X = \{2/x, 3/y, 1/z\}$ be an mset, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{X}, \widetilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ be a soft multi topology on $X$, where

$$F_1(e_1) = \{1/x\},\ F_1(e_2) = \{1/x, 1/y\},$$
$$F_2(e_1) = \{1/x\},\ F_2(e_2) = \{1/x\},$$
$$F_3(e_1) = \{1/x, 1/y\},\ F_3(e_2) = \{1/x, 1/y\}.$$

Let $(A, E)$ be a sub soft mset of $\widetilde{X}$ such that

$$A(e_1) = \{1/x, 1/z\},\ A(e_2) = \{1/x, 2/y\}.$$

Then $(A, E)$ is a $\beta$-open soft mset of $(X, \tau, E)$. But it is not a pre-open soft mset.

(4) Let $X = \{3/x, 2/y, 1/z\}$ be an mset, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{X}, \widetilde{\phi}, (F_1, E)\}$ be a soft multi topology on $X$, where $F_1(e_1) = \{2/x, 2/y\}$, $F_1(e_2) = \{2/x\}$. Let $(A, E)$ be a sub soft mset of $\widetilde{X}$ such that $A(e_1) = \{3/x, 1/z\}$, $A(e_2) = \{2/y, 1/z\}$. Then $(A, E)$ is a pre-open soft mset of $(X, \tau, E)$. But it is not an $\alpha$-open soft mset.

(5) Let $X = \{2/x, 3/y, 1/z\}$ be an mset, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{X}, \widetilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ be a soft multi topology on $X$, where

$$F_1(e_1) = \{1/x\},\ F_1(e_2) = \{1/x, 1/y\},$$
$$F_2(e_1) = \{1/x\},\ F_2(e_2) = \{1/x\},$$
$$F_3(e_1) = \{1/x, 1/y\},\ F_3(e_2) = \{1/x, 1/y\}.$$

Let $(A, E)$ be a sub soft mset of $\widetilde{X}$ such that $A(e_1) = \{1/x, 1/z\}$, $A(e_2) = \{2/y, 1/z\}$. Then $(A, E)$ is a $b$-open soft mset of $(X, \tau, E)$. But it is not a pre-open soft mset.

(6) Let $X = \{3/x, 2/y, 1/z\}$ be an mset, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{X}, \widetilde{\phi}, (F_1, E)\}$ be a soft multi topology on $X$, where $F_1(e_1) = \{2/x, 2/y\}$, $F_1(e_2) = \{2/x\}$. Let $(A, E)$ be a sub soft mset of $\widetilde{X}$ such that $A(e_1) = \{2/x, 1/y\}$, $A(e_2) = \{3/x, 1/z\}$. Then $(A, E)$ is a $b$-open soft mset of $(X, \tau, E)$. But it is not a semi-open soft mset.

(7) Let $X = \{2/x, 3/y, 1/z, 1/d\}$ be an mset, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{X}, \widetilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ be a soft multi topology on $X$, where

$$
\begin{aligned}
&F_1(e_1) = \{1/d\},\ F_1(e_2) = \phi,\\
&F_2(e_1) = \{1/x, 2/y\},\ F_2(e_2) = \{1/x, 2/y\},\\
&F_3(e_1) = \{1/x, 2/y, 1/d\},\ F_3(e_2) = \{1/x, 2/y\}.
\end{aligned}
$$

Let $(A, E)$ be a sub soft mset of $\widetilde{X}$ such that $A(e_1) = \{2/x\}$, $A(e_2) = \{2/x\}$. Then $(A, E)$ is a $\beta$-open soft mset of $(X, \tau, E)$. But it is not a $b$-open soft mset.

On accounting of Theorems 4.1, 4.2 and Examples 4.1, 4.2 we have the following corollary.

**Corollary 4.1.** *For a soft multi topological space $(X, \tau, E)$, we have the following diagram:*

$$
\begin{tikzcd}
OSM(X)_E \arrow[r] \arrow[d] & \alpha OSM(X)_E \arrow[r] \arrow[dl] & SOSM(X)_E \arrow[dl] \\
POSM(X)_E \arrow[r] & BOSM(X)_E \arrow[r] & \beta OSM(X)_E
\end{tikzcd}
$$

**Theorem 4.3.** *Let $(X, \tau, E)$ be a soft multi topological space and $N_E \in SMS(X)_E$. Then, the following statements hold:*

(1) *$N_E \in SOSM(X)_E$ if and only if $cl(N_E) = cl(int(N_E))$.*

(2) *$N_E \in SOSM(X)_E$ if and only if there exists $G_E \in OSM(X)_E$ such that $G_E \widetilde{\subseteq} N_E \widetilde{\subseteq} cl(G_E)$.*

(3) *If $N_E \in SOSM(X)_E$ and $N_E \widetilde{\subseteq} G_E \widetilde{\subseteq} cl(N_E)$, then $G_E \in SOSM(X)_E$.*

*Proof.* Immediate. □

**Theorem 4.4.** *Let $(X, \tau, E)$ be a soft multi topological space, $\gamma : SMS(X)_E \rightarrow SMS(X)_E$ be one of the operations in Definition 3.2 and $N_E \in SMS(X)_E$. Then, the following statements hold:*

(1) $\gamma SM(int(N_E^c)) = [\gamma SM(cl(N_E))]^c$.

(2) $\gamma SM(cl(N_E^c)) = [\gamma SM(int(N_E))]^c$.

*Proof.* The proof is given for the case of pre-open soft multi operator, i.e., $\gamma = int(cl)$ and the other cases are similar.

(1) $PSMcl(N_E) = \widetilde{\cap}\{G_E : N_E \widetilde{\subseteq} G_E, G_E \in PCSM(X)_E\}$.

$[PSMcl(N_E)]^c = \widetilde{\cup}\{G_E^c : G_E^c \widetilde{\subseteq} N_E^c, G_E^c \in POSM(X)_E\} = PSMint(N_E^c)$.

(2) By a similarly way. □

**Theorem 4.5.** *Let $(X, \tau, E)$ be a soft multi topological space and $N_E$, $G_E \in SMS(X)_E$. Then the following statements hold:*

(1) *$N_E \in \alpha OSM(X)_E$ if and only if there exists $H_E \in OSM(X)_E$ such that $H_E \widetilde{\subseteq} N_E \widetilde{\subseteq} int(cl(H_E))$.*

(2) *If $N_E \in \alpha OSM(X)_E$ and $N_E \widetilde{\subseteq} G_E \widetilde{\subseteq} int(cl(N_E))$, then $G_E \in \alpha OSM(X)_E$.*

*Proof.* (1)($\Rightarrow$) : Suppose that $int(N_E) = H_E \in OSM(X)_E$. Then $H_E \widetilde{\subseteq} N_E \widetilde{\subseteq} int(cl(H_E))$.

($\Leftarrow$) : Let $H_E \widetilde{\subseteq} N_E \widetilde{\subseteq} int(cl(H_E))$, where $H_E \in OSM(X)_E$. Then $int(H_E) = H_E \widetilde{\subseteq} int(N_E)$. It follows that

$$N_E \widetilde{\subseteq} int(cl(int(H_E))) \widetilde{\subseteq} int(cl(int(N_E))).$$

Thus $N_E \in \alpha OSM(X)_E$.

(2) Let $N_E \in \alpha OSM(X)_E$. Then $N_E \widetilde{\subseteq} int(cl(int(N_E)))$. Thus

$$\begin{aligned} N_E \widetilde{\subseteq} G_E \widetilde{\subseteq} & int(cl(int(cl(int(N_E))))) \\ \widetilde{\subseteq} & int(cl(int(N_E))) \\ \widetilde{\subseteq} & int(cl(int(G_E))). \end{aligned}$$

So $G_E \in \alpha OSM(X)_E$. □

**Theorem 4.6.** *Let $(X, \tau, E)$ be a soft multi topological space and $N_E \in SMS(X)_E$. Then*

(1) *$N_E \in \alpha OSM(X)_E$ if and only if $N_E \in POSM(X)_E \cap SOSM(X)_E$.*
(2) *$N_E \in \alpha CSM(X)_E$ if and only if $N_E \in PCSM(X)_E \cap SCSM(X)_E$.*

*Proof.* (1) $(\Rightarrow)$ : Let $N_E \in \alpha OSM(X)_E$. Then $N_E \widetilde{\subseteq} int(cl(int(N_E)))$. Thus

$$N_E \widetilde{\subseteq} cl(int(N_E)) \text{ and } N_E \widetilde{\subseteq} int(cl(N_E)).$$

So $N_E \in POSM(X)_E \cap SOSM(X)_E$.

$(\Leftarrow)$ : Let $N_E \in POSM(X)_E \cap SOSM(X)_E$. Then $N_E \widetilde{\subseteq} cl(int(N_E))$ and $N_E \widetilde{\subseteq} int(cl(N_E))$. Thus $N_E \widetilde{\subseteq} int(cl(cl(int(N_E)))) = int(cl(int(N_E)))$. It follows that $N_E \in \alpha OSM(X)_E$.

(2) By a similar way. □

The proof of the following proposition is straightforward, so it is omitted.

**Proposition 4.1.** *Let $(X, \tau, E)$ be a soft multi topological space and $N_E \in SMS(X)_E$. Then*

(1) *$N_E \in PCSM(X)_E$ if and only if $cl(int(N_E)) \widetilde{\subseteq} N_E$.*
(2) *$N_E \in \alpha CSM(X)_E$ if and only if $cl(int(cl(N_E))) \widetilde{\subseteq} N_E$.*
(3) *$N_E \in SCSM(X)_E$ if and only if $int(cl(N_E)) \widetilde{\subseteq} N_E$.*
(4) *$N_E \in \beta CSM(X)_E$ if and only if $int(cl(int(N_E))) \widetilde{\subseteq} N_E$.*
(5) *$N_E \in BCSM(X)_E$ if and only if $cl(int(N_E)) \widetilde{\cap} int(cl(N_E)) \widetilde{\subseteq} N_E$.*

**Theorem 4.7.** *Let $(X, \tau, E)$ be a soft multi topological space. If $N_E \in \alpha OSM(X)_E$ and $N_E^c \in POSM(X)_E$, then $N_E \in OSM(X)_E$.*

*Proof.* Let $N_E \in \alpha OSM(X)_E$ and $N_E^c \in POSM(X)_E$. Then $N_E \in PCSM(X)_E$. Thus $cl(int(N_E)) \widetilde{\subseteq} N_E \widetilde{\subseteq} int(cl(int(N_E))) \widetilde{\subseteq} cl(int(N_E))$. This means that $cl(int(N_E)$ $= N_E$. So $N_E \widetilde{\subseteq} int(cl(int(N_E))) = int(N_E)$. Hence $N_E \in OSM(X)_E$. □

## 5. Decompositions of some forms of soft multi continuity (irresolute)

In this section, some forms of soft multi continuous (irresolute) mappings are introduced. These forms are generalizations of soft multi continuous (irresolute) mappings. In addition, the relationships among them are presented in detail.

**Definition 5.1.** Let $(X, \tau_1, A)$ and $(Y, \tau_2, B)$ be two soft multi topological spaces. Let $\varphi : X^* \to Y^*$ and $\psi : A \to B$ be two functions. Let $f = (\varphi, \psi) : SMS(X)_A \to SMS(Y)_B$ be a soft multi function. Then

(i) The function $f$ is called a pre-soft multi continuous (briefly, $PSM$ $cts$) function if $f^{-1}(G, B) \in POSM(X)_A$ for all $(G, B) \in OSM(Y)_B$.

(ii) The function $f$ is called an $\alpha$-soft multi continuous (briefly, $\alpha SM$ $cts$) function if $f^{-1}(G, B) \in \alpha OSM(X)_A$ for all $(G, B) \in OSM(Y)_B$.

(iii) The function $f$ is called a semi-soft multi continuous (briefly, $SSM$ $cts$) function if $f^{-1}(G, B) \in SOSM(X)_A$ for all $(G, B) \in OSM(Y)_B$.

(iv) The function $f$ is called a $\beta$-soft multi continuous (briefly, $\beta SM$ $cts$) function if $f^{-1}(G, B) \in \beta OSM(X)_A$ for all $(G, B) \in OSM(Y)_B$.

(v) The function $f$ is called a $b$-soft multi continuous (briefly, $BSM$ $cts$) function if $f^{-1}(G, B) \in BOSM(X)_A$ for all $(G, B) \in OSM(Y)_B$.

**Theorem 5.1.** *Let $(X, \tau_1, A)$ and $(Y, \tau_2, B)$ be two soft multi topological spaces. Let $\varphi : X^* \to Y^*$ and $\psi : A \to B$ be two functions. Let $f = (\varphi, \psi) : SMS(X)_A \to SMS(Y)_B$ be a soft multi function. Then, for the classes $\gamma$-soft multi continuous functions the following are equivalent (an example is given for the class of pre-soft multi continuous functions).*

(1) *$f$ is a pre-soft multi continuous function.*
(2) *$f^{-1}(G, B) \in PCSM(X)_A$ for all $(G, B) \in CSM(Y)_B$.*
(3) *$f(PSMcl(G, A)) \widetilde{\subseteq} cl_{\tau_2}(f(G, A))$ for all $(G, A) \in SMS(X)_A$.*
(4) *$PSMcl(f^{-1}(H, B)) \widetilde{\subseteq} f^{-1}(cl_{\tau_2}(H, B))$ for all $(H, B) \in SMS(Y)_B$.*
(5) *$f^{-1}(int_{\tau_2}(H, B)) \widetilde{\subseteq} PSMint(f^{-1}(H, B))$ for all $(H, B) \in SMS(Y)_B$.*

*Proof.* (1)⇒(2) : Let $(H, B)$ be a closed soft mset over $Y$. Then $(H, B)^c \in OSM(Y)_B$ and $f^{-1}(H, B)^c \in POSM(X)_A$. Since $f^{-1}(H, B)^c = (f^{-1}(H, B))^c$, $f^{-1}(H, B) \in PCSM(X)_A$.

(2)⇒(3) : Let $(G, A) \in SMS(X)_A$. Since

$$(G, A) \widetilde{\subseteq} f^{-1}(f(G, A)) \widetilde{\subseteq} f^{-1}(cl_{\tau_2}(f(G, A))) \in PCSM(X)_A,$$

by part (2),

$$(G, A) \widetilde{\subseteq} PSMcl(G, A) \widetilde{\subseteq} f^{-1}(cl_{\tau_2}(f(G, A))).$$

Thus

$$f(PSMcl(G, A)) \widetilde{\subseteq} f(f^{-1}(cl_{\tau_2}(f(G, A)))) \widetilde{\subseteq} cl_{\tau_2}(f(G, A)).$$

So $f(PSMcl(G, A)) \widetilde{\subseteq} cl_{\tau_2}(f(G, A))$.

(3)⇒(4) : Let $(H, B) \in SMS(Y)_B$ and $(G, A) = f^{-1}(H, B)$. Then by part (3),

$$f(PSMcl(f^{-1}(H, B))) \widetilde{\subseteq} cl_{\tau_2}(f(f^{-1}(H, B))).$$

Thus

$$\begin{aligned} PSMcl(f^{-1}(H, B)) &\widetilde{\subseteq} f^{-1}(f(PSMcl(f^{-1}(H, B)))) \\ &\widetilde{\subseteq} f^{-1}(cl_{\tau_2}(f(f^{-1}(H, B)))) \widetilde{\subseteq} f^{-1}(cl_{\tau_2}(H, B)). \end{aligned}$$

So $PSMcl(f^{-1}(H, B)) \widetilde{\subseteq} f^{-1}(cl_{\tau_2}(H, B))$.

(4)⇒(2) : Let $(H,B) \in CSM(Y)_B$. Then

$$PSMcl(f^{-1}(H,B)) \widetilde{\subseteq} f^{-1}(cl_{\tau_2}(H,B)) = f^{-1}(H,B).$$

But $f^{-1}(H,B) \widetilde{\subseteq} PSMcl(f^{-1}(H,B))$. This implies that

$$f^{-1}(H,B) \in PCSM(X)_A \text{ for all } (H,B) \in CSM(Y)_B.$$

(1)⇒(5) : Let $(H,B) \in SMS(Y)_B$. Then, $f^{-1}(int_{\tau_2}(H,B)) \in POSM(X)_A$ by part (1). Thus

$$f^{-1}(int_{\tau_2}(H,B)) = PSMint(f^{-1}(int_{\tau_2}(H,B))) \widetilde{\subseteq} PSMint(f^{-1}(H,B)).$$

Hence $f^{-1}(int_{\tau_2}(H,B)) \widetilde{\subseteq} PSMint(f^{-1}(H,B))$ for all $(H,B) \in SMS(Y)_B$.

(5)⇒(1) : Let $(H,B) \in OSM(Y)_B$. Then $int_{\tau_2}(H,B) = (H,B)$. Thus $f^{-1}(int_{\tau_2}(H,B)) = f^{-1}(H,B) \widetilde{\subseteq} PSMint(f^{-1}(H,B))$ by part (5). But $PSMint(f^{-1}(H,B)) \widetilde{\subseteq} f^{-1}(H,B)$. Hence $f^{-1}(H,B) \in POSM(X)$. This means that $f$ is a pre-soft multi continuous function. □

**Theorem 5.2.** *Let $(X,\tau_1,A)$ and $(Y,\tau_2,B)$ be two soft multi topological spaces. Let $f = (\varphi,\psi) : SMS(X)_A \to SMS(Y)_B$ be a soft multi function. Then*

(1) *every soft multi continuous function is a pre-soft multi continuous function.*
(2) *every soft multi continuous function is a semi-soft multi continuous function.*
(3) *every soft multi continuous function is an $\alpha$-soft multi continuous function*
(4) *every soft multi continuous function is a $\beta$-soft multi continuous function.*
(5) *every soft multi continuous function is a b-soft multi continuous function.*

*Proof.* Immediate from Theorem 4.1. □

**Remark 5.1.** The converse of Theorem 5.2 is not true in general as shown in the following example (for part 1 of the theorem) and examples for other parts can be easily done.

**Example 5.1.** Let $X = \{2/x_1, 1/x_2\}$, $A = \{a_1,a_2\}$ and $\tau_1 = \{\widetilde{X}, \widetilde{\phi}, (F,A)\}$, where $F(a_1) = \{1/x_1, 1/x_2\}$, $F(a_2) = \{1/x_1\}$. Then, $(X,\tau_1,A)$ is a soft multi topological space. Also, let $Y = \{1/y_1, 2/y_2\}$, $B = \{b_1,b_2\}$ and $\tau_2 = \{\widetilde{Y}, \widetilde{\phi}, (G,B)\}$, where $G(b_1) = \{2/y_2\}$, $G(b_2) = \{1/y_2\}$. Then, $(Y,\tau_2,B)$ is a soft multi topological space. Assume that $\varphi : X^* \to Y^*$ and $\psi : A \to B$ are two functions defined as,

$\varphi(x_1) = y_2$, $\varphi(x_2) = y_1$,
$\psi(a_1) = b_1$, $\psi(a_2) = b_2$.

Then, the inverse image of $(G,B)$ under soft multi function $f : SMS(X)_A \to SMS(Y)_B$ is denoted by $f^{-1}(G,B)$ and defined as $C_{f^{-1}(G)(a)}(x) = C_{G(\psi(a))}(\varphi(x))$ for each $x \in X^*$, $a \in A$. Now, we want to estimate the inver image for every open soft mset in $OSM(Y)_B$. This clear that $f^{-1}(\widetilde{\phi}) = \widetilde{\phi}$ and $f^{-1}(\widetilde{Y}) = \widetilde{X}$. Also,

$C_{f^{-1}(G)(a_1)}(x_1) = C_{G(\psi(a_1))}(\varphi(x_1)) = C_{G(b_1)}(y_2) = 2,$
$C_{f^{-1}(G)(a_1)}(x_2) = C_{G(\psi(a_1))}(\varphi(x_2)) = C_{G(b_1)}(y_1) = 0,$
$C_{f^{-1}(G)(a_2)}(x_1) = C_{G(\psi(a_2))}(\varphi(x_1)) = C_{G(b_2)}(y_2) = 1,$
$C_{f^{-1}(G)(a_2)}(x_2) = C_{G(\psi(a_2))}(\varphi(x_2)) = C_{G(b_2)}(y_1) = 0.$

Then, $f^{-1}(G)(a_1) = \{2/x_1\}$ and $f^{-1}(G)(a_2) = \{1/x_1\}$. Thus, $int(cl(f^{-1}(G,B))) =$

$\widetilde{X}$. It follows that $f^{-1}(G, B)$ is a pre-open soft mset, but it is not open. So, $f$ is a pre-continuous soft multi function, but it is not a continuous soft multi function.

**Theorem 5.3.** *Let $(X, \tau_1, A)$ and $(Y, \tau_2, B)$ be two soft multi topological spaces. Let $f = (\varphi, \psi) : SMS(X)_A \to SMS(Y)_B$ be a soft multi function. Then*

(1) *Every $\alpha$-soft multi continuous function is a semi-soft multi continuous function.*

(2) *Every $\alpha$-soft multi continuous function is a pre-soft multi continuous function.*

(3) *Every pre-soft multi continuous function is a $b$-soft multi continuous function.*

(4) *Every semi-soft multi continuous function is a $b$-soft multi continuous function.*

(5) *every $b$-soft multi continuous function is a $\beta$-soft multi continuous function.*

*Proof.* Immediate from Theorem 4.2. □

**Remark 5.2.** The converse of Theorem 5.3 is not satisfied generally as shown in the following example (for part 2 of theorem) and examples for other parts can be easily done.

**Example 5.2.** From Example 5.1, $f^{-1}(G, B)$ a pre-open soft mset, but it is not $\alpha$-open. Then, $f$ is a pre-continuous soft multi function, but it is not an $\alpha$-continuous soft multi function.

**Theorem 5.4.** *Let $(X, \tau_1, A)$ and $(Y, \tau_2, B)$ be two soft multi topological spaces. Let $f = (\varphi, \psi) : SMS(X)_A \to SMS(Y)_B$ be a soft multi function. Then $f$ is an $\alpha$-soft multi continuous function if and only if it is a pre-soft multi continuous function and a semi-soft multi continuous function.*

*Proof.* Immediate from Theorem 4.6. □

On accounting of Theorems 5.2 and 5.3 we have the following corollary.

**Corollary 5.1.** *For a soft multi topological space $(X, \tau, E)$, we have the following diagram:*

$$\begin{array}{ccccc} SM\ cts & \longrightarrow & \alpha - SM\ cts & \longrightarrow & semi - SM\ cts \\ & \downarrow \quad \swarrow & & \swarrow & \\ pre - SM\ cts & \longrightarrow & b - SM\ cts & \longrightarrow & \beta - SM\ cts \end{array}$$

**Definition 5.2.** Let $(X, \tau_1, A)$ and $(Y, \tau_2, B)$ be two soft multi topological spaces. Then a function $f : SMS(X)_E \to SMS(Y)_B$ is said to be a $\gamma$-soft multi irresolute function if $f^{-1}(G, B) \in \gamma CSM(X)_E \ \forall \ (G, B) \in \gamma CSM(Y)_B$.

**Theorem 5.5.** *Every $\gamma$-soft multi irresolute mapping is a $\gamma$-soft multi continuous mapping.*

*Proof.* The proof is given for the case of $b$-soft multi irresolute mapping and the other cases are similar.
Let $f : SMS(X)_A \to SMS(Y)_B$ be a $b$-soft multi irresolute mapping. Let $(G, B)$ be a closed soft mset over $Y$. Then $(G, B)$ is a $b$-closed soft mset over $Y$. Since $f$ is a $b$-soft multi irresolute mapping, $f^{-1}(G, B)$ is a $b$-closed soft mset over $X$. Hence $f$ is a $b$-soft multi continuous function. □

**Theorem 5.6.** *Let $f : SMS(X)_A \to SMS(Y)_B$, $g : SMS(Y)_B \to SMS(Z)_E$ be two soft multi functions. Then*

*(1) $g \circ f : SMS(X)_A \to SMS(Z)_E$ is a $\gamma$-soft multi continuous function if $f$ is a $\gamma$-soft multi continuous function and $g$ is a soft multi continuous function.*

*(2) $g \circ f : SMS(X)_A \to SMS(Z)_E$ is a $\gamma$-soft multi irresolute function if $f, g$ are two $\gamma$-soft multi irresolute functions.*

*(3) $g \circ f : SMS(X)_A \to SMS(Z)_E$ is a $\gamma$-soft multi continuous function if $f$ is a $\gamma$-soft multi irresolute function and $g$ is a $\gamma$-soft multi continuous function.*

*Proof.* The proof is given for the case of $b$-soft multi continuous (irresolute) mapping and the other cases are similar.

(1) Let $(H, E)$ be a closed soft mset over $Z$. Since $g$ is a soft multi continuous function, $g^{-1}(H, E)$ is a closed soft mset over $Y$. Since $f$ is $b$-soft multi continuous function, $f^{-1}(g^{-1}(H, E)) = (g \circ f)^{-1}(H, E)$ is a $b$-closed soft mset over $X$. Thus $g \circ f$ is a $b$-soft multi continuous function.

(2) Let $(H, E)$ be a $b$-closed soft mset over $Z$. Since $g$ is a $b$-soft multi irresolute function, $g^{-1}(H, E)$ is a $b$-closed soft mset over $Y$. Since $f$ is a $b$-soft multi irresolute function, $f^{-1}(g^{-1}(H, E)) = (g \circ f)^{-1}(H, E)$ is a $b$-closed soft mset over $X$. Thus $g \circ f$ is a $b$-soft multi irresolute function.

(3) Similarly. □

## 6. Conclusions

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied and improved the mset theory and easily applied to many problems having uncertainties from social life. In this paper, we firstly gave the definition of "$\gamma$-operation" and then presented some examples of it such as pre-open soft multi operator, $\alpha$-open soft multi operator, semi open soft multi operator, b-open soft multi operator and $\beta$-open soft multi operator. Secondly, we studied the relationships among these different types of sub soft msets of soft multi topological spaces. Thirdly, we introduced the concepts of pre-soft (respectively semi-soft, $\alpha$-soft, $\beta$-soft, $b$-soft) multi continuous functions and extended the relationships among them. Finally, we presented the concepts of pre-soft (respectively semi-soft, $\alpha$-soft, $\beta$-soft, $b$-soft) multi irresolute functions and studied the composition of some generalized soft multi continuous (irresolute) functions. In the next work, the authors will introduce the separation axioms in soft multi topological space.

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## References

[1] M. Akdag and A. Ozkan, Soft $b$-open sets and soft $b$-continuous functions, J. Math. Sci. 8 (2014) 1–9.

[2] B. Chen, Soft semi-open sets and related properties in soft topological spaces, Appl. Math. Inf. Sci. 7 (2013) 287–294.

[3] S. A. El-Sheikh, R. A-K. Omar and M. Raafat, $\gamma$-operation in M-topological space, Gen. Math. Notes 27 (2015) 40–54.
[4] S. A. El-Sheikh, R. A-K. Omar and M. Raafat, Semi-compact soft multi spaces, Journal of New Theory 6 (2015) 76–87.
[5] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. A. El-Latif, $\gamma$-operation and decompositions of some forms of soft continuity in soft topological spaces, Ann. Fuzzy Math. Inform. 7 (2) (2014) 181–196.
[6] D. Tokat and I. Osmanoglu, On the soft multi topology and it's some properties, Nevsehir Universitesi Fen Bilimleri Enstitusu Dergisi Cilt 2 (2013) 109–118.
[7] D. Tokat and I. Osmanoglu, Connectedness on soft multi topological spaces, Journal of New Results in Science 2 (2013) 8–18.
[8] D. Tokat and I. Osmanoglu, Compact soft multi spaces, Eur. J. Pure Appl. Math. 7 (2014) 97–108.
[9] D. Tokat, I. Osmanoglu and Y. Ucok, On soft multi continuous functions, Journal of New Theory 1 (2015) 50–58.

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