Annals of Fuzzy Mathematics and Informatics
Volume 12, No. 1, (July 2016), pp. 129–138
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

@FMI

http://www.kyungmoon.com

Some results of mixed fuzzy soft topology and applications in Chemistry

Manash Jyoti Borah, Bipan Hazarika

Received 29 October 2015; Revised 7 January 2016; Accepted 1 February 2016

Abstract. In this paper, we introduce mixed fuzzy soft topology with separation axioms. Also we introduce soft quasi-coincident, soft quasi-neighborhood and obtain some related propositions. Finally, we provide an application of fuzzy soft sets in Chemistry.

2010 AMS Classification: 03E72

Keywords: Fuzzy soft sets, Soft quasi-coincident, Soft quasi-neighborhood.

Corresponding Author: Bipan Hazarika (bh_rgu@yahoo.co.in)

1. Introduction

Fuzzy set theory proposed by Zadeh [22] in 1965 which is a generalization of classical or crisp sets. In 1999, Molodtsov [11] introduced the theory of soft sets, which is a new mathematical approach to vagueness. In 2003, Maji, Biswas and Roy [9] studied the theory of soft sets initiated by Molodtsov [11] and developed several basic notions of soft set theory.

The notion of topological space is defined on crisp sets and hence it is affected by different generalizations of crisp sets like fuzzy sets and soft sets. In 1968, Chang [2] introduced fuzzy topological space and in 2011, subsequently Çağman and Enginoğlu [1], Shabir and Naz [15] introduced fuzzy soft topological spaces and studied basic properties. In 2012, Mahanta and Das [8], Neog, Sut and Hazarika [12] and Ray and Samanta [13] introduced fuzzy soft topological spaces in different direction. For details on soft topological spaces we refer to [5, 6, 7, 10, 14, 19, 21].

The works on mixed topology is due to Wiweger [20], Cooper [3], Das and Baishya [4], Tripathy and Ray [17, 18] and many others. The technique of mixing topologies has a number of applications in various branches of analysis, notably summability theory, measure theory, locally compact spaces, and interpolation theorems for analytic functions.

2. Preliminary results

In this section we recall some basic concepts and definitions regarding fuzzy soft sets and fuzzy soft topological space.

Definition 2.1 ([9])**.** Let U be an initial universe and F be a set of parameters. Let $\tilde{P}(U)$ denote the power set of U and A be a non-empty subset of F. Then F_A is called a fuzzy soft set over U where $F : A \to \tilde{P}(U)$ is a mapping from A into $\tilde{P}(U)$.

Definition 2.2 ([11])**.** F_E is called a soft set over U if and only if F is a mapping of E into the set of all subsets of the set U, where E is a set of parameters.

In other words, the soft set is a parameterized family of subsets of the set U. Every set $F(\epsilon)$, $\epsilon \tilde{\in} E$, from this family may be considered as the set of ϵ-element of the soft set F_E, or as the set of ϵ-approximate elements of the soft set.

Definition 2.3 ([13])**.** A fuzzy soft topology τ on (U, E) is a family of fuzzy soft sets over (U, E) satisfying the following properties :

(i) $\tilde{\phi}, \tilde{E} \tilde{\in} \tau$.
(ii) if $F_A, G_B \tilde{\in} \tau$, then $F_A \tilde{\cap} G_B \tilde{\in} \tau$.
(iii) if $F_{A_\alpha} \tilde{\in} \tau$ for all $\alpha \tilde{\in} \Delta$ an index set, then $\bigcup_{\alpha \in \Delta} F_{A_\alpha} \tilde{\in} \tau$.

Definition 2.4 ([16])**.** A fuzzy soft set F_A in a fuzzy soft topological space (U, E, τ) is a neighborhood of a fuzzy soft set G_B if and only if there exists an open fuzzy soft set H_C i.e. $H_C \tilde{\in} \tau$ such that $G_B \tilde{\subseteq} H_C \tilde{\subseteq} F_A$.

Definition 2.5 ([16])**.** Let (U, E, τ_1) and (U, E, τ_2) be two fuzzy soft topological spaces. If each $F_A \tilde{\in} \tau_1$ is in τ_2, then τ_2 is called fuzzy soft finer than τ_1, or τ_1 is fuzzy soft coarser than τ_2.

Definition 2.6 ([12])**.** The fuzzy soft set F_A over (U, E) is called a fuzzy soft point in (U, E) denoted by $e(F_A)$, if for the element $e \tilde{\in} A, F(e) \neq \bar{0}$ and $F(e^{'}) = \bar{0}$ for all $e^{'} \tilde{\in} A - \{e\}$.

Definition 2.7 ([12])**.** Let (U, E, τ) be a fuzzy soft topological space. Let F_A be a fuzzy soft set over (U, E). The fuzzy soft closure of F_A is defined as the intersection of all fuzzy soft closed sets which contained F_A and is denoted by $\bar{F_A}$. We write

$$\bar{F_A} = \tilde{\bigcap}\{G_B : G_B \text{ is fuzzy soft closed and } F_A \tilde{\subseteq} G_B\}.$$

Definition 2.8 ([12])**.** Let (U, E, τ) be a fuzzy soft topological space. Let F_A be a fuzzy soft set over (U, E). The fuzzy soft interior of F_A is defined as the union of all fuzzy soft open sets which contained F_A and is denoted by F_A°. We write

$$F_A^\circ = \tilde{\bigcup}\{G_B : G_B \text{ is fuzzy soft closed and } G_B \tilde{\subseteq} F_A\}.$$

Definition 2.9 ([12])**.** A fuzzy soft set F_A in a fuzzy soft topological space (U, E, τ) is a fuzzy soft neighborhood of the fuzzy soft point $e(G_B) \tilde{\in} (U, E)$ if there is an open fuzzy soft set H_C such that $e(G_B) \tilde{\in} H_C \tilde{\subseteq} F_A$.

Definition 2.10 ([8])**.** A fuzzy soft topological space (U, E, τ) is said to be a fuzzy soft τ_0-space if for every pair of disjoint fuzzy soft points $e(F_A), e(G_A)$ there exists a fuzzy soft open set containing one but not the other.

Proposition 2.11 ([8])**.** *A fuzzy soft subspace of a fuzzy soft τ_0-space is fuzzy soft τ_0-space.*

Definition 2.12 ([8])**.** A fuzzy soft topological space (U, E, τ) is said to be a fuzzy soft τ_1-space if for distinct pair of fuzzy soft points $e_1(F_A), e_2(G_A)$ there exists a fuzzy soft open sets H_A and K_A such that $e_1 \tilde{\in} H_A$ and $e_1 \tilde{\notin} K_A$; $e_2 \tilde{\in} H_A$ and $e_2 \tilde{\notin} K_A$.

Proposition 2.13 ([8])**.** *If every fuzzy soft point of a fuzzy soft topological space (U, E, τ) is fuzzy soft closed then (U, E, τ) is fuzzy soft τ_1-space.*

Definition 2.14 ([8])**.** A fuzzy soft topological space (U, E, τ) is said to be a fuzzy soft τ_2-space if and only if for distinct fuzzy soft points $e_1(F_A), e_2(G_A)$ on U, there exists disjoint fuzzy soft open sets H_A and K_A such that $e_1 \tilde{\in} H_A$ and $e_2 \tilde{\in} K_A$.

Definition 2.15 ([8])**.** A fuzzy soft topological space (U, E, τ) is said to be a fuzzy soft regular space if for every fuzzy soft point $e(F_A)$ and fuzzy soft closed set G_A not containing $e(F_A)$, then there exists disjoint fuzzy soft open sets L_A and M_A such that $e \tilde{\in} L_A$ and $G_A \tilde{\subseteq} M_A$.

Definition 2.16 ([8])**.** A fuzzy soft regular τ_1-space is called a fuzzy soft τ_3-space.

Definition 2.17 ([8])**.** A fuzzy soft topological space (U, E, τ) is said to be a fuzzy soft normal space if for every pair of disjoint fuzzy soft closed sets F_A and G_A, then there exists disjoint fuzzy soft open sets L_A and M_A such that $F_A \tilde{\subseteq} L_A$ and $G_A \tilde{\subseteq} M_A$.

Definition 2.18 ([8])**.** A fuzzy soft normal τ_1-space is called a fuzzy soft τ_4-space.

3. Main results

In this section we introduce the notions fuzzy soft quasi-coincident, fuzzy soft quasi-neighborhood, mixed fuzzy soft topology and prove some interesting results related to these notions.

Definition 3.1. Let μ_G be grade of membership of objects of fuzzy soft set G_A. A fuzzy soft point $e(F_A)$ in (U, E) is said to be a fuzzy soft quasi-coincident (in short soft q-coincident) with a fuzzy soft set G_A in (U, E) denoted by eqG_A if and only if $\mu_e + \mu_G \tilde{>} 1$, where $0 \tilde{<} \mu_e \tilde{\leq} 1$, $0 \tilde{<} \mu_G \tilde{\leq} 1$. If $e(F_A)$ is not fuzzy soft q-coincident with G_A, then we write $e\bar{q}G_A$.

Example 3.2. Consider $e_2(F_A)$,
$F_A = \{F(e_1) = \{(a, 0), (b, 0)\}, F(e_2) = \{(a, .6), (b, .8)\}, F(e_3) = \{(a, 0), (b, 0)\}\}$
and
$G_A = \{G(e_1) = \{(a, .5), (b, .6)\}, G(e_2) = \{(a, .8), (b, .5)\}, G(e_3) = \{(a, .9), (b, .5)\}\}$.
Then e_2qG_A.

Definition 3.3. A fuzzy soft set F_A in (U, E) is said to be a fuzzy soft quasi-coincident (in short soft q-coincident) with a fuzzy soft set G_A in (U, E) denoted by F_AqG_A if and only if $\mu_F + \mu_G \tilde{>} 1$, where $0 \tilde{<} \mu_F \tilde{\leq} 1$, $0 \tilde{<} \mu_G \tilde{\leq} 1$. If F_A is not fuzzy soft q-coincident with G_A, then we write $F_A\bar{q}G_A$.

Example 3.4. $F_A = \{F(e_1) = \{(a, .4), (b, .9)\}, F(e_2) = \{(a, .6), (b, .5)\}\}$
and

$$G_A = \{G(e_1) = \{(a, .8), (b, .3)\}, G(e_2) = \{(a, .6), (b, .6)\}\}.$$

Then F_AqG_A.

Definition 3.5. A fuzzy soft set F_A in a fuzzy soft topological space (U, E, τ) is said to be a fuzzy soft quasi-neighborhood (in short soft q-nhds) of $e(G_A)$ if and only if $H_A \tilde{\in} \tau$ such that $\bar{H}_A \tilde{\subseteq} F_A$ and eqH_A. The family of all q-nhds of $e(G_A)$ is called the system of q-nhds of $e(G_A)$.

Proposition 3.6. *Intersection of two q-nhds of $e(G_A)$ is a q-nhd.*

Proof. Obvious. □

Proposition 3.7. *Let F_A and G_A be two fuzzy soft sets in (U, E). Then the followings are true :*

(1) $F_A \tilde{\subseteq} G_A \Leftrightarrow F_A \bar{q} G_A^C$.
(2) $F_A q G_A \Rightarrow F_A \tilde{\cap} G_A \neq \tilde{\phi}$.
(3) $F_A \bar{q} F_A^C$.
(4) $F_A q G_A \Leftrightarrow$ *there exists an* $e(F_A) \tilde{\in} F_A$ *such that* eqG_A.
(5) $e(F_A) \tilde{\in} F_A^C \Leftrightarrow e\bar{q}F_A$.
(6) $F_A \tilde{\subseteq} G_A \Rightarrow eqF_A,$ *then* eqG_A.

Proof. (1)
$$\begin{aligned} F_A \tilde{\subseteq} G_A &\Leftrightarrow \mu_F \tilde{\leq} \mu_G \\ &\Leftrightarrow \mu_F - \mu_G \tilde{\leq} 0 \\ &\Leftrightarrow \mu_F + 1 - \mu_G \tilde{\leq} 1 \\ &\Leftrightarrow \mu_F + \mu_G^C \tilde{\leq} 1 \\ &\Leftrightarrow F_A \bar{q} G_A^C. \end{aligned}$$

(2) Since $F_A q G_A$, we have
$$\mu_F + \mu_G \tilde{>} 1 \Rightarrow F_A \tilde{\cap} G_A \neq \tilde{\phi}.$$

(3) Suppose $F_A q F_A^C$. Then $\mu_F + \mu_{F^C} \tilde{\ngtr} 1$. But this is not possible. Thus $F_A \bar{q} F_A^C$.
(4) $F_A q G_A \Leftrightarrow \mu_F + \mu_G \tilde{>} 1 \Leftrightarrow \mu_e + \mu_G \tilde{>} 1 \Leftrightarrow eqG_A$, because $e(F_A) \tilde{\in} F_A$.
(5) Proof is same as (1).
(6) $eqF_A \Rightarrow \mu_e + \mu_F \tilde{>} 1 \Rightarrow \mu_e + \mu_G \tilde{>} 1 \Rightarrow eqG_A$, because $F_A \tilde{\subseteq} G_A$. □

Proposition 3.8. *Let $\{F_{A_\alpha} : \alpha \tilde{\in} \Delta\}$ be a family of fuzzy soft sets in (U, E). Then a fuzzy soft point $e(G_A)$ is soft q-coincident with $\bigcup_{\alpha \tilde{\in} \Delta} F_{A_\alpha}$ if and only if there exists some $F_{A_\alpha} \tilde{\in} \{F_{A_\alpha} : \alpha \tilde{\in} \Delta\}$ such that $e(G_A) q F_{A_\alpha}$.*

Proof. Let $e(G_A) q F_{A_\alpha}$. Since $F_{A_\alpha} \tilde{\subseteq} (\bigcup_{\alpha \tilde{\in} \Delta} F_{A_\alpha})$, $e(G_A) q (\bigcup_{\alpha \tilde{\in} \Delta} F_{A_\alpha})$.

The necessary condition is easily proved by the concept of being quasi-coincident with and the help of the least upper bound properties. □

Definition 3.9. Let (U, E, τ_1) and (U, E, τ_2) be two fuzzy soft topological spaces. Consider the collection of fuzzy soft sets
$$\begin{aligned} \tau_1(\tau_2) = \{F_A \tilde{\in} I^U :\ &\text{for any fuzzy soft set } G_A \in U \text{ with } F_A q G_A, \text{ then there exists} \\ &\tau_2 - \text{ open set } H_A \text{ such that } H_A q G_A \text{ and } \tau_1 - \text{closure}, \bar{H}_A \tilde{\subseteq} F_A\}. \end{aligned}$$
Then this family of fuzzy soft sets will form a topology on (U, E) and this topology we call a mixed fuzzy soft topology on U.

Example 3.10. Suppose
$$F_A = \{F(e_1) = \{(a, .6), (b, .2)\}, F(e_2) = \{(a, .8), (b, .7)\}\},$$
$$G_A = \{G(e_1) = \{(a, .4), (b, .8)\}, G(e_2) = \{(a, .2), (b, .3)\}\},$$
and

$$\tau_1 = \{\tilde{\phi}, \tilde{E}, \tilde{F_A}\},\ \tau_2 = \{\tilde{\phi}, \tilde{E}, \tilde{G_A}\}.$$

We consider

$$H_A = \{H(e_1) = \{(a,.9),(b,.3)\}, H(e_2) = \{(a,.9),(b,.9)\}\},$$

where $G_A q H_A$ and $\tilde{E} q H_A$.
Then τ_1-closure of $G_A = \tilde{E} \tilde{\cap} G_A = G_A \tilde{\subseteq} G_A$. Thus $\tau_1(\tau_2) = \{\tilde{\phi}, \tilde{E}, \tilde{G_A}\}$.
Now we see that

(i) $\tilde{\phi}, \tilde{E} \tilde{\in} \tau$,
(ii) $\tilde{\phi} \tilde{\cap} \tilde{E} = \tilde{\phi}, \tilde{\phi} \tilde{\cap} G_A = \tilde{\phi}, \tilde{E} \tilde{\cap} G_A = G_A$,
(iii) $\tilde{\phi} \tilde{\cup} \tilde{E} = \tilde{E}, \tilde{\phi} \tilde{\cup} G_A = \tilde{G_A}, \tilde{E} \tilde{\cup} G_A = \tilde{E}, \tilde{\phi} \tilde{\cup} \tilde{E} \tilde{\cup} G_A = \tilde{E}$.

So $\tau_1(\tau_2)$ is a mixed fuzzy soft topology on (U, E), i.e., the triple $(U, E, \tau_1(\tau_2))$ is mixed fuzzy soft topological space.

Example 3.11. Consider

$$F_A = \{F(e_1) = \{(a,.8),(b,.8)\}, F(e_2) = \{(a,.6),(b,.6)\}\},$$
$$G_A = \{G(e_1) = \{(a,.2),(b,.2)\}, G(e_2) = \{(a,.4),(b,.4)\}\},$$

and

$$\tau_1 = \{\tilde{\phi}, \tilde{E}\},\ \tau_2 = \{\tilde{\phi}, \tilde{E}, \tilde{F_A}, \tilde{G_A}\}.$$

We consider

$$H_A = \{H(e_1) = \{(a,.9),(b,.9)\}, H(e_2) = \{(a,.8),(b,.7)\}\},$$

where $F_A q H_A$, $G_A q H_A$ and $\tilde{E} q H_A$.
Then

$$\tau_1\text{-closure of } F_A = \tilde{E} \tilde{\cap} G_A = G_A$$

and

$$\tau_1\text{-closure of } G_A = \tilde{E} \tilde{\cap} F_A = F_A.$$

Thus $F_A, G_A \tilde{\in} \tau_1(\tau_2)$. So $\tau_1(\tau_2) = \{\tilde{\phi}, \tilde{E}, \tilde{F_A}, \tilde{G_A}\}$.

Definition 3.12. A fuzzy soft set H_A in a fuzzy soft topological space (U, E, τ) is said to be a fuzzy soft quasi-neighborhood (in short soft q-nhds) of fuzzy soft point $e(F_A) \tilde{\in} (U, E)$ if and only if there exist $G_A \tilde{\in} \tau$ such that eqG_A and $G_A \tilde{\subseteq} H_A$.

Example 3.13. Consider
$F_A = \{F(e_1) = \{(a,0),(b,0)\}, F(e_2) = \{(a,.3),(b,.8)\}, F(e_3) = \{(a,0),(b,0)\}\}$
and
$G_A = \{G(e_1) = \{(a,.9),(b,.6)\}, G(e_2) = \{(a,.8),(b,.5)\}, G(e_3) = \{(a,.8),(b,.9)\}\}$,
where $e_2 q G_A$ and
$H_A = \{H(e_1) = \{(a,.9),(b,.7)\}, H(e_2) = \{(a,.9),(b,.8)\}, H(e_3) = \{(a,.8),(b,.9)\}\}$.
Then $G_A \tilde{\subseteq} H_A$. Thus H_A is q-nhd of $e_2(F_A)$.

Definition 3.14. Let (U, E, τ) be a fuzzy soft topological space and $e(F_A) \tilde{\in} (U, E)$ be the fuzzy soft point. Then the family $N_{e(F_A)}$ consisting of all the fuzzy soft q-nhds of $e(F_A)$ is called the system of fuzzy soft q-nhds of $e(F_A)$.

Proposition 3.15. *Let (U, E, τ) be a fuzzy soft topological space. Then for each $e(F_A)$ in U, $N_{e(F_A)}$ satisfies the following*

(i) *$e(F_A)$ is a fuzzy soft q-coincident with G_A, for every $G_A \tilde{\in} N_{e(F_A)}$.*
(ii) *If $F_A, G_A \tilde{\in} N_{e(F_A)}$ then $F_A \tilde{\cap} G_A \tilde{\in} N_{e(F_A)}$.*
(iii) *If $F_A \tilde{\in} N_{e(F_A)}$ and $F_A \tilde{\subseteq} G_A$ then $G_A \tilde{\in} N_{e(F_A)}$.*

Conversely, for each fuzzy soft point $e(F_A)$ in U, if $N_{e(F_A)}$ is the family of fuzzy soft sets in U satisfying the conditions (i), (ii), (iii), then the family τ of all fuzzy soft set G_A such that $G_A \tilde{\in} N_{e(F_A)}$ whenever eqG_A, is a fuzzy soft topology for U.

Proof. The proof is straightforward. □

Definition 3.16. Let (U, E, τ_1) and (U, E, τ_2) be two fuzzy soft topological spaces. Consider the collection of fuzzy soft sets

$$\tau_1(\tau_2) = \{F_A \tilde{\in} I^U : \text{ for every } e(G_A)qF_A, \text{ then there exists a fuzzy soft } \tau_2 - q\text{-nhd } H_A \text{ of } e(G_A) \text{ such that } \tau_1 - \text{closure}, \bar{H_A} \tilde{\subseteq} F_A\}.$$

Then $\tau_1(\tau_2)$ is fuzzy soft topology on U. This fuzzy soft topology is called a mixed fuzzy soft topology and triplet $(U, E, \tau_1(\tau_2))$ is called a mixed fuzzy soft topological space.

Proposition 3.17. *Let τ_1 and τ_2 be two fuzzy soft topologies on a soft set U. Then the mixed fuzzy soft topology $\tau_1(\tau_2)$ is coarser than τ_2, i.e. $\tau_1(\tau_2) \tilde{\subseteq} \tau_2$.*

Proof. The proof is straightforward. □

4. Mixed fuzzy soft separation axioms

Proposition 4.1. *Let (U, E, τ_1) and (U, E, τ_2) be two fuzzy soft topological spaces. If (U, E, τ_1) is fuzzy soft τ_0-space and $\tau_1 \tilde{\subseteq} \tau_2$ then $(U, E, \tau_1(\tau_2))$ is a fuzzy soft τ_0-space.*

Proof. We consider $e_1(F_A), e_2(G_A) \tilde{\in} \tau_1$. Given (U, E, τ_1) is a fuzzy soft τ_0-space, then there exists $H_A \tilde{\in} \tau_1$ such that $e_1(F_A) \tilde{\in} H_A$, $e_2(G_A) \tilde{\in} H_A$.
Let I_A be the fuzzy soft τ_1-q-nhd of $e_1(F_A)$ or $e_2(G_A)$. Since $H_A \tilde{\in} \tau_1$, $e_1 qH_A$ or $e_2 qH_A$ and $H_A \tilde{\subseteq} I_A$. Then $\mu_{e_1} + \mu_H \tilde{>} 1$ or $\mu_{e_2} + \mu_H \tilde{>} 1$. Thus $\mu_{e_1} + \mu_F \tilde{>} 1$ or $\mu_{e_2} + \mu_I \tilde{>} 1$. So $e_1 qI_A$ or $e_2 qI_A$.

Since $\tau_1 \tilde{\subseteq} \tau_2$ and $H_A \tilde{\in} \tau_1$, H_A is a τ_2-q-nhd of $e_1(F_A)$ or $e_2(G_A)$ and $\bar{H_A} \tilde{\subseteq} I_A$. Thus $I_A \tilde{\in} \tau_1(\tau_2)$. Also $e_1(F_A) \tilde{\in} H_A \tilde{\subseteq} I_A$, $e_2(G_A) \tilde{\in} H_A \tilde{\subseteq} I_A$. Thus $e_1(F_A) \tilde{\in} I_A$ or $e_2(G_A) \tilde{\in} I_A$. Hence $(U, E, \tau_1(\tau_2))$ is a fuzzy soft τ_0-space.
This complete proof of the theorem. □

Proposition 4.2. *Let (U, E, τ_1) and (U, E, τ_2) be two fuzzy soft topological spaces. If (U, E, τ_1) is fuzzy soft τ_1-space and $\tau_1 \tilde{\subseteq} \tau_2$ then $(U, E, \tau_1(\tau_2))$ is a fuzzy soft τ_1-space.*

Proof. We consider $e_1(F_A), e_2(G_A) \tilde{\in} \tau_1$. Given (U, E, τ_1) is a fuzzy soft τ_1-space then there exist $H_A, K_A \tilde{\in} \tau_1$ such that $e_1(F_A) \tilde{\in} H_A$ and $e_1(F_A) \tilde{\notin} K_A$ $e_2(G_A) \tilde{\in} K_A$ and $e_2(G_A) \tilde{\notin} H_A$.
Let L_A, M_A be the fuzzy soft τ_1-q-nhd of $e_1(F_A)$ and $e_2(G_A)$, respectively. Then $e_1 qH_A$ and $e_2 qK_A$ and $H_A \tilde{\subseteq} L_A$, $K_A \tilde{\subseteq} M_A$. Thus $\mu_{e_1} + \mu_H \tilde{>} 1$ and $\mu_{e_2} + \mu_K \tilde{>} 1$. So $\mu_{e_1} + \mu_L \tilde{>} 1$ and $\mu_{e_2} + \mu_M \tilde{>} 1$. Hence $e_1 qL_A$ and $e_1 qM_A$.

Since $\tau_1 \tilde{\subseteq} \tau_2$ and $H_A, K_A \tilde{\in} \tau_1$, $e_1 qH_A$ and $e_2 qK_A$ and $\bar{H_A} \tilde{\subseteq} L_A$, $\bar{K_A} \tilde{\subseteq} M_A$.
Thus $L_A, M_A \tilde{\in} \tau_1(\tau_2)$. Also $e_1 \tilde{\in} H_A \tilde{\subseteq} L_A$, $e_1 \tilde{\notin} K_A \tilde{\subseteq} M_A$, $e_2 \tilde{\in} K_A \tilde{\subseteq} M_A$ and $e_1 \tilde{\notin} H_A \tilde{\subseteq} L_A$. So $(U, E, \tau_1(\tau_2))$ is a fuzzy soft τ_1-space. This completes proof. □

Proposition 4.3. *Let (U, E, τ_1) and (U, E, τ_2) be two fuzzy soft topological spaces. If (U, E, τ_1) is fuzzy soft τ_2-space and $\tau_1 \tilde{\subseteq} \tau_2$, then $(U, E, \tau_1(\tau_2))$ is a fuzzy soft τ_2-space.*

Proof. We consider $e_1(F_A), e_2(G_A) \tilde{\in} \tau_1$. Given (U, E, τ_1) is a fuzzy soft τ_2-space then there exist $H_A, K_A \tilde{\in} \tau_1$ such that $e_1(F_A) \tilde{\in} H_A$ and $e_2(G_A) \tilde{\in} K_A$ and $H_A \tilde{\cap} K_A = \tilde{\phi}$. Let L_A, M_A be the fuzzy soft τ_1-q-nhd of $e_1(F_A)$ and $e_2(G_A)$, respectively. Then $e_1 q H_A$ and $e_2 q K_A$ and $H_A \tilde{\subseteq} L_A$, $K_A \tilde{\subseteq} M_A$. Thus $\mu_{e_1} + \mu_H \tilde{>} 1$ and $\mu_{e_2} + \mu_K \tilde{>} 1$. So $\mu_{e_1} + \mu_L \tilde{>} 1$ and $\mu_{e_2} + \mu_M \tilde{>} 1$. Hence $e_1 q L_A$ and $e_1 q M_A$.

Since $\tau_1 \tilde{\subseteq} \tau_2$ and $H_A, K_A \tilde{\in} \tau_1$, H_A and K_A are the τ_2-q-nhd of $e_1(F_A)$ and $e_2(G_A)$, respectively and $\bar{H_A} \tilde{\subseteq} L_A$, $\bar{K_A} \tilde{\subseteq} M_A$. This implies that $L_A, M_A \tilde{\in} \tau_1(\tau_2)$.
Also $e_1 \tilde{\in} H_A \tilde{\subseteq} L_A$, $e_2 \tilde{\in} K_A \tilde{\subseteq} M_A$ and $H_A \tilde{\cap} K_A = \tilde{\phi}$. This implies that $e_1 \tilde{\in} L_A$, $e_2 \tilde{\in} M_A$ and $L_A \tilde{\cap} M_A = \tilde{\phi}$.
Then $(U, E, \tau_1(\tau_2))$ is a fuzzy soft τ_2-space. The proof is complete. □

Proposition 4.4. *Let (U, E, τ_1) and (U, E, τ_2) be two fuzzy soft topological spaces. If (U, E, τ_1) is fuzzy soft normal space, then the mixed fuzzy topological space $(U, E, \tau_1(\tau_2))$ is a fuzzy soft normal space.*

Proof. Let $L_A \tilde{\in} (\tau_1(\tau_2))^C$, $M_A \tilde{\in} (\tau_1(\tau_2))$ and $L_A \tilde{\subseteq} M_A$. Again let $P_A \tilde{\in} (\tau_2)^C$, $R_A \tilde{\in} \tau_2$ and $P_A \tilde{\subseteq} R_A$. Then there exists $Q_A \tilde{\in} \tau_2$ such that $P_A \tilde{\subseteq} Q_A^\circ \tilde{\subseteq} \bar{Q_A} \tilde{\subseteq} R_A$. Thus $M_A \tilde{\in} \tau_2$, $Q_A \tilde{\in} (\tau_1(\tau_2))$. So $P_A \tilde{\subseteq} Q_A^\circ \tilde{\subseteq} \bar{Q_A} \tilde{\subseteq} M_A$.

Since $L_A \tilde{\in} (\tau_1(\tau_2))^C$, $1 - L_A \tilde{\in} (\tau_1(\tau_2))$. Thus $1 - L_A \tilde{\in} \tau_2$. So $L_A \tilde{\in} \tau_2^C$.
Hence $L_A \tilde{\subseteq} P_A$ and $L_A \tilde{\subseteq} Q_A^\circ \tilde{\subseteq} \bar{Q_A} \tilde{\subseteq} M_A$, i.e., the mixed fuzzy soft topological space $(U, E, \tau_1(\tau_2))$ is a fuzzy soft normal space. This completes the proof. □

Proposition 4.5. *Let (U, E, τ_1) and (U, E, τ_2) be two fuzzy soft topological spaces. A mixed fuzzy soft topological space $(U, E, \tau_1(\tau_2))$ is a fuzzy soft normal space if and only if two closed fuzzy soft sets L_A, M_A in U with $L_A \tilde{\subseteq} 1 - M_A$ then there exist $P_A, Q_A \tilde{\in} (\tau_1(\tau_2))$ such that $L_A \tilde{\subseteq} P_A$, $M_A \tilde{\subseteq} Q_A$ and $\bar{P}_A \tilde{\subseteq} 1 - \bar{Q}_A$.*

Proof. Let $L_A \tilde{\in} (\tau_1(\tau_2))^C$ and fuzzy soft open set $P_A \tilde{\in} (\tau_1(\tau_2))$ and $L_A \tilde{\subseteq} P_A$. Then, since mixed fuzzy soft topological space $(U, E, \tau_1(\tau_2))$ is a fuzzy soft normal space, there exists $Q_A \tilde{\in} (\tau_1(\tau_2))$ such that $L_A \tilde{\subseteq} Q_A^\circ \tilde{\subseteq} \bar{Q}_A \tilde{\subseteq} P_A$.

Let $M_A \tilde{\in} (\tau_1(\tau_2))^C$ and $M_A = 1 - P_A$, $L_A \tilde{\subseteq} 1 - M_A$ and $\bar{Q}_A \tilde{\subseteq} 1 - M_A$. Since $\bar{Q}_A \tilde{\subseteq} 1 - M_A$, $M_A \tilde{\subseteq} 1 - \bar{Q}_A$. Since $P_A \tilde{\in} (\tau_1(\tau_2))$, $\bar{P}_A \tilde{\in} (\tau_1(\tau_2))^C$. This implies that $\bar{P}_A \tilde{\subseteq} M_A \Rightarrow \bar{P}_A \tilde{\subseteq} 1 - \bar{Q}_A$. Also $L_A \tilde{\subseteq} 1 - M_A \Rightarrow M_A \tilde{\subseteq} 1 - L_A \tilde{\subseteq} Q_A$.

Conversely, let $L_A, M_A \tilde{\in} (\tau_1(\tau_2))^C$ and $L_A \tilde{\subseteq} 1 - M_A$. Then there exist $P_A, Q_A \tilde{\in} (\tau_1(\tau_2))$ such that $L_A \tilde{\subseteq} P_A, M_A \tilde{\subseteq} Q_A$ and $\bar{P}_A \tilde{\subseteq} 1 - \bar{Q}_A$. We consider $P_A = 1 - M_A$. Then

$$L_A \tilde{\subseteq} Q_A^\circ \tilde{\subseteq} \bar{Q}_A \tilde{\subseteq} 1 - \bar{P}_A \tilde{\subseteq} 1 - M_A = P_A, \text{ i.e., } L_A \tilde{\subseteq} Q_A^\circ \tilde{\subseteq} \bar{Q}_A \tilde{\subseteq} P_A.$$

Thus mixed fuzzy soft topological space $(U, E, \tau_1(\tau_2))$ is a fuzzy soft normal space. This completes the proof.

5. Application of fuzzy soft quasi-coincident in Chemistry

In this section, we applied soft quasi-coincident of fuzzy soft sets in Chemistry. Here we find bond strength of different hybridization.

In the chemistry, hybridization is the concept of mixing of orbitals on an atom having nearly equal energy, to produce entirely new orbitals which have same energy contents, identical shapes and are symmetrically disposed in space.

The mixing of the s and p orbitals to form sp hybridization. Similarly the mixing of the s and two p orbitals to form sp^2 hybridization and the mixing of the s and three p orbitals to form sp^3 hybridization.

We know that

(a) in the sp^3 hybridization 25% s character and 75% p character,
(b) in the sp^2 hybridization 33% s character and 66% p character,
(c) in the sp hybridization 50% s character and 50% p character.

Algorithms:

1. Construct fuzzy soft sets of different hybridization.
2. Find different grade value of different hybridization.
3. Adding 0.02 of same hybridization.
4. If the grade value of different hybridization is high then the bond strength of that hybridization is low i.e. if $\mu_F + \mu_G \tilde{>} 1$ then the strength of the hybridization is low i.e. the bond strength of the compound is so weak.
5. Finally we find the bond strength sequence of different hybridization.

Fuzzy soft sets of sp^3 hybridization is

$$F_A = \{F(e_1) = \{(s, 0.2), (p, 0.7)\}.$$

Fuzzy soft sets of sp^2 hybridization is

$$G_A = \{G(e_1) = \{(s, 0.3), (p, 0.6)\}.$$

Fuzzy soft sets of sp hybridization is

$$H_A = \{H(e_1) = \{(s, 0.5), (p, 0.5)\}.$$

Grade value of sp^3 hybridization, i.e., $\mu(F_A) = |0.2 - 0.7| = 0.5$,
Grade value of sp^2 hybridization, i.e., $\mu(G_A) = |0.3 - 0.6| = 0.3$,
Grade value of sp hybridization, i.e., $\mu(H_A) = |0.5 - 0.5| = 0.0$,
Grade value of sp^2-sp^3, i.e., $\mu(G_A, F_A) = 0.3 + 0.5 = 0.8$,
Grade value of sp^2-sp^2, i.e., $\mu(G_A, G_A) = 0.3 + 0.3 + 0.02 = 0.62$,
Grade value of sp-sp^3, i.e., $\mu(H_A, F_A) = 0.0 + 0.5 = 0.5$,
Grade value of sp^3-sp^3, i.e., $\mu(F_A, F_A) = 0.5 + 0.5 + 0.02 = 1.02 > 1$,
Grade value of sp-sp^2, i.e., $\mu(H_A, G_A) = 0.0 + 0.3 = 0.3$,
Grade value of sp-sp, i.e., $\mu(H_A, H_A) = 0.0 + 0.0 + 0.02 = 0.02$.

Therefore the sequence of bond strength is

$$sp^3 - sp^3 < sp^2 - sp^3 < sp^2 - sp^2 < sp - sp^3 < sp - sp^2 < sp - sp.$$

□

6. Conclusion

We have introduced soft quasi-coincident, soft quasi-neighborhood and mixed fuzzy soft topology which are defined over an initial universe with a fixed set of parameters. The notions of mixed fuzzy soft separation axioms are introduced and their basic properties are investigated. In this paper, we have studied a few ideas only, it will be necessary to carry out more theoretical research to establish a general framework for the practical application.

7. Competing interests

The authors declare that they have no competing interests.

8. Authors contributions

Each of the authors contributed to each part of this work equally and read and approved the final version of the manuscript.

9. Acknowlegement

The authors would like to thank Prof. Kul Hur and the referees for his/her much encouragment, support, constructive criticism, careful reading and making a useful comment which imporved the presentation and the readability of the paper.

References

[1] N. Çağman and N. S. Enginoğlu, Soft Set Theory and Uni-int Decision Making, Euro. J. Opera. Research 207 (2010) 848–855.

[2] C. L. Chang, Fuzzy Topological Spaces, J. Math. Anal. Appl. 24 (1968) 182–190.

[3] J. B. Cooper, The Strict Topology and Spaces with mixed Topologies, Proc. Amer. Math. Soc. 30 (3)(1971) 583–592.

[4] N. R. Das and P. C. Baishya, Mixed Fuzzy Topological Spaces, J. Fuzzy Math. 3 (1995) 777–784.

[5] O. Göçür and A. Kopuzlu, On soft separation axioms, Ann. Fuzzy Math. Inform. 9 (5)(2015), 817-822.

[6] A. C. Guler, G. Kale, Regularity and normality on soft ideal topological spaces, Ann. Fuzzy Math. Inform. 9 (3)(2015) 373–383.

[7] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-Latif, γ-operation and decompositions of some forms of soft continuity of soft topological spaces via soft ideals, Ann. Fuzzy Math. Inform. 9 (3)(2015) 385–402.

[8] J. Mahanta and P. K. Das, Results on Fuzzy soft topological Spaces, arXiv:1203.0634v1.

[9] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy Soft Sets, J. Fuzzy Math. 9 (3) (2001) 589–602.

[10] S. Mishar, R. Srivastava, Hausdorff fuzzy soft topological spaces, Ann. Fuzzy Math. Inform. 9 (2)(2015) 247–260.

[11] D. A. Molodstov, Soft Set Theory-First Result, Comput. Math. Appl. 37 (1999) 19–31.

[12] T. J. Neog, D. K. Sut and G. C. Hazarika, Fuzzy Soft Topological Spaces, Inter. J. Latest Trends Math. 2 (1)(2012) 54–67.

[13] S. Ray and T. K. Samanta, A note on Fuzzy Soft Topological Spaces, Ann. Fuzzy Math. Inform. 3 (2)(2012) 305–311.

[14] R. Şahin, Soft compactification of soft topological spaces: soft star topological spaces, Ann. Fuzzy Math. Inform. 10 (3) (2015) 447–464.

[15] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl. 61 (2012) 412–418.

[16] B. Tanay and M. Burç Kandemir, Topological Structure of fuzzy soft sets, Comput. Math. Appl. 61 (2011) 2952–2957.

[17] B. C. Tripathy and G. C. Ray, Mixed Fuzzy ideal topological spaces, Appl. Math. Comput. 220 (2013) 602–607.

[18] B. C. Tripathy and G. C. Ray, Weakly continuous functions on mixed fuzzy topological spaces, Acta Scientiarum Tech. 36 (2) (2014) 331–335.

[19] V. Tripathi, T. Som, The countability aspects of fuzzy soft topological spaces, Ann. Fuzzy Math. Inform. 9 (5) (2015) 835–842

[20] A. Wiweger, Linear spaces with mixed topology, Studia Math. 20 (1) (1961) 47–68.
[21] N. Xie, Soft points, the structure of soft topological spaces, Ann. Fuzzy Math. Inform. 10 (2) (2015) 309–322.
[22] L. A. Zadeh, Fuzzy Sets, Information and Control 8 (1965) 338–353.

Manash Jyoti Borah (mjyotibora9@gmail.com)
Department of Mathematics, Bahona College, Jorhat-785 101, Assam, India

Bipan Hazarika (bh_rgu@yahoo.co.in)
Department of Mathematics, Rajiv Gandhi University, Rono Hills, Doimukh-791112, Arunachal Pradesh, India