Some results of mixed fuzzy soft topology and applications in Chemistry

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Abstract. In this paper, we introduce mixed fuzzy soft topology with separation axioms. Also we introduce soft quasi-coincident, soft quasi-neighborhood and obtain some related propositions. Finally, we provide an application of fuzzy soft sets in Chemistry.

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1. Introduction


The notion of topological space is defined on crisp sets and hence it is affected by different generalizations of crisp sets like fuzzy sets and soft sets. In 1968, Chang [2] introduced fuzzy topological space and in 2011, subsequently Çağman and Enginoğlu [1], Shabir and Naz [15] introduced fuzzy soft topological spaces and studied basic properties. In 2012, Mahanta and Das [8], Neog, Sut and Hazarika [12] and Ray and Samanta [13] introduced fuzzy soft topological spaces in different direction. For details on soft topological spaces we refer to [5, 6, 7, 10, 14, 19, 21].

The works on mixed topology is due to Wiweger [20], Cooper [3], Das and Baishya [4], Tripathy and Ray [17, 18] and many others. The technique of mixing topologies has a number of applications in various branches of analysis, notably summability theory, measure theory, locally compact spaces, and interpolation theorems for analytic functions.
2. Preliminary results

In this section we recall some basic concepts and definitions regarding fuzzy soft sets and fuzzy soft topological space.

**Definition 2.1** ([9]). Let $U$ be an initial universe and $F$ be a set of parameters. Let $\hat{P}(U)$ denote the power set of $U$ and $A$ be a non-empty subset of $F$. Then $F_A$ is a fuzzy soft set over $U$, $\hat{P}(U)$ is a mapping from $A$ into $\hat{P}(U)$.

**Definition 2.2** ([11]). $F_E$ is called a soft set over $U$ if and only if $F$ is a mapping of $E$ into the set of all subsets of the set $U$, where $E$ is a set of parameters.

In other words, the soft set is a parameterized family of subsets of the set $U$. Every set $F(e), e \in E$, from this family may be considered as the set of $e$-element of the soft set $F_E$, or as the set of $e$-approximate elements of the soft set.

**Definition 2.3** ([13]). A fuzzy soft topology $\tau$ on $(U,E)$ is a family of fuzzy soft sets over $(U,E)$ satisfying the following properties:

(i) $\emptyset, \bar{E} \in \tau$.

(ii) if $F_A, G_B \in \tau$, then $F_A \cap G_B \in \tau$.

(iii) if $F_{A_\alpha} \in \tau$ for all $\alpha \in \Delta$ an index set, then $\bigcup_{\alpha \in \Delta} F_{A_\alpha} \in \tau$.

**Definition 2.4** ([16]). A fuzzy soft set $F_{A}$ in a fuzzy soft topological space $(U,E,\tau)$ is a neighborhood of a fuzzy soft set $G_{B}$ if and only if there exists an open fuzzy soft set $H_{C}$ i.e. $H_{C} \in \tau$ such that $G_{B} \subseteq H_{C} \subseteq F_{A}$.

**Definition 2.5** ([16]). Let $(U,E,\tau_1)$ and $(U,E,\tau_2)$ be two fuzzy soft topological spaces. If each $F_A \in \tau_1$ is in $\tau_2$, then $\tau_2$ is called fuzzy soft finer than $\tau_1$, or $\tau_1$ is fuzzy soft coarser than $\tau_2$.

**Definition 2.6** ([12]). The fuzzy soft set $F_A$ over $(U,E)$ is called a fuzzy soft point in $(U,E)$ denoted by $e(F_A)$, if for the element $e \in A, F(e) \neq 0$ and $F(e') = 0$ for all $e' \in A - \{e\}$.

**Definition 2.7** ([12]). Let $(U,E,\tau)$ be a fuzzy soft topological space. Let $F_A$ be a fuzzy soft set over $(U,E)$. The fuzzy soft closure of $F_A$ is defined as the intersection of all fuzzy soft closed sets which contained $F_A$ and is denoted by $\bar{F}_A$. We write

$$\bar{F}_A = \bigcap\{G_B : G_B \text{ is fuzzy soft closed and } F_A \subseteq G_B\}.$$

**Definition 2.8** ([12]). Let $(U,E,\tau)$ be a fuzzy soft topological space. Let $F_A$ be a fuzzy soft set over $(U,E)$. The fuzzy soft interior of $F_A$ is defined as the union of all fuzzy soft open sets which contained $F_A$ and is denoted by $F^o_A$. We write

$$F^o_A = \bigcup\{G_B : G_B \text{ is fuzzy soft closed and } G_B \subseteq F_A\}.$$

**Definition 2.9** ([12]). A fuzzy soft set $F_A$ in a fuzzy soft topological space $(U,E,\tau)$ is a fuzzy soft neighborhood of the fuzzy soft point $e(G_B) \in (U,E)$ if there is an open fuzzy soft set $H_C$ such that $e(G_B) \in H_C \subseteq F_A$.

**Definition 2.10** ([8]). A fuzzy soft topological space $(U,E,\tau)$ is said to be a fuzzy soft $\tau_0$-space if for every pair of disjoint fuzzy soft points $e(F_A), e(G_A)$ there exists a fuzzy soft open set containing one but not the other.
Proposition 2.11 ([8]). A fuzzy soft subspace of a fuzzy soft $\tau_0$-space is fuzzy soft $\tau_0$-space.

Definition 2.12 ([8]). A fuzzy soft topological space $(U, E, \tau)$ is said to be a fuzzy soft $\tau_1$-space if for distinct pair of fuzzy soft points $e_1(F_1), e_2(G_1)$ there exists a fuzzy soft open sets $H_1$ and $K_1$ such that $e_1 \in H_1$ and $e_1 \notin K_1$; $e_2 \in H_1$ and $e_2 \notin K_1$.

Proposition 2.13 ([8]). If every fuzzy soft point of a fuzzy soft topological space $(U, E, \tau)$ is fuzzy soft closed then $(U, E, \tau)$ is fuzzy soft $\tau_1$-space.

Definition 2.14 ([8]). A fuzzy soft topological space $(U, E, \tau)$ is said to be a fuzzy soft $\tau_2$-space if and only if for distinct fuzzy soft points $e_1(F_1), e_2(G_1)$ on $U$, there exists disjoint fuzzy soft open sets $H_1$ and $K_1$ such that $e_1 \in H_1$ and $e_2 \notin K_1$.

Definition 2.15 ([8]). A fuzzy soft topological space $(U, E, \tau)$ is said to be a fuzzy soft regular space if for every fuzzy soft point $e(F_1)$ and fuzzy soft closed set $G_1$ not containing $e(F_1)$, then there exists disjoint fuzzy soft open sets $L_1$ and $M_1$ such that $e \notin L_1$ and $G_1 \subseteq M_1$.

Definition 2.16 ([8]). A fuzzy soft regular $\tau_1$-space is called a fuzzy soft $\tau_3$-space.

Definition 2.17 ([8]). A fuzzy soft topological space $(U, E, \tau)$ is said to be a fuzzy soft normal space if for every pair of disjoint fuzzy soft closed sets $F_1$ and $G_1$, then there exists disjoint fuzzy soft open sets $L_1$ and $M_1$ such that $F_1 \subseteq L_1$ and $G_1 \subseteq M_1$.

Definition 2.18 ([8]). A fuzzy soft normal $\tau_1$-space is called a fuzzy soft $\tau_4$-space.

3. Main Results

In this section we introduce the notions fuzzy soft quasi-coincident, fuzzy soft quasi-neighborhood, mixed fuzzy soft topology and prove some interesting results related to these notions.

Definition 3.1. Let $\mu_G$ be grade of membership of objects of fuzzy soft set $G_1$. A fuzzy soft point $e(F_A)$ in $(U, E)$ is said to be a fuzzy soft quasi-coincident (in short soft $q$-coincident) with a fuzzy soft set $G_1$ in $(U, E)$ denoted by $eqG_A$ if and only if $\mu_+ + \mu_G > 1$, where $0 < \mu_+ \leq 1$, $0 < \mu_G \leq 1$. If $e(F_A)$ is not fuzzy soft $q$-coincident with $G_1$, then we write $e\bar{q}G_A$.

Example 3.2. Consider $e_2(F_A)$,

$F_A = \{F(e_1) = \{(a, 0), (b, 0)\}, F(e_2) = \{(a, 0), (b, 0)\}, F(e_3) = \{(a, 0), (b, 0)\}\}$

and

$G_A = \{G(e_1) = \{(a, 5), (b, 6)\}, G(e_2) = \{(a, 5), (b, 5)\}, G(e_3) = \{(a, 9), (b, 5)\}\}.$

Then $e_2qG_A$.

Definition 3.3. A fuzzy soft set $F_A$ in $(U, E)$ is said to be a fuzzy soft quasi-coincident (in short soft $q$-coincident) with a fuzzy soft set $G_A$ in $(U, E)$ denoted by $F_AeqG_A$ if and only if $\mu_F + \mu_G > 1$, where $0 < \mu_F \leq 1$, $0 < \mu_G \leq 1$. If $F_A$ is not fuzzy soft $q$-coincident with $G_A$, then we write $F_Ae\bar{q}G_A$.

Example 3.4. $F_A = \{F(e_1) = \{(a, 4), (b, 9)\}, F(e_2) = \{(a, 6), (b, 5)\}\}$

and

$G_A = \{G(e_1) = \{(a, 8), (b, 3)\}, G(e_2) = \{(a, 6), (b, 6)\}\}$

Then $F_AeqG_A$. 

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Definition 3.5. A fuzzy soft set $F_A$ in a fuzzy soft topological space $(U, E, \tau)$ is said to be a fuzzy soft quasi-neighborhood (in short soft $q$-nhds) of $e(G_A)$ if and only if $H_A \in \tau$ such that $H_A \subseteq F_A$ and $eqH_A$. The family of all $q$-nhds of $e(G_A)$ is called the system of $q$-nhds of $e(G_A)$.

Proposition 3.6. Intersection of two $q$-nhds of $e(G_A)$ is a $q$-nhd.

Proof. Obvious. \hfill \Box

Proposition 3.7. Let $F_A$ and $G_A$ be two fuzzy soft sets in $(U, E)$. Then the followings are true:

1. $F_A \subseteq G_A \iff F_A \bar{q} G_A^C$.
2. $F_A q G_A \Rightarrow F_A \cap G_A \neq \tilde{\phi}.$
3. $F_A \bar{q} F_A^C$.
4. $F_A q G_A \Rightarrow$ there exists an $e(F_A) \tilde{\in} F_A$ such that $eqG_A$.
5. $e(F_A) \tilde{\in} F_A^C \Rightarrow eqF_A$.
6. $F_A \tilde{\subseteq} G_A \Rightarrow eqF_A$, then $eqG_A$.

Proof. (1) $F_A \subseteq G_A \iff \mu_F \leq \mu_G$

$\iff \mu_F - \mu_G \leq 0$

$\iff \mu_F + 1 - \mu_G \leq 1$

$\iff \mu_F + \mu_G^C \leq 1$

$\iff F_A \bar{q} G_A^C.$

(2) Since $F_A q G_A$, we have

$\mu_F + \mu_G \geq 1 \Rightarrow F_A \bar{q} G_A \neq \tilde{\phi}.$

(3) Suppose $F_A q F_A^C$. Then $\mu_F + \mu_F^C \geq 1$. But this is not possible. Thus $F_A \bar{q} F_A^C$.

(4) $F_A q G_A \Rightarrow \mu_F + \mu_G > 1 \Rightarrow \mu_F > 1 \Rightarrow eqG_A$, because $e(F_A) \bar{q} F_A$.

(5) Proof is same as (1).

(6) $eqF_A \Rightarrow \mu_F + \mu_F^C > 1 \Rightarrow eqG_A$, because $F_A \tilde{\subseteq} G_A$. \hfill \Box

Proposition 3.8. Let $\{F_{A_\alpha} : \alpha \in \Delta\}$ be a family of fuzzy soft sets in $(U, E)$. Then a fuzzy soft point $e(G_A)$ is soft $q$-coincident with $\bigcup_{\alpha \in \Delta} F_{A_\alpha}$ if and only if there exists some $F_{A_\alpha} \tilde{\subseteq} (\bigcup_{\alpha \in \Delta} F_{A_\alpha})$ such that $e(G_A) q F_{A_\alpha}$.

Proof. Let $e(G_A) q F_{A_\alpha}$. Since $F_{A_\alpha} \tilde{\subseteq} (\bigcup_{\alpha \in \Delta} F_{A_\alpha})$, $e(G_A) q (\bigcup_{\alpha \in \Delta} F_{A_\alpha})$.

The necessary condition is easily proved by the concept of being quasi-coincident with and the help of the least upper bound properties. \hfill \Box

Definition 3.9. Let $(U, E, \tau_1)$ and $(U, E, \tau_2)$ be two fuzzy soft topological spaces. Consider the collection of fuzzy soft sets

$\tau_1 (\tau_2) = \{F_A \in I^U : \text{ for any fuzzy soft set } G_A \in U \text{ with } F_A q G_A, \text{ then there exists }\}$

$\tau_2 - \text{ open set } H_A \text{ such that } H_A q G_A \text{ and } \tau_1 - \text{ closure, } H_A \subseteq F_A\}$. 

Then this family of fuzzy soft sets will form a topology on $(U, E)$ and this topology we call a mixed fuzzy soft topology on $U$.

Example 3.10. Suppose

$F_A = \{F(e_1) = \{(a, .6), (b, .2)\}, F(e_2) = \{(a, .8), (b, .7)\}\},$

$G_A = \{G(e_1) = \{(a, .4), (b, .8)\}, G(e_2) = \{(a, .2), (b, .3)\}\},$

and
We consider
\[ \tau_1 = \{ \tilde{\phi}, E, \tilde{F}_A \}, \quad \tau_2 = \{ \tilde{\phi}, E, \tilde{G}_A \}. \]

Then
\[ H_A = \{ H(e_1) = \{(a, 9), (b, 9)\}, H(e_2) = \{(a, 9), (b, 9)\}\}, \]
where \( G_{AQ}H_A \) and \( E_{qH_A} \).

Then \( \tau_1 \)-closure of \( G_A = \tilde{E} \cap G_A = \tilde{G}_A \).

Thus \( \tau_1(e) = \{ \tilde{\phi}, E, \tilde{F}_A, \tilde{G}_A \} \).

Example 3.11. Consider
\[ F_A = \{ F(e_1) = \{(a, 8), (b, 8)\}, F(e_2) = \{(a, 6), (b, 6)\}\}, \]
\[ G_A = \{ G(e_1) = \{(a, 2), (b, 2)\}, G(e_2) = \{(a, 4), (b, 4)\}\}, \]
and
\[ \tau_1 = \{ \tilde{\phi}, E \}, \quad \tau_2 = \{ \tilde{\phi}, E, \tilde{F}_A, \tilde{G}_A \}. \]

We consider
\[ H_A = \{ H(e_1) = \{(a, 9), (b, 9)\}, H(e_2) = \{(a, 8), (b, 7)\}\}, \]
where \( F_{AQ}H_A, G_{AQ}H_A \) and \( E_{qH_A} \).

Then
\[ \tau_1 \)-closure of \( F_A = \tilde{E} \cap G_A = G_A \]
and
\[ \tau_1 \)-closure of \( G_A = \tilde{E} \cap F_A = F_A. \]

Thus \( F_A, G_A \in \tau_1(\tau_2) \). So \( \tau_1(\tau_2) = \{ \tilde{\phi}, E, \tilde{F}_A, \tilde{G}_A \}. \)

Definition 3.12. A fuzzy soft set \( H_A \) in a fuzzy soft topological space \((U, E, \tau)\) is said to be a fuzzy soft quasi-neighborhood (in short soft \(q\)-nhds) of fuzzy soft point \( e(F_A) \in (U, E) \) if and only if there exist \( G_A \in \tau \) such that \( eqG_A \) and \( G_A \subseteq H_A \).

Example 3.13. Consider
\[ F_A = \{ F(e_1) = \{(a, 0), (b, 0)\}, F(e_2) = \{(a, 3), (b, 3)\}, F(e_3) = \{(a, 0), (b, 0)\}\} \]
and
\[ G_A = \{ G(e_1) = \{(a, 9), (b, 5)\}, G(e_2) = \{(a, 8), (b, 9)\}, G(e_3) = \{(a, 8), (b, 9)\}\}, \]
where \( e_{2qG_A} \) and
\[ H_A = \{ H(e_1) = \{(a, 9), (b, 7)\}, H(e_2) = \{(a, 9), (b, 8)\}, H(e_3) = \{(a, 8), (b, 9)\}\}. \]

Then \( G_A \subseteq H_A \). Thus \( H_A \) is \(q\)-nhd of \( e_2(F_A) \).

Definition 3.14. Let \((U, E, \tau)\) be a fuzzy soft topological space and \( e(F_A) \in (U, E) \) be the fuzzy soft point. Then the family \( N_{e(F_A)} \) consisting of all the fuzzy soft \(q\)-nhds of \( e(F_A) \) is called the system of fuzzy soft \(q\)-nhds of \( e(F_A) \).

Proposition 3.15. Let \((U, E, \tau)\) be a fuzzy soft topological space. Then for each \( e(F_A) \) in \( U \), \( N_{e(F_A)} \) satisfies the following
The proof is straightforward.

\( \tau \)

Let \((U,E,\tau)\) be two fuzzy soft topological spaces. Consider the collection of fuzzy soft sets

\[ \tau_1(\tau_2) = \{ F_A \in I^U : \text{for every } e(G_A)qF_A, \text{ then exists a fuzzy soft } \tau_2 - q\text{-nhd } H_A \text{ of } e(G_A) \text{ such that } \tau_1 = \text{closure, } H_A \subseteq F_A \}. \]

Then \(\tau_1(\tau_2)\) is fuzzy soft topology on \(U\). This fuzzy soft topology is called a mixed fuzzy soft topology and triplet \((U,E,\tau(\tau_2))\) is called a mixed fuzzy soft topological space.

**Proposition 3.17.** Let \(\tau_1\) and \(\tau_2\) be two fuzzy soft topologies on a soft set \(U\). Then the mixed fuzzy soft topology \(\tau_1(\tau_2)\) is coarser than \(\tau_2\), i.e. \(\tau_1(\tau_2) \subseteq \tau_2\).

**Proof.** The proof is straightforward. \( \square \)

4. MIXED FUZZY SOFT SEPARATION AXIOMS

**Proposition 4.1.** Let \((U,E,\tau_1)\) and \((U,E,\tau_2)\) be two fuzzy soft topological spaces. If \((U,E,\tau_1)\) is fuzzy soft \(\tau_0\)-space and \(\tau_1 \subseteq \tau_2\) then \((U,E,\tau_1(\tau_2))\) is a fuzzy soft \(\tau_0\)-space.

**Proof.** We consider \(e_1(F_A)\), \(e_2(G_A)\) \(\in \tau_1\). Given \((U,E,\tau_1)\) is a fuzzy soft \(\tau_0\)-space, then there exists \(H_A \in \tau_1\) such that \(e_1(F_A) \in H_A\), \(e_2(G_A) \in H_A\).

Let \(I_A\) be the fuzzy soft \(\tau_1-q\)-nhd of \(e_1(F_A)\) or \(e_2(G_A)\). Since \(H_A \in \tau_1\), \(e_1qH_A\) or \(e_2qH_A\) and \(H_A \subseteq I_A\). Then \(\mu_{e_1} + \mu_{H} > 1\) or \(\mu_{e_2} + \mu_{H} > 1\). Thus \(\mu_{e_1} + \mu_{H} > 1\) or \(\mu_{e_2} + \mu_{H} > 1\).

So \(e_1qI_A\) or \(e_2qI_A\).

Since \(\tau_1 \subseteq \tau_2\) and \(H_A \in \tau_1\), \(H_A\) is a \(\tau_2-q\)-nhd of \(e_1(F_A)\) or \(e_2(G_A)\) and \(H_A \subseteq I_A\). Thus \(I_A \in \tau_1(\tau_2)\). Also \(e_1(F_A) \in H_A \subseteq I_A\), \(e_2(G_A) \in H_A \subseteq I_A\). Thus \(e_1(F_A) \in I_A\) or \(e_2(G_A) \in I_A\).

Hence \((U,E,\tau(\tau_2))\) is a fuzzy soft \(\tau_0\)-space. This complete proof of the theorem. \( \square \)

**Proposition 4.2.** Let \((U,E,\tau_1)\) and \((U,E,\tau_2)\) be two fuzzy soft topological spaces. If \((U,E,\tau_1)\) is fuzzy soft \(\tau_1\)-space and \(\tau_1 \subseteq \tau_2\) then \((U,E,\tau_1(\tau_2))\) is a fuzzy soft \(\tau_1\)-space.

**Proof.** We consider \(e_1(F_A)\), \(e_2(G_A)\) \(\in \tau_1\). Given \((U,E,\tau_1)\) is a fuzzy soft \(\tau_1\)-space then there exist \(H_A,K_A \in \tau_1\) such that \(e_1(F_A) \in H_A\) and \(e_1(F_A) \notin K_A\), \(e_2(G_A) \in K_A\) and \(e_2(G_A) \notin H_A\).

Let \(L_A,M_A\) be the fuzzy soft \(\tau_1-q\)-nhd of \(e_1(F_A)\) and \(e_2(G_A)\), respectively. Then \(e_1qH_A\) and \(e_2qK_A\) and \(H_A \subseteq L_A\), \(K_A \subseteq M_A\). Thus \(\mu_{e_1} + \mu_{H} > 1\) and \(\mu_{e_2} + \mu_{K} > 1\). So \(\mu_{e_1} + \mu_{H} > 1\) and \(\mu_{e_2} + \mu_{K} > 1\).

Hence \(e_1qM_A\) and \(e_2qL_A\).

Since \(\tau_1 \subseteq \tau_2\) and \(H_A,K_A \in \tau_1\), \(e_1qH_A\) and \(e_2qK_A\) and \(H_A \subseteq L_A\), \(K_A \subseteq M_A\). Thus \(L_A,M_A \in \tau_1(\tau_2)\). Also \(e_1 \notin H_A \subseteq L_A\), \(e_1 \notin K_A \subseteq M_A\), \(e_2 \notin K_A \subseteq M_A\) and \(e_2 \notin H_A \subseteq L_A\).

So \((U,E,\tau(\tau_2))\) is a fuzzy soft \(\tau_1\)-space. This completes proof. \( \square \)
Proposition 4.3. Let \((U, E, \tau_1)\) and \((U, E, \tau_2)\) be two fuzzy soft topological spaces. If \((U, E, \tau_1)\) is fuzzy soft \(\tau_2\)-space and \(\tau_1 \preceq \tau_2\), then \((U, E, \tau_1(\tau_2))\) is a fuzzy soft \(\tau_2\)-space.

Proof. We consider \(e_1(F_A), e_2(G_A)\) \(\in \tau_1\). Given \((U, E, \tau_1)\) is a fuzzy soft \(\tau_2\)-space then there exist \(H_A, K_A \in \tau_1\) such that \(e_1(F_A) \in H_A\) and \(e_2(G_A) \in K_A\) and \(H_A \cap K_A = \emptyset\). Let \(L_A, M_A\) be the fuzzy soft \(\tau_1\)-nhd of \(e_1(F_A)\) and \(e_2(G_A)\), respectively. Then \(e_1qH_A\) and \(e_2qK_A\) and \(H_A \subseteq L_A, K_A \subseteq M_A\). Thus \(\mu_{e_1} + \mu_H > 1\) and \(\mu_{e_2} + \mu_K > 1\). So \(\mu_{e_1} + \mu_L > 1\) and \(\mu_{e_2} + \mu_M > 1\). Hence \(e_1qL_A\) and \(e_1qM_A\).

Since \(\tau_1 \preceq \tau_2\) and \(H_A, K_A \in \tau_1\), \(H_A\) and \(K_A\) are the \(\tau_2\)-nhd of \(e_1(F_A)\) and \(e_2(G_A)\), respectively and \(H_A \subseteq L_A, K_A \subseteq M_A\). This implies that \(L_A, M_A \in \tau_1(\tau_2)\).

Also \(e_1 \in H_A \subseteq L_A\), \(e_2 \in K_A \subseteq M_A\) and \(H_A \cap K_A = \emptyset\). This implies that \(e_1 \in L_A, e_2 \in M_A\) and \(L_A \cap M_A = \emptyset\).

Then \((U, E, \tau_1(\tau_2))\) is a fuzzy soft \(\tau_2\)-space. The proof is complete.  

Proposition 4.4. Let \((U, E, \tau_1)\) and \((U, E, \tau_2)\) be two fuzzy soft topological spaces. If \((U, E, \tau_1)\) is fuzzy soft normal space, then the mixed fuzzy topological space \((U, E, \tau_1(\tau_2))\) is a fuzzy soft normal space.

Proof. Let \(L_A \in \tau_1(\tau_2)\), \(M_A \in \tau_1(\tau_2)\) and \(L_A \preceq M_A\). Again let \(P_A \in \tau_2\), \(R_A \in \tau_2\) and \(P_A \subseteq R_A\). Then there exists \(Q_A \in \tau_2\) such that \(P_A \subseteq Q_A \subseteq Q_A \subseteq R_A\). Thus \(M_A \in \tau_2\), \(Q_A \in \tau_1(\tau_2)\). So \(P_A \subseteq Q_A \subseteq Q_A \subseteq M_A\).

Since \(L_A \in \tau_1(\tau_2)\), \(1 - L_A \in \tau_1(\tau_2)\). Thus \(1 - L_A \in \tau_2\). Hence \(L_A \subseteq P_A\) and \(L_A \subseteq Q_A \subseteq Q_A \subseteq M_A\), i.e., the mixed fuzzy soft topological space \((U, E, \tau_1(\tau_2))\) is a fuzzy soft normal space. This completes the proof.

Proposition 4.5. Let \((U, E, \tau_1)\) and \((U, E, \tau_2)\) be two fuzzy soft topological spaces. A mixed fuzzy soft topological space \((U, E, \tau_1(\tau_2))\) is a fuzzy soft normal space if and only if two closed fuzzy soft sets \(L_A, M_A\) in \(U\) with \(L_A \subseteq 1 - M_A\) then there exist \(P_A, Q_A \in \tau_1(\tau_2)\) such that \(L_A \subseteq P_A, M_A \subseteq Q_A\) and \(P_A \subseteq 1 - Q_A\).

Proof. Let \(L_A \in \tau_1(\tau_2)\) and fuzzy soft open set \(P_A \in \tau_1(\tau_2)\) and \(L_A \subseteq P_A\). Then, since mixed fuzzy soft topological space \((U, E, \tau_1(\tau_2))\) is a fuzzy soft normal space, there exists \(Q_A \in \tau_1(\tau_2)\) such that \(L_A \subseteq Q_A \subseteq P_A\).

Let \(M_A \in \tau_1(\tau_2)\) and \(M_A = 1 - P_A\). Then \(L_A \subseteq 1 - M_A\) and \(Q_A \subseteq 1 - M_A\). Since \(Q_A \subseteq 1 - M_A\), \(P_A \subseteq 1 - Q_A\). Since \(P_A \subseteq Q_A \subseteq 1 - M_A\), \(P_A \subseteq Q_A \subseteq 1 - Q_A\). This implies that \(P_A \subseteq Q_A \subseteq 1 - Q_A\). Also \(L_A \subseteq 1 - M_A\) and \(L_A \subseteq Q_A\).

Conversely, let \(L_A, M_A \in \tau_1(\tau_2)\) and \(L_A \subseteq 1 - M_A\). Then there exist \(P_A, Q_A \in \tau_1(\tau_2)\) such that \(L_A \subseteq P_A, M_A \subseteq Q_A\) and \(P_A \subseteq 1 - Q_A\). We consider \(P_A = 1 - M_A\). Then \(L_A \subseteq Q_A \subseteq 1 - P_A \subseteq 1 - M_A = P_A\), i.e., \(L_A \subseteq Q_A \subseteq Q_A \subseteq P_A\).

Thus mixed fuzzy soft topological space \((U, E, \tau_1(\tau_2))\) is a fuzzy soft normal space. This completes the proof.

5. Application of fuzzy soft quasi-coincident in Chemistry

In this section, we applied soft quasi-coincident of fuzzy soft sets in Chemistry. Here we find bond strength of different hybridization.
In the chemistry, hybridization is the concept of mixing of orbitals on an atom having nearly equal energy, to produce entirely new orbitals which have same energy contents, identical shapes and are symmetrically disposed in space.

The mixing of the \( s \) and \( p \) orbitals to form \( sp \) hybridization. Similarly the mixing of the \( s \) and two \( p \) orbitals to form \( sp^2 \) hybridization and the mixing of the \( s \) and three \( p \) orbitals to form \( sp^3 \) hybridization.

We know that

(a) in the \( sp^3 \) hybridization 25% \( s \) character and 75% \( p \) character,
(b) in the \( sp^2 \) hybridization 33% \( s \) character and 66% \( p \) character,
(c) in the \( sp \) hybridization 50% \( s \) character and 50% \( p \) character.

Algorithms:

1. Construct fuzzy soft sets of different hybridization.
2. Find different grade value of different hybridization.
3. Adding 0.02 of same hybridization.
4. If the grade value of different hybridization is high then the bond strength of that hybridization is low i.e. if \( \mu_F + \mu_G > 1 \) then the strength of the hybridization is low i.e. the bond strength of the compound is so weak.
5. Finally we find the bond strength sequence of different hybridization.

Fuzzy soft sets of \( sp^3 \) hybridization is

\[ F_A = \{ F(e_1) = \{(s, 0.2), (p, 0.7)\} \}. \]

Fuzzy soft sets of \( sp^2 \) hybridization is

\[ G_A = \{ G(e_1) = \{(s, 0.3), (p, 0.6)\} \}. \]

Fuzzy soft sets of \( sp \) hybridization is

\[ H_A = \{ H(e_1) = \{(s, 0.5), (p, 0.5)\} \}. \]

Grade value of \( sp^3 \) hybridization, i.e., \( \mu(F_A) = |0.2 - 0.7| = 0.5 \),
Grade value of \( sp^2 \) hybridization, i.e., \( \mu(G_A) = |0.3 - 0.6| = 0.3 \),
Grade value of \( sp \) hybridization, i.e., \( \mu(H_A) = |0.5 - 0.5| = 0.0 \),
Grade value of \( sp^2-sp^3 \), i.e., \( \mu(G_A,F_A) = 0.3 + 0.5 = 0.8 \),
Grade value of \( sp^2-sp^2 \), i.e., \( \mu(G_A,G_A) = 0.3 + 0.3 + 0.02 = 0.62 \),
Grade value of \( sp-sp^3 \), i.e., \( \mu(H_A,F_A) = 0.0 + 0.5 = 0.5 \),
Grade value of \( sp^3-sp^3 \), i.e., \( \mu(F_A,F_A) = 0.5 + 0.5 + 0.02 = 1.02 > 1 \),
Grade value of \( sp-sp^3 \), i.e., \( \mu(H_A,F_A) = 0.0 + 0.3 = 0.3 \),
Grade value of \( sp-sp \), i.e., \( \mu(H_A,H_A) = 0.0 + 0.0 + 0.02 = 0.02 \).

Therefore the sequence of bond strength is

\[
sp^3 - sp^3 < sp^2 - sp^3 < sp^2 - sp^2 < sp - sp^3 < sp - sp^3 < sp - sp^2 < sp - sp .
\]

6. Conclusion

We have introduced soft quasi-coincident, soft quasi-neighborhood and mixed fuzzy soft topology which are defined over an initial universe with a fixed set of parameters. The notions of mixed fuzzy soft separation axioms are introduced and
their basic properties are investigated. In this paper, we have studied a few ideas only, it will be necessary to carry out more theoretical research to establish a general framework for the practical application.

7. Competing interests

The authors declare that they have no competing interests.

8. Authors contributions

Each of the authors contributed to each part of this work equally and read and approved the final version of the manuscript.

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