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Somewhat fuzzy δ-irresolute continuous mappings

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Abstract. The concept of somewhat fuzzy δ-irresolute continuous mapping and somewhat fuzzy irresolute δ-open mapping have been introduced and studied. Besides, some interesting properties of those mappings are given.

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1. Introduction

The concepts of fuzzy sets and fuzzy set operations were first introduced by L. A. Zadeh in his classical paper [14] in the year 1965. Thereafter the paper of C. L. Chang [2] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. The notion of continuity is of fundamental importance in almost all branches of Mathematics. Hence it is of considerable signifcance from applications view point, to formulate and study new variants of fuzzy continuity.

K. K. Azad [1] introduced the concept of fuzzy regular open sets and fuzzy regular closed sets in fuzzy topological spaces. Z. Petricevic [10] introduced the concept of fuzzy δ-open sets and fuzzy δ-closed sets in fuzzy topological spaces. The concept of δ-continuous functions in classical topology was introduced and studied by T. Noiri in [9]. The notion of fuzzy δ-continuous functions between fuzzy topological spaces has been introduced by Supriti Saha [11]. In classical topology, the class of somewhat continuous functions was introduced and studied by Karl. R. Gentry and Hughes B. Hoyle [3]. Later, the concept of "somewhat" in classical topology has been extended to fuzzy topological spaces. Somewhat fuzzy continuous functions, somewhat fuzzy open functions on fuzzy topological spaces were introduced and studied by G. Thangaraj and G. Balasubramanian in [12].

The concept of fuzzy irresolute continuous mappings on a fuzzy topological space was introduced and studied by M. N. Mukherjee and S. P. Shina in [7].

The concepts of somewhat fuzzy irresolute continuous mappings [4], somewhat fuzzy γ-irresolute continuous mappings [5] and somewhat fuzzy α-irresolute continuous mappings [6] on a fuzzy topological space are successively introduced and studied by Y.B. Im and others. The concepts of somewhat fuzzy δ-continuous functions and somewhat fuzzy δ-open functions are introduced and studied by G. Thangaraj and K Dinakaran in [13] and these papers have been motivation to develope this paper.

In this paper, the concepts of somewhat fuzzy δ-irresolute continuous mappings and somewhat fuzzy irresolute δ-open mappings on a fuzzy topological space are introduced and we characterize those mappings. Besides, some interesting properties of those mappings are also given.

2. Preliminaries

Throughout this paper, we denote μ^c with the complement of the fuzzy set μ on a nonempty set X, which is defined by $\mu^c(x) = (1-\mu)(x) = 1-\mu(x)$ for all $x \in X$. If μ is a fuzzy set on a nonempty set X and if ν is a fuzzy set on a nonempty set Y , then $\mu \times \nu$ is a fuzzy set on $X \times Y$, defined by $(\mu \times \nu)(x, y) = min(\mu(x), \nu(y))$ for every $(x, y) \in X \times Y$. Let $f : X \to Y$ be a mapping and let μ be a fuzzy set on X. Then $f(\mu)$ is a fuzzy set on Y defined by

$$f(\mu)(y) = \begin{cases} sup_{x \in f^{-1}(y)} \mu(x) & \text{if } f^{-1}(y) \neq \phi, y \in Y \\ 0 & \text{otherwise.} \end{cases}$$

Let ν be a fuzzy set on Y . Then $f^{-1}(\nu)$ is a fuzzy set on X, defined by $f^{-1}(\nu)(x) = \nu(f(x))$ for each $x \in X$. The graph $g : X \to X \times Y$ of f is defined by $g(x) = (x, f(x))$ for each $x \in X$. Then $g^{-1}(\mu \times \nu) = \mu \wedge f^{-1}(\nu)$. The product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ of mappings $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$ is defined by $(f_1 \times f_2)(x_1, x_2) = (f_1(x_1), f_2(x_2))$ for each $(x_1, x_2) \in X_1 \times X_2$ [1].

Now let X and Y be fuzzy topological spaces. We denote $Int(\mu)$ and $Cl(\mu)$ with the interior and with the closure of the fuzzy set μ on a fuzzy topological space X respectively. Then (i)$1 - Cl(\mu) = Int(1 - \mu)$ and (ii) $Cl(1 - \mu) = 1 - Int(\mu)$.

We say that a fuzzy topological space X is product related to a fuzzy topological space Y if, for fuzzy set μ on X and ν on Y , $\gamma \ngeq \mu$ and $\delta \ngeq \nu$ (in which case $(\gamma^c \times 1) \vee (1 \times \delta^c) \geq (\mu \times \nu)$) where γ is a fuzzy open set on X and δ is a fuzzy open set on Y ,then there exist a fuzzy open set γ_1 on X and a fuzzy open set δ_1 on Y such that $\gamma_1^c \geq \mu$ or $\delta_1^c \geq \nu$ and $(\gamma_1^c \times 1) \vee (1 \times \delta_1^c) = (\gamma^c \times 1) \times (1 \times \delta^c)$.

A fuzzy subset λ of a space X is called fuzzy regular open [1] (resp. fuzzy regular closed) if $\lambda = Int(Cl(\lambda))$ (resp.$\lambda = Cl(Int(\lambda))$). Now $Cl(\lambda)$ and $Int(\lambda)$ are defined as follows $Cl(\lambda) = \wedge\{\mu : \mu \geq \lambda, \mu \ is \ fuzzy \ closed \ in \ X\}$ and $Int(\lambda) = \vee\{\mu \leq \lambda, \mu \ is \ fuzzy \ open \ in \ X\}$. The fuzzy δ-interior of a fuzzy subset λ of X is the union of all fuzzy regular open sets contained in λ. A fuzzy subset λ is called fuzzy δ-open [8] if $\lambda = Int_\delta(\lambda)$. The complement of fuzzy δ-open set is called fuzzy δ-closed (i.e, $\lambda = Cl_\delta(\lambda)$).

Definition 2.1 ([11]). A mapping $f : X \to Y$ is called fuzzy δ-continuous if $f^{-1}(\nu)$ is a fuzzy δ-open set on X for any fuzzy open set ν on Y and a mapping $f : X \to Y$ is called fuzzy δ-open if $f(\mu)$ is a fuzzy δ-open set on Y for any fuzzy open set μ on X.

Definition 2.2 ([13]). A fuzzy set μ on a fuzzy topological space X is called fuzzy δ-dense if there exists no fuzzy δ-closed set ν in X such that $\mu < \nu < 1$. That is $Cl_\delta(\mu) = 1$.

Definition 2.3 ([13]). A mapping $f : X \to Y$ is called somewhat fuzzy δ-continuous if there exists a fuzzy δ-open set $\mu \neq 0_X$ on X such that $\mu \leq f^{-1}(\nu) \neq 0_X$ for any fuzzy open set ν on Y. It is clear that every somewhat fuzzy δ-continuous mapping is a somewhat fuzzy continuous mapping. But the converse is not true in general.

Definition 2.4 ([13]). A mapping $f : X \to Y$ is called somewhat fuzzy δ-open if there exists a fuzzy δ-open set $\nu \neq 0_Y$ on Y such that $\nu \leq f(\mu) \neq 0_Y$ for any fuzzy open set μ on X. It is clear that every fuzzy somewhat δ-open mapping is a somewhat fuzzy open mapping but the converse is not true in general.

Remark 2.5 ([13]). The fuzzy continuity and the fuzzy δ-continuity are independent notions.

3. Somewhat fuzzy δ-irresolute continuous mappings

In this section, we introduce a somewhat fuzzy δ-irresolute continuous mapping which are stronger than a somewhat fuzzy δ-continuous mapping. And we characterize a somewhat fuzzy δ-irrsesolute continuous mapping.

Definition 3.1. A mapping $f : X \to Y$ is called fuzzy δ-irresolute continuous if $f^{-1}(\nu)$ is a fuzzy δ-open set on X for any fuzzy δ-open set ν on Y.

Definition 3.2. A mapping $f : X \to Y$ is called somewhat fuzzy δ-irresolute continuous if there exists a fuzzy δ-open set $\mu \neq 0_X$ on X such that $\mu \leq f^{-1}(\nu) \neq 0_X$ for any fuzzy δ-open set $\nu \neq 0_Y$ on Y .

It is clear that every fuzzy δ-irresolute continuous mapping is a somewhat fuzzy δ-irresolute continuous mapping. And every somewhat fuzzy δ-irresolute continuous mapping is a fuzzy δ-continuous mapping. Also, every fuzzy δ-continuous mapping is a somewhat fuzzy δ-continuous mapping from the above defnition. But the converses are not true in general as the following examples show.

Example 3.3. Let λ_1,λ_2 and λ_3 be fuzzy sets on $X = \{a, b, c\}$ and let σ_1,σ_2 and σ_3 be fuzzy sets on $Y = \{x, y, z\}$. Then

$$\lambda_1 = \tfrac{0.3}{a} + \tfrac{0.3}{b} + \tfrac{0.3}{c},\ \lambda_2 = \tfrac{0.7}{a} + \tfrac{0.7}{b} + \tfrac{0.7}{c},\ \lambda_3 = \tfrac{0.5}{a} + \tfrac{0.5}{b} + \tfrac{0.5}{c}$$

and

$$\sigma_1 = \tfrac{0.2}{x} + \tfrac{0.3}{y} + \tfrac{0.2}{z},\ \sigma_2 = \tfrac{0.8}{x} + \tfrac{0.7}{y} + \tfrac{0.8}{z},\ \sigma_3 = \tfrac{0.5}{x} + \tfrac{0.5}{y} + \tfrac{0.5}{z}$$

are defined as follows: Consider $\tau = \{0_X, \lambda_1, \lambda_2, 1_X\}$, $\eta = \{0_Y, \sigma_1, \sigma_2, \sigma_3, 1_Y\}$. Then (X, τ) and (Y, η) are fuzzy topologies and $f : (X, \tau) \to (Y, \eta)$ defined by

$$f(a) = y,\ f(b) = y, f(c) = y.$$

Then we have $f^{-1}(\sigma_1) = \lambda_1$, $\lambda_1 \leq f^{-1}(\sigma_2) = \lambda_2$ and $\lambda_1 \leq f^{-1}(\sigma_3) = \lambda_3$. Since λ_1 is a fuzzy δ-open set on (X, τ), f is somewhat fuzzy δ-irresolute continuous. But $f^{-1}(\sigma_3) = \lambda_3$ is not fuzzy δ-open set on (X, τ). Thus f is not a fuzzy δ-irresolute continuous mapping.

Example 3.4. Let λ_1 and λ_2 be fuzzy sets on $X = \{a, b, c\}$ and let σ_1,σ_2 and σ_3 be fuzzy sets on $Y = \{x, y, z\}$. Then

$$\lambda_1 = \tfrac{0.2}{a} + \tfrac{0.2}{b} + \tfrac{0.2}{c},\ \lambda_2 = \tfrac{0.4}{a} + \tfrac{0.4}{b} + \tfrac{0.4}{c}$$

and

$$\sigma_1 = \tfrac{0.3}{x} + \tfrac{0.2}{y} + \tfrac{0.3}{z},\ \sigma_2 = \tfrac{0.4}{x} + \tfrac{0.4}{y} + \tfrac{0.4}{z},\ \sigma_3 = \tfrac{0.4}{x} + \tfrac{0.0}{y} + \tfrac{0.4}{z}$$

are defined as follows: Consider $\tau = \{0_X, \lambda_1, \lambda_1^c, 1_X\}$, $\eta = \{0_Y, \sigma_3, \sigma_3^c, 1_Y\}$. Then (X, τ) and (Y, η) are fuzzy topologies and $f : (X, \tau) \to (Y, \eta)$ defined by

$$f(a) = y, f(b) = y, f(c) = y.$$

Then we have $f^{-1}(\sigma_3) = 0_X$ and $f^{-1}({\sigma_3}^c) = 1_X$, f is fuzzy δ-continuous. But for a fuzzy δ-open set $\sigma_3 \neq 0_Y$ on Y, $f^{-1}(\sigma_3) = 0_X$. Thus f is not a somewhat fuzzy δ-irresolute continuous mapping.

Example 3.5. In Example 3.3, for a fuzzy open set on Y, $f^{-1}(\sigma_1) = \lambda_1$, $\lambda_1 \leq f^{-1}(\sigma_2) = \lambda_2$ and $\lambda_1 \leq f^{-1}(\sigma_3) = \lambda_3$. Since λ_1 is a fuzzy δ-open set on X, f is somewhat fuzzyδ-continuous. But $f^{-1}(\sigma_3) = \lambda_3$ is not a fuzzy δ-open set on X. Thus f is not a fuzzy δ-continuous mapping.

Theorem 3.6. *Let $f : X \to Y$ be a mapping. Then the following are equivalent :*

(1) *f is somewhat fuzzy δ-irresolute continuous.*

(2) *If ν is a fuzzy δ-closed set of Y such that $f^{-1}(\nu) \neq 1_X$ then there exists a fuzzy δ-closed set $\mu \neq 1_X$ of X such that $f^{-1}(\nu) \leq \mu$.*

(3) *If μ is a fuzzy δ-dense set on X, then $f(\mu)$ is a fuzzy δ-dense set on Y.*

Proof. (1)⇒(2): Let ν be a fuzzy δ-closed set on Y such that $f^{-1}(\nu) \neq 1_X$. Then ν^c is a fuzzy δ-open set on Y and $f^{-1}(\nu^c) = f^{-1}(\nu)^c \neq 0_X$. Since f is somewhat fuzzy δ-irresolute continuous, there exists a fuzzy δ-open set $\lambda \neq 0_X$ on X such that $\lambda \leq f^{-1}(\nu)^c$. Let $\mu = \lambda^c$. Then $\mu \neq 1_X$ is fuzzy δ-closed such that

$$f^{-1}(\nu) = 1 - f^{-1}(\nu)^c \leq 1 - \mu^c = \mu.$$

(2)⇒(3): Let μ be a fuzzy δ-dense set on X and suppose $f(\mu)$ is not fuzzy δ-dense on Y . Then there exists a fuzzy δ-closed set ν on Y such that $f(\mu) < \nu < 1$. Since $\nu < 1$ and ,$f^{-1}(\nu) \neq 1_X$, there exists a fuzzy δ-closed set $\delta \neq 1_X$ such that

$$\mu \leq f^{-1}(f(\mu)) < f^{-1}(\nu) \leq \delta.$$

This contradicts to the assumption that μ is a fuzzy δ-dense set on X. Thus $f(\mu)$ is a fuzzy δ-dense set on Y .

(3)⇒(1): Let $\nu \neq 0_Y$ be a fuzzy δ-open set on Y and $f^{-1}(\nu) \neq 0_X$. Suppose there exists no fuzzy δ-open $\mu \neq 0_X$ on X such that $\mu \leq f^{-1}(\nu)$. Then $(f^{-1}(\nu)^c)$ is a fuzzy set on X such that there is no fuzzy δ-closed set δ on X with $(f^{-1}(\nu)^c) < \delta < 1$. In fact, if there exists a fuzzy δ-open set δ^c such that $\delta^c \leq f^{-1}(\nu)$, then it is a contradiction. Thus $(f^{-1}(\nu))^c$ is a fuzzy δ-dense set on X. So $f((f^{-1}(\nu))^c)$ is a fuzzy δ-dense set on Y . But $f((f^{-1}(\nu))^c) = f((f^{-1}(\nu))^c) \neq \nu^c < 1$. This contradicts to the fact that $f((f^{-1}(\nu))^c)$ is fuzzy δ-dense on Y . Hence there exists a δ-open set $\mu \neq 0_X$ on X such that $\mu \leq f^{-1}(\nu)$. Consequently, f is somewhat fuzzy δ-irresolute continuous. □

Theorem 3.7. *Let X_1 be product related to X_2 and let Y_1 be product related to Y_2. Then the product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ of somewhat fuzzy δ-irresolute continuous mappings $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$ is also somewhat fuzzy δ-irresolute continuous.*

Proof. Let $\lambda = \vee_{i,j}(\mu_i \times \nu_j)$ be a fuzzy δ-open set on $Y_1 \times Y_2$ where $\mu_i \neq 0_{Y_1}$ and $\nu_j \neq 0_{Y_2}$ are fuzzy δ-open sets on Y_1 and Y_2 respectively. Then

$$(f_1 \times f_2)^{-1}(\lambda) = \bigvee\nolimits_{i,j}(f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j)).$$

Since f_1 is somewhat fuzzy δ-irresolute continuous, there exists a fuzzy δ-open set $\delta_i \neq 0_{X_1}$ such that $\delta_i \leq f_1^{-1}(\mu_i) \neq 0_{X_1}$. And, since f_2 is somewhat fuzzy δ-irresolute continuous, there exists a fuzzy δ-open set $\eta_j \neq 0_{X_2}$ such that $\eta_j \leq f_2^{-1}(\nu_j) \neq 0_{X_2}$. Now $\delta_i \times \eta_j \leq f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j) = (f_1 \times f_2)^{-1}(\mu_i \times \nu_j)$ and $\delta_i \times \eta_j \neq 0_{X_1} \times 0_{X_2}$ is a fuzzy δ-open set on $X_1 \times X_2$. Thus $\bigvee_{i,j}(\delta_i \times \eta_j) \neq 0_{X_1} \times 0_{X_2}$ is a fuzzy δ-open set on $X_1 \times X_2$ such that

$$\begin{aligned}\bigvee_{i,j}(\delta_i \times \eta_j) &\leq \bigvee_{i,j}(f_1^{-1}(\mu_i) \times f_2^{-1}(\mu_i)) \\ &= (f_1 \times f_2)^{-1}(\bigvee_{i,j}(\mu_i \times \nu_j)) \\ &= (f_1 \times f_2)^{-1}(\lambda) \neq 0_{X_1 \times X_2}.\end{aligned}$$

So, $f_1 \times f_2$ is somewhat fuzzy δ-irresolute continuous. □

Theorem 3.8. *Let $f : X \to Y$ be a mapping. If the graph $g : X \to X \times Y$ of f is a somewhat fuzzy δ-irresolut continuous mapping, then f is also somewhat fuzzy δ-irresolute continuous.*

Proof. Let ν be a fuzzy δ-open set on Y. Then $f^{-1}(\nu) = 1 \wedge f^{-1}(\nu) = g^{-1}(1 \times \nu)$. Since g is somewhat fuzzy δ-irresolute continuous and $(1 \times \nu)$ is a fuzzy δ-open set on $X \times Y$, there exists a fuzzy δ-open set $\mu \neq 0_X$ on X such that

$$\mu \leq g^{-1}(1 \times \nu) = f^{-1}(\nu) \neq 0_X.$$

Thus, f is somewhat fuzzy δ-irresolute continuous. □

4. Somewhat fuzzy irresolute δ-open mappings

In this section, we introduce a somewhat fuzzy irresolute δ-open mapping which are stronger than a somewhat fuzzy δ-open mapping. And we characterize a somewhat fuzzy irresolute δ-open mapping.

Definition 4.1. A mapping $f : X \to Y$ is called fuzzy irresolute δ-open if $f(\mu)$ is a fuzzy δ-open set on Y for any fuzzy δ-open set μ on X.

Definition 4.2. A fuzzy mapping $f : X \to Y$ is called somewhat fuzzy irresolute δ-open if there exists a fuzzy δ-open set $\nu \neq 0_Y$ on Y such that $\nu \leq f(\mu) \neq 0_Y$ for any fuzzy δ-open set $\mu \neq 0_X$ on X.

It is clear that every fuzzy irresolute δ-open mapping is a somewhat fuzzy irresolute δ-open mapping. And every somewhat fuzzy irresolute δ-open mapping is a fuzzy δ-open mapping. Also, every fuzzy δ-open mapping is a somewhat fuzzy δ-open mapping from the above definition. But the converses are not true in general as the following examples show.

Example 4.3. Let λ_1 and λ_2 be fuzzy sets on $X = \{a, b, c\}$ and let σ_1 and σ_2 be fuzzy sets on $Y = \{x, y, z\}$. Then

$$\lambda_1 = \tfrac{0.3}{a} + \tfrac{0.3}{b} + \tfrac{0.3}{c},\ \lambda_2 = \tfrac{0.7}{a} + \tfrac{0.7}{b} + \tfrac{0.7}{c}$$

and

$$\sigma_1 = \tfrac{0.0}{x} + \tfrac{0.3}{y} + \tfrac{0.0}{z},\ \sigma_2 = \tfrac{0.0}{x} + \tfrac{0.7}{y} + \tfrac{0.0}{z}$$

are defined as follows: Consider $\tau = \{0_X, \lambda_1, \lambda_1^c, 1_X\}$, $\eta = \{0_Y, \sigma_1, \sigma_1^c, 1_Y\}$. Then (X, τ) and (Y, η) are fuzzy topologies and $f : (X, \tau) \to (Y, \eta)$ defined by

$$f(a) = y, f(b) = y, f(c) = y.$$

Then we have $\sigma_1 \leq f(\lambda_1) = \sigma_1$ and $\sigma_1 \leq f(\lambda_2) = \sigma_2$. Since σ_1 is a fuzzy δ-open set on (Y, η), f is somewhat fuzzy irresolute δ-open. But for a fuzzy δ-open set λ_2 on X ,$f(\lambda_2) = \sigma_2$ is not a fuzzy δ-open set on (Y, η). Thus f is not a fuzzy irresolute δ-open mapping.

Example 4.4. Let λ_1,λ_2 and λ_3 be fuzzy sets on $X = \{a, b, c\}$ and let σ_1 and σ_2 be fuzzy sets on $Y = \{x, y, z\}$. Then

$$\lambda_1 = \tfrac{0.3}{a} + \tfrac{0.0}{b} + \tfrac{0.3}{c},\ \lambda_2 = \tfrac{0.4}{a} + \tfrac{0.0}{b} + \tfrac{0.4}{c},\ \lambda_3 = \tfrac{0.5}{a} + \tfrac{0.5}{b} + \tfrac{0.4}{c}$$

and

$$\sigma_1 = \tfrac{0.0}{x} + \tfrac{0.4}{y} + \tfrac{0.0}{z},\ \sigma_2 = \tfrac{0.5}{x} + \tfrac{0.5}{y} + \tfrac{0.5}{z}$$

are defined as follows: Consider $\tau = \{0_X, \lambda_1, \lambda_1^c, 1_X\}$, $\eta = \{0_Y, \sigma_2, 1_Y\}$. Then (X, τ) and (Y, η) are fuzzy topologies and $f : (X, \tau) \to (Y, \eta)$ defined by

$$f(a) = y, f(b) = y, f(c) = y.$$

Since $f(\lambda_1) = 0_Y$, $f(\lambda_1^c) = 1_Y$ and $f(\lambda_3) = \sigma_2$ are fuzzy δ-open sets on (Y, η), f is fuzzy δ-open. But for a fuzzy δ-open set λ_1, $f(\lambda_1) = 0_Y$. Thus f is not a somewhat fuzzy irresolute δ-open mapping.

Example 4.5. In Example 4.3, we have $\sigma_1 \leq f(\lambda_1) = \sigma_1$ and $\sigma_1 \leq f(\lambda_2) = \sigma_2$. Since σ_1 is a fuzzy δ-open set on X, f is somewhat fuzzy δ-open mapping. But $f(\lambda_2) = \sigma_2$ is not a fuzzy δ-open set on (Y, η). Thus f is not a fuzzy δ-open mapping.

Theorem 4.6. *Let $f : X \to Y$ be a bijection. Then the following are equivalent :*

(1) *f is somewhat fuzzy irresolute δ-open.*

(2) *If μ is a fuzzy δ-closed set on X such that $f(\mu) \neq 1_Y$, then there exists a fuzzy δ-closed set $\nu \neq 1_Y$ on Y such that $f(\mu) < \nu$.*

Proof. (1)⇒(2): Let μ be a fuzzy δ-closed set on X such that $f(\mu) \neq 1_Y$. Since f is bijective and μ^c is a fuzzy δ-open set on X, $f(\mu^c) = (f(\mu))^c \neq 0_Y$. And, since f is somewhat fuzzy irresolute δ-open, there exists a fuzzy δ-open set $\delta \neq 0_Y$ on Y such that $\delta < f(\mu^c) = (f(\mu))^c$. Consequently, $f(\mu) < \delta^c = \nu \neq 1_Y$ and ν is a fuzzy δ-closed set on Y .

(2)⇒(1): Let μ be a fuzzy δ-open set on X such that $f(\mu) \neq 0_Y$. Then μ^c is a fuzzy δ-closed set on X and $f(\mu^c) \neq 1_Y$. Thus there exists a fuzzy δ-closed set $\nu \neq 1_Y$ on Y such that $f(\nu^c) < \nu$. Since f is bijective, $f(\mu^c) = (f(\mu))^c < \nu$. So $\nu^c < f(\mu)$ and $\nu^c \neq 0_X$ is a fuzzy δ-open set on Y. Hence, f is somewhat fuzzy irresolute δ-open. □

Theorem 4.7. *Let $f : X \to Y$ be a surjection. Then the following are equivalent :*
(1) *f is somewhat fuzzy irresolute δ-open.*
(2) *If ν is a fuzzy δ-dense set on Y , then $f^{-1}(\nu)$ is a fuzzy δ-dense set on X.*

Proof. (1)⇒(2): Let ν be a fuzzy δ-dense set on Y . Suppose $f^{-1}(\nu)$ is not fuzzy δ-dense on X. Then there exists a fuzzy δ-closed set μ on X such that $f^{-1}(\nu) < \mu < 1$. Since f is somewhat fuzzy irresolute δ-open and μ^c is a fuzzy δ-open set on X, there exists a fuzzy δ-open set $\delta \neq 0_Y$ on Y such that $\delta \leq f(\text{Int}\mu^c) \leq f(\mu^c)$. Since f is surjective, $\delta \leq f(\mu^c) < f(f^{-1}(\nu^c)) = \nu^c$. Thus there exists a fuzzy δ-closed set δ^c on Y such that $\nu < \delta^c < 1$. This is a contradiction. So $f^{-1}(\nu)$ is fuzzy δ-dense on X.

(2)⇒(1): Let μ be a fuzzy δ-open set on X and $f(\mu) \neq 0_Y$. Suppose there exists no fuzzy δ-open $\nu \neq 0_Y$ on Y such that $\nu \leq f(\mu)$. Then $(f(\mu))^c$ is a fuzzy set on Y such that there exists no fuzzy δ-closed set δ on Y with $(f(\mu))^c < \delta < 1$. This means that $(f(\mu))^c$ is fuzzy δ-dense on Y . Thus $f^{-1}((f(\mu))^c)$ is fuzzy δ-dense on X. But $f^{-1}((f(\mu))^c) = (f^{-1}(f(\mu)))^c \leq \mu^c < 1$. This contradicts to the fact that $f^{-1}(f(\mu))^c$ is fuzzy δ-dense on X.SoHence there exists a fuzzy δ-open set $\nu \neq 0_Y$ on Y such that $\nu \leq f(\mu)$.Hence, f is somewhat fuzzy irresolute δ-open.

□

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