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# On neutrosophic soft function

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Abstract. In this paper, the cartesian product and the relations on neutrosophic soft sets have been defined in a new approach. Some properties of this concept have been discussed and verified with suitable real life examples. The neutrosophic soft composition has been defined and verified with the help of example. Then, some basic properties to it have been established. After that the concept of neutrosophic soft function along with some of it's basic properties have been introduced and verified by suitable examples. Injective, surjective, bijective, constant and identity neutrosophic soft functions have been defined. Finally, properties of inverse neutrosophic soft function have been discussed with proper example.

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Keywords: Neutrosophic soft set(NSS), Neutrosophic soft relations, Neutrosophic soft composition, Neutrosophic soft function, Inverse neutrosophic soft function.

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## 1. Introduction

The theory of fuzzy set was proposed by Zadeh [20] in 1965 to handle the various uncertainties in many real applications. Traditional mathematical tools like theory of probability, fuzzy set, intuitionistic fuzzy set, theory of evidence etc. have some difficulties to solve uncertain problems due to lack of adequate parametrization tools. After that, Smarandache[18] introduced the theory of Neutrosophic set (NS) that is the generalization of many theories e.g fuzzy set, intuitionistic fuzzy set[1] etc. The neutrosophic logic includes the information about the percentage of truth, indeterminacy and falsity grade in several real world problem in law, medicine, engineering, management, industrial, IT sector etc. which is not available in intuitionistic fuzzy set theory.

In 1999, Molodtsov[16] proposed the novel concept of soft set theory which is free from the parametrization inadequacy syndrome of different theories dealing with uncertainty. In soft set theory, the problem of setting the membership function among other related problems simply does not arise. This makes the theory very convenient and easy to apply in practice. In accordance of this, Maji et.al.[8, 9, 10] had given a view on fuzzy soft sets and intuitionistic fuzzy soft sets. Dinda and Samanta[5] introduced a concept on intuitionistic fuzzy soft relations. Mitra Basu et. al. [11, 12, 13, 15, 14] had introduced the concept of different operations based on fuzzy and intuitionistic fuzzy soft sets. Later, Maji[7] introduced a combined concept Neutrosophic soft set (NSS) which is developed and generalised by Deli and Broumi[4], Broumi and Smarandache[3], Sahin and Kucuk[17], Boumi[2], Wang *et. al.* [19], Maji[6] and others in different times. Presently, work on this NSS theory is progressing rapidly in different branches of Mathematics.

In this paper, the neutrosophic soft relations have been introduced in a new direction. Here also the concept of neutrosophic soft function along with some of it's basic properties has been introduced.

The organisation of this paper is as follows. Section 2 reveals some preliminary definitions of NSS which will be used in rest of this paper. In section 3, neutrosophic soft relations are defined along with some properties. Section 4 deals with the composition of neutrosophic soft relations. Then, the concept of neutrosophic soft function has been introduced along with some basic properties in section 5. Finally, section 6 presents the inverse neutrosophic soft function.

## 2. Preliminaries

For the sake of completeness, some basic definitions related to neutrosophic soft set theory are cited here.

**Definition 2.1** ([18]). A neutrosophic set (NS) on the universe of discourse U is defined as: $A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in U\}$ ,
where $T, I, F : U \to ]^-0, 1^+[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

From philosophical point of view, the neutrosophic set (NS) takes the value from real standard or nonstandard subsets of $]^-0, 1^+[$. But in real life application in scientific and engineering problems, it is difficult to use NS with value from real standard or nonstandard subset of $]^-0, 1^+[$. Hence we consider the NS which takes the value from the subset of [0,1].

**Definition 2.2** ([16]). Let U be an initial universe set and E be a set of parameters. Let P(U)denote the power set of U. Then for $A \subseteq E$, a pair $(F, A)$ is called a soft set over U, where $F : A \to P(U)$ is a mapping.

**Definition 2.3** ([7]). Let U be an initial universe set and E be a set of parameters. Let P(U) denote the set of all NS of U. Then for $A \subseteq E$, a pair $(F, A)$ is called a NSS over U, where $F : A \to P(U)$ is a mapping.

**Example 2.4.** Let U = $\{h_1, h_2, h_3\}$ be a set of houses and A= $\{e_1$(beautiful)$, e_2$(wooden), $e_3$(costly)$\}$ be a set of parameters such that $(F, A)$ refers the nature of houses. Let

$$F(e_1) = \{< h_1, (0.5, 0.6, 0.3) >, < h_2, (0.4, 0.7, 0.6) >, < h_3, (0.6, 0.2, 0.3) >\},$$
$$F(e_2) = \{< h_1, (0.6, 0.3, 0.5) >, < h_2, (0.7, 0.4, 0.3) >, < h_3, (0.8, 0.1, 0.2) >\},$$
$$F(e_3) = \{< h_1, (0.7, 0.4, 0.3) >, < h_2, (0.6, 0.7, 0.2) >, < h_3, (0.7, 0.2, 0.5) >\}.$$

Then $(F, A) = \{[e_1, F(e_1)], [e_2, F(e_2)], [e_3, F(e_3)]\}$ is a NSS over U describing the nature of houses. The tabular representation of the NSS $(F, A)$ is as :

| | $F(e_1)$ | $F(e_2)$ | $F(e_3)$ |
|---|---|---|---|
| $h_1$ | (0.5,0.6,0.3) | (0.6,0.3,0.5) | (0.7,0.4,0.3) |
| $h_2$ | (0.4,0.7,0.6) | (0.7,0.4,0.3) | (0.6,0.7,0.2) |
| $h_3$ | (0.6,0.2,0.3) | (0.8,0.1,0.2) | (0.7,0.2,0.5) |

Table 1 : Tabular form of NSS $(F, A)$

**Definition 2.5** ([7])**.** Let $(F, A)$ and $(G, B)$ be two NSSs over the common universe U. Then $(F, A)$ is said to be the neutrosophic soft subset of (G,B) if $A \subseteq B$ and $T_{F(e)}(x) \leq T_{G(e)}(x)$; $I_{F(e)}(x) \leq I_{G(e)}(x)$; $F_{F(e)}(x) \geq F_{G(e)}(x)$; $\forall e \in A$, $x \in U$.
In that case, we write $(F, A) \subseteq (G, B)$ and then (G,B) is the neutrosophic soft superset of (F,A).

**Definition 2.6** ([7])**.** Let $(F, A)$ and $(G, B)$ be two NSSs over the common universe U. Then the union of $(F, A)$ and $(G, B)$, denoted by $(F, A) \cup (G, B)$, is defined as $(F, A) \cup (G, B) = (K, C)$, where $C = A \cup B$ with, $\forall x \in U$,

$$T_{K(e)}(x) = \begin{cases} T_{F(e)}(x), \text{ if } e \in A - B \\ T_{G(e)}(x), \text{ if } e \in B - A \\ max(T_{F(e)}(x), T_{G(e)}(x)), \text{ if } e \in A \cap B, \end{cases}$$

$$I_{K(e)}(x) = \begin{cases} I_{F(e)}(x), \text{ if } e \in A - B \\ I_{G(e)}(x), \text{ if } e \in B - A \\ \frac{I_{F(e)}(x) + I_{G(e)}(x)}{2}, \text{ if } e \in A \cap B, \end{cases}$$

$$F_{K(e)}(x) = \begin{cases} F_{F(e)}(x), \text{ if } e \in A - B \\ F_{G(e)}(x), \text{ if } e \in B - A \\ min(F_{F(e)}(x), F_{G(e)}(x)), \text{ if } e \in A \cap B. \end{cases}$$

**Definition 2.7** ([7])**.** Let $(F, A)$ and $(G, B)$ be two NSSs over the common universe U. Then the intersection of $(F, A)$ and $(G, B)$, denoted by $(F, A) \cap (G, B)$ is defined by $(F, A) \cap (G, B) = (K, C)$, where $C = A \cap B$ with, $\forall e \in C$, $\forall x \in U$,

$$T_{K(e)}(x) = min(T_{F(e)}(x), T_{G(e)}(x)),$$
$$I_{K(e)}(x) = \frac{I_{F(e)}(x) + I_{G(e)}(x)}{2},$$
$$F_{K(e)}(x) = max(F_{F(e)}(x), F_{G(e)}(x)).$$

## 3. Relations on NSS

The notion of neutrosophic soft relations was first given by Deli and Broumi[4]. In this section, the cartesian product and the relations on NSS are modified in a new direction to establish some properties.

**Definition 3.1.** Let $(F, A)$ and $(G, B)$ be two NSSs over the common universe U. Then their cartesian product is another NSS $(K, C) = (F, A) \times (G, B)$, where $C = A \times B$ and $K(a, b) = F(a) \times G(b)$. The truth,indeterminacy and falsity membership of (K,C) are given by $\forall a \in A$, $\forall b \in B$, $\forall x \in U$,

$$T_{K(a,b)}(x) = min(T_{F(a)}(x), T_{G(b)}(x)),$$
$$I_{K(a,b)}(x) = I_{F(a)}(x).I_{G(b)}(x),$$
$$F_{K(a,b)}(x) = max(F_{F(a)}(x), F_{G(b)}(x)).$$

This definition can be extended for more than two NSSs.

**Example 3.2.** Let U= $\{x_1, x_2, x_3\}$ be a universe. Suppose A= $\{a_1, a_2, a_3\}$ and B= $\{b_1, b_2\}$ are two parametric sets.
Let the tabular representation of $(F, A)$ be :

| | $F(a_1)$ | $F(a_2)$ | $F(a_3)$ |
|---|---|---|---|
| $x_1$ | (0.7,0.6,0.7) | (0.5,0.7,0.8) | (0.3,0.5,0.6) |
| $x_2$ | (0.4,0.2,0.8) | (0.5,0.9,0.3) | (0.7,0.5,0.4) |
| $x_3$ | (0.9,0.1,0.5) | (0.5,0.6,0.8) | (0.8,0.6,0.9) |

Table 2: Tabular form of NSS $(F, A)$

Let the tabular representation of $(G, B)$ be :

| | $G(b_1)$ | $G(b_2)$ |
|---|---|---|
| $x_1$ | (0.8,0.9,0.6) | (0.3,0.3,0.6) |
| $x_2$ | (0.7,0.8,0.8) | (0.6,0.2,0.8) |
| $x_3$ | (0.5,0.6,0.4) | (0.4,0.7,0.5) |

Table- 3: Tabular form of NSS $(G, B)$

Then the tabular representation of $(K, C) = (F, A) \times (G, B)$ is given by the following tables.

| | $K(a_1, b_1)$ | $K(a_1, b_2)$ | $K(a_2, b_1)$ |
|---|---|---|---|
| $x_1$ | (0.7,0.54,0.7) | (0.3,0.18,0.7) | (0.5,0.63,0.8) |
| $x_2$ | (0.4,0.16,0.8) | (0.4,0.04,0.8) | (0.5,0.72,0.8) |
| $x_3$ | (0.5,0.06,0.5) | (0.4,0.07,0.5) | (0.5,0.36,0.8) |

| | $K(a_2, b_2)$ | $K(a_3, b_1)$ | $K(a_3, b_2)$ |
|---|---|---|---|
| $x_1$ | (0.3,0.21,0.8) | (0.3,0.45,0.6) | (0.3,0.15,0.6) |
| $x_2$ | (0.5,0.18,0.8) | (0.7, 0.4,0.8) | (0.6,0.1,0.8) |
| $x_3$ | (0.4,0.42,0.8) | (0.5,0.36,0.9) | (0.4,0.42,0.9) |

Table 4: Tabular form of NSS $(K, C)$

**Definition 3.3.** A neutrosophic soft relation $R$ between two NSSs $(F, A)$ and $(G, B)$ over the common universe U is the neutrosophic soft subset of $(F, A) \times (G, B)$. Clearly, it is another NSS $(R, C)$ where $C \subseteq A \times B$ and $R(a, b) = F(a) \times G(b)$ for $(a, b) \in C$.

**Example 3.4.** Considering the example 3.2, we define a neutrosophic soft relation $(R, C)$ as follows :

| | $R(a_1, b_2)$ | $R(a_2, b_2)$ | $R(a_3, b_1)$ |
|---|---|---|---|
| $x_1$ | (0.3,0.18,0.7) | (0.3,0.21,0.8) | (0.3,0.45,0.6) |
| $x_2$ | (0.4,0.04,0.8) | (0.5,0.18,0.8) | (0.7, 0.4,0.8) |
| $x_3$ | (0.4,0.07,0.5) | (0.4,0.42,0.8) | (0.5,0.36,0.9) |

Table 5: Tabular form of neutrosophic soft relation R

**Definition 3.5.** Let $(F, A)$ and $(G, B)$ be two NSSs over the common universe U and let $R$ be a neutrosophic soft relation from $(F, A)$ to $(G, B)$. Then $R^{-1}$ is the inverse relation from $(G, B)$ to $(F, A)$ and is defined as $R^{-1}(b, a) = R(a, b)$ for $(a, b) \in A \times B$.

**Example 3.6.** From example 3.4, we get $R^{-1}$ as :

| | $R^{-1}(b_2, a_1)$ | $R^{-1}(b_2, a_2)$ | $R^{-1}(b_1, a_3)$ |
|---|---|---|---|
| $x_1$ | (0.3,0.18,0.7) | (0.3,0.21,0.8) | (0.3,0.45,0.6) |
| $x_2$ | (0.4,0.04,0.8) | (0.5,0.18,0.8) | (0.7,0.4,0.8) |
| $x_3$ | (0.4,0.07,0.5) | (0.4,0.42,0.8) | (0.5,0.36,0.9) |

Table 6: Tabular form of $R^{-1}$

**Theorem 3.7.** *If R be a neutrosophic soft relation from (F,A) to (G,B) over U, then $R^{-1}$ is a neutrosophic soft relation from (G,B) to (F,A).*

*Proof.* For $a \in A, b \in B, x \in U$, we have :

$T_{R^{-1}(b,a)}(x) = min(T_{G(b)}(x), T_{F(a)}(x)) = min(T_{F(a)}(x), T_{G(b)}(x)) = T_{R(a,b)}(x)$,

$I_{R^{-1}(b,a)}(x) = I_{G(b)}(x).I_{F(a)}(x) = I_{F(a)}(x).I_{G(b)}(x) = I_{R(a,b)}(x)$,

and

$F_{R^{-1}(b,a)}(x) = max(F_{G(b)}(x), F_{F(a)}(x)) = max(F_{F(a)}(x), F_{G(b)}(x)) = F_{R(a,b)}(x)$.

This completes the theorem. □

**Theorem 3.8.** *Let $R_1$ and $R_2$ be two neutrosophic soft relations. Then*

(1) $(R_1^{-1})^{-1} = R_1$.

(2) *If $R_1 \subseteq R_2$, then $R_1^{-1} \subseteq R_2^{-1}$.*

*Proof.* Straightforward. □

## 4. Composition of neutrosophic soft relations

**Definition 4.1.** Let $(F, A)$, $(G, B)$ and $(H, C)$ be three NSSs over the common universe U so that $Z \subseteq (F, A) \times (G, B)$ and $R \subseteq (G, B) \times (H, C)$ be two neutrosophic soft relations. Then their composition (briefly, neutrosophic soft composition) is denoted by $RoZ$ and is defined as

$RoZ = \{< x, T_{(RoZ)(a,c)}(x), I_{(RoZ)(a,c)}(x), F_{(RoZ)(a,c)}(x) >_{x \in U}: (a, c) \in A \times C\}$,

where

$$T_{(RoZ)(a,c)}(x) = max\{min_{b \in B}(T_{Z(a,b)}(x), T_{R(b,c)}(x))\},$$
$$I_{(RoZ)(a,c)}(x) = max_{b \in B}\{I_{Z(a,b)}(x).I_{R(b,c)}(x)\},$$
$$F_{(RoZ)(a,c)}(x) = min\{max_{b \in B}(F_{Z(a,b)}(x), F_{R(b,c)}(x))\}.$$

**Example 4.2.** Let U=$\{s_1, s_2, s_3\}$ be a set of students. We consider three NSSs $(F, A)$, $(G, B)$ and $(H, C)$ over U. Suppose

$(F, A)$ describes 'the universities from which students acquire degree',

$(G, B)$ describes 'the degrees, students may gain',

$(H, C)$ describes 'the professions, students may be engaged after acquiring degrees'.

Let

A= {Havard, USA($a_1$); J. N. U, Delhi($a_2$); Presidency, Kolkata($a_3$); Oxford, London($a_4$)},

B= {MSC($b_1$); BDS($b_2$); MBA($b_3$); B. Tech($b_4$)},

and

C= {Professor in IIM($c_1$); Architect($c_2$); Lecturer in general degree college($c_3$); Doctor($c_4$)}.

We define two neutrosophic soft relations $Z \subseteq (F, A) \times (G, B)$ and $R \subseteq (G, B) \times (H, C)$ as $Z$ : Post graduate student from foreign university; $R$ : Post graduate student engaged in teaching profession. Then $RoZ$ : Student from foreign university engaged in teaching profession.

Let the tabular representations of $Z$ and $R$ be respectively :

| | $Z(a_1, b_1)$ | $Z(a_1, b_3)$ | $Z(a_4, b_1)$ | $Z(a_4, b_3)$ |
|---|---|---|---|---|
| $s_1$ | (0.7,0.5,0.2) | (0.6,0.7,0.3) | (0.5,0.5,0.6) | (0.8,0.3,0.4) |
| $s_2$ | (0.5,0.7,0.5) | (0.3,0.8,0.6) | (0.6,0.6,0.4) | (0.5,0.4,0.6) |
| $s_3$ | (0.8,0.2,0.2) | (0.9,0.4,0.1) | (0.7,0.6,0.5) | (0.8,0.7,0.2) |

Table 7: Tabular form of neutrosophic soft relation $Z$

| | $R(b_1, c_1)$ | $R(b_1, c_3)$ | $R(b_3, c_1)$ | $R(b_3, c_3)$ |
|---|---|---|---|---|
| $s_1$ | (0.2,0.6,0.7) | (0.6,0.6,0.5) | (0.3,0.7,0.7) | (0.7,0.3,0.3) |
| $s_2$ | (0.8,0.4,0.2) | (0.5,0.6,0.7) | (0.4,0.4,0.7) | (0.9,0.2,0.1) |
| $s_3$ | (0.7,0.6,0.3) | (0.8,0.6,0.3) | (0.7,0.7,0.3) | (0.4,0.7,0.5) |

Table 8: Tabular form of neutrosophic soft relation $R$

Then $RoZ$ (post graduate student in teaching profession) is given by the following table.

| | $(RoZ)(a_1, c_1)$ | $(RoZ)(a_1, c_3)$ | $(RoZ)(a_4, c_1)$ | $(RoZ)(a_4, c_3)$ |
|---|---|---|---|---|
| $s_1$ | (0.3,0.49,0.7) | (0.6,0.3,0.3) | (0.3,0.3,0.7) | (0.7,0.3,0.4) |
| $s_2$ | (0.5,0.32,0.5) | (0.5,0.42,0.6) | (0.6,0.24,0.4) | (0.5,0.36,0.6) |
| $s_3$ | (0.7,0.28,0.3) | (0.8,0.28,0.3) | (0.7,0.49,0.3) | (0.7,0.49,0.5) |

Table 9: Tabular form of neutrosophic soft composition $RoZ$

Here the scheme of $RoZ \subseteq (F, A) \times (H, C)$ is given as following :

$(< (a_1, b_1), (b_1, c_1) >; < (a_1, b_3), (b_3, c_1) >) \Rightarrow (a_1, c_1)$,
$(< (a_1, b_1), (b_1, c_3) >; < (a_1, b_3), (b_3, c_3) >) \Rightarrow (a_1, c_3)$,
$(< (a_4, b_1), (b_1, c_1) >; < (a_4, b_3), (b_3, c_1) >) \Rightarrow (a_4, c_1)$,
$(< (a_4, b_1), (b_1, c_3) >; < (a_4, b_3), (b_3, c_3) >) \Rightarrow (a_4, c_3)$.

Thus we provide one calculation in convenience of the table of $RoZ$.

$$T_{(RoZ)(a_1,c_1)}(s_1)$$
$$= max\{min(T_{Z(a_1,b_1)}(s_1), T_{R(b_1,c_1)}(s_1)), min(T_{Z(a_1,b_3)}(s_1), T_{R(b_3,c_1)}(s_1))\}$$
$$= max\{min(0.7, 0.2), min(0.6, 0.3)\}$$
$$= max(0.2, 0.3)$$
$$= 0.3,$$
$$I_{(RoZ)(a_1,c_1)}(s_1)$$
$$= max\{(I_{Z(a_1,b_1)}(s_1).I_{R(b_1,c_1)}(s_1)), (I_{Z(a_1,b_3)}(s_1).I_{R(b_3,c_1)}(s_1))\}$$
$$= max\{(0.5).(0.6), (0.7).(0.7)\}$$
$$= max(0.30, 0.49)$$
$$= 0.49,$$
$$F_{(RoZ)(a_1,c_1)}(s_1)$$
$$= min\{max(F_{Z(a_1,b_1)}(s_1), F_{R(b_1,c_1)}(s_1)), max(F_{Z(a_1,b_3)}(s_1), F_{R(b_3,c_1)}(s_1))\}$$
$$= min\{max(0.2, 0.7), max(0.3, 0.7)\}$$
$$= min(0.7, 0.7)$$
$$= 0.7.$$

**Theorem 4.3.** *Let (F,A), (G,B) and (H,C) be three NSSs over the universal set U. If Z⊆ (F,A) × (G,B) and R⊆ (G,B) × (H,C) be two neutrosophic soft relations then RoZ ⊆ (F,A) × (H,C).*

*Proof.* The structure of $Z, R, RoZ$ are given in definition. Now, for $(a,c) \in A \times C$ and $x \in U$,

$$\begin{aligned}
T_{(RoZ)(a,c)}(x) &= max\{min_{b\in B}(T_{Z(a,b)}(x), T_{R(b,c)}(x))\} \\
&= max\{min_{b\in B}\{min(T_{F(a)}(x), T_{G(b)}(x)), min(T_{G(b)}(x), T_{H(c)}(x))\}\} \\
&= max\{min_{b\in B}(T_{F(a)}(x), T_{G(b)}(x), T_{H(c)}(x))\} \\
&\leq max\{min_{b\in B}(T_{F(a)}(x), 1, T_{H(c)}(x))\} \\
&= max\{min(T_{F(a)}(x), T_{H(c)}(x))\} \\
&= min(T_{F(a)}(x), T_{H(c)}(x)).
\end{aligned}$$

(4.3.1) $$T_{(RoZ)(a,c)}(x) \leq min(T_{F(a)}(x), T_{H(c)}(x)).$$

$$\begin{aligned}
I_{(RoZ)(a,c)}(x) &= max_{b\in B}(I_{Z(a,b)}(x).I_{R(b,c)}(x)) \\
&= max_{b\in B}\{(I_{F(a)}(x).I_{G(b)}(x)).(I_{G(b)}(x).I_{H(c)}(x))\} \\
&= max_{b\in B}(I_{F(a)}(x).I^2_{G(b)}(x).I_{H(c)}(x)) \\
&\leq max_{b\in B}(I_{F(a)}(x).1^2.I_{H(c)}(x)) \\
&= I_{F(a)}(x).I_{H(c)}(x).
\end{aligned}$$

Thus

(4.3.2) $$I_{(RoZ)(a,c)}(x) \leq I_{F(a)}(x).I_{H(c)}(x)).$$

$$\begin{aligned}
F_{(RoZ)(a,c)}(x) &= min\{max_{b\in B}(F_{Z(a,b)}(x), F_{R(b,c)}(x))\} \\
&= min\{max_{b\in B}\{max(F_{F(a)}(x), F_{G(b)}(x)), max(F_{G(b)}(x), F_{H(c)}(x))\}\} \\
&= min\{max_{b\in B}(F_{F(a)}(x), F_{G(b)}(x), F_{H(c)}(x))\} \\
&\geq min\{max_{b\in B}(F_{F(a)}(x), 0, F_{H(c)}(x))\} \\
&= min\{max(F_{F(a)}(x), F_{H(c)}(x))\} \\
&= max(F_{F(a)}(x), F_{H(c)}(x)).
\end{aligned}$$

Thus

(4.3.3) $$F_{(RoZ)(a,c)}(x) \geq max(F_{F(a)}(x), F_{H(c)}(x)).$$

So the theorem follows from (4.3.1),(4.3.2), and (4.3.3). □

**Remark 4.4.** The Theorem 4.3 does not hold if the relations on NSS are defined as viewed by Deli and Broumi[4]. The inequality (4.3.2) of Theorem 4.3 does not hold in that case.

There is another critical point. If $(RoZ)$(a,c) holds for $Z(a,b_1), R(b_1,c)$ and $Z(a,b_2), R(b_2,c)$ conjugately, for $b_1, b_2 \in B$ then 'composition of relations' on NSS defined by Deli and Broumi [4] does not meet the fact (see Example 4.2).

So, we implement the definition of cartesian product of two NSS as well as the relations on NSS and then generalise the composition of relations on NSS.

**Proposition 4.5.** *Let $\Gamma, Z \subseteq (F,A)\times(G,B)$ and $R \subseteq (G,B)\times(H,C)$ be three neutrosophic soft relations defined over $U$. Then $Ro(\Gamma\cup Z) = (Ro\Gamma)\cup(RoZ)$.*

*Proof.* Clearly $Ro(\Gamma\cup Z), (Ro\Gamma)\cup(RoZ) \subseteq (F,A)\times(H,C)$. (by Theorem 4.3) If $\Gamma\cap Z = \phi$ and $(Ro\Gamma)\cap(RoZ) = \phi$, the proposition is obvious.

Let $\Gamma\cap Z \neq \phi$. Now, for $(a,c)\in A\times C, x\in U$,

$$\begin{aligned}
&T_{(Ro(\Gamma\cup Z))(a,c)}(x)\\
&= max\{min_{b\in B}[T_{(\Gamma\cup Z)(a,b)}(x), T_{R(b,c)}(x)]\}\\
&= max\{min_{b\in B}[max(T_{\Gamma(a,b)}(x), T_{Z(a,b)}(x)), T_{R(b,c)}(x)]\}\\
&= max\{min_{b\in B}[max(min\{T_{F(a)}(x), T_{G(b)}(x)\}, min\{T_{F(a)}(x), T_{G(b)}(x)\}),\\
&\quad min\{T_{G(b)}(x), T_{H(c)}(x)\}]\}\\
&= max\{min_{b\in B}[min(T_{F(a)}(x), T_{G(b)}(x)), min(T_{G(b)}(x), T_{H(c)}(x))]\}\\
&= max\{min_{b\in B}[T_{F(a)}(x), T_{G(b)}(x), T_{H(c)}(x)]\}.
\end{aligned}$$

Thus

$$T_{(Ro(\Gamma\cup Z))(a,c)}(x) = max\{min_{b\in B}[T_{F(a)}(x), T_{G(b)}(x), T_{H(c)}(x)]\}. \tag{4.5.1}$$

$$\begin{aligned}
&I_{(Ro(\Gamma\cup Z))(a,c)}(x)\\
&= max_{b\in B}[I_{(\Gamma\cup Z)(a,b)}(x).I_{R(b,c)}(x)]\\
&= max_{b\in B}[\frac{1}{2}\{I_{\Gamma(a,b)}(x) + I_{Z(a,b)}(x)\}.I_{R(b,c)}(x)]\\
&= max_{b\in B}[\frac{1}{2}\{I_{F(a)}(x).I_{G(b)}(x) + I_{F(a)}(x).I_{G(b)}(x)\}.\{I_{G(b)}(x).I_{H(c)}(x)\}]\\
&= max_{b\in B}[\{I_{F(a)}(x).I_{G(b)}(x)\}.\{I_{G(b)}(x).I_{H(c)}(x)\}].
\end{aligned}$$

Thus

$$I_{(Ro(\Gamma\cup Z))(a,c)}(x) = max_{b\in B}[\{I_{F(a)}(x).I_{G(b)}(x)\}.\{I_{G(b)}(x).I_{H(c)}(x)\}]. \tag{4.5.2}$$

$$\begin{aligned}
&F_{(Ro(\Gamma\cup Z))(a,c)}(x)\\
&= min\{max_{b\in B}[F_{(\Gamma\cup Z)(a,b)}(x), F_{R(b,c)}(x)]\}\\
&= min\{max_{b\in B}[min(F_{\Gamma(a,b)}(x), F_{Z(a,b)}(x)), F_{R(b,c)}(x)]\}\\
&= min\{max_{b\in B}[min(max\{F_{F(a)}(x), F_{G(b)}(x)\}, max\{F_{F(a)}(x), F_{G(b)}(x)\}),\\
&\quad max\{F_{G(b)}(x), F_{H(c)}(x)\}]\}\\
&= min\{max_{b\in B}[max(F_{F(a)}(x), F_{G(b)}(x)), max(F_{G(b)}(x), F_{H(c)}(x))]\}\\
&= min\{max_{b\in B}[F_{F(a)}(x), F_{G(b)}(x), F_{H(c)}(x))]\}
\end{aligned}$$

Thus

$$I_{(Ro(\Gamma\cup Z))(a,c)}(x) = max_{b\in B}[\{I_{F(a)}(x).I_{G(b)}(x)\}.\{I_{G(b)}(x).I_{H(c)}(x)\}]. \tag{4.5.3}$$

Further, let $(Ro\Gamma)\cap(RoZ)\neq\phi$. Then

$$\begin{aligned}
&T_{[(Ro\Gamma)\cup(RoZ)](a,c)}(x)\\
&= max[T_{(Ro\Gamma)(a,c)}(x), T_{(RoZ)(a,c)}(x)]\\
&= max[max\{min_{b\in B}(T_{\Gamma(a,b)}(x), T_{R(b,c)}(x))\}, max\{min_{b\in B}(T_{Z(a,b)}(x), T_{R(b,c)}(x))\}]\\
&= max[max\{min_{b\in B}[min(T_{F(a)}(x), T_{G(b)}(x)), min(T_{G(b)}(x), T_{H(c)}(x))]\},\\
&\quad max\{min_{b\in B}[min(T_{F(a)}(x), T_{G(b)}(x)), min(T_{G(b)}(x), T_{H(c)}(x))]\}]\\
&= max\{min_{b\in B}[min(T_{F(a)}(x), T_{G(b)}(x)), min(T_{G(b)}(x), T_{H(c)}(x))]\}\\
&= max\{min_{b\in B}(T_{F(a)}(x), T_{G(b)}(x), T_{H(c)}(x))\}.
\end{aligned}$$

$$(4.5.4)\qquad T_{[(Ro\Gamma)\cup(RoZ)](a,c)}(x) = max\{min_{b\in B}(T_{F(a)}(x), T_{G(b)}(x), T_{H(c)}(x))\}.$$

$$\begin{aligned}
&I_{[(Ro\Gamma)\cup(RoZ)](a,c)}(x)\\
&= \frac{1}{2}[I_{(Ro\Gamma)(a,c)}(x) + I_{(RoZ)(a,c)}(x)]\\
&= \frac{1}{2}[max_{b\in B}\{I_{\Gamma(a,b)}(x).I_{R(b,c)}(x)\} + max_{b\in B}\{I_{Z(a,b)}(x).I_{R(b,c)}(x)\}]\\
&= \frac{1}{2}[max_{b\in B}\{(I_{F(a)}(x).I_{G(b)}(x)).(I_{G(b)}(x).I_{H(c)}(x))\}\\
&\quad + max_{b\in B}\{(I_{F(a)}(x).I_{G(b)}(x)).(I_{G(b)}(x).I_{H(c)}(x))\}]\\
&= \frac{1}{2}[2\ max_{b\in B}\{(I_{F(a)}(x).I_{G(b)}(x)).(I_{G(b)}(x).I_{H(c)}(x))\}]\\
&= max_{b\in B}\{(I_{F(a)}(x).I_{G(b)}(x)).(I_{G(b)}(x).I_{H(c)}(x))\}.
\end{aligned}$$

Thus

$$(4.5.5)\qquad I_{[(Ro\Gamma)\cup(RoZ)](a,c)}(x) = max_{b\in B}\{(I_{F(a)}(x).I_{G(b)}(x)).(I_{G(b)}(x).I_{H(c)}(x))\}.$$

$$\begin{aligned}
&= min[F_{(Ro\Gamma)(a,c)}(x), F_{(RoZ)(a,c)}(x)]\\
&= min[min\{max_{b\in B}(F_{\Gamma(a,b)}(x), F_{R(b,c)}(x))\}, min\{max_{b\in B}(F_{Z(a,b)}(x), F_{R(b,c)}(x))\}]\\
&= min[min\{max_{b\in B}[max(F_{F(a)}(x), F_{G(b)}(x)), max(F_{G(b)}(x), F_{H(c)}(x))]\},\\
&\quad min\{max_{b\in B}[max(F_{F(a)}(x), F_{G(b)}(x)), max(F_{G(b)}(x), F_{H(c)}(x))]\}]\\
&= min\{max_{b\in B}[max(F_{F(a)}(x), F_{G(b)}(x)), max(F_{G(b)}(x), F_{H(c)}(x))]\}\\
&= min\{max_{b\in B}(F_{F(a)}(x), F_{G(b)}(x), F_{H(c)}(x))\}.
\end{aligned}$$

Thus

$$(4.5.6)\qquad F_{[(Ro\Gamma)\cup(RoZ)](a,c)}(x) = \{max_{b\in B}(F_{F(a)}(x), F_{G(b)}(x), F_{H(c)}(x))\}.$$

Hence, by (4.5.1),(4.5.2),(4.5.3),(4.5.4),(4.5.5), and (4.5.6), the proposition is proved. □

**Proposition 4.6.** *Let $Z\subseteq(F,A)\times(G,B)$ and $R\subseteq(G,B)\times(H,C)$ be two neutrosophic soft relations defined over U. Then $(RoZ)^{-1}=Z^{-1}oR^{-1}$.*

*Proof.* Here $(RoZ) \subseteq (F,A) \times (H,C); (RoZ)^{-1} \subseteq (H,C) \times (F,A); R^{-1} \subseteq (H,C) \times (G,B); Z^{-1} \subseteq (G,B) \times (F,A); Z^{-1}oR^{-1} \subseteq (H,C) \times (F,A)$.
Now, for $(a,c) \in A \times C, x \in U$,

$$\begin{aligned} T_{(RoZ)^{-1}(c,a)}(x) &= T_{(RoZ)(a,c)}(x) \\ &= max\{min_{b\in B}(T_{Z(a,b)}(x), T_{R(b,c)}(x))\}. \end{aligned}$$

Thus

$$T_{(RoZ)^{-1}(c,a)}(x) = max\{min_{b\in B}(T_{Z(a,b)}(x), T_{R(b,c)}(x))\}. \tag{4.6.1}$$

$$\begin{aligned} I_{(RoZ)^{-1}(c,a)}(x) &= I_{(RoZ)(a,c)}(x) \\ &= max_{b\in B}(I_{Z(a,b)}(x).I_{R(b,c)}(x)). \end{aligned}$$

Thus

$$I_{(RoZ)^{-1}(c,a)}(x) = max_{b\in B}(I_{Z(a,b)}(x).I_{R(b,c)}(x)). \tag{4.6.2}$$

$$\begin{aligned} F_{(RoZ)^{-1}(c,a)}(x) &= F_{(RoZ)(a,c)}(x) \\ &= min\{max_{b\in B}(F_{Z(a,b)}(x), F_{R(b,c)}(x))\}. \end{aligned}$$

Thus

$$F_{(RoZ)^{-1}(c,a)}(x) = min\{max_{b\in B}(F_{Z(a,b)}(x), F_{R(b,c)}(x))\}. \tag{4.6.3}$$

Further,

$$\begin{aligned} T_{Z^{-1}oR^{-1}(c,a)}(x) &= max\{min_{b\in B}(T_{R^{-1}(c,b)}(x), T_{Z^{-1}(b,a)}(x))\} \\ &= max\{min_{b\in B}(T_{R(b,c)}(x), T_{Z(a,b)}(x))\} \\ &= max\{min_{b\in B}(T_{Z(a,b)}(x), T_{R(b,c)}(x))\}. \end{aligned}$$

Thus

$$T_{Z^{-1}oR^{-1}(c,a)}(x) = max\{min_{b\in B}(T_{Z(a,b)}(x), T_{R(b,c)}(x))\}. \tag{4.6.4}$$

$$\begin{aligned} I_{Z^{-1}oR^{-1}(c,a)}(x) &= max_{b\in B}(I_{R^{-1}(c,b)}(x).I_{Z^{-1}(b,a)}(x))\} \\ &= max_{b\in B}(I_{R(b,c)}(x).I_{Z(a,b)}(x))\} \\ &= max_{b\in B}(I_{Z(a,b)}(x).I_{R(b,c)}(x))\}. \end{aligned}$$

Thus

$$T_{Z^{-1}oR^{-1}(c,a)}(x) max\{min_{b\in B}(T_{Z(a,b)}(x), T_{R(b,c)}(x))\}. \tag{4.6.5}$$

$$\begin{aligned} F_{Z^{-1}oR^{-1}(c,a)}(x) &= min\{max_{b\in B}(F_{R^{-1}(c,b)}(x), F_{Z^{-1}(b,a)}(x))\} \\ &= min\{max_{b\in B}(F_{R(b,c)}(x), F_{Z(a,b)}(x))\} \\ &= min\{max_{b\in B}(F_{Z(a,b)}(x), F_{R(b,c)}(x))\}. \end{aligned}$$

Thus

$$F_{Z^{-1}oR^{-1}(c,a)}(x) = min\{max_{b\in B}(F_{Z(a,b)}(x), F_{R(b,c)}(x))\}. \tag{4.6.6}$$

Hence, by (4.6.1),(4.6.2),(4.6.3),(4.6.4),(4.6.5),(4.6.6), the proposition is ended. □

## 5. Neutrosophic soft function

**Definition 5.1.** Let $(F, A), (G, B)$ be two NSSs over the universal set U and $f$ be a neutrosophic soft relation defined on $(F, A) \times (G, B)$. Then $f$ is called neutrosophic soft function if $f$ associates each element of $(F, A)$ with the unique element of $(G, B)$. We write $f : (F, A) \to (G, B)$ as a neutrosophic soft function or a mapping. For $F(a) \in (F, A)$ and $G(b) \in (G, B)$ when $F(a) \times G(b) \in f$, we denote it by $f(F(a)) = G(b)$. Here $(F, A)$ and $(G, B)$ are called domain and codomain respectively and $G(b)$ is the image of $F(a)$ under $f$.

**Remark 5.2.** ( Mathematical representation of neutrosophic soft function )
Let $f : (F, A) \to (G, B)$ be a neutrosophic soft function over U, i.e., $f$ is a special type of relation of the form $(H, C)$, a NSS over U, where $C \subseteq A \times B$ and $H(a, b) \subseteq (F, A) \times (G, B)$, $\forall (a, b) \in C$. The truth, indeterminacy, falsity membership of $(H, C)$ are given by

$$T_{H(a,b)}(x) = min(T_{F(a)}(x), T_{G(b)}(x)),$$
$$I_{H(a,b)}(x) = (I_{F(a)}(x).I_{G(b)}(x)),$$
$$F_{H(a,b)}(x) = max(F_{F(a)}(x), F_{G(b)}(x)).$$

Then $H(a, b) = \{< x, T_{H(a,b)}(x), I_{H(a,b)}(x), F_{H(a,b)}(x) >: x \in U\}$ is the supporting NSS of $f[F(a)] = G(b)$.

**Example 5.3.** (1) Let U$= \{s_1, s_2, s_3\}$ be a universal set of students. We consider two parametric sets A and B given by A={MA,MSC,MBA,MD}$= \{a, s, b, d\}$ and B={professor in general degree college, lecturer in IIM, doctor in hospital, ADO}$= \{p, l, h, o\}$, say. Suppose $(F, A)$ and $(G, B)$ denote respectively 'the qualification of student'and 'student may be engaged in the profession'. They are given by

$$\begin{aligned} & (F, A) \\ = \; & \{F(a) = \{< s_1, (0.2, 0.5, 0.9) >, < s_2, (0.6, 0.4, 0.5) >, < s_3, (0.8, 0.5, 0.9) >\}; \\ & F(s) = \{< s_1, (0.7, 0.4, 0.2) >, < s_2, (0.4, 0.5, 0.5) >, < s_3, (0.5, 0.7, 0.4) >\}; \\ & F(b) = \{< s_1, (0.8, 0.5, 0.9) >, < s_2, (0.6, 0.5, 0.3) >, < s_3, (0.3, 0.5, 0.8) >\}; \\ & F(d) = \{< s_1, (0.7, 0.5, 0.5) >, < s_2, (0.9, 0.4, 0.2) >, < s_3, (0.4, 0.7, 0.6) >\}\} \end{aligned}$$

and

$$\begin{aligned} & (G, B) \\ = \; & \{G(p) = \{< s_1, (0.5, 0.6, 0.3) >, < s_2, (0.4, 0.3, 0.7) >, < s_3, (0.5, 0.4, 0.6) >\}; \\ & G(l) = \{< s_1, (0.7, 0.4, 0.3) >, < s_2, (0.2, 0.8, 0.6) >, < s_3, (0.9, 0.5, 0.1) >\}; \\ & G(h) = \{< s_1, (0.8, 0.4, 0.4) >, < s_2, (0.5, 0.4, 0.3) >, < s_3, (0.7, 0.5, 0.6) >\}; \\ & G(o) = \{< s_1, (0.4, 0.3, 0.5) >, < s_2, (0.8, 0.5, 0.5) >, < s_3, (0.2, 0.5, 0.8) >\}\} \end{aligned}$$

Now $f$ is defined as 'a postgraduate student will be engaged either in teaching profession or in govt sector'. Let $f = \{F(a) \times G(p), F(s) \times G(p), F(b) \times G(l), F(d) \times G(h)\}$ i,e $f(F(a)) = G(p); f(F(s)) = G(p); f(F(b)) = G(l); f(F(d)) = G(h)$.
The supporting NSS of $f$ is given by the following table.

| | $F(a)\times G(p)$ | $F(s)\times G(p)$ | $F(b)\times G(l)$ | $F(d)\times G(h)$ |
|---|---|---|---|---|
| $s_1$ | (0.2,0.3,0.9) | (0.5,0.24,0.3) | (0.7,0.2,0.9) | (0.7,0.2,0.5) |
| $s_2$ | (0.4,0.12,0.7) | (0.4,0.15,0.7) | (0.2,0.4,0.6) | (0.5,0.16,0.3) |
| $s_3$ | (0.5,0.35,0.9) | (0.5,0.28,0.6) | (0.3,0.25,0.8) | (0.4,0.35,0.6) |

Table 10: Tabular form of the supporting NSS of $f$

Thus $f$ is a neutrosophic soft function from $(F, A)$ to $(G, B)$ over U.

(2) Let $(F, A)$, a NSS, denote the intensity of colour of flowers over the set of flowers U= $\{f_1, f_2, f_3\}$ where A= {brown,red,white} = {b,r,w} is the set of parameter. We consider a relation $f$ on $(F, A) \times (F, A)$ as 'both flowers are of different bright colours'. Then $f = \{F(b) \times F(r), F(r) \times F(b)\}$. It is not a mapping as F(w) has no image. If we consider A$^*$ =A-{w}, then $f$ is a neutrosophic soft function on $(F, A^*) \times (F, A^*)$.

**Definition 5.4.** (i) (Injective function) A neutrosophic soft function $f : (F, A) \to (G, B)$ is injective if $F(a_1) \neq F(a_2) \Rightarrow f(F(a_1) \neq f(F(a_2))$ for $a_1, a_2 \in A$. Mathematically, $F(a_1) \neq F(a_2)$ holds if at least one of truth, indeterminacy, falsity membership values of at least one x in between $F(a_1)$ and $F(a_2)$ are different. In (2) of Example 5.3, $f : (F, A^*) \to (F, A^*)$ is obviously injective.

(ii) (Surjective function) A neutrosophic soft function $f : (F, A) \to (G, B)$ is surjective if $f(F, A) = (G, B)$, i.e., every element in $(G, B)$ has at least one preimage in $(F, A)$. In (1) of Example 5.3, $f : (F, A) \to (G, B^*)$ is obviously surjective, where B$^*$ =B-{o}.

(iii) (Bijective function) An injective and surjective neutrosophic soft function together is called bijective. In (1) of Example 5.3, $f : (F, A^*) \to (F, A^*)$ is obviously bijective.

(iv) (Constant function) The constant neutrosophic soft function gives same image for every element in a domain. Let $(F, A)$ and $(G, B)$ be two NSS over a set of persons representing 'the diseases'and 'the origin of diseases'respectively, where A= {malaria, dengue, filaria}= $\{m, d, r\}$ and B={mosquito, food, hard labour} = $\{M, D, H\}$ are two parametric sets. Let $f : (F, A) \to (G, B)$ be given by 'diseases are originated from mosquito only'. Then $f$=$\{F(m) \times G(M), F(d) \times G(M), F(r) \times G(M)\}$ is a constant neutrosophic soft function.

(v) (Identity function) An identity neutrosophic soft function $f : (F, A) \to (F, A)$ is defined by $f(F(a)) = F(a)\ \forall a \in A$ and is denoted by $I_{(F,A)}$. Let $(F, A)$ be a NSS describing 'the colour of flowers' over a set of flowers U, where A={brown, red, white}= $\{b, r, w\}$ is the set of parameters. We consider a neutrosophic soft function $f : (F, A) \to (F, A)$ by 'both the flowers are of same colours'. Then $f$= $\{F(b) \times F(b), F(r) \times F(r), F(w) \times F(w)\}$ is an identity neutrosophic soft function.

**Theorem 5.5.** *Let $f : (F, A) \to (G, B)$ be a neutrosophic soft function over U and $(F, A_1), (F, A_2)$ are the neutrosophic soft subsets of (F,A). Then*

(1) $(F, A_1) \subseteq (F, A_2) \Rightarrow f(F, A_1) \subseteq f(F, A_2)$,

(2) $f[(F, A_1) \cup (F, A_2)] = f(F, A_1) \cup f(F, A_2)$,

(3) $f[(F, A_1) \cap (F, A_2)] \subseteq f(F, , A_1) \cap f(F, A_2)$, *equality holds if f is injective.*

*Proof.* (1) Let $G(b) \in f(F, A_1)$. Then $\exists F(a) \in (F, A_1) \subseteq (F, A_2)$. Thus

$$f(F(a)) = G(b) \Rightarrow G(b) \in f(F, A_2) \Rightarrow f(F, A_1) \subseteq f(F, A_2).$$

(2) Let $G(b) \in f[(F, A_1) \cup (F, A_2)]$. Then

$$\begin{aligned}
& f(F(a)) = G(b) \text{ for } F(a) \in [(F, A_1) \cup (F, A_2)] \\
\Rightarrow\ & G(b) = f(F(a)) \text{ for } F(a) \in (F, A_1) \text{ or } F(a) \in (F, A_2) \\
\Rightarrow\ & G(b) = f(F(a)) \text{ for } F(a) \in (F, A_1) \text{ or} \\
& G(b) = f(F(a)) \text{ for } F(a) \in (F, A_2) \\
\Rightarrow\ & G(b) \in f(F, A_1) \text{ or } G(b) \in f(F, A_2) \\
\Rightarrow\ & G(b) \in f(F, A_1) \cup f(F, A_2).
\end{aligned}$$

Thus $f[(F, A_1) \cup (F, A_2)] \subseteq f(F, A_1) \cup f(F, A_2)$.
Next, since

$$(F, A_1) \subseteq (F, A_1) \cup (F, A_2) \text{ and } (F, A_2) \subseteq (F, A_1) \cup (F, A_2),$$

$$f(F, A_1) \subseteq f[(F, A_1) \cup (F, A_2)] \text{ and } f(F, A_2) \subseteq f[(F, A_1) \cup (F, A_2)].$$

So $f(F, A_1) \cup f(F, A_2) \subseteq [(F, A_1) \cup (F, A_2)]$.

(3) Let $G(b) \in f[(F, A_1) \cap (F, A_2)]$. Then

$$\begin{aligned}
& f(F(a)) = G(b) \text{ for } F(a) \in [(F, A_1) \cap (F, A_2)] \\
\Rightarrow\ & G(b) = f(F(a)) \text{ for } F(a) \in (F, A_1) \text{ and } F(a) \in (F, A_2) \\
\Rightarrow\ & G(b) = f(F(a)) \text{ for } F(a) \in (F, A_1) \text{ and} \\
& G(b) = f(F(a)) \text{ for } F(a) \in (F, A_2) \\
\Rightarrow\ & G(b) \in f(F, A_1) \text{ and } G(b) \in f(F, A_2) \\
\Rightarrow\ & G(b) \in f(F, A_1) \cap f(F, A_2).
\end{aligned}$$

Thus

$$f[(F, A_1) \cap (F, A_2)] \subseteq f(F, A_1) \cap f(F, A_2). \tag{5.5.1}$$

Finally, let $G(b) \in f(F, A_1) \cap f(F, A_2)$. Then

$$\begin{aligned}
& G(b) \in f(F, A_1) \text{ and } G(b) \in f(F, A_2) \\
\Rightarrow\ & f(F(a_1)) = G(b) \text{ for } F(a_1) \in (F, A_1) \text{ and} \\
& f(F(a_2)) = G(b) \text{ for } F(a_2) \in (F, A_2) \\
& \text{But, } F(a_1) = F(a_2) = F(a), \text{say, as } f \text{ is injective.} \\
& \text{So, } F(a) \in (F, A_1) \cap (F, A_2) \\
\Rightarrow\ & f(F(a)) \in f[(F, A_1) \cap (F, A_2)] \\
\Rightarrow\ & G(b) \in f[(F, A_1) \cap (F, A_2)].
\end{aligned}$$

So

$$f(F, A_1) \cap f(F, A_2) \subseteq f[(F, A_1) \cap (F, A_2)]. \tag{5.5.2}$$

Hence, by (5.5.1) and (5.5.2), the last part is proved. □

**Definition 5.6.** Let $f : (F, A) \to (G, B)$ and $g : (G, B) \to (H, C)$ be two neutrosophic soft functions over the universal set U. Then the composition of $f$ and $g$, denoted by $gof : (F, A) \to (H, C)$, is a neutrosophic soft function over U and defined by $(gof)(F(a)) = g(f(F(a))),\ a \in A$.

**Example 5.7.** Let $(F, A), (G, B)$ and $(H, C)$ be three NSSs over the universal set U= $\{s_1, s_2, s_3, s_4\}$, a set of Indian students. Suppose

$(F, A)$ describes a set of foreign universities giving some degrees to these students,

$(G, B)$ describes a set of degrees which the students may gain,

$(H, C)$ refers a set of pay scale which the students may gain in professional period.

Let A={California,Havard,Oxford,Cambridge}= $\{c, h, o, r\}$,

B={MSC,B.Tech,MD,MCA}= $\{s, t, d, m\}$,

C={35000-45000,45000-60000}= $\{a, b\}$.

Now we define two neutrosophic soft functions $f : (F, A) \to (G, B)$ as 'each foreign university offers a specific postgraduate degree to the students'and $g : (G, B) \to (H, C)$ by 'a = undergraduate scale, b = post graduate scale'.

Let $f = \{F(c) \times G(m), F(h) \times G(s), F(o) \times G(d), F(r) \times G(s)\}$ and

$g = \{G(s) \times H(b), G(t) \times H(a), G(d) \times H(b), G(m) \times H(b)\}$.

Then $gof = \{F(c) \times H(b), F(h) \times H(b), F(o) \times H(b), F(r) \times H(b)\}$.

**Theorem 5.8.** *Let $f : (F, A) \to (G, B)$; $g : (G, B) \to (H, C)$ and $h : (H, C) \to (E, D)$ be three neutrosophic soft functions over the universal set U. Then $ho(gof) = (hog)of$.*

*Proof.* Clearly $ho(gof), (hog)of : (F, A) \to (E, D)$.
Let $F(a) \in (F, A)$, $G(b) \in (G, B)$, $H(c) \in (H, C)$, and $E(d) \in (E, D)$. Then

$$f(F(a)) = G(b),\ g(G(b)) = H(c),\ h(H(c)) = E(d).$$

Now,

$$\begin{aligned}\{ho(gof)\}F(a) &= h\{(gof)(F(a))\}\\ &= h\{g(f(F(a)))\} = h\{g(G(b))\}\\ &= h\{H(c)\} = E(d).\end{aligned}$$

Furthermore,

$$\begin{aligned}\{(hog)of\}F(a) &= (hog)\{f(F(a))\}\\ &= (hog)G(b) = h\{g(G(b))\}\\ &= h\{H(c)\} = E(d).\end{aligned}$$

So $\{ho(gof)\}F(a) = \{(hog)of\}F(a)$. Hence $ho(gof) = (hog)of$. □

**Theorem 5.9.** *Let $f : (F, A) \to (G, B)$ and $g : (G, B) \to (H, C)$ be two neutrosophic soft functions over the universal set U so that they are both injective. Then their composite mapping $gof : (F, A) \to (H, C)$ is also injective.*

*Proof.* Let $F(a_1), F(a_2) \in (F, A)$, $G(b_1), G(b_2) \in G, B)$, and $H(c_1), H(c_2) \in (H, C)$. Then

$$f(F(a_1)) = G(b_1), f(F(a_2)) = G(b_2) \text{ and } g(G(b_1)) = H(c_1), g(G(b_2)) = H(c_2).$$

Since $f$ and $g$ are injective, $F(a_1) \neq F(a_2)$. Thus $G(b_1) \neq G(b_2)$. So $H(c_1) \neq H(c_2)$. Now,

$$(gof)(F(a_1)) = g(f(F(a_1))) = g(G(b_1)) = H(c_1)$$

and

$$(gof)(F(a_2)) = g(f(F(a_2))) = g(G(b_2)) = H(c_2).$$

This shows that $(gof)(F(a_1) \neq (gof)(F(a_2)$ for $F(a_1) \neq F(a_2)$.
Hence $gof$ is injective.
**Alternative:** We may prove it by truth, indeterminacy and falsity membership values of neutrosophic soft function.
Let $F(a_1), F(a_2) \in (F, A)$; $G(b_1), G(b_2) \in (G, B)$; $H(c_1), H(c_2) \in (H, C)$. Then

$$f(F(a_1)) = G(b_1), f(F(a_2)) = G(b_2), g(G(b_1)) = H(c_1), g(G(b_2)) = H(c_2).$$

Now, $F(a_i) = \{< x, T_{F(a_i)}(x), I_{F(a_i)}(x), F_{F(a_i)}(x) >: x \in U\}$ and similarly $G(b_i), H(c_i)$ for $i = 1, 2$. Since $f$ is injective, $T_{F(a_1)}(x) \neq T_{F(a_2)}(x)$. Then $T_{f(F(a_1))}(x) \neq T_{f(F(a_2))}(x)$. Thus $T_{G(b_1)}(x) \neq T_{G(b_2)}(x)$.

Similarly for $g$, $T_{G(b_1)}(x) \neq T_{G(b_2)}(x)$. Then $T_{g(G(b_1))}(x) \neq T_{g(G(b_2))}(x)$. Thus $H(c_1)(x) \neq T_{H(c_2)}(x)$.

For injective neutrosophic soft function , here we accept only the inequality of truth membership values of each x w.r.t two different parameters(from definition of injective function).

Finally, we see that

$$\begin{aligned} & T_{F(a_1)}(x) \neq T_{F(a_2)}(x) \\ \Rightarrow\ & T_{H(c_1)}(x) \neq T_{H(c_2)}(x) \\ \Rightarrow\ & T_{g(G(b_1))}(x) \neq T_{g(G(b_2))}(x) \\ \Rightarrow\ & T_{g(f(F(a_1)))}(x) \neq T_{g(f(F(a_2)))}(x) \\ \Rightarrow\ & T_{(gof)(F(a_1))}(x) \neq T_{(gof)(F(a_2))}(x). \end{aligned}$$

Hence $(gof)(F(a_1) \neq (gof)(F(a_2)$ for $F(a_1) \neq F(a_2)$. □

**Example 5.10.** Let $(F, A), (G, B)$ and $(H, C)$ be three NSSs over the universal set U, a set of flowers. Suppose $(F, A)$ denotes the 'size of flowers', $(G, B)$ denotes the 'colour of flowers'and $(H, C)$ denotes the 'fragrance of flowers'.
Let A= { small,large}= $\{s, l\}$, B={ brown,pale yellow}= $\{b, p\}$;, and C= { fragrant, dis-fragrant}= $\{r, d\}$.
Let $f : (F, A) \to (G, B)$ be defined as 'only the large flower belongs to bright colour catagory'and $g : (G, B) \to (H, C)$ be defined as 'only the light coloured flower has some fragrance'.
Then $f = \{F(l) \times G(b), F(s) \times G(p)\}$, $g = \{G(p) \times H(r), G(b) \times H(d)\}$, and
$gof = \{F(l) \times H(d), F(s) \times H(r)\} \subseteq (F, A) \times (H, C)$.
Obviously $f, g, gof$ are all injective here.

**Remark 5.11.** Converse of Theorem 5.9 is not true, i.e., if $gof$ is injective then $f$ is injective but $g$ need not be. In Example 5.10 if we consider B= {brown,pale yellow,off white}= $\{b, p, w\}$ and $f : (F, A) \to (G, B)$ as 'only the large flower belongs to bright colour category and small flower falls into any one light category'.
Then $f = \{F(l) \times G(b), F(s) \times G(w)\}$, say $g = \{G(b) \times H(d), G(p) \times H(r), G(w) \times H(r)\}$ and so, $gof = \{F(l) \times H(d), F(s) \times H(r)\}$.
Here $gof, f$ are both injective but $g$ is not.

**Theorem 5.12.** *Let $f : (F, A) \to (G, B)$ and $g : (G, B) \to (H, C)$ be two neutrosophic soft functions over the universal set U so that they are both surjective. Then their composite mapping $gof : (F, A) \to (H, C)$ is also surjective.*

*Proof.* Let $H(c) \in (H, C)$. Since $g$ is onto, there exit $G(b) \in (G, B)$ such that $g(G(b)) = H(c)$. Since $f$ is onto, there exists $F(a) \in (F, A)$ such that $f[F(a)] = G(b)$. Now $H(c) = g(G(b)) = g[f(F(a))] = (gof)(F(a))$. This shows that an arbitrary element $H(c)$ in codomain set $(H, C)$ has a pre-image $F(a)$ in the domain set $(F, A)$ under the mapping $gof$. Hence $gof$ is also onto. □

**Example 5.13.** We consider the students of class vii in a school. Let $(F, A), (G, B), (H, C)$ indicate respectively 'the age of students', 'the intelligence of students', 'the financial status of the family of students'.
Suppose A = { 12,13,14} =$\{a_1, a_2, a_3\}$;, B = { moderate,high}= $\{b_1, b_2\}$ and
C = { needy,moderate} = $\{c_1, c_2\}$.
Let $f : (F, A) \to (G, B)$ be defined as 'students of age 13 or above are of high intelligent and rest of are moderate'and $g : (G, B) \to (H, C)$ be treated as 'only the high intelligent students come from moderate family'.
Then $f = \{F(a_1)\times G(b_1), F(a_2)\times G(b_2), F(a_3)\times G(b_2)\}$; $g = \{G(b_1)\times H(c_1), G(b_2)\times H(c_2)\}$; and $gof = \{F(a_1) \times H(c_1), F(a_2) \times H(c_2), F(a_3) \times H(c_2)\}$.
Clearly $f, g, gof$ are all onto.

**Remark 5.14.** Converse of Theorem 5.12 is not true i.e if gof is surjective then g is surjective but f need not be.
There is a collection of different sweets in a shop. We like to buy some sweets on the basis of following criteria. Let $(F, A)$ denote the colour of sweets, $(G, B)$ denote the price of sweets, (H,C)denote the taste of sweets. Suppose A= { white,brown} = $\{w, b\}$, B= {Rs.10,Rs.7,Rs.4}= $\{t, s, r\}$ and C= {high,medium}= $\{h, m\}$ be three parametric sets. Let $f : (F, A) \to (G, B)$ be defined as 'the price of white sweet is Rs.10 and that of brown sweet is less than Rs.10 but only one price category' and $g : (G, B) \to (H, C)$ be treated as 'the sweet of Rs.10 is of high taste only and others are medium'.
Then $f = \{F(w) \times G(t), F(b) \times G(s)\}$, say $g = \{G(t) \times H(h), G(s) \times H(m), G(r) \times H(m)\}$ and $gof = \{F(w) \times H(h), F(b) \times H(m)\}$.
Clearly $g, gof$ both are onto but f is not.

**Theorem 5.15.** *Let f : (F,A)→(G,B) and g : (G,B)→(H,C) be two neutrosophic soft functions over the universal set U so that they are both bijective. Then their composite mapping gof:(F,A)→ (H,C) is also bijective.*

*Proof.* It is in combination of Theorems 5.9 and 5.12. □

**Remark 5.16.** Converse of Theorem 5.15 is not true i.e if $gof$ is bijective then $f$ is injective but $g$ is surjective, e.g. example of remark 5.11.

## 6. Inverse neutrosophic soft function

**Definition 6.1.** Let $f : (F, A) \to (G, B)$ be a neutrosophic soft function over the universal set U. If there exists another neutrosophic soft function $g : (G, B) \to (F, A)$ with $gof : (F, A) \to (F, A)$ and $fog : (G, B) \to (G, B)$ such that $gof = I_{(F,A)}$ and $fog = I_{(G,B)}$ then $g$ is called the inverse neutrosophic soft function of $f$. It is denoted by $f^{-1}$ and is defined as : $F(a) \times G(b) \in f^{-1}$ iff $G(b) \times F(a) \in f$.

**Example 6.2.** For A= $\{a_1, a_2, a_3\}$ and B= $\{b_1, b_2, b_3\}$, let $f : (F, A) \to (G, B)$ over U= $\{x_1, x_2, x_3\}$ be given by

| | $F(a_1) \times G(b_1)$ | $F(a_2) \times G(b_3)$ | $F(a_3) \times G(b_2)$ |
|---|---|---|---|
| $x_1$ | (0.7,0.6,0.7) | (0.5,0.7,0.8) | (0.8,0.6,0.9) |
| $x_2$ | (0.4,0.2,0.8) | (0.5,0.9,0.3) | (0.5,0.9,0.9) |
| $x_3$ | (0.9,0.2,0.5) | (0.5,0.6,0.8) | (0.3,0.7,0.6) |

Table 11: Tabular form of neutrosophic soft function $f$

Then the tabular representation of $f^{-1}$ is as :

| | $G(b_1) \times F(a_1)$ | $G(b_3) \times F(a_2)$ | $G(b_2) \times F(a_3)$ |
|---|---|---|---|
| $x_1$ | (0.7,0.6,0.7) | (0.5,0.7,0.8) | (0.8,0.6,0.9) |
| $x_2$ | (0.4,0.2,0.8) | (0.5,0.9,0.3) | (0.5,0.9,0.9) |
| $x_3$ | (0.9,0.2,0.5) | (0.5,0.6,0.8) | (0.3,0.7,0.6) |

Table 12: Tabular form of $f^{-1}$

**Theorem 6.3.** *A neutrosophic soft function $f : (F, A) \to (G, B)$ is invertible iff $f$ is bijective.*

*Proof.* First suppose, $f$ is invertible. Then there exists another neutrosophic soft function $g : (G, B) \to (F, A)$ such that $gof = I_{(F,A)}$ and $fog = I_{(G,B)}$. Since identity mapping is always bijective, $f$ is injective in $gof$ and is surjective in $fog$. Thus $f$ is bijective in combination of these.

Next suppose $f$ is bijective. Then $f$ is surjective and so for $G(b) \in (G, B)$ there exists $F(a) \in (F, A)$ such that $f[F(a)] = G(b)$. We consider another mapping $g : (G, B) \to (F, A)$ given by $g[G(b)] = F(a)$.
Then

$$(gof)[F(a)] = g[f(F(a))] = g(G(b)) = F(a)$$

and

$$(fog)[G(b)] = f[g(G(b))] = f(F(a)) = G(b).$$

So $f$ is invertible. □

**Theorem 6.4.** *Let $f : (F, A) \to (G, B)$ be a neutrosophic soft bijective mapping. Then $f^{-1} : (G, B) \to (F, A)$ is also bijective and $(f^{-1})^{-1} = f$.*

*Proof.* Since $f$ is bijective, $f$ is invertible and $fof^{-1} = I_{(G,B)}$, $f^{-1}of = I_{(F,A)}$. Since identity mapping is always bijective, $f^{-1}$ is bijective.

Next, since $f^{-1}$ is bijective, $(f^{-1})^{-1} : (F, A) \to (G, B)$ exists and is also bijective. For $F(a) \in (F, A)$, let there exist $G(b) \in (G, B)$ such that $f^{-1}(G(b)) = F(a)$. Then

$$(f^{-1})^{-1}[F(a)] = (f^{-1})^{-1}[f^{-1}(G(b))] = [(f^{-1})^{-1}of^{-1}](G(b)) = G(b) = f[F(a)].$$

Hence $(f^{-1})^{-1} = f$. □

**Theorem 6.5.** *Let $f : (F, A) \to (G, B)$ and $g : (G, B) \to (H, C)$ be two neutrosophic soft bijective functions over the universal set U. Then $gof : (F, A) \to (H, C)$ is also bijective and $(gof)^{-1} = f^{-1}og^{-1}$.*

*Proof.* 1st part is already proved in Theorem 5.15.

Next, here $f^{-1}, g^{-1}, (gof)^{-1}$ all exist.

For $F(a) \in (F, A), G(b) \in (G, B), H(c) \in (H, C)$, let

$$f^{-1}[G(b)] = F(a), g^{-1}[H(c)] = G(b), (gof)^{-1}[H(c)] = F(a).$$

Then $(gof)^{-1}[H(c)] = F(a) = f^{-1}[G(b)] = f^{-1}[g^{-1}(H(c))] = (f^{-1}og^{-1})[H(c)]$. Thus $(gof)^{-1} = f^{-1}og^{-1}$. □

**Theorem 6.6.** *Let $f : (F, A) \to (G, B)$ be a neutrosophic soft function over U which is surjective and $(G, B_1), (G, B_2)$ be two subsets of (G,B). Then*
(1) *If $(G, B_1) \subseteq (G, B_2)$, then $f^{-1}(G, B_1) \subseteq f^{-1}(G, B_2)$.*
(2) *$f^{-1}[(G, B_1) \cup (G, B_2)] = f^{-1}(G, B_1) \cup f^{-1}(G, B_2)$.*
(3) *$f^{-1}[(G, B_1) \cap (G, B_2)] = f^{-1}(G, B_1) \cap f^{-1}(G, B_2)$.*

*Proof.* Here $f^{-1} : (G, B) \to (F, A)$ and as f is onto then for $G(b) \in (G, B)$ there exist $F(a) \in (F, A)$ such that $f(F(a)) = G(b)$.
(1) Let $F(a) \in f^{-1}(G, B_1)$. Then

$$\begin{aligned} & f(F(a)) \in (G, B_1) \subseteq (G, B_2) \\ \Rightarrow\ & f(F(a)) \in (G, B_2) \\ \Rightarrow\ & F(a) \in f^{-1}(G, B_2) \\ \Rightarrow\ & f^{-1}(G, B_1) \subseteq f^{-1}(G, B_2). \end{aligned}$$

(2) Let $F(a) \in f^{-1}[(G, B_1) \cup (G, B_2)]$. Then

$$\begin{aligned} & f(F(a)) \in (G, B_1) \cup (G, B_2) \\ \Leftrightarrow\ & f(F(a)) \in (G, B_1) \text{ or } f(F(a)) \in (G, B_2) \\ \Leftrightarrow\ & F(a) \in f^{-1}(G, B_1) \text{ or } F(a) \in f^{-1}(G, B_2) \\ \Leftrightarrow\ & F(a) \in f^{-1}(G, B_1) \cup f^{-1}(G, B_2). \end{aligned}$$

(3) Let $F(a) \in f^{-1}[(G, B_1) \cap (G, B_2)]$. Then

$$\begin{aligned} & f(F(a)) \in (G, B_1) \cap (G, B_2) \\ \Leftrightarrow\ & f(F(a)) \in (G, B_1) \text{ and } f(F(a)) \in (G, B_2) \\ \Leftrightarrow\ & F(a) \in f^{-1}(G, B_1) \text{ and } F(a) \in f^{-1}(G, B_2) \\ \Leftrightarrow\ & F(a) \in f^{-1}(G, B_1) \cap f^{-1}(G, B_2). \end{aligned}$$

□

## 7. Conclusions

In the present paper the theoretical point of view of neutrosophic soft function has been discussed. The neutrosophic soft relations have been introduced in a new direction by implementing the indeterminacy membership value. We extend this concepts of relation and functions in NSS theory context. The new structure of indeterminacy membership will bring a new opportunity in research and development of NSS theory. The concept of neutrosophic soft function will also be helpful to solve many real life problems.

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