Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 12, No. 1, (July 2016), pp. 1–14
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
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Pattern recognition based on normalized Euclidean distance of interval valued intuitionistic fuzzy soft sets of root type

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Received 11 November 2015; Revised 11 December 2015; Accepted 13 January 2016

Abstract. In this paper we show that the notion of interval valued intuitionistic fuzzy soft set of root type (IVIFSSRT) is a generalization of interval valued fuzzy soft set and discuss some of its properties. We introduce the concept of normalized Euclidean distance between IVIFSSRT and establish some of its interesting theoretical properties. We also develop a pattern recognition algorithm based on this distance measure and illustrate it by an example.

2010 AMS Classification: 94D05, 62P10

Keywords: Interval valued intuitionistic fuzzy soft set of root type, Normalized Euclidean distance, Pattern recognition.

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1. Introduction

Pattern recognition problems in fuzzy environment have been widely studied in different angles by a number of authors. In the literature relating to studies of pattern recognition problems involving IVIFSs, a number of techniques have been developed to identify the best alternative and among these the notion of distance measure is popularly used by a good number of authors. This paper analyzes some of the important contributions in this direction. It is noticed that there are some distance measures that fail to identify the best alternative when the alternatives are specified by data of intuitionistic fuzzy set (IFS) or IVIFS types. Thus there is a need for new distance measure which could identify the best alternatives in all cases.

In the definition of IFS the membership and non membership functions are not completely independent which is overcome in intuitionistic fuzzy set of root type (IFSRT) as it preserves the independence of membership and non membership functions. Also in IFS, the membership and non membership values are specified by single values from the unit interval. But, whereas for most of the linguistic expressions fuzzy membership values can be better specified by intervals rather than single values as explained by Zadeh [26]. Many authors have also combined fuzzy sets with soft sets and contributed to the study of fuzzy soft sets. These are the motivating factors to study IVIFSSRT.

In this study a new normalized Euclidean distance between IVIFSSRT is defined and some of its theoretical properties are established. A new method for solving pattern recognition problems in fuzzy environment using this distance measure is explained. An algorithm is developed for identifying the alternatives of pattern recognition problems in fuzzy environment and its working is explained by means of an example.

2. Literature review

The theory of intuitionistic fuzzy set was introduced by Atanassov [3, 4, 7]. The concept of interval valued intuitionistic fuzzy set was developed by the same author [5]. Palaniappan et al. [20] introduced some operations on intuitionistic fuzzy sets of root type. Soft set theory was first introduced by Molodtsov [17]. Motivated by these theories, the theory of fuzzy soft set [12, 13, 21] and the theory of intuitionistic fuzzy soft set [14, 15] have been developed. Yang et al. [24] presented the concept of interval valued fuzzy soft sets by combining the interval valued fuzzy set and soft set models [9, 10]. By combining the concepts of interval valued intuitionistic fuzzy set, fuzzy soft set and intuitionistic fuzzy set of root type Anita Shanthi and Vadivel Naidu [1] introduced the notion of interval valued intuitionistic fuzzy soft set of root type (IVIFSSRT) and defined some operators. The similarity measure between IVIFSSRT was introduced by the same authors [2].

Atanassov [6] was the first one to introduce the notion of different types of similarity measures on interval valued intuitionistic fuzzy sets. A technique for pattern recognition problem using similarity measure based on Hausdorff distance in intuitionistic fuzzy set was developed by Hung and Yang [11]. Majumdar and Samanta [16] have studied the similarity measure based on distance between intuitionistic fuzzy soft sets. A distance measure on interval valued intuitionistic fuzzy set for solving group decision making problems was proposed by Xu [23]. Recently, Zhang and Yu [25] introduced new distance measures between intuitionistic fuzzy sets and interval valued fuzzy sets. Deli and Cagman [8] proposed the distance based similarity measure on intuitionistic fuzzy soft sets. The similarity measure based on normalized Euclidean distance in intuitionistic fuzzy sets was introduced by Szmidt [22]. The concept of similarity measure for interval valued intuitionistic fuzzy soft sets based on set theoretic approach was introduced by Mukerjee and Sarkar [18]. The same authors [19] defined three types of similarity measure for interval valued fuzzy soft sets based on matching function, distance and set theoretic approach.

The rest of this paper is organized as follows: The basic definitions needed are provided in Section 2. In Section 3, we define two topological operators on interval valued intuitionistic fuzzy soft set of root type and study some of the properties of these operators. In Section 4, we define the normalized Euclidean distance on interval valued intuitionistic fuzzy soft set of root type. We develop an algorithm for pattern recognition problems based on this distance measure. An example is given to illustrate the working of this algorithm. In the last section we present a brief conclusion.

3. Preliminaries

In this section we recall some definitions and results needed for our study.

Definition 3.1 ([4])**.** Let X be a non empty set. An intuitionistic fuzzy set A in X is an object of the form $A = \{(x, \mu_A(x), \nu_A(x)); x \in X\}$, where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define the degree of membership and degree of non-membership of the element $x \in X$ respectively, and for every $x \in X$,
$0 < \mu_A(x) + \nu_A(x) \leq 1$.

Remark 3.2. The original definition of intuitionistic fuzzy set proposed by Atanassov assumes that the membership and non membership degrees $\mu_A(x)$ and $\nu_A(x)$ are assigned independently for each $x \in X$, subject to the condition $0 < \mu_A(x)+\nu_A(x) \leq 1$. This definition imposes a restriction that $\nu_A(x) \leq 1 - \mu_A(x)$. This poses a major threat to the independence of $\mu_A(x)$ and $\nu_A(x)$. If we allow $\mu_A(x)$ and $\nu_A(x)$ to be assigned independently then there is always a possibility that $\mu_A(x) + \nu_A(x) \geq 1$. This deficiency in the definition of intuitionistic fuzzy set has necessitated the need for a new type of intuitionistic fuzzy set called intuitionistic fuzzy set of root type (IFSRT) in which $\mu_A(x)$ and $\nu_A(x)$ are assigned totally independent of each other. Hence the possibility of $\mu_A(x) + \nu_A(x) \geq 1$. To make the definition of IFSRT to be more meaningful and useful, we impose the condition $\sqrt{\mu_{\mathbb{A}}(x)} + \sqrt{\nu_{\mathbb{A}}(x)} \leq 2$.

Definition 3.3 ([20])**.** Let X be a non empty set. An intuitionistic fuzzy set of root type $\mathbb{A}$ in X is an object of the form $\mathbb{A} = \{\langle x, \mu_{\mathbb{A}}(x), \nu_{\mathbb{A}}(x)\rangle; x \in X\}$, where the functions $\mu_{\mathbb{A}} : X \to [0,1]$ and $\nu_{\mathbb{A}} : X \to [0,1]$ define the degree of membership and non-membership of the element $x \in X$, respectively and for every $x \in X$,
$0 < \frac{1}{2}\sqrt{\mu_{\mathbb{A}}(x)} + \frac{1}{2}\sqrt{\nu_{\mathbb{A}}(x)} \leq 1$.

Definition 3.4 ([5])**.** An interval valued intuitionistic fuzzy set on a non empty set X is an object of the form $\mathscr{A} = \{\langle x, \mu_{\mathscr{A}}(x), \nu_{\mathscr{A}}(x)\rangle; x \in X\}$, where
$\mu_{\mathscr{A}}(x) = [\underline{\mu}_{\mathscr{A}}(x), \overline{\mu}_{\mathscr{A}}(x)]$, $\nu_{\mathscr{A}}(x) = [\underline{\nu}_{\mathscr{A}}(x), \overline{\nu}_{\mathscr{A}}(x)]$, where $\mu_{\mathscr{A}}(x), \nu_{\mathscr{A}}(x) : \mathscr{A} \to D([0,1])$. $D([0,1])$ stands for the set of all closed subintervals of $[0,1]$ which satisfy the condition, $0 < \overline{\mu}_{\mathscr{A}}(x) + \overline{\nu}_{\mathscr{A}}(x) \leq 1$.

Let U be the universe of objects and E the set of parameters in relation to objects in U. Parameters are often attributes, characteristics or properties of objects.

Definition 3.5 ([13])**.** Let $\mathscr{F}(U)$ be the set of all fuzzy subsets of U and $A \subseteq E$. The pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by $F : A \to \mathscr{F}(U)$.

For any parameter $e \in A$, $F(e)$ is a fuzzy subset of U and it is called fuzzy value set of the parameter e. $F(e) = \{(x, \mu_{F(e)}(x)); x \in U\}$. $\mu_{F(e)}(x)$ denotes the membership degree that an object x holds on the parameter e, $x \in U$, $e \in A$.

Definition 3.6 ([14])**.** Let U be an universe and E a set of parameters. Let $P(U)$ denote the set of all intuitionistic fuzzy subsets of U and $A \subseteq E$.

A pair (F, A) is called an intuitionistic fuzzy soft set over U, where F is a mapping given by $F : A \to P(U)$.

4. Interval valued intuitionistic fuzzy soft set of root type

In this section, we discuss the notion of IVIFSSRT and define two topological operators on IVIFSSRT and discuss some properties of these operators.

Definition 4.1 ([1])**.** Let U be an universe and E a set of parameters. Let $IVIFSRT(U)$ denote the set of all interval valued intuitionistic fuzzy sets of root type over U and $A \subseteq E$. A pair (F, A) is an IVIFSSRT over U, where F is a mapping given by $F : A \to IVIFSRT(U)$ and

$$(F, A) = \{\langle x, [\underline{\mu}_{F(e)}(x), \overline{\mu}_{F(e)}(x)], [\underline{\nu}_{F(e)}(x), \overline{\nu}_{F(e)}(x)]\rangle; x \in U,\ e \in A\}.$$

For any parameter $e \in A$, $F(e)$ is an IVIFSRT.

Definition 4.2 ([1])**.** The necessity operator on an IVIFSSRT (F, A) denoted by $\Box(F, A)$ is defined as

$$\Box(F, A) = \left\{\left\langle x, \left[\underline{\mu}_{F(e)}(x), \overline{\mu}_{F(e)}(x)\right], \left[\left(1 - \sqrt{\overline{\mu}_{F(e)}(x)}\right)^2, \left(1 - \sqrt{\underline{\mu}_{F(e)}(x)}\right)^2\right]\right\rangle; x \in U,\ e \in A\right\}.$$

Definition 4.3 ([1])**.** The possibility operator on an IVIFSSRT (F, A) denoted by $\Diamond(F, A)$ is defined as

$$\Diamond(F, A) = \left\{\left\langle x, \left[\left(1 - \sqrt{\overline{\nu}_{F(e)}(x)}\right)^2, \left(1 - \sqrt{\underline{\nu}_{F(e)}(x)}\right)^2\right], \left[\underline{\nu}_{F(e)}(x), \overline{\nu}_{F(e)}(x)\right]\right\rangle; x \in U,\ e \in A\right\}.$$

Definition 4.4. The complement of an IVIFSSRT (F, A) denoted by $(F, A)^c$ is defined as $(F, A)^c = \{\langle x, [\underline{\nu}_{F(e)}(x), \overline{\nu}_{F(e)}(x)], [\underline{\mu}_{F(e)}(x), \overline{\mu}_{F(e)}(x)]\rangle; x \in U,\ e \in A\}$.

Definition 4.5. Let $A, B \subseteq E$. (F, A) is an interval valued intuitionistic fuzzy soft subset of root type of (G, B) denoted by $(F, A) \Subset (G, B)$ if and only if

(i) $A \subseteq B$,

(ii) $\forall\ e \in A$, $F(e)$ is an interval valued intuitionistic fuzzy soft subset of root type of $G(e)$. i.e, $\forall\ x \in U$, $e \in A$, $\underline{\mu}_{F(e)}(x) \leq \underline{\mu}_{G(e)}(x)$, $\overline{\mu}_{F(e)}(x) \leq \overline{\mu}_{G(e)}(x)$, $\underline{\nu}_{F(e)}(x) \geq \underline{\nu}_{G(e)}(x)$ and $\overline{\nu}_{F(e)}(x) \geq \overline{\nu}_{G(e)}(x)$.

Further (G, B) is called an interval valued intuitionistic fuzzy superset of root type (F, A) and is denoted by $(G, B) \Supset (F, A)$.

Definition 4.6. The degree of non-determinacy of an element $x \in U$, $e \in A$ to the IVIFSSRT (F, A) is defined as

$$\underline{\pi}_{F(e)}(x) = \left(1 - \sqrt{\underline{\mu}_{F(e)}(x)} - \sqrt{\underline{\nu}_{F(e)}(x)}\right)^2$$

and

$$\overline{\pi}_{F(e)}(x) = \left(1 - \sqrt{\overline{\mu}_{F(e)}(x)} - \sqrt{\overline{\nu}_{F(e)}(x)}\right)^2.$$

Definition 4.7. Let $\alpha \in [0, 1]$ be a fixed number. Given an IVIFSSRT (F, A), the operator $\mathbb{D}_\alpha$ is defined as

$$\mathbb{D}_\alpha(F, A) = \left\{\left\langle x, \left[\left(\sqrt{\underline{\mu}_{F(e)}(x)} + \alpha\sqrt{\underline{\pi}_{F(e)}(x)}\right)^2, \left(\sqrt{\overline{\mu}_{F(e)}(x)} + \alpha\sqrt{\overline{\pi}_{F(e)}(x)}\right)^2\right],\right.\right.$$

$$\Big[\Big(\sqrt{\underline{\nu}_{F(e)}(x)}+(1-\alpha)\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\nu}_{F(e)}(x)}+(1-\alpha)\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big]\Big\rangle; \quad x \in U,\ e \in A\Big\}.$$

Proposition 4.8. *For every IVIFSSRT* (F, A) *and* $\alpha, \beta \in [0, 1]$,

(1) *If* $\alpha \leq \beta$ *then* $\mathbb{D}_\alpha(F, A) \Subset \mathbb{D}_\beta(F, A)$,

(2) $\mathbb{D}_0(F, A) = \Box(F, A)$,

(3) $\mathbb{D}_1(F, A) = \Diamond(F, A)$.

Proof. (1) $\mathbb{D}_\alpha(F, A)$

$$= \Big\{\Big\langle x, \Big[\Big(\sqrt{\underline{\mu}_{F(e)}(x)} + \alpha\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\mu}_{F(e)}(x)} + \alpha\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big],$$
$$\Big[\Big(\sqrt{\underline{\nu}_{F(e)}(x)}+(1-\alpha)\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\nu}_{F(e)}(x)}+(1-\alpha)\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big]\Big\rangle; \quad x \in U,\ e \in A\Big\}$$

and

$$\mathbb{D}_\beta(F, A) = \Big\{\Big\langle x, \Big[\Big(\sqrt{\underline{\mu}_{F(e)}(x)} + \beta\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\mu}_{F(e)}(x)} + \beta\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big],$$
$$\Big[\Big(\sqrt{\underline{\nu}_{F(e)}(x)}+(1-\beta)\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\nu}_{F(e)}(x)}+(1-\beta)\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big]\Big\rangle; \quad x \in U,\ e \in A\Big\}.$$

$$\begin{aligned}\Big(\sqrt{\underline{\mu}_{F(e)}(x)} + \alpha\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2 &= \underline{\mu}_{F(e)}(x) + \alpha^2\underline{\pi}_{F(e)}(x) + 2\alpha\sqrt{\underline{\mu}_{F(e)}(x)}\sqrt{\underline{\pi}_{F(e)}(x)}\\ &\leq \underline{\mu}_{F(e)}(x) + \beta^2\underline{\pi}_{F(e)}(x) + 2\beta\sqrt{\underline{\mu}_{F(e)}(x)}\sqrt{\underline{\pi}_{F(e)}(x)}\\ &= \Big(\sqrt{\underline{\mu}_{F(e)}(x)} + \beta\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2.\end{aligned}$$

Similarly, $\Big(\sqrt{\overline{\mu}_{F(e)}(x)} + \alpha\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2 \leq \Big(\sqrt{\overline{\mu}_{F(e)}(x)} + \beta\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2$.

Since $\alpha < \beta,\ (1-\alpha)^2 > (1-\beta)^2$ and

$$\begin{aligned}\Big(\sqrt{\underline{\nu}_{F(e)}(x)} + (1-\beta)\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2 &= \underline{\nu}_{F(e)}(x) + (1-\beta)^2\underline{\pi}_{F(e)}(x)\\ &\quad +2\sqrt{\underline{\nu}_{F(e)}(x)}\ (1-\beta)\sqrt{\underline{\pi}_{F(e)}(x)}\\ &\leq \underline{\nu}_{F(e)}(x) + (1-\alpha)^2\underline{\pi}_{F(e)}(x) + 2\sqrt{\underline{\nu}_{F(e)}(x)}\\ &\quad +(1-\alpha)\sqrt{\underline{\pi}_{F(e)}(x)}\\ &= \Big(\sqrt{\underline{\nu}_{F(e)}(x)} + (1-\alpha)\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2.\end{aligned}$$

Similarly,

$$\Big(\sqrt{\overline{\nu}_{F(e)}(x)} + (1-\beta)\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2 \leq \Big(\sqrt{\overline{\nu}_{F(e)}(x)} + (1-\alpha)\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2.$$

Hence it follows that $\mathbb{D}_\alpha(F, A) \Subset \mathbb{D}_\beta(F, A)$.

(2) $$\mathbb{D}_0(F, A) = \Big\{\Big\langle x, \Big[\Big(\sqrt{\underline{\mu}_{F(e)}(x) + 0}\Big)^2, \Big(\sqrt{\overline{\mu}_{F(e)}(x) + 0}\Big)^2\Big],$$
$$\Big[\Big(\sqrt{\underline{\nu}_{F(e)}(x)} + \sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\nu}_{F(e)}(x)} + \sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big]\Big\rangle; \quad x \in U,\ e \in A\Big\}$$

$$= \Big\{\Big\langle x, \Big[\underline{\mu}_{F(e)}(x), \overline{\mu}_{F(e)}(x)\Big], \Big[\Big(\sqrt{\underline{\nu}_{F(e)}(x)} + 1 - \sqrt{\underline{\mu}_{F(e)}(x)} - \sqrt{\underline{\nu}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\nu}_{F(e)}(x)} + 1 - \sqrt{\overline{\mu}_{F(e)}(x)} - \sqrt{\overline{\nu}_{F(e)}(x)}\Big)^2\Big]\Big\rangle; x \in U,\ e \in A\Big\}$$

$$= \Big\{\Big\langle x, \Big[\underline{\mu}_{F(e)}(x), \overline{\mu}_{F(e)}(x)\Big], \Big[\Big(1 - \sqrt{\underline{\mu}_{F(e)}(x)}\Big)^2, \Big(1 - \sqrt{\overline{\mu}_{F(e)}(x)}\Big)^2\Big]\Big\rangle; x \in U,\ e \in A\Big\}$$

$$= \Box(F, A).$$

(3) $$\mathbb{D}_1(F, A) = \Big\{\Big\langle x, \Big[\Big(\sqrt{\underline{\mu}_{F(e)}(x)} + \sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\mu}_{F(e)}(x)} + \sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big], \Big[\Big(\sqrt{\underline{\nu}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\nu}_{F(e)}(x)}\Big)^2\Big]\Big\rangle; x \in U,\ e \in A\Big\}$$

$$= \Big\{\Big\langle x, \Big[\Big(\sqrt{\underline{\mu}_{F(e)}(x)} + \sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\mu}_{F(e)}(x)} + \sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big], \Big[\Big(\sqrt{\underline{\nu}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\nu}_{F(e)}(x)}\Big)^2\Big]\Big\rangle; x \in U,\ e \in A\Big\}$$

$$= \Big\{\Big\langle x, \Big[\Big(\sqrt{\underline{\mu}_{F(e)}(x)} + 1 - \sqrt{\underline{\mu}_{F(e)}(x)} - \sqrt{\underline{\nu}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\mu}_{F(e)}(x)} + 1 - \sqrt{\overline{\mu}_{F(e)}(x)} - \sqrt{\overline{\nu}_{F(e)}(x)}\Big)^2\Big], \Big[\Big(\sqrt{\underline{\nu}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\nu}_{F(e)}(x)}\Big)^2\Big]\Big\rangle; x \in U,\ e \in A\Big\}$$

$$= \Big\{\Big\langle x, \Big[\Big(1 - \sqrt{\underline{\nu}_{F(e)}(x)}\Big)^2, \Big(1 - \sqrt{\overline{\nu}_{F(e)}(x)}\Big)^2\Big], \Big[\Big(\sqrt{\underline{\nu}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\nu}_{F(e)}(x)}\Big)^2\Big]\Big\rangle; x \in U,\ e \in A\Big\}$$

$$= \Diamond(F, A).$$ □

Remark 4.9. The operator $\mathbb{D}_\alpha$ is an extension of the operators $\Box$ and $\Diamond$.

Definition 4.10. For $\alpha, \beta \in [0, 1],\ \alpha + \beta \leq 1$ the operator $\mathbb{F}_{\alpha,\beta}$ for an IVIFSSRT (F, A) is defined as

$$\mathbb{F}_{\alpha,\beta}(F, A) = \Big\{\Big\langle x, \Big[\Big(\sqrt{\underline{\mu}_{F(e)}(x)} + \alpha\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\mu}_{F(e)}(x)} + \alpha\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big], \Big[\Big(\sqrt{\underline{\nu}_{F(e)}(x)} + \beta\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\nu}_{F(e)}(x)} + \beta\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big]\Big\rangle; x \in U,\ e \in A\Big\}.$$

Theorem 4.11. *For IVIFSSRT (F, A) and $\forall\, \alpha, \beta, \gamma \in [0, 1]$ such that $\alpha + \beta \leq 1$, the following hold :*

(1) $\mathbb{F}_{\alpha,\beta}(F, A)$ *is an IVIFSSRT,*
(2) *If* $0 \leq \gamma \leq \alpha$ *then* $\mathbb{F}_{\gamma,\beta}(F, A) \Subset \mathbb{F}_{\alpha,\beta}(F, A)$,
(3) *If* $0 \leq \gamma \leq \beta$ *then* $\mathbb{F}_{\alpha,\beta}(A) \Subset \mathbb{F}_{\alpha,\gamma}(F, A)$,
(4) $\mathbb{D}_\alpha(F, A) = \mathbb{F}_{\alpha,1-\alpha}(F, A)$,
(5) $\Box(F, A) = \mathbb{F}_{0,1}(F, A)$,
(6) $\Diamond(F, A) = \mathbb{F}_{1,\,0}(F, A)$,
(7) $(\mathbb{F}_{\alpha,\,\beta}(F, A)^c)^c = \mathbb{F}_{\beta,\,\alpha}(F, A)$.

Proof. (1) Consider, $\dfrac{\sqrt{\overline{\mu}_{F(e)}(x)} + \alpha\sqrt{\overline{\pi}_{F(e)}(x)}}{2} + \frac{\sqrt{\overline{\nu}_{F(e)}(x)} + \beta\sqrt{\overline{\pi}_{F(e)}(x)}}{2}$

$$= \frac{\sqrt{\overline{\mu}_{F(e)}(x)}}{2} + \frac{\sqrt{\overline{\nu}_{F(e)}(x)}}{2} + (\alpha+\beta)\frac{\sqrt{\overline{\pi}_{F(e)}(x)}}{2}$$

$$\leq \frac{\sqrt{\overline{\mu}_{F(e)}(x)}}{2} + \frac{\sqrt{\overline{\nu}_{F(e)}(x)}}{2} + \frac{1-\sqrt{\overline{\mu}_{F(e)}(x)}-\sqrt{\overline{\nu}_{F(e)}(x)}}{2}$$

$$\leq \tfrac{1}{2} < 1.$$

Hence $\mathbb{F}_{\alpha,\beta}(F,A)$ is an IVIFSSRT.

(2) We have

$$\mathbb{F}_{\gamma,\,\beta}(F,A) = \Big\{\Big\langle x, \Big[\Big(\sqrt{\underline{\mu}_{F(e)}(x)} + \gamma\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\mu}_{F(e)}(x)} + \gamma\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big], \Big[\Big(\sqrt{\underline{\nu}_{F(e)}(x)} + \beta\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\nu}_{F(e)}(x)} + \beta\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big]\Big\rangle; x \in U,\ e \in A\Big\}$$

$$\mathbb{F}_{\alpha,\,\beta}(F,A) = \Big\{\Big\langle x, \Big[\Big(\sqrt{\underline{\mu}_{F(e)}(x)} + \alpha\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\mu}_{F(e)}(x)} + \alpha\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big], \Big[\Big(\sqrt{\underline{\nu}_{F(e)}(x)} + \beta\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\nu}_{F(e)}(x)} + \beta\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big]\Big\rangle; x \in U,\ e \in A\Big\}$$

Now, $\Big(\sqrt{\underline{\mu}_{F(e)}(x)}+\gamma\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2 = \underline{\mu}_{F(e)}(x)+\gamma^2\underline{\pi}_{F(e)}(x)+2\gamma\sqrt{\underline{\mu}_{F(e)}(x)}\sqrt{\underline{\pi}_{F(e)}(x)}$

$$\leq \underline{\mu}_{F(e)}(x) + \alpha^2\underline{\pi}_{F(e)}(x) + 2\alpha\sqrt{\underline{\mu}_{F(e)}(x)}\sqrt{\underline{\pi}_{F(e)}(x)}$$

$$= \Big(\sqrt{\underline{\mu}_{F(e)}(x)} + \alpha\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2, \text{ since } \gamma \leq \alpha.$$

Similarly, we have $\Big(\sqrt{\overline{\mu}_{F(e)}(x)} + \gamma\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2 \leq \Big(\sqrt{\overline{\mu}_{F(e)}(x)} + \alpha\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2$.
Hence it follows that $\mathbb{F}_{\gamma,\,\beta}(F,A) \Subset \mathbb{F}_{\alpha,\,\beta}(F,A)$.

(3) Proof is similar to (2).

(4) $\mathbb{F}_{\alpha,1-\alpha}(F,A)$

$$= \Big\{\Big\langle x, \Big[\Big(\sqrt{\underline{\mu}_{F(e)}(x)} + \alpha\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\mu}_{F(e)}(x)} + \alpha\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big], \Big[\Big(\sqrt{\underline{\nu}_{F(e)}(x)}+(1-\alpha)\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\nu}_{F(e)}(x)}+(1-\alpha)\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big]\Big\rangle; x \in U,\ e \in A\Big\}$$

$$= \mathbb{D}_\alpha(F,A).$$

(5) $\mathbb{F}_{\alpha,1}(F,A)$

$$= \Big\{\Big\langle x, \Big[\Big(\sqrt{\underline{\mu}_{F(e)}(x)} + \alpha\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2, \Big(\sqrt{\overline{\mu}_{F(e)}(x)} + \alpha\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big],$$

$$\Big[\Big(\sqrt{\underline{\nu}_{F(e)}(x)}+\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2,\Big(\sqrt{\overline{\nu}_{F(e)}(x)}+\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big]\Big\rangle;$$
$$x\in U,\ e\in A\Big\}$$
$$=\Big\{\Big\langle x,\Big[\underline{\mu}_{F(e)}(x),\overline{\mu}_{F(e)}(x)\Big],\Big[\Big(1-\sqrt{\underline{\mu}_{F(e)}(x)}\Big)^2,\Big(1-\sqrt{\overline{\mu}_{F(e)}(x)}\Big)^2\Big]\Big\rangle;$$
$$x\in U,\ e\in A\Big\}$$
$$=\Box(F,A).$$

(6) $\mathbb{F}_{1,\,0}(F,A)$
$$=\Big\{\Big\langle x,\Big[\Big(\sqrt{\underline{\mu}_{F(e)}(x)}+\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2,\Big(\sqrt{\overline{\mu}_{F(e)}(x)}+\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big],$$
$$\Big[\Big(\sqrt{\underline{\nu}_{F(e)}(x)}\Big)^2,\Big(\sqrt{\overline{\nu}_{F(e)}(x)}\Big)^2\Big]\Big\rangle;x\in U,\ e\in A\Big\}$$
$$=\Big\{\Big\langle x,\Big[\Big(1-\sqrt{\underline{\nu}_{F(e)}(x)}\Big)^2,\Big(1-\sqrt{\overline{\nu}_{F(e)}(x)}\Big)^2\Big],\Big[\underline{\nu}_{F(e)}(x),\overline{\nu}_{F(e)}(x)\Big]\Big\rangle;$$
$$x\in U,\ e\in A\Big\},$$
$$=\Diamond(F,A).$$

(7) $\mathbb{F}_{\alpha,\,\beta}(F,A)^c$
$$=\Big\{\Big\langle x,\Big[\Big(\sqrt{\underline{\nu}_{F(e)}(x)}+\alpha\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2,\Big(\sqrt{\overline{\nu}_{F(e)}(x)}+\alpha\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big],$$
$$\Big[\Big(\sqrt{\underline{\mu}_{F(e)}(x)}+\beta\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2,\Big(\sqrt{\overline{\mu}_{F(e)}(x)}+\beta\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big]\Big\rangle;$$
$$x\in U,\ e\in A\Big\}.$$

Then

$(\mathbb{F}_{\alpha,\,\beta}(F,A)^c)^c$
$$=\Big\{\Big\langle x,\Big[\Big(\sqrt{\underline{\mu}_{F(e)}(x)}+\beta\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2,\Big(\sqrt{\overline{\mu}_{F(e)}(x)}+\beta\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big],$$
$$\Big[\Big(\sqrt{\underline{\nu}_{F(e)}(x)}+\alpha\sqrt{\underline{\pi}_{F(e)}(x)}\Big)^2,\Big(\sqrt{\overline{\nu}_{F(e)}(x)}+\alpha\sqrt{\overline{\pi}_{F(e)}(x)}\Big)^2\Big]\Big\rangle;$$
$$x\in U,\ e\in A\Big\}$$
$$=\mathbb{F}_{\beta,\,\alpha}(F,A).$$
□

Remark 4.12. If $\alpha+\beta=1$, then $\mathbb{F}_{\alpha,\,\beta}$ coincides with $\mathbb{D}_\alpha$.

Definition 4.13. Let $\alpha,\beta\in[0,1]$. Given an IVIFSSRT (F,A), the operator $\mathbb{G}_{\alpha,\,\beta}$ is defined as
$\mathbb{G}_{\alpha,\,\beta}(F,A)=\{\langle x,[\alpha^2\underline{\mu}_{F(e)}(x),\alpha^2\overline{\mu}_{F(e)}(x)],[\beta^2\underline{\nu}_{F(e)}(x),\beta^2\overline{\nu}_{F(e)}(x)]\rangle;$
$x\in U,\ e\in A\}.$
Obviously, $\mathbb{G}_{1,1}(F,A)=(F,A)$ and $\mathbb{G}_{0,0}(F,A)=\phi$.

Theorem 4.14. *For every IVIFSSRT (F, A) and $\alpha, \beta, \gamma \in [0, 1]$,*
(1) $\mathbb{G}_{\alpha,\, \beta}(F, A)$ *is an IVIFSSRT,*
(2) *If $\alpha \leq \gamma$ then $\mathbb{G}_{\alpha,\, \beta}(F, A) \Subset \mathbb{G}_{\gamma,\, \beta}(F, A)$,*
(3) *If $\beta \leq \gamma$ then $\mathbb{G}_{\alpha,\, \beta}(F, A) \Supset \mathbb{G}_{\alpha,\, \gamma}(F, A)$,*
(4) *If $\delta \in [0, 1]$ then $\mathbb{G}_{\alpha,\, \beta}(\mathbb{G}_{\gamma,\, \delta}(F, A)) = \mathbb{G}_{\alpha\gamma,\, \beta\delta}(F, A) = \mathbb{G}_{\gamma,\, \delta}(\mathbb{G}_{\alpha,\, \beta}(F, A))$,*
(5) $(\mathbb{G}_{\alpha,\, \beta}(F, A)^c)^c = \mathbb{G}_{\beta,\, \alpha}(F, A)$.

Proof. (1) Clearly $\mathbb{G}_{\alpha,\beta}(F, A)$ is an IVIFSSRT.

(2) $$\mathbb{G}_{\alpha,\, \beta}(F, A) = \{\langle x, [\alpha^2 \underline{\mu}_{F(e)}(x), \alpha^2 \overline{\mu}_{F(e)}(x)], [\beta^2 \underline{\nu}_{F(e)}(x), \beta^2 \overline{\nu}_{F(e)}(x)]\rangle;\ x \in U,\ e \in A\},$$
$$\mathbb{G}_{\gamma,\, \beta}(F, A) = \{\langle x, [\gamma^2 \underline{\mu}_{F(e)}(x), \gamma^2 \overline{\mu}_{F(e)}(x)], [\beta^2 \underline{\nu}_{F(e)}(x), \beta^2 \overline{\nu}_{F(e)}(x)]\rangle; x \in U,\ e \in A\}.$$
Since $\alpha \leq \gamma,\ \alpha^2 \underline{\mu}_{F(e)}(x) \leq \gamma^2 \underline{\mu}_{F(e)}(x)$ and $\alpha^2 \overline{\mu}_{F(e)}(x) \leq \gamma^2 \overline{\mu}_{F(e)}(x)$.
Then $\mathbb{G}_{\alpha,\, \beta}(F, A) \Subset \mathbb{G}_{\gamma,\, \beta}(F, A)$.

(3) $$\mathbb{G}_{\alpha,\, \beta}(F, A) = \{\langle x, [\alpha^2 \underline{\mu}_{F(e)}(x), \alpha^2 \overline{\mu}_{F(e)}(x)], [\beta^2 \underline{\nu}_{F(e)}(x), \beta^2 \overline{\nu}_{F(e)}(x)]\rangle; x \in U,\ e \in A\},$$
$$\mathbb{G}_{\alpha,\, \gamma}(F, A) = \{\langle x, [\alpha^2 \underline{\mu}_{F(e)}(x), \alpha^2 \overline{\mu}_{F(e)}(x)], [\gamma^2 \underline{\nu}_{F(e)}(x), \gamma^2 \overline{\nu}_{F(e)}(x)]\rangle; x \in U,\ e \in A\}.$$
Since $\beta \leq \gamma,\ \beta^2 \underline{\nu}_{F(e)}(x) \leq \gamma^2 \underline{\nu}_{F(e)}(x),\ \beta^2 \overline{\nu}_{F(e)}(x) \leq \gamma^2 \overline{\nu}_{F(e)}(x)$.
Then $\mathbb{G}_{\alpha,\, \beta}(F, A) \Supset \mathbb{G}_{\alpha,\, \gamma}(F, A)$.

(4) $$\begin{aligned}\mathbb{G}_{\alpha,\, \beta}(\mathbb{G}_{\gamma,\, \delta}(F, A)) &= \{\langle x, [\alpha^2\gamma^2 \underline{\mu}_{F(e)}(x), \alpha^2\gamma^2 \overline{\mu}_{F(e)}(x)], \\ &\qquad [\beta^2\delta^2 \underline{\nu}_{F(e)}(x), \beta^2\delta^2 \overline{\nu}_{F(e)}(x)]\rangle; x \in U,\ e \in A\} \\ &= \{\langle x, [(\alpha\gamma)^2 \underline{\mu}_{F(e)}(x), (\alpha\gamma)^2 \overline{\mu}_{F(e)}(x)], \\ &\qquad [(\beta\delta)^2 \underline{\nu}_{F(e)}(x), (\beta\delta)^2 \overline{\nu}_{F(e)}(x)]\rangle; x \in U,\ e \in A\} \\ &= \mathbb{G}_{\alpha\gamma,\, \beta\delta}(F, A), \qquad (1)\end{aligned}$$

$$\begin{aligned}\mathbb{G}_{\gamma,\, \delta}(\mathbb{G}_{\alpha,\, \beta}(F, A)) &= \{\langle x, [\gamma^2\alpha^2 \underline{\mu}_{F(e)}(x), \gamma^2\alpha^2 \overline{\mu}_{F(e)}(x)], \\ &\qquad [\delta^2\beta^2 \underline{\nu}_{F(e)}(x), \delta^2\beta^2 \overline{\nu}_{F(e)}(x)]\rangle; x \in U,\ e \in A\} \\ &= \{\langle x, [(\gamma\alpha)^2 \underline{\mu}_{F(e)}(x), (\gamma\alpha)^2 \overline{\mu}_{F(e)}(x)], \\ &\qquad [(\delta\beta)^2 \underline{\nu}_{F(e)}(x), (\delta\beta)^2 \overline{\nu}_{F(e)}(x)]\rangle; x \in U,\ e \in A\} \\ &= \{\langle x, [(\alpha\gamma)^2 \underline{\mu}_{F(e)}(x), (\alpha\gamma)^2 \overline{\mu}_{F(e)}(x)], \\ &\qquad [(\beta\delta)^2 \underline{\nu}_{F(e)}(x), (\beta\delta)^2 \overline{\nu}_{F(e)}(x)]\rangle; x \in U,\ e \in A\} \\ &= \mathbb{G}_{\alpha\gamma,\, \beta\delta}(F, A). \qquad (2)\end{aligned}$$

From Eqs.(1) and (2), it follows that

$$\mathbb{G}_{\alpha,\,\beta}(\mathbb{G}_{\gamma,\,\delta}(F,A)) = G_{\alpha\gamma,\,\beta\delta}(F,A) = \mathbb{G}_{\gamma,\,\delta}(\mathbb{G}_{\alpha,\,\beta}(F,A)).$$

(5) $(F,A)^c = \{\langle x, [\underline{\nu}_{F(e)}(x), \overline{\nu}_{F(e)}(x)], [\underline{\mu}_{F(e)}(x), \overline{\mu}_{F(e)}(x)]\rangle; x \in U,\ e \in A\}$,

$$\mathbb{G}_{\alpha,\,\beta}(F,A)^c = \{\langle x, [\alpha^2\underline{\nu}_{F(e)}(x), \alpha^2\overline{\nu}_{F(e)}(x)], [\beta^2\underline{\mu}_{F(e)}(x), \beta^2\overline{\mu}_{F(e)}(x)]\rangle; x \in U,\ e \in A\},$$

$$(\mathbb{G}_{\alpha,\,\beta}(F,A)^c)^c = \{\langle x, [\beta^2\underline{\mu}_{F(e)}(x), \beta^2\overline{\mu}_{F(e)}(x)], [\alpha^2\underline{\nu}_{F(e)}(x), \alpha^2\overline{\nu}_{F(e)}(x)]\rangle; x \in U,\ e \in A\}$$

$$= \mathbb{G}_{\beta,\,\alpha}(F,A).$$

Then $(\mathbb{G}_{\alpha,\,\beta}(F,A)^c)^c = \mathbb{G}_{\beta,\,\alpha}(F,A)$.
Thus the proof is completed. □

5. Normalized Euclidean distance between IVIFSSRT

In this section we define the normalized Euclidean distance of IVIFSSRT and establish that it is a metric.

Definition 5.1. Let $U = \{x_1, x_2, ..., x_n\}$ be an universal set, $E = \{e_1, e_2, ..., e_m\}$ be a set of parameters and (F,A), (G,B) two IVIFSSRT on U. Then the normalized Euclidean distance between (F,A) and (G,B) is defined as

$$\mathcal{D}_{\mathcal{E}}\langle (F,A),(G,B)\rangle$$

$$= \Big\{\frac{1}{4mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\Big[(\underline{\mu}_{F(e_i)}(x_j) - \underline{\mu}_{G(e_i)}(x_j))^2 + (\overline{\mu}_{F(e_i)}(x_j) - \overline{\mu}_{G(e_i)}(x_j))^2 + (\underline{\nu}_{F(e_i)}(x_j) - \underline{\nu}_{G(e_i)}(x_j))^2 + (\overline{\nu}_{F(e_i)}(x_j) - \overline{\nu}_{G(e_i)}(x_j))^2 + (\underline{\pi}_{F(e_i)}(x_j) - \underline{\pi}_{G(e_i)}(x_j))^2 + (\overline{\pi}_{F(e_i)}(x_j) - \overline{\pi}_{G(e_i)}(x_j))^2\Big]\Big\}^{\frac{1}{2}}.$$

Theorem 5.2. *Let $IVIFSSRT(U)$ be the set of all IVIFSSRT over U. Then the distance function $\mathcal{D}_{\mathcal{E}}$ from $IVIFSSRT(U)$ to the set of non negative real numbers is a metric.*

Proof. Let (F,A), (G,B) and (H,C) be three IVIFSSRT over U.
(i) $\mathcal{D}_{\mathcal{E}}\langle (F,A),(G,B)\rangle > 0$ follows from Definition 5.1.
(ii) $\mathcal{D}_{\mathcal{E}}\langle (F,A),(G,B)\rangle = 0$

$$\Leftrightarrow (\underline{\mu}_{F(e_i)}(x_j) - \underline{\mu}_{G(e_i)}(x_j))^2 + (\overline{\mu}_{F(e_i)}(x_j) - \overline{\mu}_{G(e_i)}(x_j))^2 + (\underline{\nu}_{F(e_i)}(x_j) - \underline{\nu}_{G(e_i)}(x_j))^2 + (\overline{\nu}_{F(e_i)}(x_j) - \overline{\nu}_{G(e_i)}(x_j))^2 + (\underline{\pi}_{F(e_i)}(x_j) - \underline{\pi}_{G(e_i)}(x_j))^2 + (\overline{\pi}_{F(e_i)}(x_j) - \overline{\pi}_{G(e_i)}(x_j))^2 = 0$$

$$\Leftrightarrow \underline{\mu}_{F(e_i)}(x_j) = \underline{\mu}_{G(e_i)}(x_j),\ \overline{\mu}_{F(e_i)}(x_j) = \overline{\mu}_{G(e_i)}(x_j),$$

$$\underline{\nu}_{F(e_i)}(x_j) = \underline{\nu}_{G(e_i)}(x_j),\ \overline{\nu}_{F(e_i)}(x_j) = \overline{\nu}_{G(e_i)}(x_j)$$
$$\underline{\pi}_{F(e_i)}(x_j) = \underline{\pi}_{G(e_i)}(x_j) \text{ and } \overline{\pi}_{F(e_i)}(x_j) = \overline{\pi}_{G(e_i)}(x_j)$$
$$\Leftrightarrow (F, A) = (G, B).$$

(iii) Clearly, $\mathcal{D}_{\mathcal{E}}\langle (F, A), (G, B)\rangle = \mathcal{D}_{\mathcal{E}}\langle (G, B), (F, A)\rangle$.

(iv) Assume that (F, A), (G, B) and (H, C) are IVIFSSRT over U. Then for all $i \in \{1, 2, .., m\}$, $j \in \{1, 2, ..., n\}$,

$$\begin{aligned}
&(\underline{\mu}_{F(e_i)}(x_j) - \underline{\mu}_{G(e_i)}(x_j))^2 + (\overline{\mu}_{F(e_i)}(x_j) - \overline{\mu}_{G(e_i)}(x_j))^2 \\
&\quad +(\underline{\nu}_{F(e_i)}(x_j) - \underline{\nu}_{G(e_i)}(x_j))^2 + (\overline{\nu}_{F(e_i)}(x_j) - \overline{\nu}_{G(e_i)}(x_j))^2 \\
&\quad +(\underline{\pi}_{F(e_i)}(x_j) - \underline{\pi}_{G(e_i)}(x_j))^2 + (\overline{\pi}_{F(e_i)}(x_j) - \overline{\pi}_{G(e_i)}(x_j))^2 \\
&= (\underline{\mu}_{F(e_i)}(x_j) - \underline{\mu}_{H(e_i)}(x_j) + \underline{\mu}_{H(e_i)}(x_j) - \underline{\mu}_{G(e_i)}(x_j))^2 \\
&\quad +(\overline{\mu}_{F(e_i)}(x_j) - \overline{\mu}_{H(e_i)}(x_j) + \overline{\mu}_{H(e_i)}(x_j) - \overline{\mu}_{G(e_i)}(x_j))^2 \\
&\quad +(\underline{\nu}_{F(e_i)}(x_j) - \underline{\nu}_{H(e_i)}(x_j) + \underline{\nu}_{H(e_i)}(x_j) - \underline{\nu}_{G(e_i)}(x_j))^2 \\
&\quad +(\overline{\nu}_{F(e_i)}(x_j) - \overline{\nu}_{H(e_i)}(x_j) + \overline{\nu}_{H(e_i)}(x_j) - \overline{\nu}_{G(e_i)}(x_j))^2 \\
&\quad +(\underline{\pi}_{F(e_i)}(x_j) - \underline{\pi}_{H(e_i)}(x_j) + \underline{\pi}_{H(e_i)}(x_j) - \underline{\pi}_{G(e_i)}(x_j))^2 \\
&\quad +(\overline{\pi}_{F(e_i)}(x_j) - \overline{\pi}_{H(e_i)}(x_j) + \overline{\pi}_{H(e_i)}(x_j) - \overline{\pi}_{G(e_i)}(x_j))^2 \\
&\leq (\underline{\mu}_{F(e_i)}(x_j) - \underline{\mu}_{H(e_i)}(x_j))^2 + (\underline{\mu}_{H(e_i)}(x_j) - \underline{\mu}_{G(e_i)}(x_j))^2 \\
&\quad +(\overline{\mu}_{F(e_i)}(x_j) - \overline{\mu}_{H(e_i)}(x_j))^2 + (\overline{\mu}_{H(e_i)}(x_j) - \overline{\mu}_{G(e_i)}(x_j))^2 \\
&\quad +(\underline{\nu}_{F(e_i)}(x_j) - \underline{\nu}_{H(e_i)}(x_j))^2 + (\underline{\nu}_{H(e_i)}(x_j) - \underline{\nu}_{G(e_i)}(x_j))^2 \\
&\quad +(\overline{\nu}_{F(e_i)}(x_j) - \overline{\nu}_{H(e_i)}(x_j))^2 + (\overline{\nu}_{H(e_i)}(x_j) - \overline{\nu}_{G(e_i)}(x_j))^2 \\
&\quad +(\underline{\pi}_{F(e_i)}(x_j) - \underline{\pi}_{H(e_i)}(x_j))^2 + (\underline{\pi}_{H(e_i)}(x_j) - \underline{\pi}_{G(e_i)}(x_j))^2 \\
&\quad +(\overline{\pi}_{F(e_i)}(x_j) - \overline{\pi}_{H(e_i)}(x_j))^2 + (\overline{\pi}_{H(e_i)}(x_j) - \overline{\pi}_{G(e_i)}(x_j))^2 \\
&= \mathcal{D}_{\mathcal{E}}\langle (F, A), (H, C)\rangle + \mathcal{D}_{\mathcal{E}}\langle (H, C), (G, B)\rangle.
\end{aligned}$$

Thus, $\mathcal{D}_{\mathcal{E}}\langle (F, A), (G, B)\rangle \leq \mathcal{D}_{\mathcal{E}}\langle (F, A), (H, C)\rangle + \mathcal{D}_{\mathcal{E}}\langle (H, C), (G, B)\rangle$.
This shows that $\mathcal{D}_{\mathcal{E}}$ satisfies the triangle inequality.
So $\mathcal{D}_{\mathcal{E}}$ is a metric. □

6. Pattern recognition problem

In this section, we develop a pattern recognition problem based on distance measure of IVIFSSRT. We present an example to illustrate the working of this algorithm.

Step 1. Construct an IVIFSSRT $(\mathcal{S}, E)$ over U based on expert evaluation and this can be considered as the known pattern.

Step 2. Construct an IVIFSSRT (F_i, E), $\forall\ i = 1, 2, 3, ...$ over U based on the data available for the unknown pattern.

Step 3. Calculate the normalized Euclidean distance between $(\mathcal{S}, E)$ and (F_i, E).

Step 4. The pattern with less normalized Euclidean distance between $(\mathcal{S}, E)$ and (F_i, E) the pattern is the best suitable pattern.

Example 6.1. Four teams U = {Team-1, Team-2, Team-3, Team-4} of an organization are to assess three institutions A, B and C using the parameters $E = \{e_1, e_2, e_3, e_4, e_5\}$ where e_1 = board of studies, e_2 = academic audit report, e_3 =

internal assessment marks, e_4 = library facilities and e_5 = computers and internet details. The organization has to select the best institution depending upon the above parameters.

Step 1. The construction of known pattern is as follows (Table 1).

U	Team-1	Team-2	Team-3	Team-4
e_1	$[0.58, 0.63], [0.36, 0.42]$	$[0.61, 0.66], [0.31, 0.37]$	$[0.51, 0.57], [0.43, 0.46]$	$[0.79, 0.83], [0.28, 0.35]$
e_2	$[0.57, 0.62], [0.34, 0.39]$	$[0.59, 0.64], [0.46, 0.49]$	$[0.53, 0.59], [0.44, 0.51]$	$[0.54, 0.62], [0.35, 0.41]$
e_3	$[0.44, 0.54], [0.46, 0.49]$	$[0.42, 0.49], [0.56, 0.59]$	$[0.54, 0.59], [0.39, 0.45]$	$[0.46, 0.56], [0.38, 0.46]$
e_4	$[0.66, 0.72], [0.27, 0.29]$	$[0.63, 0.66], [0.35, 0.39]$	$[0.65, 0.74], [0.24, 0.27]$	$[0.59, 0.62], [0.36, 0.39]$
e_5	$[0.54, 0.63], [0.31, 0.39]$	$[0.59, 0.63], [0.29, 0.39]$	$[0.61, 0.66], [0.29, 0.35]$	$[0.67, 0.72], [0.25, 0.29]$

TABLE 1. IVIFSSRT $(\mathcal{S}, E)$ over U is the data from the previous records of the organization for the best institution.

Step 2. The construction of unknown patterns are as follows (Tables 2, 3 and 4).

U	Team-1	Team-2	Team-3	Team-4
e_1	$[0.57, 0.62], [0.38, 0.44]$	$[0.6, 0.65], [0.33, 0.39]$	$[0.5, 0.56], [0.45, 0.48]$	$[0.78, 0.82], [0.3, 0.37]$
e_2	$[0.56, 0.61], [0.36, 0.41]$	$[0.58, 0.63], [0.48, 0.51]$	$[0.52, 0.58], [0.46, 0.53]$	$[0.53, 0.61], [0.37, 0.43]$
e_3	$[0.43, 0.53], [0.48, 0.51]$	$[0.41, 0.48], [0.58, 0.61]$	$[0.53, 0.58], [0.41, 0.47]$	$[0.45, 0.55], [0.4, 0.48]$
e_4	$[0.65, 0.71], [0.29, 0.31]$	$[0.62, 0.65], [0.37, 0.41]$	$[0.64, 0.73], [0.26, 0.29]$	$[0.58, 0.61], [0.38, 0.41]$
e_5	$[0.53, 0.62], [0.33, 0.41]$	$[0.58, 0.62], [0.31, 0.41]$	$[0.6, 0.65], [0.31, 0.37]$	$[0.66, 0.71], [0.27, 0.31]$

TABLE 2. IVIFSSRT (F_1, E) over U gives the data collected by the organization for the Institution-A.

U	Team-1	Team-2	Team-3	Team-4
e_1	$[0.56, 0.61], [0.37, 0.43]$	$[0.59, 0.64], [0.32, 0.38]$	$[0.49, 0.55], [0.44, 0.47]$	$[0.77, 0.81], [0.29, 0.36]$
e_2	$[0.55, 0.6], [0.35, 0.4]$	$[0.57, 0.62], [0.47, 0.5]$	$[0.51, 0.57], [0.45, 0.52]$	$[0.52, 0.6], [0.36, 0.42]$
e_3	$[0.42, 0.52], [0.47, 0.5]$	$[0.4, 0.47], [0.57, 0.6]$	$[0.52, 0.57], [0.4, 0.46]$	$[0.44, 0.54], [0.39, 0.47]$
e_4	$[0.64, 0.7], [0.28, 0.3]$	$[0.61, 0.64], [0.36, 0.4]$	$[0.63, 0.72], [0.25, 0.28]$	$[0.57, 0.6], [0.37, 0.4]$
e_5	$[0.52, 0.61], [0.32, 0.4]$	$[0.57, 0.61], [0.3, 0.4]$	$[0.59, 0.64], [0.3, 0.36]$	$[0.65, 0.7], [0.26, 0.3]$

TABLE 3. IVIFSSRT (F_2, E) over U gives the data collected by the organization for the Institution-B.

U	Team-1	Team-2	Team-3	Team-4
e_1	$[0.54, 0.59], [0.41, 0.47]$	$[0.57, 0.62], [0.36, 0.42]$	$[0.47, 0.53], [0.48, 0.51]$	$[0.75, 0.79], [0.33, 0.4]$
e_2	$[0.53, 0.58], [0.39, 0.44]$	$[0.55, 0.6], [0.51, 0.54]$	$[0.49, 0.55], [0.49, 0.56]$	$[0.5, 0.58], [0.4, 0.46]$
e_3	$[0.4, 0.5], [0.51, 0.54]$	$[0.38, 0.45], [0.61, 0.64]$	$[0.5, 0.55], [0.44, 0.5]$	$[0.42, 0.52], [0.43, 0.51]$
e_4	$[0.62, 0.68], [0.32, 0.34]$	$[0.59, 0.62], [0.4, 0.44]$	$[0.61, 0.7], [0.29, 0.32]$	$[0.55, 0.58], [0.41, 0.44]$
e_5	$[0.5, 0.59], [0.36, 0.44]$	$[0.55, 0.59], [0.34, 0.44]$	$[0.57, 0.62], [0.34, 0.4]$	$[0.63, 0.68], [0.3, 0.34]$

TABLE 4. IVIFSSRT (F_3, E) over U gives the data collected by the organization for the Institution-C.

Step 3. The normalized Euclidean distance between $\mathcal{D}_{\mathcal{E}}\langle(\mathcal{S}, E), (F_i, E)\rangle$ is calculated using Definition 5.1. The values evaluated are as follows:

$$\mathcal{D}_{\mathcal{E}}\langle(\mathcal{S}, E), (F_1, E)\rangle = 0.0167,$$
$$\mathcal{D}_{\mathcal{E}}\langle(\mathcal{S}, E), (F_2, E)\rangle = 0.0161,$$
$$\mathcal{D}_{\mathcal{E}}\langle(\mathcal{S}, E), (F_3, E)\rangle = 0.0451.$$

Step 4. We observe that, $\mathcal{D}_{\mathcal{E}}\langle(\mathcal{S}, E), (F_2, E)\rangle$ is the least distance. Hence Institution-B is the best.

7. Conclusion

In this paper, we have defined two new operators on interval valued intuitionistic fuzzy soft set of root type and discussed the properties of these operators. We have introduced the notion of normalized Euclidean distance between interval valued intuitionistic fuzzy soft sets of root type and developed a new decision making technique for solving pattern recognition problems. Finally, we have provided an example for illustrating this new technique.

Acknowledgement: The second author is thankful to the University Grants Commission, New Delhi-110021, India, for providing BSR fellowship under grant no: F.4-1/2006(BSR)/7-254/2009 (BSR) dated 26.02.2013

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