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Homeomorphism on intuitionistic topological spaces

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Abstract. The purpose of this paper is to show the existence of open and closed maps in intuitionistic topological spaces. Also intuitionistic generalized preregular homeomorphism and intuitionistic generalized preregular *- homeomorphism were introduced and several properties are outlined.

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Keywords: Igpr homeomorphism, Igpr*-homeomorphism, Igpr open map, Igpr closed maps.

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1. Introduction

After the introduction of the concept of fuzzy set by Zadeh [11], Atanassov [1] proposed the concept of intuitionistic fuzzy sets. Coker [2] introduced the concept of intuitionistic sets and intuitionistic points in 1996 and intuitionistic fuzzy topological spaces [3] in 1997. Also in 2000, Coker [4] developed the concept of intuitionistic topological spaces (also called intuitionistic fuzzy special topological space by Selma Ozcag and Dogan Coker [6] in 1998) with intuitionistic sets and investigated basic properties of continuous functions and compactness. Selma Ozcag and Dogan Coker [6, 7] also examined connectedness in intuitionistic topological spaces. Later, several researchers [10] studied some weak forms of intuitionistic topological spaces. Since, homeomorphism plays an important role in topology, in this paper, we introduce and study few properties of Igpr homeomorphism and Igpr*-homeomorphism in intuitionistic topological space. Also the relation between Igpr open maps, Igpr closed maps and Igpr homeomorphism were studied.

2. Preliminaries

We recall some definitions and results which are useful for this sequel. Throughout the present study, a space X means an intuitionistic topological space (X, τ) and Y means an intuitionistic topological space (Y, σ) unless otherwise mentioned.

Definition 2.1 ([2]). Let X be a nonempty set. An intuitionistic set (IS for short) A is an object having the form A $= < X, A_1, A_2 >$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of members of A, while A_2 is called the set of non-members of A.

Definition 2.2 ([2]). Let X be a nonempty set and let A,B are intuitionistic sets in the form A $= < X, A_1, A_2 >$, B $= < X, B_1, B_2 >$ respectively. Then
(i) A $\subseteq$ B iff $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$.
(ii) A = B iff A $\subseteq$ B and B $\subseteq$ A.
(iii) $\overline{A} = < X, A_2, A_1 >$.
(iv) A - B = $A \cap \overline{B}$.
(v) $\underset{\sim}{\phi} = < X, \phi, X >$, $\underset{\sim}{X} = < X, X, \phi >$.
(vi) A $\cup$ B $= < X, A_1 \cup B_1, A_2 \cap B_2 >$.
(vii) A $\cap$ B $= < X, A_1 \cap B_1, A_2 \cup B_2 >$.
Furthermore, let $\{A_i : i \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_i = < X, A_i^{(1)}, A_i^{(2)} >$. Then
(viii) $\bigcap A_i = < X, \cap A_i^{(1)}, \cup A_i^{(2)} >$.
(xi) $\bigcup A_i = < X, \cup A_i^{(1)}, \cap A_i^{(2)} >$.

Definition 2.3 ([4]). An intuitionistic topology (IT for short) on a nonempty set X is a family τ of IS's in X containing $\underset{\sim}{\phi}$, $\underset{\sim}{X}$ and closed under finite infima and arbitrary suprema. The pair (X, τ) is called an intuitionistic topological space $(ITS\ for\ short)$. Any intuitionistic set in τ is known as an intuitionistic open set $(IOS\ for\ short)$ in X and the complement of IOS is called intuitionistic closed set $(ICS\ for\ short)$.

Definition 2.4 ([2]). Let (X, τ) be an ITS and A $= < X, A_1, A_2 >$ be an IS in X. Then the closure and interior of A are defined as

$$\text{Icl(A)} = \bigcap \{\text{K : K is an ICS in X and A} \subseteq \text{K}\}$$

and

$$\text{Iint(A)} = \bigcup \{\text{G : G is an IOS in X and G} \subseteq \text{A}\}.$$

It can be shown that Icl(A)is an ICS and Iint(A) is an IOS in X and A is an ICS in X iff Icl(A)= A and is an IOS in X iff Iint(A) = A.

Definition 2.5. Let (X, τ) be an ITS. An intuitionistic set A of X is said to be intuitionistic regular open (intuitionistic regular closed) if A = Iint$(Icl(A))$ $(A = Icl(Iint(A)))$.

Definition 2.6 ([4]). Let X,Y be two non-empty sets and f :X $\to$ Y be a function.
(i) If B $= < X, B_1, B_2 >$ is an IS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$,is the IS in X defined by $f^{-1}(B) = < X, f^{-1}(B_1), f^{-1}(B_2) >$.

(ii) If A = $< X, A_1, A_2 >$ is an IS in X, then the image of A under f, denoted by $f(A)$, is the IS in Y defined by $f(B) = < Y, f(A_1), f_-(A_2) >$, where $f_-(A_2) = (f((A_2)^c))^c$.

Definition 2.7 ([4]). Let (X, τ) and (Y, σ) be two intuitionistic topological spaces and f :X → Y be a function. Then f is said to be continuous iff the preimage of each ICS in Y is intuitionistic closed in X.

Corollary 2.8 ([3, 4]). *Let A, A_i $(i \in J)$ be IS's in X, B, B_j $(j \in K)$ be IS's in Y and f : $(X, \tau) \to (Y, \sigma)$ be a function. Then*

(1) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$, $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$.

(2) $A \subseteq f^{-1}(f(A))$ *and if f is injective, then* $A = f^{-1}(f(A))$.

(3) $f(f^{-1}(B)) \subseteq B$ *and if f is surjective, then* $f(f^{-1}(B)) = B$.

(4) $f(\cup A_i) = \cup f(A_i)$, $f(\cap A_i) \subseteq \cap f(A_i)$ *and if f is injective, then* $f(\cap A_i) = \cap f(A_i)$.

(5) $f^{-1}(\cup B_i) = \cup f^{-1}(B_i)$, $f^{-1}(\cap B_i) \subseteq \cap f^{-1}(B_i)$.

(6) *If f is surjective, then* $\overline{f(A)} \subseteq f(\overline{A})$. *Further if f is injective then* $\overline{f(A)} = f(\overline{A})$.

(7) $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$.

Proposition 2.9 ([4]). *The following statements are equivalent :*

(1) *f :* $(X, \tau) \to (Y, \sigma)$ *is continuous.*

(2) $f^{-1}(int(B)) \subseteq int(f^{-1}(B))$ *for each IS B in Y.*

(3) $cl(f^{-1}(int(B))) \subseteq f^{-1}(cl(B))$ *for each IS B in Y.*

Definition 2.10 ([5]). Let (X, τ) be an ITS and let $A = < X, A_1, A_2 >$ be an intuitionistic set. Then A is said to be intuitionistic generalized pre regular closed (in short, Igpr closed) if Ipcl(A) ⊆ U whenever A ⊆ U and U is intuitionistic regular open in X. The class of all Igpr closed subsets of (X, τ) is denoted by IGPRC(τ).

The complement of intuitionistic generalized pre regular closed sets are intuitionistic generalized pre regular open (in short, Igpr open) and the class of all Igpr open sets of (X, τ) is denoted by IGPRO(τ).

Definition 2.11 ([5]). Let (X, τ) be an ITS and let A = $< X, A_1, A_2 >$ be the subset of X . Then

$$\text{Igprcl}(A) = \cap\{F : F \text{ is Igpr closed in } X \text{ and } A \subseteq F\}$$

and

$$\text{Igprint}(A) = \cup\{G : G \text{ is Igpr open in } X \text{ and } G \subseteq A\}.$$

Definition 2.12 ([8, 9]). Let (X, τ) and (Y, σ) be two intuitionistic topological spaces and f :X → Y be a function. Then f is said to be Igpr continuous if the preimage of every intuitionistic closed set of Y is Igpr closed set in X.

Definition 2.13 ([8]). Let (X, τ) and (Y, σ) be two intuitionistic topological spaces and f :X → Y be a function. Then f is said to be Igpr-irresolute if the preimage of every Igprclosed set of Y is Igpr closed in X.

3. Igpr open and Igpr closed maps

Definition 3.1. A map $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called intuitionistic open (closed) if the image $f(A)$ is intuitionistic open (closed) in Y for every intuitionistic open (closed) set in X .

Definition 3.2. A map $f : (X, \tau) \longrightarrow (Y, \sigma)$ is said to be intuitionistic generalized pre regular closed (Igpr closed) if the image $f(A)$ is intuitionistic generalized pre regular closed (Igpr closed) in Y for every intuitionistic closed set in X .

Example 3.3. Let X = $\{a, b\}$ = Y, τ =$\{\underset{\sim}{\phi}, \underset{\sim}{X}, < X, \{a\}, \phi >, < X, \phi, \{b\} >\}$ and σ =$\{\underset{\sim}{\phi}, \underset{\sim}{X}, < X, \{b\}, \phi >, < X, \phi, \{a\} >\}$. Define $f : (X, \tau) \longrightarrow (Y, \sigma)$ by $f(a) = b$, $f(b) = a$. Then the map f is Igpr open.

Remark 3.4. If (Y, σ) is an intuitionistic discrete space, then f is a closed map of all types.(intuitionistic semi closed map, intuitionistic pre closed map,...)

Remark 3.5. If (Y, σ) is an intuitionistic indiscrete space, then f is an Igpr closed map.

Proof. If (Y, σ) is an intuitionistic indiscrete space, then all the intuitionistic subsets of Y are Igpr open and thus Igpr closed. So f is an Igpr closed map. □

Theorem 3.6. *A map $f : (X, \tau) \longrightarrow (Y, \sigma)$ is Igpr closed if and only if for each intuitionistic subset A of (Y, σ) and for each intuitionistic open set U containing $f^{-1}(A)$ there is an Igpr open set V of (Y, σ) such that $A \subset V$ and $f^{-1}(A) \subset U$.*

Proof. Suppose that f is an Igpr closed map. Let A ⊂ Y and U be an intuitionistic open subset of (X, τ) such that $f^{-1}(A) \subset$ U. Then V = $(f(U^c))^c$ is an Igpr open set containing A such that $f^{-1}(A) \subset$U.

Conversely, let A be an intuitionistic closed set of (X, τ). Then $f^{-1}((f(A))^c) \subset A^c$ and A^c is intuitionistic open. By assumption, there exists an Igpr open set V of (Y, σ) such that $f(A)^c \subset$ V and $f^{-1}(V) \subset A^c$. Thus A $\subset (f^{-1}(V))^c$. So $V^c \subset f(A) \subset f(f^{-1}(V)^c) \subset V^c$. Hence $f(A) = V^c$. Since V^c is Igpr closed in (Y, σ), $f(A)$ is Igpr closed in (Y, σ). Therefore f is Igpr closed. □

Theorem 3.7. *A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is Igpr open if and only if for any intuitionistic subset B of (Y, σ) and for any intuitionistic closed set S containing $f^{-1}(B)$, there exists an Igpr closed set A of (Y, σ) containing B such that $f^{-1}(A) \subseteq S$.*

Proof. Similar to Theorem 3.6. □

Theorem 3.8. *A map $f : X \longrightarrow Y$ is Igpr closed iff $Igprcl(f(A)) \subset f(Icl(A))$.*

Proof. Let A ⊂ X and f : $X \longrightarrow Y$ be Igpr closed. Then $f(Icl(A))$ is Igpr closed in Y. Thus $Igprcl(f(Icl(A))) = f(Icl(A))$. Since $f(A) \subset f(Icl(A))$, $Igprcl(f(A)) \subset Igprcl(f(Icl(A))) \subset f(Icl(A))$ for every intuitionistic subset A of X.

Conversely, let A be any intuitionistic closed set in (X, τ). Then A = $Icl(A)$. Thus $f(A)$= $f(Icl(A)) \supseteq Igprcl(f(A))$, by hypothesis. Since $f(A) \subset Igprcl(f(A))$, $f(A) = Igprcl(f(A))$. So $f(A)$ is Igpr closed. Hence f is Igpr closed. □

Theorem 3.9. *If $f : X \longrightarrow Y$ be Igpr open, then $f(Iint(A)) \subset Igprint(f(A))$.*

Proof. Let A $\subset$ X and f : $X \longrightarrow Y$ be Igpr open map. Then $f(Iint(A))$ is Igpr open in Y. Thus $Igprint(f(Iint(A))) \subseteq Igprint(f(A))$. Since $f(Iint(A)$ is Igpr open in Y, $Igprint(f(IintA)) \subset f(Iint(A))$. So $f(Iint(A)) \subset Igprint(f(A))$. □

Proposition 3.10. *If for every intuitionistic subset A of X, $Icl(Iint(f(A))) \subset f(Icl(A))$, then $f : (X, \tau) \longrightarrow (Y, \sigma)$ is Igpr closed.*

Proof. If $Icl(Iint(f(A))) \subset f(Icl(A))$ for every intuitionistic subset A of X, then f is intuitionistic preclosed and thus Igpr closed. □

4. Igpr homeomorphism in intuitionistic topological spaces

Definition 4.1. A bijection $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called intuitionistic homeomorphism if f is both intuitionistic continuous and intuitionistic open.

Definition 4.2. A bijection $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called intuitionistic Igpr homeomorphism (resp. Igp homeomorphism, Ig homeomorphism) if f is both Igpr continuous (resp. Igp continuous, Ig continuous) and Igpr open (resp. Igp open, Ig open).

Example 4.3. Let X = $\{a, b\}$ = Y, $\tau =\{\underset{\sim}{\phi}, \underset{\sim}{X}, < X, \{a\}, \phi >, < X, \phi, \{b\} >\}$ and $\sigma =\{\underset{\sim}{\phi}, \underset{\sim}{X}, < X, \{b\}, \phi >, < X, \phi, \{a\} >\}$. Define f: $(X, \tau) \longrightarrow (Y, \sigma)$ by $f(a) = b$, $f(b) = a$. Then the map f is bijective, Igpr continuous and Igpr open. So f is Igpr homeomorphism.

Remark 4.4. Every intuitionistic homeomorphism is an Igpr homeomorphism but the converse is not true.

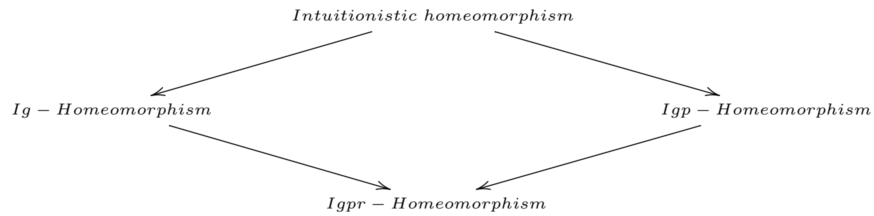
Example 4.5. Let X = $\{a, b\}$ = Y, $\tau =\{\underset{\sim}{\phi}, \underset{\sim}{X}, < X, \{a\}, \phi >, < X, \{a\}, \{b\} >\}$ and $\sigma =\{\underset{\sim}{\phi}, \underset{\sim}{Y}, < Y, \{b\}, \{a\} >, < Y, \{a\}, \{b\} >\}$. Define f: $(X, \tau) \longrightarrow (Y, \sigma)$ by $f(a) = b$, $f(b) = a$. Since the pre-image of $< Y, \{a\}, \{b\} >$ in (Y, σ) is not open in (X, τ) under f, it is not intuitionistic homeomorphism but Igpr homeomorphism.

Remark 4.6. (1) Every Igp homeomorphism is Igpr homeomorphism but the converse need not be true.

(2) Every Ig homeomorphism is Igpr homeomorphism but the converse need not be true.

Example 4.7. Let X = $\{a, b\}$ = Y, $\tau = \{\underset{\sim}{\phi}, \underset{\sim}{X}, < X, \{a\}, \phi >, < X, \{b\}, \phi >, < X, \phi, \phi >\}$ and $\sigma = \{\underset{\sim}{\phi}, \underset{\sim}{Y}, < Y, \{a\}, \phi >, < Y, \{b\}, \phi >, < Y, \{b\}, \{a\} >, < Y, \phi, \{a\}, >\}$. Define $f : (X, \tau) \longrightarrow (Y, \sigma)$ by $f(a) = a$, $f(b) = b$. Since IGPRC(X, τ) = IGPRC(Y, σ) = $\mathbb{IP}(X)$, f is Igpr homeomorphism but the intuitionistic set $< Y, \{a\}, \phi >$ which is Igp closed and Ig closed in (Y, σ) has no pre-image in (X, τ). Thus it is not Ig homeomorphism and Igp homeomorphism.

Hence, we have the followind diagram.



But none of the reverse implication is true.

Proposition 4.8. *For a bijective map* $f:(X,\tau)\longrightarrow(Y,\sigma)$*, the following are equivalent :*

(1) f *is Igpr open.*
(2) f *is Igpr closed.*
(3) $f^{-1}:(Y,\sigma)\longrightarrow(X,\tau)$ *is Igpr continuous.*

Proof. (1) $\Longrightarrow$ (2): Let $A=<X,A_1,A_2>$ be intuitionistic closed in (X,τ). Then $X-A=<X,A_2,A_1>$ is intuitionistic open in X. Since f is an Igpr open map, f$(X-A)$ is Igpr open in (Y,σ). Thus

$$\text{f}(<X,A_2,A_1>)=<Y,f(A_2),f_{-}(A_1)>=<Y,f(A_2),Y-f(X-A_1)>$$

is Igpr open in Y. So $<Y,Y-f(X-A_1),f(A_2)>$ is Igpr closed in Y. Since $Y-f(X-A_1)=A_1, <Y,Y-f(X-A_1),f(A_2)>=<Y,f(A_1),f(A_2)>)$ is Igpr closed in Y. Hence f is an Igpr closed map.

(2) $\Longrightarrow$ (3): Let A be intuitionistic closed in (X,τ). Since f is Igpr closed, f(A) is Igpr closed in (Y,σ). Since f is bijective $f(A)=(f^{-1})^{-1}(A)$, f^{-1} is Igpr continuous.

(3) $\Longrightarrow$ (1): Let A be intuitionistic open in (X,τ). By hypothesis, $(f^{-1})^{-1}(A)$ is Igpr open in Y. That is, $f(A)$ is Igpr open in Y. □

Proposition 4.9. *For a bijective Igpr continuous map* $f:(X,\tau)\longrightarrow(Y,\sigma)$*, the following are equivalent:*

(1) f *is Igpr open.*
(2) f *is Igpr homeomorphism.*
(3) f *is Igpr closed.*

Proof. (1) $\Longrightarrow$ (2): Since f is intuitionistic bijective, Igpr continuous and Igpr open map, by definition, f is an Igpr homeomorphism.

(2) $\Longrightarrow$ (3): Let f be an Igpr homeomorphism. Then f is Igpr open. By proposition 4.8, f is Igpr closed.

(3) $\Longrightarrow$ (1): Follows from proposition 4.8 □

Definition 4.10. A map f : $(X,\tau)\longrightarrow(Y,\sigma)$ is called intuitionistic generalised pre regular *-open ($resp.\ Igpr^*-open$) if $f(U)$ is Igpr open in (Y,σ) for every Igpr open set U of (X,τ). A map f : $(X,\tau)\longrightarrow(Y,\sigma)$ is called $Igpr^*$-closed if $f(U)$ is Igpr closed in (Y,σ) for every Igpr closed set U of (X,τ).

Proposition 4.11. *For a bijective map* $f:(X,\tau)\longrightarrow(Y,\sigma)$*, the following statements are equivalent.*

(1) $f^{-1}:(X,\tau)\longrightarrow(Y,\sigma)$ *is Igpr-irresolute.*

(2) *f is $Igpr^*$-open.*
(3) *f is $Igpr^*$-closed.*

Proof. (1) $\Longrightarrow$ (2): Let $A = < X, A_1, A_2 >$ be Igpr open in (X, τ). By (1), $(f^{-1})^{-1}(A) = f(A)$ is Igpr open in (Y, σ). So, f is an $Igpr^*$-open map.

(2) $\Longrightarrow$ (3): Let A be Igpr closed in (X, τ). Then $X - A = < X, A_2, A_1 >$ is Igpr open in X. By (2),

$$\begin{aligned} f(X - A) &= f(< X, A_2, A_1 > \\ &=< Y, f(A_2), f_-(A_1) > \\ &=< Y, f(A_2), Y - f(X - A_1) > \end{aligned}$$

is Igpr open in Y. Thus $< Y, Y - f(X - A_1), f(A_2) >$ is Igpr closed in Y. Since $Y - f(X - A_1) = A_1$, $< Y, Y - f(X - A_1), f(A_2) > = < Y, f(A_1), f(A_2) >$). So $f(A)$ is Igpr closed in (Y, σ). Hence f is an $Igpr^*$-closed map.

(3) $\Longrightarrow$ (1): Let A be Igpr closed in (X, τ). By (3), $f(A)$ is Igpr closed in (Y, σ). But $f(A) = (f^{-1})^{-1}(A)$. Thus f^{-1} is Igpr-irresolute. □

Definition 4.12. A bijection $f : (X, \tau) \longrightarrow (Y, \sigma)$ is said to be Igpr*-homeomorphism if f and f^{-1} are Igpr-irresolute.

Proposition 4.13. *Every Igpr*-homeomorphism is Igpr homeomorphism but not conversely.*

Example 4.14. Let X $= \{a, b, c\} =$ Y, $\tau = \{\underset{\sim}{\phi}, \underset{\sim}{X}, < X, \phi, \{a\} >\}$ and $\sigma = \{\underset{\sim}{\phi}, \underset{\sim}{Y}, < Y, \phi, \{a\} >, < Y, \{a\}, \phi >\}$. Define $f : (X, \tau) \longrightarrow (Y, \sigma)$ by $f(a) = b$, $f(b) = c$ and $f(c) = a$. Then IGPRC$(X, \tau) = \mathbb{IP}(X)$ except $< X, \phi, \{a\} >$ and IGPRC$(Y, \sigma) = \mathbb{IP}(X)$. Then f is Igpr continuous and Igpr open but is not Igpr-irresolute under f because $< X, \phi, \{b\} >$ has no preimage in (X, τ) . Thus Igpr homeomorphism does not implies $Igpr^*$-homeomorphism.

Remark 4.15. Igpr*-homeomorphism and intuitionistic homeomorphism are independent.

Example 4.16. Let X $= \{a, b, c\} =$ Y, $\tau = \{\underset{\sim}{\phi}, \underset{\sim}{X}, < X, \{a\}, \{b, c\} >\}$ and $\sigma = \{\underset{\sim}{\phi}, \underset{\sim}{Y}, < Y, \phi, \{a\} >, < Y, \{a\}, \phi >\}$. Define $f : (X, \tau) \longrightarrow (Y, \sigma)$ by $f(a) = b$, $f(b) = c$ and $f(c) = a$. Since IGPRC$(X, \tau) =$ IGPRC$(Y, \tau) = \mathbb{IP}(X)$, f is $Igpr^*$-homeomorphism but not intuitionistic homeomorphism.

The intuitionistic topological space (X, τ) and (Y, σ) are Igpr homeomorphic if there exist an Igpr homeomorphism from (X, τ) onto (Y, σ). The family of all Igpr homeomorphisms (resp. $Igpr^*$-homeomorphism, Intuitionistic homeomorphism) from (X, τ) onto itself is denoted by Igprh(X, τ) (resp. $Igpr^*h(X, \tau), Ih(X, \tau)$).

Proposition 4.17. *For any intuitionistic topological space (X, τ), $Igpr^*h(X, \tau) \subset Igprh(X, \tau)$.*

Proof. Since every Igpr-irresolute function is Igpr continuous and every $Igpr^*$-open map is Igpr open, $Igpr^*h(X, \tau) \subset Igprh(X, \tau)$. □

Proposition 4.18. *If $f : (X, \tau) \longrightarrow (Y, \sigma)$ is Igpr-irresolute, then for every intuitionistic subset A of X, $f(Igprcl(A)) \subset Ipcl(f(A))$.*

Proof. For every A ⊂ X, $Ipcl(f(A))$ is Igpr closed in Y. By hypothesis, $f^{-1}(Ipcl(f(A)))$ is Igpr closed in X. Since A ⊂ $f^{-1}(f(A)) \subset f^{-1}(Ipcl(f(A)))$, by the definition, we have $Igprcl(A) \subset f^{-1}(Ipcl(f(A)))$. Thus $f(Igprcl(A)) \subset Ipcl(f(A))$. □

Proposition 4.19. *If* $f : (X, \tau) \longrightarrow (Y, \sigma)$ *is Igpr-irresolute, then for every intuitionistic subset B of Y,* $Igprcl(f^{-1}(B)) \subset f^{-1}(Ipcl(f(B)))$.

Proof. Since $Ipcl(B)$ is Igpr closed in Y, by hypothesis, $f^{-1}(Ipcl(B))$ is Igpr closed in X. Since $f^{-1}(B) \subset f^{-1}(Ipcl(B))$, $Igprcl(f^{-1}(B) \subset f^{-1}(Ipcl(B))$. □

Proposition 4.20. *If* $f : (X, \tau) \longrightarrow (Y, \sigma)$ *is* $Igpr^*$*-closed and* $IGPRC(X)$ *is Igpr closed under intersections, then for every* $A \subset X$, $Igprcl(f(A)) \subseteq f(Igprcl(A))$.

Proof. For every A ⊂ X, Igprcl(A) is Igpr closed in X, by hypothesis. Since f is $Igpr^*$-closed, $f(Igprcl(A))$ is Igpr closed in Y. Since $f(A) \subset f(Igprcl(A))$, $Igprcl(f(A)) \subseteq f(Igprcl(A))$. □

Theorem 4.21. *Let* (X, τ) *be an intuitionistic topological space such that* $\tau_g = \{V \subset X / Igprcl(X - V) = X - V\}$. *Then every Igpr closed set is intuitionistic preclosed if and only if* $\tau_g = IPO(X, \tau)$ *holds.*

Proposition 4.22. *If* $f : (X, \tau) \longrightarrow (Y, \sigma)$ *is* $Igpr^*$*-homeomorphism with the property* $\sigma_g = IPO(Y, \sigma)$ *and* $IGPRC(X)$ *is Igpr closed under intersections, then* $Igprcl(f(A)) = f(Igprcl(A))$.

Proof. Since f is $Igpr^*$-closed, by Proposition 4.20, $Igprcl(f(A)) \subseteq f(Igprcl(A))$. Since f is Igpr-irresolute, $f(Igprcl(A)) \subset Ipcl(f(A))$, by Proposition 4.18. Thus $Igprcl(f(A)) \subset$ Ipcl$(f(A))$. By hypothesis, $Ipcl(f(A)) = Igprcl(f(A))$ (by Theorem 4.21). So $Igprcl(f(A)) \subset f(Igprcl(A)) \subset Igprcl(f(A))$. Hence $Igprcl(f(A)) = f(Igprcl(A))$. □

Corollary 4.23. *If* $f : (X, \tau) \longrightarrow (Y, \sigma)$ *is* $Igpr^*$*-homeomorphism with the property* $\sigma_g = IPO(Y, \sigma)$ *and* $IGPRC(X)$ *is Igpr closed under intersections, then* $Igprint(f(A)) = f(Igprint(A))$ *for all* $A \subset X$.

Proof. For any intuitionistic set A ⊂ X, $Igprint(A) = (Igprcl(A^c))^c$. Then

$$\begin{aligned} f(Igprint(A)) &= f(Igprcl(A^c))^c \\ &= [f(Igprcl(A^c))]^c = [Igprcl(f(A^c))]^c \\ &= [Igprcl(f(A))^c]^c = Igprint(f(A)). \end{aligned}$$

□

Corollary 4.24. *If* $f : (X, \tau) \longrightarrow (Y, \sigma)$ *is* $Igpr^*$*-homeomorphism, then* $f^{-1}(Igprint(A)) = Igprint(f^{-1}(A))$ *for all* $A \subset X$.

Proof. Since f^{-1}:$(Y, \sigma) \longrightarrow (X, \tau)$ is $Igpr^*$-homeomorphism, the proof follows from previous corollary. □

Proposition 4.25. *If* $f : (X, \tau) \longrightarrow (Y, \sigma)$ *is* $Igpr^*$*-closed , Igpr homeomorphism and* $IGPRC(X)$ *is Igpr closed under intersections with* $\sigma_g = \sigma$*,then* $Igprcl(f(A)) = f(Igprcl(A))$.

Proof. Since f is Igpr continuous, $f(Igprcl(A)) \subset \text{Icl}(f(A))$. Since f is $Igpr^*$-closed by Proposition 4.20, $Igprcl(f(A)) \subseteq f(Igprcl(A)) \subset Icl(f(A))$. And $\sigma_g = \sigma$ implies $Icl(f(A)) = Igprcl(f(A))$. Thus $Igprcl(f(A)) \subset f(Igprcl(A)) \subset Igprcl(f(A))$. So $Igprcl(f(A)) = f(Igprcl(A))$. □

Proposition 4.26. *If $f : (X, \tau) \longrightarrow (Y, \sigma)$ be $Igpr^*$-homeomorphism and $IGPRC(X)$ and $IGPRC(Y)$ are Igpr closed under intersections respectively, then $Igprcl(f^{-1}(A))$ $= f^{-1}(Igprcl(A))$ for all $A \subset Y$.*

Proof. Since f is $Igpr^*$-homeomorphism, f is Igpr-irresolute. Since $Igprcl(A)$ is Igpr closed in (Y, σ), $f^{-1}(Igprcl(f(A)))$ is Igpr closed in (X, τ). Now, $f^{-1}(A) \subset f^{-1}(Igprcl(A))$ and thus $Igprcl(f^{-1}(A)) \subseteq f^{-1}(Igprcl(A))$.

Conversely, since f is an $Igpr^*$-homeomorphism, f^{-1} is an Igpr-irresolute map. Since $Igprcl(f^{-1}(A)$ is Igpr closed in (X, τ), $(f^{-1})^{-1}(Igprcl(f^{-1}(A))) = f(Igprcl(f^{-1}(A))$ is Igpr closed in (Y, σ). On the other hand,

$$\begin{aligned} A &\subseteq (f^{-1})^{-1}(f(A)) \\ &\subseteq (f^{-1})^{-1}(Igprcl(f^{-1}(A))) \\ &= f(Igprcl(f^{-1}(A))). \end{aligned}$$

Thus $Igprcl(A) \subseteq f(Igprcl(f^{-1}(A)))$. So

$$f^{-1}(Igprcl(A)) \subseteq f^{-1}(f(Igprcl(f^{-1}(A)))) \subseteq Igprcl(f^{-1}(A)).$$

Hence $Igprcl(f^{-1}(A)) = f^{-1}(Igprcl(A))$ for all A $\subset$ Y. □

Definition 4.27. An intuitionistic topological space (X, τ) having no proper intuitionistic regular open set is called IR space.

Example 4.28. Let X = {a,b,c} with $\tau = \{\underset{\sim}{\phi}, \underset{\sim}{X}, < X, \{a\}, \phi >\}$ be an intuitionistic topological space. Then (X, τ) is an IR space.

Definition 4.29. An intuitionistic topological space (X, τ) is called an IGPR space if every intuitionistic subset of X is Igpr closed (Igpr open).

Example 4.30. Every intuitionistic door space, intuitionistic submaximal space are all IGPR space.

Remark 4.31. Every IR space is always IGPR space but every IGPR space need not be an IR space.

Example 4.32. Let X = {a,b,c} with $\tau = \{\underset{\sim}{\phi}, \underset{\sim}{X}, < X, \phi, \{a, b\} >\}$ be an intuitionistic topological space. Then (X, τ) is an IGPR space but not IR Space.

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