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Generalization of t-intuitionistic fuzzy subring

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Abstract. In this paper, the notion of (t_1, t_2)-intuitionistic fuzzy subring (normal subring and ideals) are defined and discussed. The homomorphic behaviour of (t_1, t_2)-intuitionistic fuzzy subring (normal subring and ideals) and their inverse images has been obtained. Some related results have been derived.

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1. Introduction

The concept of the fuzzy set was introduced by Zadeh [13]. Since then, many papers on fuzzy sets appeared showing the importance of the concept and its applications to logic, set theory, group theory, real analysis, measure theory, topology etc. The concept of fuzzy subgroups was introduced by Rosenfeld [6]. The notion of intuitionistic fuzzy sets introduced by Atanassov [1]. Biswas [2] applied the concept of intuitionistic fuzzy sets to the theory of groups and studied intuitionistic fuzzy subgroups of a group. Marashdeh and Salleh [3] introduced the notion of intuitionistic fuzzy rings based on the notion of fuzzy space. Wang Lin and Yin [12] defined intuitionistic fuzzy ideals with thresholds (α, β) of rings. K. Meena and K.V. Thomas [4] defined intuitionistic L-fuzzy subring and N. Palaniappan, K. Arjunan and M. Palanivelrajan [5] defined intuitionistic fuzzy L-subring. P.K. Shaema [7, 8] defined translates of intuitionistic fuzzy subring and on intuitionistic fuzzy magnified translation in rings,. The notion of t-intuitionistic fuzzy subgroups and t-intuitionistic fuzzy quotient group has already been introduced by P.K. Sharma [9, 10]. Also he [11] introduced the notion of t-intuitionistic fuzzy subring of a ring. Here we introduce the notion of (t_1, t_2)-intuitionistic fuzzy set and then define (t_1, t_2)-intuitionistic fuzzy subrings (normal subrings and ideals) of a ring R and study their properties.

2. Preliminaries

Definition 2.1 ([10]). Let R be a ring. An IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in R\}$ of R is said to be intuitionistic fuzzy subring of R (in short IFSR) if

(i) $\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\}$,
(ii) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$,
(iii) $\nu_A(x-y) \leq \max\{\nu_A(x), \nu_A(y)\}$,
(iv) $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$, for all $x, y \in R$.

Definition 2.2 ([11]). Let R be a ring. An IFSR of R is said to be intuitionistic fuzzy normal subring (in short IFNSR) of R if

(i) $\mu_A(xy) = \mu_A(yx)$,
(ii) $\nu_A(xy) = \nu_A(yx)$, for all $x, y \in R$.

Definition 2.3 ([11]). An IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in R\}$ of a ring R is said to be

(a) Intuitionistic fuzzy left ideal of R (in short IFLI) if
  (i) $\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\}$,
  (ii) $\mu_A(xy) \geq \mu_A(y)$,
  (iii) $\nu_A(x-y) \leq \max\{\nu_A(x), \nu_A(y)\}$,
  (iv) $\nu_A(xy) \leq \nu_A(y)$, for all $x, y \in R$.
(b) Intuitionistic fuzzy right ideal of R (in short IFRI) if
  (i) $\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\}$,
  (ii) $\mu_A(xy) \geq \mu_A(x)$,
  (iii) $\nu_A(x-y) \leq \max\{\nu_A(x), \nu_A(y)\}$,
  (iv) $\nu_A(xy) \leq \nu_A(y)$, for all $x, y \in R$.
(c) Intuitionistic fuzzy ideal of R (in short IFI) if
  (i) $\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\}$,
  (ii) $\mu_A(xy) \geq \max\{\mu_A(x), \mu_A(y)\}$,
  (iii) $\nu_A(x-y) \leq \max\{\nu_A(x), \nu_A(y)\}$,
  (iv) $\nu_A(xy) \leq \min\{\nu_A(x), \nu_A(y)\}$, for all $x, y \in R$.

Theorem 2.4 ([10]). *Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in R\}$ be IFSR of ring R. Then*
(1)*(i)* $\mu_A(0) \geq \mu_A(x)$ *and* $\nu_A(0) \leq \nu_A(x)$.
(2) $\mu_A(-x) = \mu_A(x)$ *and* $\nu_A(-x) = \nu_A(x)$, *for all* $x, y \in R$.
(3) *If R is a ring with unity 1, then* $\mu_A(1) \leq \mu_A(x)$ *and* $\nu_A(1) \geq \nu_A(x)$, *for all* $x \in R$.

Definition 2.5 ([12]). Let X and Y be two non-empty sets and $f : X \to Y$ be a mapping. Let A and B be IFS's of X and Y respectively. Then the image of A under the map f is denoted by $f(A)$ and is defined as

$$f(A)(y) = (\mu_{f(A)}(y), \nu_{f(A)}(y)),$$

where $\mu_{f(A)}(y) = \begin{cases} \nu\{\mu_A(x) : & x \in f^{-1}(y)\} \\ 0, & \text{otherwise} \end{cases}$

and

$$\nu_{f(A)}(y) = \begin{cases} \mu\{\nu_A(x) : & x \in f^{-1}(y) \\ 1, & \text{otherwise.} \end{cases}$$

Also the pre-image of B under f is denoted by $f^{-1}(B)$ and is defined as

$$f^{-1}(B)(x) = (\mu_{f^{-1}(B)}(x), \nu_{f^{-1}(B)}(x)),$$

where $\mu_{f^{-1}(B)}(x) = \mu_B(f(x))$ and $\nu_{f^{-1}(B)}(x) = \nu_B(f(x))$.

Definition 2.6 ([13]). The mapping $f : R_1 \to R_2$ from the ring R_1 into a ring R_2 is called ring homomorphism if

(i) $f(x + y) = f(x) + f(y)$,
(ii) $f(xy) = f(x)f(y)$, for all $x, y \in R_1$.

Definition 2.7 ([9]). Let A be a IFS of a ring R. Let $t \in [0, 1]$. Then the IFS A^t of R is called the t-intuitionistic fuzzy subset (t$-IFS$) of R w.r.t. IFS A and is defined as

$$A^t = (\mu_{A^t}, \nu_{A^t}),$$

where $\mu_{A^t}(x) = \min\{\mu_A(x), t\}$ and $\nu_{A^t}(x) = \max\{\nu_A(x), 1 - t\}$, for all $x \in R$.

3. (t_1, t_2)-intuitionistic fuzzy subring

Definition 3.1. Let A be a IFS of a ring R. Let $t_1, t_2 \in [0, 1]$ and $t_2 \le 1 - t_1$. Then the IFS A' of R is called the (t_1, t_2)-intuitionistic fuzzy subset ((t_1, t_2)-IFS) of R w.r.t. IFS A and is defined as $A' = (\mu_{A'}, \nu_{A'})$,
where $\mu_{A'}(x) = \min(\mu_A(x), t_1)$ and $\nu_{A'}(x) = \max(\nu_A(x), t_2)$, for all $x \in R$.

Remark 3.2. When $t_2 = 1-t_1$, then (t_1, t_2)- intuitionistic fuzzy set coincide with t_1-intuitionistic fuzzy set. Thus every t-intuitionistic fuzzy set is $(t, 1-t)$-intuitionistic fuzzy sets and vice a versa.

Result 3.3. *Let $A' = (\mu_{A'}, \nu_{A'})$ and $B' = (\mu_{B'}, \nu_{B'})$ be two (t_1, t_2)-IFS of a ring R. Then $(A \cap B)' = A' \cap B'$.*

Proof. Let $x \in R$ be any element. Then

$$\begin{aligned}\mu_{(A\cap B)'}(x) &= \min\{\mu_{(A\cap B)}(x), t_1\} = \min\{\min\{\mu_A(x), \mu_B(x)\}, t_1\}\\ &= \min\{\min\{\mu_A(x), t_1\}, \min\{\mu_B(x), t_1\}\}\\ &= \min\{\mu_{A'}(x), \mu_{B'}(x)\} = \mu_{A'\cap B'}(x).\end{aligned}$$

Similarly, we can show that $\nu_{(A\cap B)'}(x) = \nu_{A'\cap B'}(x)$.
Thus $(A \cap B)' = A' \cap B'$. □

Result 3.4. *Let $f : X \to Y$ be a mapping. Let A and B be two IFS of X and Y respectively. Then*

(1) $f^{-1}(B') = (f^{-1}(B))'$.
(2) $f(A') = (f(A))'$, *for all* $t_1, t_2 \in [0, 1]$, *where* $t_2 \le 1 - t_1$.

Proof. (1)

$$\begin{aligned}f^{-1}(B')(x) &= B'(f(x))\\ &= (\mu_{B'}(f(x)), \nu_{B'}(f(x)))\\ &= (\min\{\mu_B(f(x)), t_1\}, \max\{\nu_B(f(x)), t_2\})\\ &= (\min\{\mu_{f^{-1}(B)}(x), t_1\}, \max\{\nu_{f^{-1}(B)}(x), t_2\})\\ &= (f^{-1}(B))'(x).\end{aligned}$$

Then $f^{-1}(B') = (f^{-1}(B))'$.

(2)

$$\begin{aligned} f(A')(y) &= (\vee\{\mu_{A'}(x); f(x) = y\}, \wedge\{\nu_{A'}(x); f(x) = y\}) \\ &= (\vee\{\min\{\mu_A(x), t_1\}; f(x) = y\}, \\ &\qquad \wedge \{\max\{\nu_A(x), t_2\}; f(x) = y\}) \\ &= (\min\{\vee\{\mu_A(x); f(x) = y\}, t_1\}, \\ &\qquad \max\{\wedge\{\nu_A(x); f(x) = y\}, t_2\}) \\ &= (\min\{\mu_{f(A)}(y), t_1\}, \max\{\nu_{f(A)}(y), t_2\}) \\ &= (\mu_{(f(A))'}(y), \nu_{(f(A))'}(y)) \\ &= (f(A))'(y). \end{aligned}$$

Then $f(A') = (f(A))'$. □

Definition 3.5. Let A be a IFS of a ring R. Let $t_1, t_2 \in [0,1]$ and $t_2 \leq 1 - t_1$. Then A is called (t_1, t_2)-intuitionistic fuzzy subring (inshort (t_1, t_2)-IFSR) of R if A' hold the following conditions :

(i) $\mu_{A'}(x - y) \geq \min\{\mu_{A'}(x), \mu_{A'}(y)\}$,
(ii) $\mu_{A'}(xy) \geq \min\{\mu_{A'}(x), \mu_{A'}(y)\}$,
(iii) $\nu_{A'}(x - y) \leq \max\{\nu_{A'}(x), \nu_{A'}(y)\}$,
(iv) $\nu_{A'}(xy) \leq \max\{\nu_{A'}(x), \nu_{A'}(y)\}$, for all $x, y \in R$.

Note 3.1. If A is (t_1, t_2)- intuitionistic fuzzy subring of R for all $t_1, t_2 \in [0, 1]$ with $t_2 \leq 1 - t_1$, then A is t-intuitionistic fuzzy subring for all $t \in [0,1]$ (on taking $t = t_1, t_2 = 1 - t$). However, if A is t-intuitionistic fuzzy subring of R for some $t \in [0, 1]$, then it may not be necessarily that A is (t_1, t_2)- intuitionistic fuzzy subring of R, when $t = t_1$ and $t_2 \neq 1 - t_1$, as the following example shows.

Example 3.6. Consider the ring $(Z_4, \oplus, \odot)$, where $Z_4 = \{0, 1, 2, 3\}$. Define the IFS A of Z_4 by

$$\mu_A(x) = \begin{cases} 0.6; & \text{if } x = 0 \\ 0.5, & \text{if } x = 2 \\ 0.3, & \text{if } x = 1, 3 \end{cases}$$

and

$$\nu_A(x) = \begin{cases} 0.4; & \text{if } x = 0 \\ 0.5, & \text{if } x = 1, 3 \\ 0.6, & \text{if } x = 2. \end{cases}$$

If we take $t_1 = 0.4$, then

$$\mu_{A'}(x) = \begin{cases} 0.4; & \text{for } x = 0, 2 \\ 0.3, & \text{if } x = 1, 3 \end{cases}$$

and $\nu_{A'}(x) = 0.6$ for all $x \in Z_4$. Clearly, A is 0.4-IFSR of Z_4.

If we take $t_1 = 0.4$ and $t_2 = 0.45$, then

$$\mu_{A'}(x) = \begin{cases} 0.4; & \text{for } x = 0, 2 \\ 0.3, & \text{if } x = 1, 3 \end{cases}$$

and

$$\nu_{A'}(x) = \begin{cases} 0.45; & \text{for } x = 0 \\ 0.5; & \text{for } x = 1, 3 \\ 0.6, & \text{if } x = 2. \end{cases}$$

A is not a $(0.4, 0.45)$-IFSR of Z_4 as $\nu_{A'}(1-3) \not\leq \max\{\nu_{A'}(1),\ \nu_{A'}(3)\}$ as $\nu_{A'}(1-3) = 0.6$ where as $\max\{\nu_{A'}(1),\ \nu_{A'}(3)\} = 0.5$
Thus A is 0.4-IFSR of Z_4, but A is not a $(0.4, 0.45)$-IFSR of Z_4.

Proposition 3.7. *If A is IFSR of a ring R, then A is also (t_1, t_2)-IFSR of R where $t_2 \leq 1 - t_1$ and $t_1, t_2 \in [0, 1]$.*

Proof. Let $x, y \in R$ be any element of the ring R. Then

$$\begin{aligned} \mu_{A'}(x-y) &= \min(\mu_A(x-y), t_1) \\ &\geq \min[\min\{\mu_A(x), \mu_A(y)\}, t_1] \\ &= \min[\min\{\mu_A(x), t\}, \min\{\mu_A(y), t_1\}] \\ &= \min\{\mu_{A'}(x), \mu_{A'}(y)\}. \end{aligned}$$

Thus $\mu_{A'}(x-y) \geq \min\{\mu_{A'}(x), \mu_{A'}(y)\}$.

Similarly, we can show that $\mu_{A'}(xy) \geq \min\{\mu_{A'}(x), \mu_{A'}(y)\}$. On the one hand,

$$\begin{aligned} \nu_{A'}(x-y) &= \max(\nu_A(x-y), t_2) \\ &\leq \max[\max\{\nu_A(x), \nu_A(y)\}, t_2] \\ &= \max\{\max\{\nu_A(x), t_2\}; \max\{\nu_A(y), t_2\}\} \\ &= \max\{\nu_{A'}(x), \nu_{A'}(y)\}. \end{aligned}$$

Thus $\nu_{A'}(x-y) \leq \max\{\nu_{A'}(x), \nu_{A'}(y)\}$.

Similarly, we can show that $\nu_{A'}(xy) \leq \max\{\nu_{A'}(x), \nu_{A'}(y)\}$.
So A is (t_1, t_2)-IFSR of R. □

Note 3.2. The converse of the proposition 3.7 need not be true as the following example shows.

Example 3.8. Consider the ring $(Z_5, +_5, \times_5)$, where $Z_5 = \{0, 1, 2, 3, 4\}$. Define the IFS A of Z_5 by

$$\mu_A(x) = \begin{cases} 0.7; & \text{if } x = 0 \\ 0.5, & \text{if } x = 1, 3 \\ 0.4, & \text{if } x = 2, 4 \end{cases}$$

and

$$\nu_A(x) = \begin{cases} 0.3; & \text{if } x = 0 \\ 0.4, & \text{if } x = 1, 3 \\ 0.5, & \text{if } x = 2, 4. \end{cases}$$

It is easy to verify that A is not IFSR of Z_5 as $\mu(3-1) \ngeq \mu(3) \wedge \mu(1)$ as $\mu(3-1) = 0.4$ and $\mu(3) = 0.5, \quad \mu(1) = 0.5$.

If we take $t_1 = 0.3$ and $t_2 = 0.6$, then $\mu_{A'}(x) = 0.3$ and $\nu_{A'}(x) = 0.6$ for all $x \in Z_5$. Now, it can be easily proved that A' is IFSR of Z_5 and thus A is $(0.3, 0.6)$-IFSR of Z_5.

Proposition 3.9. *Let A be a IFS of the ring R. Let $t_1 < \min\{p, 1-q\}$ and $t_2 = 1-t_1$, where $p = \min\{\mu_A(x);$ for all $x \in R\}$ and $q = \max\{\nu_A(x);$ for all $x \in R\}$. Then A is (t_1, t_2)-IFSR of R.*

Proof. Since $t_1 < \min\{p, 1-q\}$ and $t_2 = 1-t_1$, $p > t_1$ and $q < 1-t_1 = t_2$. Then

$$\min\{\mu_A(x); \text{ for all } x \in R\} > t_1$$

and

$$\max\{\nu_A(x); \text{ for all } x \in R\} < 1 - t_1 = t_2.$$

Thus $\mu_A(x) > t_1$ for all $x \in R$ and $\nu_A(x) < t_2$ for all $x \in R$. So

$$\mu_{A'}(x-y) \geq \min\{\mu_{A'}(x), \mu_{A'}(y)\},\ \mu_{A'}(xy) \geq \min\{\mu_{A'}(x), \mu_{A'}(y)\}$$

and

$$\nu_{A'}(x-y) \leq \max\{\nu_{A'}(x), \nu_{A'}(y)\},\ \nu_{A'}(xy) \leq \max\{\nu_{A'}(x), \nu_{A'}(y)\}.$$

So A is (t_1, t_2)-IFSR of R. □

Proposition 3.10. *The intersection of two (t_1, t_2)-intuitionistic fuzzy subring of a ring R is also (t_1, t_2)-IFSR of R.*

Proof. Let $x, y \in R$ be any element of the ring R. Then

$$\begin{aligned}
\mu_{(A\cap B)'}(x-y) &= \min\{\mu_{A\cap B}(x-y), t_1\}\\
&= \min\{\min\{\mu_A(x-y), \mu_B(x-y)\}, t_1\}\\
&= \min\{\min\{\mu_A(x-y), t_1\}, \min\{\mu_B(x-y), t_1\}\}\\
&= \min\{\mu_{A'}(x-y), \mu_{B'}(x-y)\}\\
&\geq \min\{\min\{\mu_{A'}(x), \mu_{A'}(y), \min\{\mu_{B'}(x), \mu_{B'}(y)\}\}\\
&= \min\{\min\{\mu_{A'}(x), \mu_{B'}(x)\}, \min\{\mu_{A'}(y), \mu_{B'}(y)\}\\
&= \min\{\mu_{A'\cap B'}(x), \mu_{A'\cap B'}(y)\}\\
&= \min\{\mu_{(A\cap B)'}(x), \mu_{(A\cap B)'}(y)\}.
\end{aligned}$$

Thus $\mu_{(A\cap B)'}(x-y) \geq \min\{\mu_{(A\cap B)'}(x), \mu_{(A\cap B)'}(y)\}$.

Similarly, we can show that

$$\mu_{(A\cap B)'}(xy) \geq \min\{\mu_{(A\cap B)'}(x), \mu_{(A\cap B)'}(y)\}.$$

On the one hand,

$$\begin{aligned}
\nu_{(A\cap B)'}(x-y) &= \max\{\nu_{A\cap B}(x-y), t_2\}\\
&= \max\{\max\{\nu_A(x-y), \mu_B(x-y)\}, t_2\}\\
&= \max\{\max\{\nu_A(x-y), t_2\}, \max\{\nu_B(x-y), t_2\}\}\\
&= \max\{\nu_{A'}(x-y), \nu_{B'}(x-y)\}\\
&\leq \max\{\max\{\nu_{A'}(x), \nu_{A'}(y), \max\{\nu_{B'}(x), \nu_{B'}(y)\}\}\\
&= \max\{\max\{\nu_{A'}(x), \nu_{B'}(x)\}, \max\{\nu_{A'}(y), \nu_{B'}(y)\}\\
&= \max\{\nu_{A'\cap B'}(x), \nu_{A'\cap B'}(y)\}\\
&= \max\{\nu_{(A\cap B)'}(x), \nu_{(A\cap B)'}(y)\}.
\end{aligned}$$

So $\nu_{(A\cap B)'}(x-y) \leq \max\{\nu_{(A\cap B)'}(x), \nu_{(A\cap B)'}(y)\}$.

Similarly, we can show that

$$\nu_{(A\cap B)'}(xy) \leq \max\{\nu_{(A\cap B')}(x), \nu_{(A\cap B)'}(y)\}.$$

Hence $(A \cap B)$ is (t_1, t_2)-IFSR of R. □

Proposition 3.11. *Let A be IFNSR of a ring R. Then A is also (t_1, t_2)-IFNSR of R, where $t_2 \leq 1 - t_1$.*

Proof. Let $x, y \in R$ be any element of the ring R. Then

$$\mu_{A'}(xy) = \min\{\mu_A(xy), t_1\} = \min\{\mu_A(yx), t_1\} = \mu_{A'}(yx).$$

Similarly,

$$\nu_{A'}(xy) = \max\{\mu_A(xy), t_2\} = \max\{\mu_A(yx), t_2\} = \mu_{A'}(yx).$$

Hence A is (t_1, t_2)-IFNSR of R. □

Definition 3.12. Let A be a IFS of a ring R. Let $t_1, t_2 \in [0, 1]$ and $t_2 \leq 1 - t_1$. Then A is called

(a) (t_1, t_2)-Intuitionistic fuzzy left ideal of R (in short (t_1, t_2)-IFLI) of R if
  (i) $\mu_{A'}(x-y) \geq \min\{\mu_{A'}(x), \mu_{A'}(y)\}$,
  (ii) $\mu_{A'}(xy) \geq \mu_{A'}(y)$,
  (iii) $\nu_{A'}(x-y) \leq \max\{\nu_{A'}(x), \nu_{A'}(y)\}$,
  (iv) $\nu_{A'}(xy) \leq \nu_{A'}(y)$, for all $x, y \in R$.

(b) (t_1, t_2)-Intuitionistic fuzzy right ideal of R (in short (t_1, t_2)-IFRI) of R if
  (i) $\mu_{A'}(x-y) \geq \min\{\mu_{A'}(x), \mu_{A'}(y)\}$,
  (ii) $\mu_{A'}(xy) \geq \mu_{A'}(x)$,
  (iii) $\nu_{A'}(x-y) \leq \max\{\nu_{A'}(x), \nu_{A'}(y)\}$,
  (iv) $\nu_{A'}(xy) \leq \nu_{A'}(y)$, for all $x, y \in R$.

(c) (t_1, t_2)-Intuitionistic fuzzy ideal of R (in short (t_1, t_2)-IFI) of R if
  (i) $\mu_{A'}(x-y) \geq \min\{\mu_{A'}(x), \mu_{A'}(y)\}$,
  (ii) $\mu_{A'}(xy) \geq \max\{\mu_{A'}(x), \mu_{A'}(y)\}$,
  (iii) $\nu_{A'}(x-y) \leq \max\{\nu_{A'}(x), \nu_{A'}(y)\}$,
  (iv) $\nu_{A'}(xy) \leq \min\{\nu_{A'}(x), \nu_{A'}(y)\}$, for all $x, y \in R$.

Proposition 3.13. *If A is IFLI of a ring R, then A is also (t_1, t_2)-IFLI of R.*

Proof. In view of Proposition 3.7, we need to prove that for all $x, y \in R$

$$\mu_{A'}(xy) \geq \mu_{A'}(y)$$

and

$$\nu_{A'}(xy) \leq \nu_{A'}(y), \text{ hold for all } x, y \in R,$$

Let $x, y \in R$. Then

$$\mu_{A'}(xy) = \min\{\mu_A(xy), t_1\} \geq \min\{\mu_A(y), t_1\} = \mu_{A'}(y).$$

Thus $\mu_{A'}(xy) \geq \mu_{A'}(y)$.

Similarly, we can show that $\nu_{A'}(xy) \leq \nu_{A'}(y)$. Thus A is (t_1, t_2)-IFLI of R. □

Proposition 3.14. *If A is IFRI of a ring R, then A is also (t_1, t_2)-IFRI of R.*

Proof. In view of Proposition 3.7, we need to prove that for all $x, y \in R$,

$$\mu_{A'}(xy) \geq \mu_{A'}(x)$$

and

$$\nu_{A'}(xy) \leq \nu_{A'}(x), \text{ hold for all } x, y \in R.$$

Let $x, y \in R$. Then

$$\mu_{A'}(xy) = \min\{\mu_A(xy), t_1\} \geq \min\{\mu_A(x), t_1\} = \mu_{A'}(x).$$

Thus $\mu_{A'}(xy) \geq \mu_{A'}(x)$.

Similarly, we can show that $\nu_{A'}(xy) \leq \nu_{A'}(x)$. So A is (t_1, t_2)-IFRI of R. □

Proposition 3.15. *If A is IFI of a ring R, then A is also (t_1, t_2)-IFI of R.*

Proof. Follows from Proposition 3.13 and Proposition 3.14. □

Remark 3.16. The converse of the Proposition 3.15 need not be true as the following example shows.

Example 3.17. Consider the ring $(Z_4, +_4, X_4)$, where $Z_4 = \{0, 1, 2, 3\}$. Define the IFS A of Z_4 by

$$\mu_A(x) = \begin{cases} 0.7; & \text{if } x = 0 \\ 0.4, & \text{if } x = 1, 3 \\ 0.3, & \text{if } x = 2 \end{cases}$$

and

$$\nu_A(x) = \begin{cases} 0.2; & \text{if } x = 0 \\ 0.5, & \text{if } x = 1, 3 \\ 0.6, & \text{if } x = 2. \end{cases}$$

It is easy to verify that A is not IFI of Z_4. However, if we take $t_1 = 0.2$ and $t_2 = 0.7$, then $\mu_{A'}(x) = 0.2$ and $\nu_{A'}(x) = 0.7$ for all $x \in Z_4$. Now, it can be easily proved that A' is IFI of Z_4 and hence A is $(0.2, 0.7)$-IFI of Z_4.

4. Homomorphism of (t_1, t_2)- intuitionistic fuzzy subrings

Theorem 4.1. *Let $f : R_1 \to R_2$ be a ring homomorphism from the ring R_1 into a ring R_2. Let B be (t_1, t_2)-IFSR of R_2. Then $f^{-1}(B)$ is (t_1, t_2)-IFSR of R_1.*

Proof. Let B be (t_1, t_2)-IFSR of R_2. Let $x_1, x_2 \in R_1$ be any element. Then clearly

$$f^{-1}(B')(x_1 - x_2) = (\mu_{f^{-1}(B')}(x_1 - x_2), \nu_{f^{-1}(B')}(x_1 - x_2)).$$

On the one hand,

$$\begin{aligned} \mu_{f^{-1}(B')}(x_1 - x_2) &= \mu_{B'}(f(x_1 - x_2)) \\ &= \mu_{B'}(f(x_1) - f(x_2)) \\ &\geq \min\{\mu_{B'}(f(x_1)), \mu_{B'}(f(x_2))\} \\ &= \min\{\mu_{f^{-1}(B')}(x_1), \mu_{f^{-1}(B')}(x_2)\} \end{aligned}$$

and

$$\begin{aligned}\mu_{f^{-1}(B')}(x_1x_2) &= \mu_{B'}(f(x_1x_2))\\ &= \mu_{B'}(f(x_1)f(x_2))\\ &\geq \min\{\mu_{B'}(f(x_1)), \mu_{B'}(f(x_2))\}\\ &= \min\{\mu_{f^{-1}(B')}(x_1), \mu_{f^{-1}(B')}(x_2)\}.\end{aligned}$$

Thus

$$\mu_{f^{-1}(B')}(x_1 - x_2) \geq \min\{\mu_{f^{-1}(B')}(x_1), \mu_{f^{-1}(B')}(x_2)\}$$

and

$$\mu_{f^{-1}(B')}(x_1x_2) \geq \min\{\mu_{f^{-1}(B')}(x_1), \mu_{f^{-1}(B')}(x_2)\}.$$

On the other hand,

$$\begin{aligned}\nu_{f^{-1}(B')}(x_1 - x_2) &= \nu_{B'}(f(x_1 - x_2))\\ &= \nu_{B'}(f(x_1) - f(x_2))\\ &\leq \max\{\nu_{B'}(f(x_1)), \nu_{f^{-1}(B')}(f(x_2))\}\\ &= \max\{\nu_{f^{-1}(B')}(x_1), \nu_{f^{-1}(B')}(x_2)\}.\end{aligned}$$

Thus $\nu_{f^{-1}(B')}(x_1 - x_2) \geq \max\{\nu_{f^{-1}(B')}(x_1), \nu_{f^{-1}(B')}(x_2)\}$.

Similarly, we can show that

$$\nu_{f^{-1}(B')}(x_1x_2) \leq \max\{\nu_{f^{-1}(B')}(x_1), \nu_{f^{-1}(B')}(x_2)\}.$$

So $f^{-1}(B') = (f^{-1}(B))'$ is IFSR of R_1. Hence $f^{-1}(B)$ is (t_1, t_2)-IFSR of R_1. □

Theorem 4.2. *Let $f : R_1 \to R_2$ be a ring homomorphism from the ring R_1 into a ring R_2. Let B be (t_1, t_2)-IFNSR of R_2. Then $f^{-1}(B)$ is (t_1, t_2)-IFNSR of R_1.*

Proof. Let B be (t_1, t_2)-IFNSR of R_2. Let $x_1, x_2 \in R_1$ be any element. Since

$$f^{-1}(B')(x_1x_2) = (\mu_{f^{-1}(B')}(x_1x_2), \nu_{f^{-1}(B')}(x_1x_2)),$$

it is enough to show that

$$\mu_{f^{-1}(B')}(x_1x_2) = \mu_{B'}(f(x_1x_2))$$

and

$$\nu_{f^{-1}(B')}(x_1x_2) = \nu_{B'}(f(x_2x_1)).$$

Now,

$$\begin{aligned}\mu_{f^{-1}(B')}(x_1x_2) &= \mu_{B'}(f(x_1x_2))\\ &= \mu_{B'}(f(x_1)f(x_2))\\ &= \mu_{B'}(f(x_2)f(x_1))\\ &= \mu_{B'}(f(x_2x_1))\\ &= \mu_{f^{-1}(B')}(x_2x_1)\end{aligned}$$

and

$$\begin{aligned}\nu_{f^{-1}(B')}(x_1x_2) &= \nu_{B'}(f(x_1x_2))\\ &= \nu_{B'}(f(x_1)f(x_2))\\ &= \nu_{B'}(f(x_2)f(x_1))\\ &= \nu_{B'}(f(x_2x_1))\\ &= \nu_{f^{-1}(B')}(x_2x_1).\end{aligned}$$

Thus $f^{-1}(B') = (f^{-1}(B))'$ is IFNSR of R_1 So $f^{-1}(B')$ is (t_1, t_2)-IFNSR of R_1. □

Theorem 4.3. *Let $f : R_1 \to R_2$ be a ring homomorphism from the ring R_1 into a ring R_2. Let B be (t_1, t_2)-IFLI of R_2. Then $f^{-1}(B)$ is (t_1, t_2)-IFLI of R_1.*

Proof. Let B be (t_1, t_2)-IFLI of R_2. Let $x_1, x_2 \in R_1$ be any element. Then in view of Proposition 4.1, we need to prove that

$$\mu_{f^{-1}(B')}(x_1x_2) \geq \mu_{f^{-1}(B')}(x_2)$$

and

$$\nu_{f^{-1}(B')}(x_1x_2) \leq \nu_{f^{-1}(B')}(x_2).$$

Now,

$$\begin{aligned}\mu_{f^{-1}(B')}(x_1x_2) &= \mu_{B'}(f(x_1x_2))\\ &= \mu_{B'}(f(x_1)f(x_2))\\ &\geq \mu_{B'}(f(x_2))\\ &= \mu_{f^{-1}(B')}(x_2).\end{aligned}$$

Thus $\mu_{f^{-1}(B')}(x_1x_2) \geq \mu_{f^{-1}(B')}(x_2)$.

Similarly, we can show that $\nu_{f^{-1}(B')}(x_1x_2) \leq \nu_{f^{-1}(B')}(x_2)$.
So $f^{-1}(B') = (f^{-1}(B))'$ is IFLI of R_1. Hence $f^{-1}(B)$ is (t_1, t_2)-IFLI of R_1. □

Theorem 4.4. *Let $f : R_1 \to R_2$ be a ring homomorphism from the ring R_1 into a ring R_2. Let B be (t_1, t_2)-IFRI of R_2. Then $f^{-1}(B)$ is (t_1, t_2)-IFRI of R_1.*

Proof. It can be obtained similar to Theorem 4.3. □

Theorem 4.5. *Let $f : R_1 \to R_2$ be a ring homomorphism from the ring R_1 into a ring R_2. Let B be (t_1, t_2)-IFI of R_2. Then $f^{-1}(B)$ is (t_1, t_2)-IFI of R_1.*

Proof. It follows from Theorem 4.3 and Theorem 4.4. □

Theorem 4.6. *Let $f : R_1 \to R_2$ be surjective ring homomorphism and A be (t_1, t_2)-IFSR of R_1. Then $f(A)$ is (t_1, t_2)-IFSR of R_2*

Proof. Let $y_1, y_2 \in R_2$ be any element. Then there exist some $x_1, x_2 \in R_1$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Clearly

$$f(A')(y_1y_2) = (\mu_{f(A')}(y_1y_2), \nu_{f(A')}(y_1y_2)).$$

On the one hand,

$$\begin{aligned}
\mu_{f(A')}(y_1 - y_2) &= \mu_{(f(A))'}(y_1 - y_2) \\
&= \min\{\mu_{f(A)}(f(x_1) - f(x_2), t_1\}) \\
&= \min\{\mu_{f(A)}(f(x_1 - x_2)), t_1\} \\
&\geq \min\{\mu_A(x_1 - x_2), t_1\} \\
&= \mu_{A'}(x_1 - x_2) \\
&\geq \min\{\mu_{A'}(x_1), \mu_{A'}(x_2)\},\ \textit{for all } x_1, x_2 \in R_1 \\
&\qquad \textit{such that } f(x_1) = y_1 \textit{ and } f(x_2) = y_2 \\
&= \min\{\vee\{\mu_{A'}(x_1) : f(x_1) = y_1\}, \\
&\qquad \vee\{\mu_{A'}(x_2) : f(x_2) = y_2\}\} \\
&= \min\{\mu_{f(A')}(y_1), \mu_{f(A')}(y_2)\}.
\end{aligned}$$

Thus $\mu_{f(A')}(y_1 - y_2) \geq \min\{\mu_{f(A')}(y_1), \mu_{f(A')}(y_2)\}$.

Similarly we can show that

$$\mu_{f(A')}(y_1 y_2) \geq \min\{\mu_{f(A')}(y_1), \mu_{f(A')}(y_2)\}.$$

On the other hand,

$$\begin{aligned}
\nu_{f(A')}(y_1 - y_2) &= \nu_{(f(A))'}(y_1 - y_2) \\
&= \max\{\nu_{f(A)}(f(x_1) - f(x_2)), t_2\} \\
&= \max\{\nu_{f(A)}(f(x_1 - x_2)), t_2\} \\
&\leq \max\{\nu_A(x_1 - x_2), t_2\} \\
&= \nu_{A'}(x_1 - x_2) \\
&\leq \max\{\nu_{A'}(x_1), \nu_{A'}(x_2)\},\ \textit{for all } x_1, x_2 \in R_1 \\
&\qquad \textit{such that } f(x_1) = y_1 \textit{ and } f(x_2) = y_2 \\
&= \max\{\wedge\{\nu_{A'}(x_1) : f(x_1) = y_1\}, \\
&\qquad \wedge\{\nu_{A'}(x_2) : f(x_2) = y_2\}\} \\
&= \max\{\nu_{f(A')}(y_1), \nu_{f(A')}(y_2)\}.
\end{aligned}$$

Thus $\nu_{f(A')}(y_1 - y_2) \leq \max\{\nu_{f(A')}(y_1), \nu_{f(A')}(y_2)\}$.

Similarly we can show that

$$\nu_{f(A')}(y_1 y_2) \leq \max\{\nu_{f(A')}(y_1), \nu_{f(A')}(y_2)\}.$$

So $f(A') = (f(A))'$ is IFSR of R_2. Hence $f(A)$ is (t_1, t_2)-IFSR of R_2. □

Theorem 4.7. *Let $f : R_1 \to R_2$ be surjective ring homomorphism and A be (t_1, t_2)-IFNSR of R_1. Then $f(A)$ is (t_1, t_2)-IFNSR of R_2*

Proof. Let A be (t_1, t_2)-IFNSR of R_1 and let $y_1, y_2 \in R_2$ be any element. Then there exist some $x_1, x_2 \in R_1$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Clearly

$$f(A')(y_1 y_2) = (\mu_{f(A)}(y_1 y_2), \nu_{f(A)}(y_1 y_2)).$$

In view of Proposition 4.6, we need only to prove that

$$\mu_{f(A')}(y_1y_2) = \mu_{f(A')}(y_2y_1)$$

and

$$\nu_{f(A')}(y_1y_2) = \nu_{f(A')}(y_2y_1).$$

On the none hand,

$$\begin{aligned}
\mu_{(f(A))'}(y_1y_2) &= \mu_{f(A')}(f(x_1)f(x_2)) \\
&= \mu_{f(A')}(f(x_1x_2)) \\
&= \vee\{\mu_{A'}(x_1x_2) : f(x_1x_2) = y_1y_2\} \\
&= \vee\{\mu_{A'}(x_2x_1) : f(x_1x_2) = y_1y_2\} \\
&= \mu_{f(A')}(f(x_2x_1)) \\
&= \mu_{f(A')}(f(x_2)f(x_1)) \\
&= \mu_{(f(A))'}(y_2y_1)
\end{aligned}$$

and

$$\begin{aligned}
\nu_{(f(A))'}(y_1y_2) &= \nu_{f(A')}(f(x_1)f(x_2)) \\
&= \nu_{f(A')}(f(x_1x_2)) \\
&= \wedge\{\nu_{A'}(x_1x_2) : f(x_1x_2) = y_1y_2\} \\
&= \wedge\{\nu_{A'}(x_2x_1) : f(x_1x_2) = y_1y_2\} \\
&= \nu_{f(A')}(f(x_2x_1)) \\
&= \nu_{f(A')}(f(x_2)f(x_1)) \\
&= \nu_{(f(A))'}(y_2y_1).
\end{aligned}$$

Thus $f(A') = (f(A))'$ is IFNSR of R_2. So $f(A)$ is (t_1, t_2)-IFNSR of R_2. □

Theorem 4.8. *Let $f : R_1 \to R_2$ be bijective ring homomorphism and A be (t_1, t_2)-IFLI of R_1. Then $f(A)$ is (t_1, t_2)-IFLI of R_2 where $t_2 \leq 1 - t_1$.*

Proof. Let A is (t_1, t_2)-IFLI of R_1 and let $y_1, y_2 \in R_2$ be any element. Then there exist some $x_1, x_2 \in R_1$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Clearly

$$(f(A))'(y_1y_2) = (\mu_{(f(A))'}(y_1y_2), \nu_{(f(A))'}(y_1y_2)).$$

In view of Proposition 4.6, we need only to prove that

$$\mu_{(f(A))'}(y_1y_2) \geq \mu_{(f(A))'}(y_2)$$

and

$$\nu_{(f(A))'}(y_1y_2) \leq \nu_{(f(A))'}(y_2).$$

On the one hand,

$$\begin{aligned}\mu_{(f(A))'}(y_1y_2) &= \min\{\mu_{f(A)}(f(x_1)f(x_2)), t_1\}\\ &= \min\{\mu_{f(A)}(f(x_1x_2)), t_1\}\\ &= \min\{\mu_A(x_1x_2), t\}\\ &= \mu_{A'}(x_2x_1)\\ &\geq \mu_{A'}(x_2)\\ &= \min\{\mu_A(x_2), t_1\}\\ &= \min\{\mu_{f(A)}(f(x_2)), t_1\}\\ &= \min\{\mu_{f(A)}(y_2), t_1\}\\ &= \mu_{(f(A))'}(y_2).\end{aligned}$$

Thus $\mu_{f(A)'}(y_1y_2) \geq \mu_{(f(A))'}(y_2)$.

Similarly, we can show that

$$\nu_{(f(A))'}(y_1y_2) \leq \nu_{(f(A))'}(y_2).$$

So $(f(A))'$ is IFLI of R_2. Hence $(f(A))$ is (t_1, t_2)-IFLI of R. □

Theorem 4.9. *Let $f : R_1 \to R_2$ be bijective ring homomorphism and A be (t_1, t_2)-IFRI of R_1. Then $f(A)$ is (t_1, t_2)-IFRI of R_2.*

Proof. It can be obtained similar to Theorem 4.8. □

Theorem 4.10. *Let $f : R_1 \to R_2$ be bijective ring homomorphism and A be (t_1, t_2)-IFI of R_1. Then $f(A)$ is (t_1, t_2)-IFI.*

Proof. It follows from Theorem 4.8 and Theorem 4.9. □

Definition 4.11. Let A' be (t_1, t_2)-IFS of R w.r.t IFS A. Then (α, β)-cut of A' is a crisp subset $C_{\alpha,\beta}(A')$ of the IFS A' is given by

$$C_{\alpha,\beta}(A') = \{x \in R, \mu_{A'}(x) \geq \alpha, \nu_{A'}(x) \leq \beta\},$$

where $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$ and $\alpha \leq t_1, \beta \geq t_2$.

Theorem 4.12. *Let A be intuitionistic fuzzy subset of R. If A is (t_1, t_2)-IFSR of R if and only if $C_{\alpha,\beta}(A')$ is a subring of R. for all $\alpha, \beta \in [0, 1]$ such that $\alpha + \beta \leq 1$ and $\alpha \leq t_1$, $\beta \geq t_2$.*

Proof. Consider A is a (t_1, t_2)-IFSR of R. Let $x, y \in C_{\alpha,\beta}(A')$. Then

$$\mu_{A'}(x), \mu_{A'}(y) \geq \alpha$$

and

$$\nu_{A'}(x), \nu_{A'}(y) \leq \beta.$$

On the one hand,

$$\mu_{A'}(x - y) \geq \mu_{A'}(x) \wedge \mu_{A'}(y) \; [\because A \text{ is } (t_1, t_2)\text{-IFSR}]$$

$$\geq \alpha \wedge \alpha = \alpha$$

and

$$\nu_{A'}(x-y) \leq \nu_{A'}(x) \vee \nu_{A'}(y)$$

$$\leq \beta \vee \beta = \beta.$$

Thus $x - y \in C_{\alpha,\beta}(A')$.
On the other hand,

$$\mu_{A'}(xy) \geq \mu_{A'}(x) \wedge \mu_{A'}(y) \ [\because A \text{ is } (t_1,t_2)\text{-IFSR}]$$

$$\geq \alpha \wedge \alpha = \alpha$$

and

$$\nu_{A'}(xy) \leq \nu_{A'}(x) \vee \nu_{A'}(y)$$

$$\leq \beta \vee \beta = \beta.$$

So $xy \in C_{\alpha,\beta}(A')$. Hence $C_{\alpha,\beta}(A')$ is a subring of R.

Conversely, suppose $C_{\alpha,\beta}(A')$ is a subring of R. Let $x, y \in R$ and $\alpha = \mu_{A'}(x) \wedge \mu_{A'}(y)$ and $\beta = \nu_{A'}(x) \vee \nu_{A'}(y)$. Then $\mu_{A'}(x) \geq \alpha$, $\nu_{A'}(x) \leq \beta$ and $\mu_{A'}(y) \geq \alpha$, $\nu_{A'}(y) \leq \beta$. Thus $x, y \in C_{\alpha,\beta}(A')$. Since $C_{\alpha,\beta}(A')$ is a subring of R such that $\alpha \leq t$ and $\beta \geq t_2$, $x - y \in C_{\alpha,\beta}(A')$.
So

$$\mu_{A'}(x-y) \geq \alpha = \mu_{A'}(x) \wedge \mu_{A'}(y)$$

and

$$\nu_{A'}(x-y) \leq \beta = \nu_{A'}(x) \vee \nu_{A'}(y).$$

Also

$$\mu_{A'}(xy) \geq \alpha = \mu_{A'}(x) \wedge \mu_{A'}(y)$$

and

$$\nu_{A'}(xy) \leq \beta = \nu_{A'}(x) \vee \nu_{A'}(y).$$

Hence A is a (t_1, t_2)-IFSR of R. □

Example 4.13. Consider the ring $(Z_6, +_6, \odot_6)$ where $Z_6 = \{0, 1, 2, 3, 4, 5\}$. Define the IFS A of Z_6 by

$$\mu_A(x) = \begin{cases} 0.7; & \text{if } x = 0 \\ 0.6; & \text{if } x = 1, 3, 5 \\ 0.4; & \text{if } x = 2, 4 \end{cases}$$

and

$$\nu_A(x) = \begin{cases} 0.3; & \text{if } x = 0 \\ 0.5; & \text{if } x = 1, 3, 5 \\ 0.6; & \text{if } x = 2, 4. \end{cases}$$

If we take $t_1 = 0.3$ and $t_2 = 0.7$, then $\mu_{A'}(x) = 0.3$ and $\nu_{A'}(x) = 0.7$ for all $x \in Z_6$. Now if we take $\alpha = 0.2$ and $\beta = 0.8$ then it can be easily proved that $C_{\alpha,\beta}(A')$ is a subring of R_1 but A=$< \mu_A, \nu_A >$ is not a IFSR of R.

Theorem 4.14. *Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two rings. If $f : R \to R'$ is a surjective ring homomorphism. The homomorphic image of a level set which is a subring of a (t_1,t_2)-IFSR of R is again a level set which is a subring of (t_1,t_2)-IFSR of R'.*

Proof. Let A be a (t_1,t_2)-IFSR of R. and let $x_1, x_2 \in R$. Then clearly $f(A)$ is a (t_1,t_2)-IFSR of R' and $f(x_1) = y_1, f(x_2) = y_2 \in R'$. Let $C_{\alpha,\beta}(A')$ be a level set of A. Then clearly $C_{\alpha,\beta}(A')$ is a subring of R. Suppose $x_1, x_2 \in C_{\alpha,\beta}(A')$. Then

$$\begin{aligned}\mu_{f(A')}(f(x_1)-f(x_2)) &= \mu_{f(A')}(f(x_1-x_2))\\ &= \sup\{\mu_{A'}(x_1-x_2)/x_1-x_2 \in f^{-1}(y_1-y_2)\}\\ &\geq \sup\{\mu_{A'}(x_1)\wedge\mu_{A'}(x_2)/x_1 \in f^{-1}(y_1),\\ &\qquad\qquad x_2 \in f^{-1}(y_2)\}\\ &= \sup\{\mu_{A_1}(x_1)/x_1 \in f^{-1}(y_1)\}\\ &\qquad \wedge \sup\{\mu_{A'}(x_2)/x_2 \in f^{-1}(y_2)\}\\ &\geq \alpha\wedge\alpha = \alpha\end{aligned}$$

and

$$\begin{aligned}\nu_{f(A')}(f(x_1)-f(x_2)) &= \nu_{f(A')}(f(x_1-x_2))\\ &= \inf\{\nu_{A'}(x_1-x_2)/x_1-x_2 \in f^{-1}(y_1-y_2)\}\\ &\leq \inf\{\nu_{A'}(x_1)\vee\nu_{A'}(x_2)/x_1 \in f^{-1}(y_1),\\ &\qquad\qquad x_2 \in f^{-1}(y_2)\}\\ &= \inf\{\nu_{A_1}(x_1)/x_1 \in f^{-1}(y_1)\}\\ &\qquad \vee \inf\{\nu_{A'}(x_2)/x_2 \in f^{-1}(y_2)\}\\ &\leq \beta\vee\beta = \beta.\end{aligned}$$

Thus

$$\mu_{f(A')}(f(x_1)-f(x_2)) \geq \alpha \tag{4.1}$$

and

$$\nu_{f(A')}(f(x_1)-f(x_2)) \leq \beta. \tag{4.2}$$

From (4.1) and (4.2), $f(x_1)-f(x_2) \in f(C_{\alpha,\beta}(A'))$.
On the one hand,

$$\begin{aligned}\mu_{f(A')}(f(x_1)f(x_2)) &= \mu_{f(A')}(f(x_1x_2))\\ &= \sup\{\mu_{A'}(x_1x_2)/x_1x_2 \in f^{-1}(y_1y_2)\}\\ &\geq \sup\{\mu_{A'}(x_1)\wedge\mu_{A'}(x_2)/x_1 \in f^{-1}(y_1),\\ &\qquad\qquad x_2 \in f^{-1}(y_2)\}\\ &= \sup\{\mu_{A_1}(x_1)/x_1 \in f^{-1}(y_1)\}\\ &\qquad \wedge \sup\{\mu_{A'}(x_2)/x_2 \in f^{-1}(y_2)\}\\ &\geq \alpha\wedge\alpha = \alpha\end{aligned}$$

and

$$\begin{aligned}\nu_{f(A')}(f(x_1)f(x_2)) &= \nu_{f(A')}(f(x_1x_2)) \\ &= \inf\{\nu_{A'}(x_1x_2)/x_1x_2 \in f^{-1}(y_1y_2)\} \\ &\leq \inf\{\nu_{A'}(x_1) \vee \nu_{A'}(x_2)/x_1 \in f^{-1}(y_1), \\ &\qquad\qquad\qquad x_2 \in f^{-1}(y_2)\} \\ &= \inf\{\nu_{A_1}(x_1)/x_1 \in f^{-1}(y_1)\} \\ &\qquad \vee \inf\{\nu_{A'}(x_2)/x_2 \in f^{-1}(y_2)\} \\ &\leq \beta \vee \beta = \beta.\end{aligned}$$

So

$$\mu_{f(A')}(f(x_1)f(x_2)) \geq \alpha \tag{4.3}$$

and

$$\nu_{f(A')}(f(x_1)f(x_2)) \leq \beta. \tag{4.4}$$

From (4.3) and (4.4), $f(x_1)f(x_2) \in f(C_{\alpha,\beta}(A'))$.
Hence $f(C_{\alpha,\beta}(A'))$ is a subring in R'. □

Theorem 4.15. *Let $(R, +, \cdot)$ and $R', +, \cdot$ be any two rings. If $f : R \to R'$ is a surjective ring homomorphism. The homomorphic pre-image of a level set which is a subring of a (t_1, t_2)-IFSR of R' is again a level set which is a subring of (t_1, t_2)-IFSR of R.*

Proof. Let B be a (t_1, t_2) intuitionistic fuzzy subring of R'. Then clearly $f^{-1}(B)$ is a (t_1, t_2)-IFSR of R. Let $f(x_1), f(x_2) \in B$. and suppose $f(x_1), f(x_2) \in C_{\alpha,\beta}(B')$. Then $\mu_{B'}(f(x_1)) \geq \alpha$, $\nu_{B'}(f(x_1)) \leq \beta$ and $\mu_{B'}(f(x_2)) \geq \alpha$, $\nu_{B'}(f(x_2)) \leq \beta$.
Now,

$$\begin{aligned}\mu_{f^{-1}(B')}(x_1 - x_2) &= \mu_{B'}(f(x_1 - x_2)) = \mu_{B'}(f(x_1) - f(x_2)) \\ &\geq \mu_{B'}(f(x_1)) \wedge \mu_{B'}(f(x_2)) \geq \alpha \wedge \alpha = \alpha\end{aligned}$$

and

$$\begin{aligned}\nu_{f^{-1}(B')}(x_1 - x_2) &= \nu_{B'}(f(x_1 - x_2)) = \nu_{B'}(f(x_1) - f(x_2)) \\ &\leq \nu_{B'}(f(x_1)) \wedge \nu_{B'}(f(x_2)) \leq \beta \wedge \beta = \beta.\end{aligned}$$

Thus $\mu_{f^{-1}(B')}(x_1 - x_2) \geq \alpha$ and $\nu_{f^{-1}(B')}(x_1 - x_2) \leq \beta$.
On the one hand,

$$\begin{aligned}\mu_{f^{-1}(B')}(x_1x_2) &= \mu_{B'}(f(x_1x_2)) = \mu_{B'}(f(x_1)f(x_2)) \\ &\geq \mu_{B'}(f(x_1)) \wedge \mu_{B'}(f(x_2)) \geq \alpha \wedge \alpha = \alpha\end{aligned}$$

and

$$\begin{aligned}\nu_{f^{-1}(B')}(x_1x_2) &= \nu_{B'}(f(x_1x_2)) = \nu_{B'}(f(x_1)f(x_2)) \\ &\leq \nu_{B'}(f(x_1)) \wedge \nu_{B'}(f(x_2)) \leq \beta \wedge \beta = \beta.\end{aligned}$$

So $\mu_{f^{-1}(B')}(x_1x_2) \geq \alpha$ and $\nu_{f^{-1}(B')}(x_1x_2) \leq \beta$.
Hence $f^{-1}(C_{\alpha,\beta}(B'))$ is a subring in R. □

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