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A note on intutionisticI fuzzy weakly Voltera spaces

Srinivasan Soundararajan, Ganesan Thangaraj, Shanmugam Balakrishnan

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Abstract. In this paper we investigate several characterizations of intuitionistic fuzzy weakly Volterra spaces and study the conditions underwhich an intuitionistic fuzzy topological space becomes an intuitionistic fuzzy weakly Volterra space.

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Keywords: Intuitionistic fuzzy nowhere dense set, Intuitionistic fuzzy dense set, Intuitionistic fuzzy first category, Intuitionistic fuzzy second category, Intuitionistic fuzzy D-Baire, Intuitionistic fuzzy weakly Volterra spaces.

Corresponding Author: S. Soundararajan (soundar.rajan2002@yahoo.com)

1. Introduction

The fuzzy concept has invaded almost all branches of mathematics ever since the introduction of fuzzy set by Zadeh [17]. The theory of fuzzy topological spaces was introduced and developed by Chang [7]. The idea of "Intuitionistic fuzzy set" was first published by Atanassov [1, 2, 4]. Later, in [5] this concept was generalised to "Intuitionistic L-fuzzy set" by Atanassov and Stoneva. The concept of volterra spaces have been studied extensively in classical topology [6, 10, 11, 12]. The concept of fuzzy Volterra space is introduced and studied by Thangaraj and Soundararajan [16]. The concept of intuitionistic fuzzy weakly Volterra space was introduced and studied by Soundararajan, Rizwan and Syed Tahir Hussainy [13]. In this paper, we study the characterizations of intuitionistic fuzzy weakly Volterra spaces and study the conditions underwhich an intuitionistic fuzzy topological space becomes an intuitionistic fuzzy weakly Volterra space. Intuitionistic fuzzy second category space, intuitionistic fuzzy $\sigma-$second category space, intuitionistic fuzzy submaximal space, intuitionistic fuzzy almost resolvable space, intuitionistic fuzzy D-Baire space and intuitionistic fuzzy $P-$space are considered for this work.

2. Preliminaries

Definition 2.1 ([3])**.** Let X be a non-empty set. An Intuitionistic Fuzzy Set (IFS) A in X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where $\mu_A(x) : X \to [0\ ,1\ ]$ and $\nu_A(x) : X \to [0,1]$ denote the membership and non-membership functions of A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$, for each $x \in X$.

Definition 2.2 ([3])**.** Let A and B be two IFSs of the non-empty set X such that

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

and

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}.$$

We define the following basic operations on A and B.
(i) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x),\ \forall x \in X$.
(ii) $A \supseteq B$ iff $\mu_A(x) \geq \mu_B(x)$ and $\nu_A(x) \leq \nu_B(x), \forall x \in X$.
(iii) $A = B$ iff $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x), \forall x \in X$.
(iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x),\ \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$.
(v) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x),\ \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.
(vi) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$.

Definition 2.3 ([8])**.** An Intuitionistic fuzzy topology (IFT) on X is a family T of IFSs in X satisfying the following axioms :
(i) $\tilde{0}, \tilde{1} \in T$,
(ii) $G_1 \cap G_2 \in T$, for any $G_1, G_2 \in T$,
(iii) $\cup G_i \in T$ for any family $\{G_i / i \in J\} \subseteq T$.
In this case the pair (X, T) is called an Intuitionistic fuzzy topological space (IFTS) and any IFS in T is known as Intuitionistic fuzzy open set (IFOS) in X.
The complement A^c of an IFOS A in an IFTS (X, T) is called an Intuitionistic fuzzy colsed set (IFCS) in X.

Definition 2.4 ([8])**.** Let (X, T) be an IFTS and $A = \langle X, \mu_A, \nu_A\rangle$ be an IFS in X. Then the Intuitionistic fuzzy interior and an Intuitionistic fuzzy closure are defined by

$$IFint(A) = \cup\{G/G \text{ is an IFOS in } X \text{ and } G \subseteq A\}$$

and

$$IFcl(A) = \cap\{K/K \text{ is an IFCS in } X \text{ and } A \subseteq K\}.$$

Proposition 2.5 ([8])**.** *Let (X, T) be any Intuitionistic fuzzy topological space. Let A be an IFS in (X, T). Then*

(i) $\tilde{1} - IFcl(A) = IFint(\tilde{1} - A)$.

(ii) $\tilde{1} - IFint(A) = IFcl(\tilde{1} - A)$.

Definition 2.6 ([9])**.** An Intuitionistic Fuzzy set A in an Intuitionistic fuzzy topological space(X, T) is called an intuitionistic fuzzy dense if there exists no Intuitionistic fuzzy closed set B in (X, T) such that $A \subset B \subset \tilde{1}$.

Definition 2.7 ([15]). An Intuitionistic Fuzzy set A in an Intuitionistic fuzzy topological space(X,T) is called an intuitionistic fuzzy nowhere dense set if there exists no Intuitionistic fuzzy open set U in (X,T) such that $U \subset IFclA$. That is, if $IFintIFcl(A) = \tilde{0}$.

Theorem 2.8 ([14]). *If A is an intuitionistic fuzzy nowhere dense set in IFTS (X,T), then $\tilde{1} - A$ is an intuitionistic fuzzy dense set in (X,T).*

Theorem 2.9 ([14]). *Let (X,T) be an IFTS. An intuitionistic fuzzy set A is an intuitionistic fuzzy dense and intuitionistic fuzzy open set in (X,T), then $\tilde{1} - A$ is an intuitionistic fuzzy nowhere dense set in (X,T).*

Definition 2.10 ([13]). An Intuitionistic fuzzy set A in an Intuitionistic fuzzy topological space an Intuitionistic fuzzy $G_\delta-$set in (X,T) if $A = \bigcap_{i=1}^{\infty} A_i$ where $A_i \in T, \forall i$

Definition 2.11 ([13]). An Intuitionistic fuzzy set A in an Intuitionistic fuzzy topological space (X,T) is called an Intuitionistic fuzzy $F_\sigma-$set in (X,T)) if $A = \bigcup_{i=1}^{\infty} A_i$ where $\tilde{1} - A_i \in T, \forall i$.

Lemma 2.12 ([13]). *A is an Intuitionistic fuzzy $G_\delta-$set in an Intuitionistic fuzzy topological space (X,T) if and only if $\tilde{1} - A$ an Intuitionistic fuzzy $F_\sigma-$set in (X,T).*

3. Intuitionistic fuzzy weakly Volterra spaces

Definition 3.1 ([13]). An Intuitionistic fuzzy topological space (X,T) (IFTS) is called an Intuitionistic fuzzy Volterra space if $IFCl\left(\bigcap_{i=1}^{\infty} A_i\right) = \tilde{1}$, where A_i's are Intuitionistic fuzzy dense and Intuitionistic fuzzy $G_\delta-$sets in (X,T).

Definition 3.2 ([13]). Let (X,T) be an IFTS. Then (X,T) is called an Intuitionistic fuzzy weakly Volterra space if $IFCl\left(\bigcap_{i=1}^{\infty} A_i\right) \neq \tilde{0}$, where A_i's are Intuitionistic fuzzy dense and Intuitionistic fuzzy $G_\delta-$sets in (X,T).

Definition 3.3 ([13]). Let (X,T) be an IFTS. An intitionistic fuzzy set A in (X,T) is called an intitionistic fuzzy $\sigma-$nowhere dense set if A is an intitionistic fuzzy $F_\sigma-$set such that $IFint(A) = \tilde{0}$.

Theorem 3.4 ([13]). *In a IFTS (X,T), an Intuitionistic fuzzy set A is Intuitionistic fuzzy $\sigma-$nowhere dense if and only if $\tilde{1} - A$ is an Intuitionistic fuzzy dense and Intuitionistic fuzzy $G_\delta-$set.*

Definition 3.5. An IFTS (X,T) is an intuitionistic fuzzy almost resolvable space if $\bigcup_{\alpha=1}^{\infty} A_i = \tilde{1}$, where the IFS, A_i's in (X,T) are such that $IFint(A_i) = \tilde{0}$.

Otherwise, (X,T) is called an intuitionistic fuzzy almost irresolvable.

Proposition 3.6. *If the IFTS (X,T) is an intuitionistic fuzzy almost irresolvable space, then (X,T) is an intuitionistic fuzzy weakly Volterra space.*

Proof. Let A_i's $(i = 1, 2, \ldots, N)$ be intuitionistic fuzzy dense and intuitionistic fuzzy G_δ sets in (X,T).

Now
$$IFCl(A_i) = \tilde{1} \Rightarrow IFint(\tilde{1} - A_i) = \tilde{0}.$$
Since (X,T) is an intuitionistic fuzzy almost irresolvable space, $\bigcup_{i=1}^{\infty} B_i \neq \tilde{1}$, where the intuitionistic fuzzy sets B_i's in (X,T) are such that $IFintB_i = \tilde{0}$.

Let us take the first N (B_i)'s as $(\tilde{1}-A_i)$'s in (X,T).

Now, $\bigcup_{i=1}^{\infty} B_i \neq \tilde{1}$ implies that $\tilde{1}-\bigcup_{i=1}^{\infty} B_i \neq \tilde{0}$. This implies that $\bigcap_{i=1}^{\infty}(\tilde{1}-B_i) \neq \tilde{0}$ and thus $IFCl\left(\bigcap_{i=1}^{\infty}(\tilde{1}-B_i)\right) \neq \tilde{0}$. Since $\tilde{0} \neq IFCl\left(\bigcap_{i=1}^{\infty}(\tilde{1}-B_i)\right) \subseteq IFCl\left(\bigcap_{i=1}^{N}(\tilde{1}-B_i)\right)$, $IFCl\left(\bigcap_{i=1}^{N}(\tilde{1}-B_i)\right) \neq \tilde{0}$.
Hence $IFCl\left(\bigcap_{i=1}^{N}(\tilde{1}-(\tilde{1}-A_i))\right) \neq \tilde{0}$ replacing B_i by $\tilde{1}-A_i,\ i=1,2,\ldots,N$.
This implies $IFCl\left(\bigcap_{i=1}^{N} A_i\right) \neq \tilde{0}$. Therefore (X,T) is an intuitionistic fuzzy weakly Volterra space. □

Definition 3.7 ([13])**.** An IFS A is an IFTS (X,T) is called an Intuitionistic fuzzy first category set if $A=\bigcup_{i=1}^{\infty} A_i$, where A_i's are intuitionistic fuzzy nowhere dense sets in (X,T). Any other intuitionistic fuzzy set in (X,T) is said to be of second category.

Definition 3.8 ([13])**.** An IFTS (X,T) is called an Intuitionistic fuzzy first category space if $\tilde{1}=\bigcup_{i=1}^{\infty} A_i$, where A_i's are intuitionistic fuzzy nowhere dense sets in (X,T). (X,T) is called an intuitionistic fuzzy second category space if it is not an intuitionistic fuzzy first category space.

Definition 3.9 ([13])**.** An IFTS (X,T) is called an intuitionistic fuzzy $p-$space if countable intersection of intuitionistic fuzzy open sets in (X,T) is intuitionistic fuzzy open in (X,T).

Proposition 3.10. *If the IFTS (X,T) is an intuitionistic fuzzy secone category and intuitionistic fuzzy $p-$space, then (X,T) is an intuitionistic fuzzy weakly Volterra space.*

Proof. Let A_i's $(i=1,2,\ldots,N)$ be intuitionistic fuzzy dense and intuitionistic fuzzy G_δ sets in (X,T). Since (X,T) is an intuitionistic fuzzy $p-$space, then intuitionistic fuzzy G_δ sets A_i's are intuitionistic fuzzy open sets in (X,T). Then A_i's $(i=1,2,\ldots,N)$ be intuitionistic fuzzy dense and intuitionistic fuzzy open sets in (X,T). Then by theorem 2.9, $(\tilde{1}-A_i)$'s are intuitionistic fuzzy nowhere dense sets in (X,T). Since (X,T) is an intuitionistic fuzzy second category space, $\bigcup_{i=1}^{\infty} B_i \neq \tilde{1}$, where B_i's are intuitionistic fuzzy nowhere dense sets in (X,T). Let us take the first N (B_i)'s as $(\tilde{1}-A_i)$'s in (X,T). Then

$$\bigcup_{i=1}^{N}(\tilde{1}-A_i)=\bigcup_{i=1}^{N} B_i \subseteq \bigcup_{i=1}^{\infty} B_i \neq \tilde{1}.$$

This implies that

$$\tilde{1}-\bigcup_{i=1}^{N}(\tilde{1}-A_i) \neq \tilde{0} \quad \Rightarrow \quad \bigcap_{i=1}^{N} A_i \neq \tilde{0}.$$

Thus

$$IFCl\left(\bigcap_{i=1}^{N} A_i\right) \neq \tilde{0},$$

where A_i's are intuitionistic fuzzy dense and intuitionistic fuzzy $G_\delta-$sets in (X,T). So (X,T) is an intuitionistic fuzzy weakly Volterra space. □

Definition 3.11 ([9]). An Intuitionistic fuzzy topological space (X, T) is called Intuitionistic fuzzy submaximal space if for each Intuitionistic fuzzy set A in (X, T) such that $IFcl(A) = \tilde{1}$, then $A \in T$.

Proposition 3.12. *If the IFTS (X, T) is an intuitionistic fuzzy second category and intuitionistic fuzzy submaximal space, then (X, T) is an intuitionistic fuzzy weakly Volterra space.*

Proof. Let A_i's $(i = 1, 2, \ldots, N)$ be intuitionistic fuzzy dense and intuitionistic fuzzy $G_\delta-$sets in (X, T). Since (X, T) is an intuitionistic fuzzy submaximal space, the intuitionistic fuzzy dense sets A_i's are intuitionistic fuzzy open sets in (X, T). Then the proof is same as in the previous proof. □

Proposition 3.13. *If A is an intuitionistic fuzzy dense and intuitionstic fuzzy $G_\delta-$set in an intuitionistic fuzzy topological space (X, T), then $\tilde{1} - A$ is an intuitionistic fuzzy first category set in (X, T).*

Proof. Since A is an intuitionistic fuzzy $G_\delta-$set in (X, T) and $A = \bigcap_{i=1}^{\infty} A_i$, where $A_i \in T$ and since A is an intuitionistic fuzzy dense set in (X, T), $IFcl(A) = \tilde{1}$.
Then $IFcl\left(\bigcap_{i=1}^{\infty} A_i\right) = \tilde{1}$. But $IFcl\left(\bigcap_{i=1}^{\infty} A_i\right) \subseteq \bigcap_{i=1}^{\infty} IFcl(A_i)$.
Thus $\tilde{1} \subseteq \bigcap_{i=1}^{\infty} IFcl(A_i)$. That is $\bigcap_{i=1}^{\infty} IFcl(A_i) = \tilde{1}$.
This implies $IFcl(A_i) = \tilde{1}, \forall A_i \in T$ and thus $IFcl(IFint(A_i)) = \tilde{1} \ \ \forall i$.
So $IFint(IFcl(\tilde{1} - A_i)) = \tilde{0} \ \ \forall i$.
Hence $(\tilde{1} - A_i)$'s are intuitionistic fuzzy nowhere dense sets in (X, T).
Om the other hand,

$$\tilde{1} - A = \tilde{1} - \bigcap_{i=1}^{\infty} A_i = \bigcup_{i=1}^{\infty} (\tilde{1} - A_i),$$

where $(\tilde{1} - A_i)$'s are intuitionistic fuzzy nowhere dense sets in (X, T).
Therefore $(\tilde{1} - A)$ is an intuitionistic fuzzy first category sets in (X, T). □

Proposition 3.14. *If each intuitionistic fuzzy first category set is an intuitionistic fuzzy closed set in an intuitionistic fuzzy second category space (X, T) then (X, T) is an intuitionistic fuzzy weakly Volterra space.*

Proof. Let A_i's $(i = 1, 2, \ldots, N)$ be intuitionistic fuzzy dense and intuitionistic fuzzy $G_\delta-$sets in (X, T). Then by proposition 3.13, $(\tilde{1} - A_i)$'s are intuitionistic fuzzy first category sets in (X, T).

By hypothesis $(\tilde{1} - A_i)$'s are intuitionistic fuzzy closed sets and hence A_i's are intuitionistic fuzzy open sets in (X, T). Hence A_i's are intuitionistic fuzzy dense and intuitionistic fuzzy open sets in (X, T).

Then the proof is same as in the previous proof. □

Definition 3.15. An IFS A is an IFTS (X, T) is called an Intuitionistic fuzzy $\sigma-$first category set if $A = \bigcup_{i=1}^{\infty} A_i$, where A_i's are intuitionistic fuzzy $\sigma-$nowhere dense sets in (X, T). Any other intuitionistic fuzzy set in (X, T) is said to be of second category.

Definition 3.16. An IFTS (X, T) is called an Intuitionistic fuzzy $\sigma-$first category space if $\tilde{1} = \bigcup_{i=1}^{\infty} A_i$, where A_i's are intuitionistic fuzzy $\sigma-$nowhere dense sets in (X, T). (X, T) is called an intuitionistic fuzzy $\sigma-$second category space if it is not an intuitionistic fuzzy $\sigma-$first category space.

Proposition 3.17. *If the IFTS (X, T) is an intuitionistic fuzzy $\sigma-$second category space, then (X, T) is an Intuitionistic fuzzy weakly Volterra space.*

Proof. Let A_i's $(i = 1, 2, \ldots, N)$ be intuitionistic fuzzy and intuitionistic fuzzy G_δ sets in (X, T). Then by theorem 3.4, $(\tilde{1} - A_i)$'s are intuitionistic fuzzy $\sigma-$nowhere dense sets in (X, T). Let B_i $(i = 1$ to $\infty)$ be intuitionistic fuzzy $\sigma-$nowhere dense sets in (X, T) in which the first N (B_i)'s are $(\tilde{1} - A_i)$'s. Since (X, T) is an intuitionistic fuzzy $\sigma-$second category space, $\bigcup_{i=1}^{\infty} B_i \neq \tilde{1}$. Then $\tilde{1} - \bigcup_{i=1}^{\infty} B_i \neq \tilde{0}$.

$$\bigcap_{i=1}^{\infty}(\tilde{1} - B_i) \neq \tilde{0} \Rightarrow IFCl \bigcap_{i=1}^{\infty}(\tilde{1} - B_i) \neq \tilde{0} \Rightarrow IFCl \bigcap_{i=1}^{N}(\tilde{1} - B_i) \neq \tilde{0},$$

since $IFCl \bigcap_{i=1}^{\infty}(\tilde{1} - B_i) \subseteq IFCl \bigcap_{i=1}^{N}(\tilde{1} - B_i)$.
So $IFCl \left[\bigcap_{i=1}^{N}(\tilde{1} - (\tilde{1} - A_i))\right] \neq \tilde{0}$, replacing B_i's by $\tilde{1} - A_i$'s. $IFCl \bigcap_{i=1}^{N} A_i \neq \tilde{0}$, where A_i's are intuitionistic fuzzy dense and intuitionistic fuzzy G_δ sets in (X, T). Therefore (X, T) is an intuitionistic fuzzy weakly Volterra space. □

Proposition 3.18. *If the intuitionistic fuzzy topological space (X, T) is not an intuitionistic fuzzy weakly Volterra space, then (X, T) is an intuitionistic fuzzy $\sigma-$first category space.*

Proof. Let B_i's $(i = 1$ to $\infty)$ intuitionistic fuzzy $\sigma-$nowhere dense sets in an intuitionistic fuzzy topological space (X, T) which is not an intuitionistic fuzzy weakly Volterra space.

Now, we claim that $\bigcup_{i=1}^{\infty} B_i = \tilde{1}$.

Suppose that $\bigcup_{i=1}^{\infty} B_i \neq \tilde{1}$. Then $\bigcap_{i=1}^{\infty}(\tilde{1} - B_i) \neq \tilde{0}$.

Since B_i's are intuitionistic fuzzy $\sigma-$nowhere dense sets in (X, T), by theorem $(\tilde{1} - B_i)$'s are intuitionistic fuzzy dense and intuitionistic fuzzy G_δ sets in (X, T).

Now, $\tilde{0} \neq \bigcap_{i=1}^{\infty}(\tilde{1} - B_i) \subseteq \bigcap_{i=1}^{N}(\tilde{1} - B_i)$ implies that $\bigcap_{i=1}^{N}(\tilde{1} - B_i) \neq \tilde{0}$.

Let $A_i = \tilde{1} - B_i$, then $\bigcap_{i=1}^{N} A_i \neq \tilde{0}$ implies that $IFCl\left(\bigcap_{i=1}^{N} A_i\right) \neq \tilde{0}$, where A_i's are intuitionistic fuzzy dense and intuitionistic fuzzy G_δ sets in (X, T).

But this is a contradiction, since (X, T) is not an intuitionistic fuzzy weakly Volterra space.

Hence $\bigcup_{i=1}^{\infty} B_i = \tilde{1}$. Therefore, (X, T) is an intuitionistic fuzzy $\sigma-$first category space. □

Definition 3.19. An IFTS (X, T) is called an intuitionistic fuzzy $D-$Baire space if every intuitionistic fuzzy first category set in (X, T) is an intuitionistic fuzzy nowhere dense in (X, T).

Proposition 3.20. *If the intuitionistic fuzzy $D-$Baire space (X, T) is an intuitionistic fuzzy second category space, then (X, T) is an intuitionistic fuzzy weakly Volterra space.*

Proof. Let A_i's $(i = 1, 2, \ldots, N)$ be intuitionistic fuzzy dense and intuitionistic fuzzy $G_\delta-$sets in (X, T).

Then, by proposition 3.13, $(\tilde{1} - A_i)$'s are intuitionistic fuzzy first category sets in (X, T). Since (X, T) is an intuitionistic fuzzy $D-$Baire space $(\tilde{1} - A_i)$'s are intuitionistic fuzzy nowhere dense sets in (X, T). Since (X, T) is an intuitionistic fuzzy second category space $\bigcup_{i=1}^{\infty} B_i \neq \tilde{1}$, where B_i's are intuitionistic fuzzy nowhere dense sets in (X, T).

Let us take the first N B_i's as $(\tilde{1} - A_i)$'s in (X, T). Then

$$\bigcup_{i=1}^{N} (\tilde{1} - A_i) = \bigcup_{i=1}^{N} B_i \subseteq \bigcup_{i=1}^{\infty} B_i \neq \tilde{1}.$$

Thus

$$\tilde{1} - \bigcup_{i=1}^{N} (\tilde{1} - A_i) \neq \tilde{0} \quad \text{or} \quad \bigcap_{i=1}^{N} A_i \neq \tilde{0}.$$

So

$$IFCl\left(\bigcap_{i=1}^{N} A_i\right) \neq \tilde{0}.$$

Hence (X, T) is an intuitionistic fuzzy weakly Volterra space. □

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S. Soundararajan (soundar.rajan2002@yahoo.com)
Department of Mathematics, Islamiah College, Vaniyambadi, India. Postal Code - 635 752

G. Thangaraj (g.thangaraj@rediffmail.com)
Professor and Head, Department of Mathemathics, Thiruvalluvar University, Vellore, Tamil Nadu, India

S. Balakrishnan (s_krishnan1982@yahoo.co.in)
Department of Mathematics, Islamiah College, Vaniyambadi, India. Postal Code - 635 752