Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 11, No. 6, (June 2016), pp. 1003–1009
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Multiple criteria decision making based on bipolar valued fuzzy sets

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Received 10 July 2015; Revised 4 November 2015; Accepted 17 December 2015

Abstract. This paper solve multiple criteria decision making (MCDM) problems, in which all information provided by decision-maker (DM) in Bipolar valued fuzzy sets (BVFSs) decision matrix. We apply the proposed function and then calculate importance of each alternative. The alternative(s) with maximum value of importance is best choice(s).

2010 AMS Classification: 03B52, 03E72, 90C70

Keywords: Fuzzy set(s), Bipolar Valued Fuzzy Set(s), Multiple decision Making, Score function.

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1. Introduction

Decision support systems are dominant tackles integrating scientific methods for supporting multifarious decisions with methods established in information science, and are achievement an improved reputation in many areas. Decision support systems help to human intellectual insufficiencies by integrating different causes of information, providing intellectual entrance to significant information, supporting the process of organizing, and enhancing decisions. There are different types of decision support systems and different tools are using for such decision making. Several types of theories and mathematical models have been used as tools for decision support systems but they all failed to explain the complicated problem solution. Researchers have used some new theories as mathematical tool for decision support system and decision making systems i.e fuzzy set theory (Zadeh 1965 [21]), rough set theory (Pawlak 1982 [15]), soft set theory (Molodtsov 1999 [12]). They are well-know and effective mathematical tools for enhancing the solution of decision making systems and decision support systems. It is essential to discriminate between good decisions and good outcomes. By a stroke of good luck a pitiable decision can indication to a very worthy outcome. Likewise, a very worthy decision can be monitored by a ruthless outcome. Supporting decisions means assisting the decision making process so that good decisions are prepared. Good decisions can be anticipated to indication to best outcomes.

Decision making is the procedure to find the best alternative among a set of feasible alternatives. Decision making is a most important scientific, social and economic endeavor. By researchers Chen 2000 [4], Hwang et al 1981 [7], Jahanshahloo et al. 2006 [8] and Wang et al. 2007 [19] are classified these problems into two categories. 1) Multiple Criteria Decision Making (MCDM) in which number of alternative is finite, 2) Multiple Objective Decision Making (MODM) in which number of alternative is infinite.

Zadeh in 1965 [21] proposed fuzzy set theory, which is an extension of classical sets by enlarging the truth value set from $\{0, 1\}$ to $[0, 1]$. Fuzzy set is a class of object with a continuum of membership grades to each object. After initation many researcher extent fuzzy set to decision making problems. Technique for order preference by similarity to an ideal solution (TOPSIS) method, developed by Hwang and Yoon in 1981 [7], is a well known multicriteria decision making method. Triantaphyllou et al. in 1996 [17], develop a TOPSIS fuzzy version. Jahanshahloo et al. in 2006 [8], Extension TOPSIS method for decision making problems with fuzzy data. Chen in 2000 [4], extends TOPSIS to fuzzy group decision making problems. Tsaur et al. in 2002 [18], convert a fuzzy decision making problem into a crisp one and then solve by TOPSIS approach.

Intuitionistic fuzzy sets (Atanassov 1986 [1]), interval valued intuitionistic fuzzy sets (Atanassov et al. 1987 [2]) and vague sets (Gau et al. 1993 [6]) etc, these are extension of the fuzzy set in fuzzy set theory and applied them in decision making problems. Chen in 2000 [4], introduced SAW-based and TOPSIS-based methods through score functions and weight constraints under an interval-valued fuzzy framework. Li in 2012 [10], using the score function and the accuracy function to rank the Intuitionistic fuzzy sets. Ye in 2007 [20], improved algorithm for multicriteria decision making problems based on the score function of vague sets and using an improved accuracy function of vague sets. Bai in 2013 [3], extent TOPSIS Method Based on an Improved Score Function to Interval Valued Intuitionistic Fuzzy.

As corresponding to each property there exists counter-property. Lee in 2000 [11] initiated an other extension of fuzzy set in which the range $[0, 1]$ enlarges to $[-1, 1]$ named Bipolar valued fuzzy sets (BVFSs), Which give grade property of an object and also to counter-property of an object. Therefore, at time of initiation many researchers are engage in BVFSs. Mahmood et al. in 2013 [13], introduced Bipolar fuzzy subgroup. Saeid in 2009 [16], introduce Bipolar-valued fuzzy BCK/BCI-algebra. Jun et al. in 2009 [9], work on Filters of BCH-algebras based on bipolar valued fuzzy sets. Nagarajan et al. in 2014 [14], improve Bipolar Fuzzy Soft H-Ideals over Hemi Rings to Socialistic Decision Making Approach. Da Silva Neves et al. in 2008 [5], discuss Bipolarity in human reasoning and affective decision making.

In this paper we proposed score function, improve score function and double improve score function solve MCDM problems by assign a real value to each BVFS value where nature order is define. The remaining part of this paper is organized as, in section 2, we recall some definition like Fuzzy sets and Bipolar valued fuzzy sets. In section 3, we proposed score function, improve score function and double improve score function some of its properties and present some example to show effectiveness of proposed function. In section 4, we define algorithm to solve MCDM problems by using proposed function. In section 5, we demonstrate two examples to illustrate the effectiveness of proposed method. In section 6, conclusion of this paper is given.

2. Preliminaries

Definition 2.1. Let us consider that $M \neq \phi$, and h be a function from M into $[0,1]$, then Fuzzy set is an object define and denoted as,$(M,h) = \{(m, h(m)) : m \in M\}$.Where $h(m)$ is degree of membership of m in (M,h).

Remark 2.2. If $h(m) = 0$, then m not included in fuzzy set (M,h), if $h(m) = 1$, then m fully including in fuzzy set (M,h), if $h(m) \in (0,1)$, then m is a member of fuzzy set (M,h).

Definition 2.3. Let us consider that $M \neq \phi$, then Bipolar valued fuzzy set is an object define and denoted as, $B = \{(m, \mu_B^+(m), \mu_B^-(m)) : m \in M\}$. Where μ_B^+ and μ_B^- are functions from M into $[0,1]$ and $[-1,0]$ respectively, Where $\mu_B^+(m)$ is degree of positive membership denotes the satisfaction degree of an element m to the property corresponding to a Bipolar valued fuzzy set B and $\mu_B^-(m)$ is degree of negative membership denotes counter satisfaction degree of an element m to the property corresponding to a Bipolar valued fuzzy set B.

Remark 2.4. If $\mu_B^-(m) = 0$ and $\mu_B^+(m) \neq 0$, then m is only positive satisfaction for B. If $\mu_B^-(m) \neq 0$ and $\mu_B^+(m) = 0$, then m is only negative satisfaction for B. If $\mu_B^-(m) \neq 0$ and $\mu_B^+(m) \neq 0$ is a situation, when m is positive satisfaction as well as negative satisfaction i.e the membership function of the property overlaps that of its counter property over some portion of M.

For our simplicity from now to onward instead of writing $B = \{(m, \mu^+(m), \nu^-(m)) : m \in M\}$, we will write $B =< \mu^+, \nu^- >$.

3. BVFS value score function

In below discussion $\wp = \{B_i =< \mu_{B_i}^+, \mu_{B_i}^- > |i \in I\}$ be a collection of BVFS values.

Definition 3.1. Let $\wp$, then score function is real valued function define and denoted as

$$s(B_i) = \mu_{B_i}^+ + \mu_{B_i}^-, \tag{3.1}$$

where $s(B_i) \in [-1,1]$ and $s(B_i) = 0$ if and only if $\mu_{B_i}^+ = -\mu_{B_i}^-$.

Remark 3.2. Let $B_1 = \langle \mu_{B_1}^+, \mu_{B_1}^- \rangle$ and $B_2 = \langle \mu_{B_2}^+, \mu_{B_2}^- \rangle$ be two BVFS values and let $s(B_1) = \mu_{B_1}^+ + \mu_{B_1}^-$ and $s(B_2) = \mu_{B_2}^+ + \mu_{B_2}^-$ be the score of B_1 and B_2 respectively. Then

If $s(B_1) < s(B_2)$, then B_1 is smaller than B_2 denoted by $B_1 < B_2$.

If $s(B_1) > s(B_2)$, then B_1 is greater than B_2 denoted by $B_1 > B_2$.

Example 3.3. Let $A_1 = \langle 0.5, -0.4 \rangle$ and $A_2 = \langle 0.2, -0.5 \rangle$ be two BVFS values, then $s(A_1) = 0.1$ and $s(A_2) = -0.3$. So, clearly $A_1 > A_2$. Let $B_1 = \langle 0.9, -1 \rangle$ and $B_2 = \langle 1, -0.4 \rangle$ then, $s(B_1) = -.01$ and $s(B_2) = 0.6$. So, clearly $B_1 < B_2$. Let

$C_1 = \langle 0.9, -0.2\rangle$ and $C_2 = \langle 1, -0.3\rangle$ be two BVFS values then, $s(C_1) = 0.7$ and $s(C_2) = 0.7$. Here, it is impossible to decide either $C_1 < C_2$ or $C_1 > C_2$.

Definition 3.4. Improve score function is real valued function on $\wp$ define and denoted as

$$I(B_i) = (\mu^+_{B_i})^2 s(B_i) + (\mu^-_{B_i})^2 s(B_i) - (\mu^+_{B_i}\mu^-_{B_i})s(B_i). \tag{3.2}$$

Remark 3.5. Let $A = \langle \mu^+_{B_1}, \mu^-_{B_1}\rangle$ and $B = \langle \mu^+_{B_2}, \mu^-_{B_2}\rangle$ be two BVFS values let $I(A) = (\mu^+_A)^3 + (\mu^-_A)^3$ and $I(B) = (\mu^+_B)^3 + (\mu^-_B)^3$ be the improve score of A and B respectively. Then

If $I(A) < I(B)$, then A is smaller than B denoted by $A < B$.

If $I(A) > I(B)$, then A is greater than B denoted by $A > B$.

Consider example 3.3. Then $I(A_1) = 0.061$ and $I(A_2) = -0.117$. Thus $A_1 > A_2$, $I(B_1) = -0.271$ and $I(B_2) = 0.936$. So $B_1 < B_2$ and $I(C_1) = 0.721$ and $I(C_2) = 0.973$. Hence $C_1 < C_2$.

Let $D_1 = \langle 0.965489, -0.5848\rangle$ and $D_2 = \langle 1, -0.66943\rangle$ be two BVFS values. Then $I(D_1) = 0.7$ and $I(D_2) = 0.7$. Here it is impossible to decide either $D_1 < D_2$ or $D_1 > D_2$.

Definition 3.6. Double Improve score function is real valued function on $\wp$ define and denoted as

$$D(B_i) = s(B_i) + I(B_i). \tag{3.3}$$

Remark 3.7. Let $A = \langle \mu^+_{B_1}, \mu^-_{B_1}\rangle$ and $B = \langle \mu^+_{B_2}, \mu^-_{B_2}\rangle$ be two BVFS values, then $D(A)$ and $D(B)$, be the double improve score of A and B respectively. Then

If $D(A) < D(B)$, then A is smaller than B denoted by $A < B$.

If $D(A) > D(B)$, then A is greater than B denoted by $A > B$.

If $D(A) = D(B)$, then A is equal to B denoted by $A = B$.

Example 3.8. Let $D(A_1) = 0.161$ and $D(A_2) = -0.417$. Then $A_1 > A_2$, $D(B_1) = -0.371$ and $D(B_2) = 1.536$. Thus $B_1 < B_2$ and $D(C_1) = 1.421$ and $D(C_2) = 1.673$. So $C_1 < C_2$. $D(D_1) = 1.080691564$ and $D(D_2) = 1.030573966$. Hence $D_1 > D_2$.

4. Algorithm for MCDM based on BVFSs score function

In this section, we develop algroithm to solve MCDM problems by using proposed score function in which all information provided by DM is express in Bipolar valued fuzzy Decision matrix. Where each of the element is characterized by BVFS value, and criterion weight is also given by DM.

For this paper different decision factors for MCDM problem and their representative symbols are as follows: (1) Alternatives by set $M = \{m_i : 1 \leq i \leq n\}$.(2) Criteria associated to each alternative $C = \{c_j : 1 \leq j \leq n\}$.

The " Importance"and "Quality" of alternative m_i corresponding to criteria c_j which are given by the decision makers are assigned in fuzzy literature which are defined using BVFSs. Table 1 gives an example of the term measure on " Importance"and "Quality"on different levels.

Level	"Imoprtance"Measure	"Quality"Measure	BVF values)
L1	Extremely Important (EI)	Extremely Positive (EP)	$\langle 1, 0\rangle$
L2	Great Important (GI)	Absolutely Positive (AP)	$\langle 0.8, -0.2\rangle$
L3	Very Important (VI)	Very Very Positive (VVP)	$\langle 0.6, -0.4\rangle$
L4	Important (I)		$\langle 0.6, -0.5\rangle$
L5	Medium (M)	Positive (P)	$\langle 0.5, -0.5\rangle$
L6	Unimportant (UI)	Negative (N)	$\langle 0.4, -0.6\rangle$
L7	Not Important (NI)	Very Negative (VN)	$\langle 0.2, -0.6\rangle$
L8	Extremely Not Important (ENI)	Extremely Negative (EN)	$\langle 0, -1\rangle$

Tabel 1

Information provide by decision maker about alternative m_i and criteria c_j in Decision matrix in which each entry $x_{ij} =< \mu^+_{x_{ij}}, \mu^-_{x_{ij}} >$ is characteristics by BVFS value. Using equation 3.1 and 3.2, we get

$$\begin{aligned} R_1 &= [s(x_{ij})]_{m\times n}, \\ &\text{and} \\ R_2 &= [I(x_{ij})]_{m\times n}. \end{aligned}$$

Then

$$R = [D(x_{ij})]_{m\times n}.$$

Best alternative is given by, $best(m_i) = \max\{imp(m_i)|i = 1, 2, 3, ..., m\}$, where, $imp(m_i) = \sum_{j=1}^{n} D(x_{ij})$, (where $i = 1, 2, ..., m$).

5. Illustrated examples

In this section, we demonstrate a example to illustrate the effectiveness of proposed method.

Example 5.1. A company announced vacant post of office representative in customer care centre. Six candidates applied for job. Let $M = \{m_1, m_2, m_3, m_4, m_5, m_6\}$ be the set of candidates applied for a job. Selection board decid to select a candidate according to the criteria c_1 = Hard Working, c_2 = Decisiveness,c_3 = Politeness, c_4 = Self-confidence, c_5 = Flexibility, c_6 = Optimism. "Imoprtance" and "Quality" of each candidates provide by selection board corresponding to criteria from table 1.

$$D(x_{ij}) = \begin{bmatrix} \text{GI} & \text{M} & \text{ENI} & \text{I} & \text{UI} & \text{NI} \\ \text{M} & \text{I} & \text{VI} & \text{GI} & \text{EI} & \text{M} \\ \text{EI} & \text{UI} & \text{NI} & \text{I} & \text{UI} & \text{GI} \\ \text{I} & \text{VI} & \text{M} & \text{M} & \text{VI} & \text{EI} \\ \text{EI} & \text{M} & \text{NI} & \text{UI} & \text{EI} & \text{I} \\ \text{M} & \text{ENI} & \text{I} & \text{UI} & \text{VI} & \text{NI} \end{bmatrix}.$$

Best alternative is given by, $best(m_i) = \max\{imp(m_i)|i = 1, 2, 3, 4, 5, 6\}$, where $imp(m_i)$ is given in table 2.

m_i	$imp(m_i)$
m_1	-2.161
m_2	3.647
m_3	1.487
m_4	2.895
m_5	2.735
m_6	-2.913

Table 2

So, among these available four candidates best candidate is m_2.

6. Conclusions

From our study we conclude that corresponding to each property there exist counter property in decision making problems focus on both property and counter property of object.

In this paper, we proposed score function, improve score and double improve score function to solve MCDM problems. In which all preference information provide by Decision maker in Decision matrix, where each of the elements is characterized by BVFS value. We apply the proposed function to assign real number to each entry in decision matrix, then calculate sum of each row to compute importance of each alternative. Alternative(s) with maximum value of importance is best choice(s).

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