Similarity measure of IFS-sets and its application in medical diagnosis

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ABSTRACT. In this article, we first give definitions of some types of distances between two intuitionistic fuzzy soft (IFS) sets and then proposed similarity measures of two IFS-sets. We then construct a decision method which is based on similarity measures of IFS-sets. We finally give two simple applications which show the possibility of using this method for diagnosis of diseases could be improved by incorporating clinical results and other competing diagnosis.

1. Introduction

In 1999, Molodtsov [36] has introduced the concept of soft sets. The soft set theory successfully models the problems which contains uncertainties. In literature, there are theories, such as probability, fuzzy sets [47], intuitionistic fuzzy sets [8], rough sets [41] that are dealing with the uncertain data.

In this work we use soft set theory. The operations (e.g. [16, 19, 33, 43]) and applications (e.g. [2, 3, 4, 5, 7, 15, 17, 38, 42, 45]) on soft set theory have been studied by some researcher. In recent years, many decision making on soft set theory have been expanded by embedding the ideas of fuzzy sets (e.g. [1, 6, 10, 11, 18, 20, 21, 22, 23, 29, 32, 35, 40, 46, 44]), intuitionistic fuzzy sets (e.g. [8, 9, 12, 13, 14, 27, 30, 31, 37, 39]) and rough sets [6, 24].

Majumdar and Samanta[34] give two types of similarity measure between soft sets and have shown an application of this similarity measure of soft sets. Kharal [26] give counterexamples to show that Definition 2.7 and Lemma 3.5 contain errors in [34]. In [26], a new measures have been presented and this measures have been applied to the problem of financial diagnosis of firms. Also, Jiang et al. [28] further studied on intuitionistic fuzzy soft sets based on distance measure and entropy measure.

In this paper, we first present the basic definitions and theorem of soft sets, fuzzy sets, intuitionistic fuzzy sets and intuitionistic fuzzy soft sets that are useful for subsequent discussions. We then define distances and similarity measures between two intuitionistic fuzzy soft (IFS) sets. By using the similarity we construct a decision making method. We finally give an application, which shows that the similarity measures can be successfully applied to a medical diagnosis problem that contains uncertainties.

2. Preliminary

In this section, we present the basic definitions of soft set theory [19, 36], fuzzy set theory [47], intuitionistic fuzzy set theory [8] and intuitionistic fuzzy soft set theory [14] that are useful for subsequent discussions.

Definition 2.1 ([19]). Let U be a universe, $P(U)$ be the power set of U, E be a set of parameters that are describe by the elements of U, and $A \subseteq E$. Then, a soft set F_A over U is a set defined by a set valued function f_A representing a mapping

$$(2.1) \qquad f_A : E \to P(U) \text{ such that } f_A(x) = \varnothing \text{ if } x \in E - A,$$

where f_A is called approximate function of the soft set F_A. In other words, the soft set is a parametrized family of subsets of the set U, and therefore it can be written a set of ordered pairs

$$F_A = \{(x, f_A(x)) : x \in E, f_A(x) = \varnothing \text{ if } x \in E - A\}.$$

Definition 2.2 ([47]). Let U be a universe. Then a fuzzy set X over U is defined as;

$$X = \{(\mu_X(u)/u) : u \in U\},$$

where $\mu_X : U \to [0.1]$.

Here, μ_X is called membership function of X, and the value $\mu_X(u)$ is called the grade of membership of $u \in U$ in the fuzzy set X. The value represents the degree of u belonging to the fuzzy set X.

Definition 2.3 ([8]). Let E be a universe. An intuitionistic fuzzy set A on E can be defined as follows:

$$A = \{< x, \mu_A(x), \nu_A(x) >: \ x \in E\}$$

where, $\mu_A : E \rightarrow [0,1]$ and $\nu_A : E \rightarrow [0,1]$ such that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for any $x \in E$. Here, $\mu_A(x)$ and $\nu_A(x)$ is the degree of membership and degree of non-membership of the element x, respectively.

Definition 2.4 ([8]). $A = \{< x, \mu_A(x), \nu_A(x) >: \ x \in E\}$ and $B = \{< x, \mu_B(x), \nu_B(x) >: x \in E\}$ are two intuitionistic fuzzy sets on E. Then

    (1) $A \subset B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ $\forall x \in E$.
    (2) $A = B$ if and only if $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$ $\forall x \in E$.
    (3) $A^c = \{< x, \nu_A(x), \mu_A(x) >: \ x \in E\}$.
    (4) $A \cup B = \{< x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x) >: \ x \in E\}$.
    (5) $A \cap B = \{< x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x) >: \ x \in E\}$.
    (6) $A + B = \{< x, \mu_X(x) + \mu_Y(x) - \mu_X(x)\mu_Y(x), \nu_X(x)\nu_Y(x) >: \ x \in E\}$.
    (7) $A \cdot B = \{< x, \mu_A(x)\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x) >: \ x \in E\}$.

Definition 2.5 ([14]). Let U be a initial universe, $\hat{F}$ be a intuitionistic fuzzy subset of U, E be a set of all parameters. Then an intuitionistic fuzzy soft set(or namely IFS-set), denoted Γ_A, is defined as

$$\Gamma_A = \{(x, \gamma_A(x)) : x \in E, \gamma_A(x) \in \hat{F}(U)\},$$

where $\gamma_A : E \rightarrow \hat{F}(U)$ such that $\gamma_A(x) = \hat{\varnothing}$ if $x \notin A$ and $\hat{\varnothing}$ is intuitionistic fuzzy empty set. Moreover $\gamma_A(x)$ is an intuitionistic fuzzy set. So it is denoted by

$$\gamma_A(x) = \{(u, \mu_A(u), \nu_A(u)) : u \in U\}$$

for all $x \in E$. Moreover, $\mu_A : U \rightarrow [0,1]$ and $\nu_A : U \rightarrow [0,1]$ with the condition $0 \leq \mu_A(u) + \nu_A(u) \leq 1$, for all $u \in U$. The numbers $\mu_A(u)$ and $\nu_A(u)$ denote the membership degree end non-membership degree of $u \in U$ to the intuitionistic fuzzy set $\gamma_A(x)$, respectively.

Example 2.6. Suppose that there are five cars in the universe $U = \{u_l, u_2, u_3, u_4, u_5\}$ under consideration "x_1 =large", "x_2 =costly", "x_3 =secure", "x_4 =strong", "$x_5 =$ economic" and", "$x_6 =$ repair". Therefore parameter set is $E = \{x_1, x_2, x_3, x_4, x_5, x_6\}$. Let $A = \{x_1, x_2, x_3, x_4\}$.

A customer to select a car from the auto agent, can construct a IFS-set Γ_A that describes the characteristic of cars according to own requests. Assume that

$$\gamma_A(x_1) = \{(u_1, 0.5, 0.2), (u_2, 0.5, 0.2), (u_3, 0.5, 0.2), (u_4, 0.5, 0.2)\},$$

$$\gamma_A(x_2) = \{(u_1, 0.6, 0.4), (u_2, 0.9, 0.1), (u_3, 0.5, 0.3), (u_4, 0.1, 0.9)\},$$

$$\gamma_A(x_3) = \{(u_1, 0.7, 0.2), (u_2, 0.8, 0.1), (u_3, 0.2, 0.16), (u_4, 0.4, 0.5)\},$$

$$\gamma_A(x_4) = \{(u_1, 0.4, 0.3), (u_2, 0.2, 0.7), (u_3, 0.8, 0.2), (u_4, 0.2, 0.1)\}.$$

Then we have

$$\Gamma_A = \left\{ \begin{array}{l} (x_1, \{(u_1, 0.5, 0.2), (u_2, 0.5, 0.2), (u_3, 0.5, 0.2), (u_4, 0.5, 0.2)\}, \\[4pt] (x_2, \{(u_1, 0.6, 0.4), (u_2, 0.9, 0.1), (u_3, 0.5, 0.3), (u_4, 0.1, 0.9)\}, \\[4pt] (x_3, \{(u_1, 0.7, 0.2), (u_2, 0.8, 0.1), (u_3, 0.2, 0.16), (u_4, 0.4, 0.5)\}, \\[4pt] (x_4, \{(u_1, 0.4, 0.3), (u_2, 0.2, 0.7), (u_3, 0.8, 0.2), (u_4, 0.2, 0.1)\} \end{array} \right\}.$$

3. Similarity Measures of IFS-Sets

In this section, we first present the basic definitions of distances between two intuitionistic fuzzy sets [9] and two soft sets [34] that are useful for subsequent discussions. We then define some distances and similarity measures of IFS-sets.

Definition 3.1 ([9])**.** Let $U = \{x_1, x_2, x_3, ..., x_n\}$ be a universe and A, B be two intuitionistic fuzzy sets over U with their membership functions μ_A, μ_B and non-membership functions ν_A, ν_B, respectively. Then the distances of A and B are defined as,

(1) Hamming distance(HD);

$$d(A, B) = \frac{1}{2} \sum_{i=1}^{n} [|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|].$$

(2) Normalized Hamming distance(NHD);

$$l(A, B) = \frac{1}{2n} \sum_{i=1}^{n} [|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|].$$

(3) Euclidean distance(ED);

$$e(A, B) = \sqrt{\frac{1}{2} \sum_{i=1}^{n} [(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2]}.$$

(4) Normalized Euclidean distance(NED);

$$q(A, B) = \sqrt{\frac{1}{2n} \sum_{i=1}^{n} [(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2]}.$$

Definition 3.2 ([34])**.** Let $U = \{u_1, u_2, u_3, ...\}$ be a universe, $E = \{x_1, x_2, x_3, ...\}$ be a set of parameters, $A, B \subseteq E$, and F_A and G_B be two soft sets on U with their approximate functions f_A and g_B, respectively.

If $A = B$, then similarity between F_A and G_B is defined by

$$S(F_A, G_B) = \frac{\sum_{i=1} \overrightarrow{f_A(x_i)} \cdot \overrightarrow{g_B(x_i)}}{\sum_{i=1} max[\overrightarrow{f_A(x_i)}^2, \overrightarrow{g_B(x_i)}^2]},$$

where

$$\overrightarrow{f_A(x_i)} = (\chi_{f_A(x_i)}(u_1), \chi_{f_A(x_i)}(u_2), \chi_{f_A(x_i)}(u_3), ...),$$
$$\overrightarrow{g_B(x_i)} = (\chi_{g_B(x_i)}(u_1), \chi_{g_B(x_i)}(u_2), \chi_{g_B(x_i)}(u_3), ...)$$

and

$$\chi_{f_A(x_i)}(u_j) = \left\{ \begin{array}{l} 1, u_j \in f_A(x_i) \\ 0, u_j \notin f_A(x_i) \end{array} \right. , \quad \chi_{g_B(x_i)}(u_j) = \left\{ \begin{array}{l} 1, u_j \in g_B(x_i) \\ 0, u_j \notin g_B(x_i). \end{array} \right.$$

If $A \neq B$ and $C = A \cap B \neq \varnothing$, then $\overrightarrow{f_A(x_i)} = 0$ for $x_i \in B/C$ and $\overrightarrow{g_B(x_i)} = 0$ for $x_i \in A/C$.

If $A \cap B = \varnothing$, then $S(F_A, G_B) = 0$ and $S(F_A, F_A^c) = 0$ as $\overrightarrow{f_A(x_i)} \cdot \overrightarrow{f_A^c(x_i)} = 0$ for all i.

Definition 3.3 ([34]). Let F_A and G_B be two soft sets over U. Then, F_A and G_B are said to be α-similar, denoted as $F_A \approx^\alpha G_B$, if and only if $S(F_A, G_B) \geq \alpha$ for $\alpha \in (0, 1)$.

Definition 3.4 ([34]). Let $U = \{u_1, u_2, u_3, ...\}$ be a universe, $E = \{x_1, x_2, x_3, ...\}$ be a set of parameters, $A, B \subseteq E$ and F_A, G_B be two soft sets on U with their approximate functions f_A and g_B, respectively. Then, the distances of F_A and G_B are defined as,

(1) HD;

$$d(F_A, G_B) = \frac{1}{m} \left\{ \sum_{i=1}^m \sum_{j=1}^n |f_A(x_i)(u_j) - g_B(x_i)(u_j)| \right\}.$$

(2) NHD;

$$l(F_A, G_B) = \frac{1}{mn} \left\{ \sum_{i=1}^m \sum_{j=1}^n |f_A(x_i)(u_j) - g_B(x_i)(u_j)| \right\}.$$

(3) ED;

$$e(F_A, G_B) = \sqrt{\frac{1}{m} \sum_{i=1}^m \sum_{j=1}^n (f_A(x_i)(u_j) - g_B(x_i)(u_j))^2}.$$

(4) NED;

$$q(F_A, G_B) = \sqrt{\frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n (f_A(x_i)(u_j) - g_B(x_i)(u_j))^2}.$$

Definition 3.5 ([34]). Let F_A and G_B be two soft sets over U. Then, by using the ED, similarity measure of F_A and G_B is defined as,

$$s'(F_A, G_B) = \frac{1}{1 + e(F_A, G_B)}.$$

Another similarity measure of F_A and G_B can be defined as,

$$s''(F_A, G_B) = e^{-\alpha e(F_A, G_B)},$$

where α is a positive real number called the steepness measure.

Definition 3.6. Let $U = \{u_1, u_2, ..., u_n\}$ be a universe, $E = \{x_1, x_2, ..., x_m\}$ be a set of parameters, $A, B \subseteq E$ and Γ_A, Λ_B be two IFS-sets on U with their intuitionistic fuzzy approximate functions $\gamma_A(x_i) = \{(u, \mu_A(u), \nu_A(u)) : u \in U\}$ and $\lambda_B(x_i) = \{(u, \mu_B(u), \nu_B(u)) : u \in U\}$, respectively.

If $A = B$ and $\mu_A(x_i)(u_j) - \nu_A(x_i)(u_j) \neq 0$ or $\mu_B(x_i)(u_j) - \nu_B(x_i)(u_j) \neq 0$ for at least one $i \in \{1, 2, ..., n\}$ and $j \in \{1, 2, ..., m\}$, then similarity between Γ_A and Λ_B is defined by

$$S_{IFS}(\Gamma_A, \Lambda_B) =$$

$$\frac{\sum_{i=1}^{m} \sum_{j=1}^{n} |(\overrightarrow{\mu_A(x_i)(u_j)} - \overrightarrow{\nu_A(x_i)(u_j)}) \cdot (\overrightarrow{\mu_B(x_i)(u_j)} - \overrightarrow{\nu_B(x_i)(u_j)})|}{\sum_{i=1}^{m} \sum_{j=1}^{n} max\{||\overrightarrow{\mu_A(x_i)(u_j)} - \overrightarrow{\nu_A(x_i)(u_j)}||^2, ||\overrightarrow{\mu_B(x_i)(u_j)} - \overrightarrow{\nu_B(x_i)(u_j)}||^2\}},$$

where

$$
\begin{aligned}
\overrightarrow{\mu_A(x_i)(u_j)} &= (\mu_A(x_i)(u_1), \mu_A(x_i)(u_2), ..., \mu_A(x_i)(u_n)), \\
\overrightarrow{\nu_A(x_i)(u_j)} &= (\nu_A(x_i)(u_1), \nu_A(x_i)(u_2), ..., \nu_A(x_i)(u_n)), \\
\overrightarrow{\mu_B(x_i)(u_j)} &= (\mu_B(x_i)(u_1), \mu_B(x_i)(u_2), ..., \mu_B(x_i)(u_n)), \\
\overrightarrow{\nu_B(x_i)(u_j)} &= (\nu_B(x_i)(u_1), \nu_B(x_i)(u_2), ..., \nu_B(x_i)(u_n)).
\end{aligned}
$$

If $A = B$ and $\mu_A(x_i)(u_j) - \nu_A(x_i)(u_j) = 0$ and $\mu_B(x_i)(u_j) - \nu_B(x_i)(u_j) = 0$ for all $i \in \{1, 2, ..., n\}$ and $j \in \{1, 2, ..., m\}$, then $S_{IFS}(\Gamma_A, \Lambda_B) = 1$.

Example 3.7. Assume that $U = \{u_1, u_2, u_3, u_4\}$ is a universal set, $E = \{x_1, x_2, x_3, x_4\}$ is a set of parameters, $A = \{x_1, x_2, x_4\}$, $B = \{x_1, x_2, x_4\}$ are subsets of E. If two IFS-sets Γ_A and Λ_B over U are contracted as follows;

$$
\begin{aligned}
\Gamma_A = \big\{ &(x_1, \{(u_1, 0.5, 0.5), (u_2, 0.4, 0.5), (u_3, 0.7, 0.2), (u_4, 0.8, 0.1)\}), \\
&(x_2, \{(u_1, 0.4, 0.6), (u_2, 0.2, 0.7), (u_3, 0.2, 0.8), (u_4, 0.2, 0.2)\}), \\
&(x_4, \{(u_1, 0.2, 0.7), (u_2, 0.1, 0.9), (u_3, 0.5, 0.4), (u_4, 0.7, 0.2)\}) \big\},
\end{aligned}
$$

$$
\begin{aligned}
\Lambda_B = \big\{ &(x_1, \{(u_1, 0.2, 0.7), (u_2, 0.1, 0.9), (u_3, 0.5, 0.4), (u_4, 0.4, 0.4)\}), \\
&(x_2, \{(u_1, 0.5, 0.5), (u_2, 0.4, 0.5), (u_3, 0.3, 0.6), (u_4, 0.4, 0.5)\}), \\
&(x_4, \{(u_1, 0.4, 0.6), (u_2, 0.2, 0.7), (u_3, 0.2, 0.8), (u_4, 0.2, 0.5)\}) \big\}.
\end{aligned}
$$

Then we can obtain
$$\overrightarrow{\mu_A(x_1)(u_j)} = (0.5, 0.4, 0.7, 0.8), \quad \overrightarrow{\nu_A(x_1)(u_j)} = (0.5, 0.5, 0.2, 0.1),$$
$$\overrightarrow{\mu_A(x_2)(u_j)} = (0.4, 0.2, 0.2, 0.2), \quad \overrightarrow{\nu_A(x_2)(u_j)} = (0.6, 0.7, 0.8, 0.2),$$
$$\overrightarrow{\mu_A(x_4)(u_j)} = (0.2, 0.1, 0.5, 0.7), \quad \overrightarrow{\nu_A(x_4)(u_j)} = (0.7, 0.9, 0.4, 0.2),$$
$$\overrightarrow{\mu_B(x_1)(u_j)} = (0.2, 0.1, 0.5, 0.4), \quad \overrightarrow{\nu_B(x_1)(u_j)} = (0.7, 0.9, 0.4, 0.4),$$
$$\overrightarrow{\mu_B(x_2)(u_j)} = (0.5, 0.4, 0.3, 0.4), \quad \overrightarrow{\nu_B(x_2)(u_j)} = (0.5, 0.5, 0.6, 0.5),$$
$$\overrightarrow{\mu_B(x_4)(u_j)} = (0.4, 0.2, 0.2, 0.2), \quad \overrightarrow{\nu_B(x_4)(u_j)} = (0.6, 0.7, 0.8, 0.5)$$
and
$$(\overrightarrow{\mu_A(x_1)(u_j)} - \overrightarrow{\nu_A(x_1)(u_j)}) = (0.0, -0.1, 0.5, 0.7),$$

$$\overrightarrow{(\mu_A(x_2)(u_j)} - \overrightarrow{\nu_A(x_2)(u_j)} = (-0.2, -0.5, -0.6, 0.0),$$
$$\overrightarrow{(\mu_A(x_4)(u_j)} - \overrightarrow{\nu_A(x_4)(u_j)} = (-0.5, -0.8, 0.1, 0.5),$$
$$\overrightarrow{(\mu_B(x_1)(u_j)} - \overrightarrow{\nu_B(x_1)(u_j)} = (-0.5, -0.8, 0.1, 0.0),$$
$$\overrightarrow{(\mu_B(x_2)(u_j)} - \overrightarrow{\nu_B(x_2)(u_j)} = (0.0, -0.1, -0.3, -0.1),$$
$$\overrightarrow{(\mu_B(x_4)(u_j)} - \overrightarrow{\nu_B(x_4)(u_j)} = (-0.2, -0.5, -0.6, -0.3).$$

Now the similarity between Γ_A and Λ_B is calculated as

$$S_{IFS}(\Gamma_A, \Lambda_B) = 0.31.$$

Theorem 3.8. *Let E be a parameter set, $A, B \subseteq E$ and Γ_A and Λ_B be two IFS-sets over U. Then the followings hold :*
(i) $S_{IFS}(\Gamma_A, \Lambda_B) = S_{IFS}(\Lambda_B, \Gamma_A)$.
(ii) $0 \leq S_{IFS}(\Gamma_A, \Lambda_B) \leq 1$.
(iii) $S_{IFS}(\Gamma_A, \Gamma_A) = 1$.

Proof. Proof easly can be made by using Definition 3.6. □

Definition 3.9 ([28])**.** Let $U = \{u_1, u_2, ..., u_n\}$ be a universe, $E = \{x_1, x_2, ..., x_m\}$ be a set of parameters, $A, B \subseteq E$ and Γ_A, Λ_B be two IFS-sets on U with their intuitionistic fuzzy approximate functions $\gamma_A(x_i) = \{(u, \mu_A(u), \nu_A(u)) : u \in U\}$ and $\lambda_B(x_i) = \{(u, \mu_B(u), \nu_B(u)) : u \in U\}$, respectively. Then the distances of Γ_A and Λ_B are defined as,

(1) HD,

$$d^s_{IFS}(\Gamma_A, \Lambda_B) =$$
$$\frac{1}{2} \left\{ \sum_{i=1}^m \sum_{j=1}^n |\mu_A(x_i)(u_j) - \mu_B(x_i)(u_j)| + |\nu_A(x_i)(u_j) - \nu_B(x_i)(u_j)| \right\}.$$

(2) NHD,

$$l^s_{IFS}(\Gamma_A, \Lambda_B) =$$
$$\frac{1}{2mn} \left\{ \sum_{i=1}^m \sum_{j=1}^n |\mu_A(x_i)(u_j) - \mu_B(x_i)(u_j)| + |\nu_A(x_i)(u_j) - \nu_B(x_i)(u_j)| \right\}.$$

(3) ED,

$$e^s_{IFS}(\Gamma_A, \Lambda_B) =$$
$$\left(\frac{1}{2} \sum_{i=1}^m \sum_{j=1}^n [(\mu_A(x_i)(u_j) - \mu_B(x_i)(u_j))^2 + (\nu_A(x_i)(u_j) - \nu_B(x_i)(u_j))^2] \right)^{\frac{1}{2}}.$$

(4) NED,

$$q^s_{IFS}(\Gamma_A, \Lambda_B) =$$
$$\left(\frac{1}{2mn} \sum_{i=1}^m \sum_{j=1}^n [(\mu_A(x_i)(u_j) - \mu_B(x_i)(u_j))^2 + (\nu_A(x_i)(u_j) - \nu_B(x_i)(u_j))^2] \right)^{\frac{1}{2}}.$$

Now we give distance measure of IFS-sets with propositions by adding a weighted value for element of parameter set in Jiang et al. [28].

Definition 3.10. Let $U = \{u_1, u_2, ..., u_n\}$ be a universe, $E = \{x_1, x_2, ..., x_m\}$ be a set of parameters, $A, B \subseteq E$ and Γ_A, Λ_B be two IFS-sets on U with their intuitionistic fuzzy approximate functions $\gamma_A(x_i) = \{(u, \mu_A(u), \nu_A(u)) : u \in U\}$ and $\lambda_B(x_i) = \{(u, \mu_B(u), \nu_B(u)) : u \in U\}$, respectively. Then the distances of Γ_A and Λ_B are defined as,

(1) weighted HD,

$$d^w_{IFS}(\Gamma_A, \Lambda_B) =$$
$$\frac{1}{2}\left\{\sum_{i=1}^m w_i \sum_{j=1}^n |\mu_A(x_i)(u_j) - \mu_B(x_i)(u_j)| + |\nu_A(x_i)(u_j) - \nu_B(x_i)(u_j)|\right\}.$$

(2) weighted NHD,

$$l^w_{IFS}(\Gamma_A, \Lambda_B) =$$
$$\frac{1}{2mn}\left\{\sum_{i=1}^m w_i \sum_{j=1}^n |\mu_A(x_i)(u_j) - \mu_B(x_i)(u_j)| + |\nu_A(x_i)(u_j) - \nu_B(x_i)(u_j)|\right\}.$$

(3) weighted ED,

$$e^w_{IFS}(\Gamma_A, \Lambda_B) =$$
$$\left(\frac{1}{2}\sum_{i=1}^m w_i \sum_{j=1}^n [(\mu_A(x_i)(u_j) - \mu_B(x_i)(u_j))^2 + (\nu_A(x_i)(u_j) - \nu_B(x_i)(u_j))^2]\right)^{\frac{1}{2}}.$$

(4) weighted NED,

$$q^w_{IFS}(\Gamma_A, \Lambda_B) =$$
$$\left(\frac{1}{2mn}\sum_{i=1}^m w_i \sum_{j=1}^n [(\mu_A(x_i)(u_j) - \mu_B(x_i)(u_j))^2 + (\nu_A(x_i)(u_j) - \nu_B(x_i)(u_j))^2]\right)^{\frac{1}{2}},$$

where $w_i \in [0, 1]$ is called weight of x_i.

Example 3.11. Let us consider the Example 3.7 and $w_i = 1$ for all parameter x_i. Then, the distances of Γ_A and Λ_B are calculated as follows;

$$\begin{aligned}
d^w_{IFS}(\Gamma_A, \Lambda_B) &= 0.07, \\
l^w_{IFS}(\Gamma_A, \Lambda_B) &= 0.37, \\
e^w_{IFS}(\Gamma_A, \Lambda_B) &= 0.28, \\
q^w_{IFS}(\Gamma_A, \Lambda_B) &= 0.19.
\end{aligned}$$

Theorem 3.12. *Let E be a parameter set, $A, B \subseteq E$ and Γ_A and Λ_B be two IFS-sets over U. Then the followings hold :*
(i) $d^w_{IFS}(\Gamma_A, \Lambda_B) \leq mn$.
(ii) $l^w_{IFS}(\Gamma_A, \Lambda_B) \leq 1$.
(iii) $e^w_{IFS}(\Gamma_A, \Lambda_B) \leq \sqrt{mn}$.

(iv) $q_{IFS}^w(\Gamma_A, \Lambda_B) \leq 1$.

Proof. Proof easily can be made by using Definition 3.10. $\square$

Theorem 3.13. *Let $IFS(U)$ be a set of all IFS-sets over U. Then the distances functions d_{IFS}^w, I_{IFS}^w, e_{IFS}^w, and q_{IFS}^w defined from $IFS(U)$ to the non-negative real number R^+, are metric.*

Proof. We give only proof for d_{IFS}^w. If Γ_A, Λ_B and $\Upsilon_C \in IFS(U)$, then

- $d_{IFS}^w(\Gamma_A, \Lambda_B) \geq 0 \ \forall i = \{1, 2, ..., m\}, j = \{1, 2, ..., n\}$.
  Suppose $d_{IFS}^w(\Gamma_A, \Lambda_B) = 0$. Then

  $$|\mu_A(x_i)(u_j) - \mu_B(x_i)(u_j)| + |\nu_A(x_i)(u_j) - \nu_B(x_i)(u_j)| = 0.$$

  Thus

  $$\mu_A(x_i)(u_j) = \mu_B(x_i)(u_j) \wedge \nu_A(x_i)(u_j) = \nu_B(x_i)(u_j).$$

  So $\Gamma_A = \Lambda_B$.
  Conversely, suppose $\Gamma_A = \Lambda_B$. Then

  $$\mu_A(x_i)(u_j) = \mu_B(x_i)(u_j) \wedge \nu_A(x_i)(u_j) = \nu_B(x_i)(u_j).$$

  Thus

  $$|\mu_A(x_i)(u_j) - \mu_B(x_i)(u_j)| + |\nu_A(x_i)(u_j) - \nu_B(x_i)(u_j)| = 0.$$

  So $d_{IFS}^w(\Gamma_A, \Lambda_B) = 0$.
- Clearly, $d_{IFS}^w(\Gamma_A, \Lambda_B) = d_{IFS}^w(\Lambda_B, \Gamma_A)$.
- Triangle inequality follows easily from the observation that for any three IFS-sets $\Gamma_A. \Lambda_B$ and $\Upsilon_C, \forall i = \{1, 2, ..., m\}, j = \{1, 2, ..., n\}$. For

  $$|\mu_A(x_i)(u_j) - \mu_B(x_i)(u_j)| + |\nu_A(x_i)(u_j) - \nu_B(x_i)(u_j)|$$
  $$= |\mu_A(x_i)(u_j) - \mu_C(x_i)(u_j) + \mu_C(x_i)(u_j) - \mu_B(x_i)(u_j)|$$
  $$+ |\nu_A(x_i)(u_j) - \nu_C(x_i)(u_j) + \nu_C(x_i)(u_j) - \nu_B(x_i)(u_j)|$$
  $$\leq |\mu_A(x_i)(u_j) - \mu_C(x_i)(u_j)| + |\mu_C(x_i)(u_j) - \mu_B(x_i)(u_j)|$$
  $$+ |\nu_A(x_i)(u_j) - \nu_C(x_i)(u_j)| + |\nu_C(x_i)(u_j) - \nu_B(x_i)(u_j)|.$$

  Then, we have

  $$d_{IFS}^w(\Gamma_A, \Lambda_B) \leq d_{IFS}^w(\Gamma_A, \Upsilon_C) + d_{IFS}^w(\Upsilon_C, \Lambda_B).$$

The others proofs can made similarly. $\square$

Definition 3.14. Let Γ_A and Λ_B be two IFS-sets over U. Then, by using the Hamming distance, similarity measure of Γ_A and Λ_B is defined as

$$S'_{IFS}(\Gamma_A, \Lambda_B) = \frac{1}{1 + d_{IFS}^w(\Gamma_A, \Lambda_B)}.$$

Another similarity measure of F_A and G_B can be defined as,

$$S''_{IFS}(\Gamma_A, \Lambda_B) = e^{-\alpha d_{IFS}^w(\Gamma_A, \Lambda_B)}$$

where α is a positive real number called the steepness measure.

Definition 3.15. Let Γ_A and Λ_B be two IFS-sets over U. Then, Γ_A and Λ_B are said to be α-similar, denoted as $\Gamma_A \approx^\alpha \Lambda_B$, if and only if $S'(\Gamma_A, \Lambda_B) \geq \alpha$ for $\alpha \in (0, 1)$.
We call the two IFS-sets significantly similar if $S'_{IFS}(\Gamma_A, \Lambda_B) > \frac{1}{2}$.

Example 3.16. Let us consider the Example 3.11. Similarity measure of Γ_A and Λ_B is obtained as

$$S'_{IFS}(\Gamma_A, \Lambda_B) = \frac{1}{1 + d^w_{IFS}(\Gamma_A, \Lambda_B)} = 0.73.$$

Γ_A and Λ_B is significantly similar because $S'_{IFS}(\Gamma_A, \Lambda_B) = 0.73 > \frac{1}{2}$.

Theorem 3.17. *Let E be a parameter set, $A, B \subseteq E$ and Γ_A and Λ_B be two IFS-sets over U. Then the followings hold :*
(i) $0 \leq S'_{IFS}(\Gamma_A, \Lambda_B) \leq 1$.
(ii) $S'_{IFS}(\Gamma_A, \Lambda_B) = S'_{IFS}(\Lambda_B, \Gamma_A)$.
(iii) $S'_{IFS}(\Gamma_A, \Lambda_B) = 1 \Leftrightarrow \Gamma_A = \Lambda_B$.

Proof. Proof easily can be made by using Definition 3.14. □

4. Decision Making Method

In this section, we construct a decision making method that is based on the similarity measure of two IFS-sets. The algorithm of decision making method can be given as;

Step 1: Constructs a IFS-set Γ_A over U based on an expert,
Step 2: Constructs a IFS-set Λ_B over U based on a responsible person for the problem,
Step 3: Calculate the distances of Γ_A and Λ_B,
Step 4: Calculate the similarity measure of Γ_A and Λ_B,
Step 5: Estimate result by using the similarity.

Now, we can give an application for the decision making method. By using the weighted HD, similarity measure of two IFS-sets can be applied to detect whether an ill person is suffering from a certain disease or not.

5. Application

In this applications, we will try to estimate the possibility that an ill person having certain visible symptoms is suffering from cancer. For this, we first construct a IFS-set for the illness and a IFS-set for the ill person. We then find the similarity measure of these two IFS-sets. If they are significantly similar, then we conclude that the person is possibly suffering from cancer. [It is adopted from [34]]

Example 5.1. Assume that our universal set contain only two elements cancer and not cancer, i.e. $U = \{u_1, u_2\}$. Here the set of parameters $A = B = E$ is the set of certain visible symptoms, let us say, $E = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$ where $x_1 =$ jaundice, $x_2 =$ bone pain, $x_3 =$ headache, $x_4 =$ loss of appetite, $x_5 =$ weight loss, $x_6 =$ heal wounds , $x_7 =$ handle and shoulder pain, $x_8 =$ lump anywhere on the body for no reason and $x_9 =$ chest pain and also $w_i = 1$ for all $i = 1, 2, ..., 9$.

Step 1: Constructs a IFS-set Γ_A over U for cancer is given below and this can be prepared with the help of a medical person:

$$\Gamma_A = \Big\{ \quad (x_1, \{(u_1, 0.5, 0.5), (u_2, 0.4, 0.5)\}), (x_2, \{(u_1, 0.7, 0.2), (u_2, 0.8, 0.1)\}),$$
$$(x_3, \{(u_1, 0.4, 0.6), (u_2, 0.2, 0.7)\}), (x_4, \{(u_1, 0.2, 0.8), (u_2, 0.2, 0.2)\}),$$
$$(x_5, \{(u_1, 0.2, 0.7), (u_2, 0.1, 0.9)\}), (x_6, \{(u_3, 0.5, 0.4), (u_4, 0.7, 0.2)\}),$$
$$(x_7, \{(u_1, 0.3, 0.7), (u_2, 0.4, 0.4)\}), (x_8, \{(u_1, 0.5, 0.2), (u_2, 0.7, 0.1)\}),$$
$$(x_9, \{(u_1, 0.3, 0.4), (u_2, 0.7, 0.1)\}) \Big\}.$$

Step 2 : Constructs a IFS-set Λ_B over U based on data of ill person:

$$\Lambda_B = \Big\{ \quad (x_1, \{(u_1, 0.9, 0.1), (u_2, 0.9, 0.0)\}), (x_2, \{(u_1, 0.1, 0.9), (u_2, 0.1, 0.8)\}),$$
$$(x_3, \{(u_1, 0.7, 0.1), (u_2, 0.8, 0.9)\}), (x_4, \{(u_1, 0.9, 0.1), (u_2, 0.9, 0.8)\}),$$
$$(x_5, \{(u_1, 0.9, 0.1), (u_2, 0.9, 0.2)\}), (x_6, \{(u_3, 0.1, 0.9), (u_4, 0.1, 0.8)\})$$
$$(x_7, \{(u_1, 0.9, 0.1), (u_2, 0.7, 0.9)\}), (x_8, \{(u_1, 0.9, 0.9), (u_2, 0.1, 0.9)\}),$$
$$(x_9, \{(u_1, 0.8, 0.1), (u_2, 0, 1)\}) \Big\}.$$

Step 3 : Calculate weighted HD of Γ_A and Λ_B,
$$d^w_{IFS}(\Gamma_A, \Lambda_B) \cong 1.1.$$

Step 4 : Calculate the similarity measure of Γ_A and Λ_B,
$$S'_{IFS}(\Gamma_A, \Lambda_B) = \frac{1}{1 + d^w_{IFS}(\Gamma_A, \Lambda_B)} \cong 0.48 < \frac{1}{2}.$$

Step 5 : Hence the two IFS-sets, i.e. two symptoms Γ_A and Λ_B are not significantly similar. Therefore, we conclude that the person is not possibly suffering from cancer.

Example 5.2. Let us consider Example 5.1 with different ill person.

Step 1 : Constructs a IFS-set for cancer Γ_A is in the Example 5.1.

Step 2 : A person suffering from the following symptoms whose corresponding IFS-set Υ_C is given below:

$$\Upsilon_C = \Big\{ \quad (x_1, \{(u_1, 0.5, 0.4), (u_2, 0.4, 0.4)\}), (x_2, \{(u_1, 0.7, 0.1), (u_2, 0.8, 0.1)\}),$$
$$(x_3, \{(u_1, 0.4, 0.5), (u_2, 0.2, 0.6)\}), (x_4, \{(u_1, 0.2, 0.7), (u_2, 0.2, 0.1)\}),$$
$$(x_5, \{(u_1, 0.2, 0.6), (u_2, 0.1, 0.8)\}), (x_6, \{(u_3, 0.5, 0.3), (u_C, 0.7, 0.1)\})$$
$$(x_7, \{(u_1, 0.2, 0.6), (u_2, 0.1, 0.8)\}), (x_8, \{(u_1, 0.5, 0.3), (u_2, 0.7, 0.1)\}),$$
$$(x_9, \{(u_1, 0.5, 0.3), (u_2, 0.7, 0.1)\}) \Big\}.$$

Step 3 : Calculate weighted HD of Γ_A and Λ_B,
$$d^w_{IFS}(\Gamma_A, \Lambda_B) \cong 0, 41.$$

Step 4. : Find the similarity measure of these two IFS-sets as:
$$S'_{IFS}(\Gamma_A, \Upsilon_C) = \frac{1}{1 + d^w_{IFS}(\Gamma_A, \Upsilon_C)} \cong 0.71 > \frac{1}{2}.$$

Step 5 : Here the two IFS-sets, i.e. two symptoms Γ_A and Υ_C are significantly similar. Therefore, we conclude that the person is possibly suffering from cancer.

6. Conclusion

In this paper, we have defined four types of distances between two IFS-sets and proposed similarity measures of two IFS-sets. Then, we construct a decision making method based on the similarity measures. Finally, we give two simple examples to show the possibility of using this method by using Hamming distance for diagnosis of diseases. In these example, if we use the other distances, we can obtain similar result. The method can be applied to problems that contain uncertainty such as problems in social, economic systems, pattern recognition, medical diagnosis, game theory, coding theory and so on.

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