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On fuzzy $^{*}\mathrm{G}_{\delta}\beta$ continuity in fuzzy *matroids

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Abstract. In this paper, fuzzy *matroids are introduced via fuzzy *flat axioms. The concepts of fuzzy $^{*}\mathrm{G}_{\delta}\beta$ open sets and fuzzy $^{*}\mathrm{G}_{\delta}\beta$ continuous maps are introduced. Also the concepts of fuzzy $^{*}\mathrm{G}_{\delta}\beta$ connectedness, fuzzy $^{*}\mathrm{G}_{\delta}\beta$ compactness and fuzzy $^{*}\mathrm{G}_{\delta}\beta$ normal matroids are introduced and studied.

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1. Introduction

Zadeh introduced the fundamental concepts of fuzzy sets in his classical paper [21]. Fuzzy sets have applications in many fields such as information [15] and control [16]. In mathematics, topology provided the most natural framework for the concepts of fuzzy sets to flourish. Chang [6] introduced and developed the concept of fuzzy topological spaces. Matroids were introduced by Whitney in 1935 as a generalization of both graphs and vector spaces. Matroid theory has several interesting applications in combinatorics, graph theory, discrete optimization, algebraic geometry and many other fields of science. The notion of fuzzy matroids was first introduced by Geotschel and Voxman in their landmark paper [7], using the notion of fuzzy independent sets. In [9], an attempt was made to define a closure operator which explored another way to determine Goetschel and Voxman fuzzy matroid (GV fuzzy matroid for short) and presented an application of the notion co-tower in GV fuzzy matroids. Fu-Gui Shi in [13], based on the idea of [20], defined a closed fuzzy pre-matroid as a fuzzifying matroid and generalized it to L-fuzzy set theory. An extension of matroids was introduced in lattice-valued fuzzy set theory by Fu-Gui Shi [14]. In [14], the notion of (L, M)-matroids which is a generalization of L-matroids and M-fuzzifying matroids was introduced. In [17], the notion of closure operators of matroids was generalized to an L-fuzzy setting. Lu and Zheng [10], studied the categorical relations between Goetschel-Voxman's fuzzy matroids and Shi's fuzzifying matroids. In [18], the notions of perfect [0, 1]-matroids and closed [0, 1]-matroids were introduced and characterized by means of its [0, 1]-fuzzy rank function. Wei Yao and Fu-gui Shi [19], besides showing that weighted graphs are examples of fuzzifying matroids, established the axioms of bases and that of circuits are established for fuzzifying matroids. In [2], Talal Al-Hawary introduced the notion of fuzzy flats. The notion of flats [1, 11] in traditional matroids is one of the significant notions that plays a very important rule in characterizing strong maps. The concepts of fuzzy G_δ sets and fuzzy β open sets were introduced by G. Balasubramanian in [3] and [4], respectively. In [12], the notion of soft quasi coincidence for fuzzy soft sets was introduced by Serkan Atmaca and Idris Zorlutuna.

In this paper, fuzzy ${}^*G_\delta\beta$ open sets are introduced and studied. Some interesting properties are discussed. Several characterisations of fuzzy ${}^*G_\delta\beta$ continuous maps are established. In this connection, some of their interrelations with other related sets are discussed and their counter examples are provided wherever necessary. Further, characterizations of fuzzy ${}^*G_\delta\beta$ connected, fuzzy ${}^*G_\delta\beta$ compact and fuzzy ${}^*G_\delta\beta$ normal matroids are introduced and their properties are discussed.

2. Preliminaries

Definition 2.1 ([2])**.** Let E be a finite set and let $\mathfrak{F}$ be a family of fuzzy sets of E satisfying the following three conditions:

(i) $1^E \in \mathfrak{F}$.

(ii) If μ_1, $\mu_2 \in \mathfrak{F}$, then $\mu_1 \wedge \mu_2 \in \mathfrak{F}$.

(iii) If $\mu \in \mathfrak{F}$ and μ_1, μ_2,......,μ_n are all members of $\mathfrak{F}$ such that $\mu \prec \mu_i$ for all i = 1, 2,...., n, then the fuzzy union of μ_1, μ_2,......,μ_n is equal to 1^E (i.e. $\vee_{i=1}^{n}\mu_i = 1^E$). Then the system FM = (E, $\mathfrak{F}$) is called a fuzzy matroid and the elements of $\mathfrak{F}$ are fuzzy flats of FM. The complement of a fuzzy flat of FM is called a fuzzy open set of FM.

Definition 2.2 ([2])**.** Let FM = (E, $\mathfrak{F}$) be a fuzzy matroid and μ be a fuzzy set. Then the fuzzy closure of μ is $\bar{\mu} = \bigwedge_{\lambda\in\mathfrak{F};\mu\leq\lambda} \lambda$.

Definition 2.3 ([3])**.** Let (X, T) be a fuzzy topological space. Let λ be any fuzzy set. Then λ is said to be a fuzzy G_δ set if $\lambda = \bigwedge_{i=1}^{\infty} \mu_i$, where each μ_i is fuzzy open set. The complement of a fuzzy G_δ set is fuzzy F_σ.

Definition 2.4 ([4])**.** Let (X, T) be a fuzzy topological space. Let λ be any fuzzy set. Then λ is said to be a fuzzy β open set if $\lambda \leq \text{cl}(\text{int}(\text{cl}(\lambda)))$. The complement of a fuzzy β open set is fuzzy β closed.

Definition 2.5 ([5])**.** Let (X, T) be a fuzzy topological space. Let λ be any fuzzy set. Then λ is said to be a fuzzy α open set if $\lambda \leq \text{int}(\text{cl}(\text{int}(\lambda)))$. The complement of a fuzzy α open set is fuzzy α closed.

Definition 2.6 ([5])**.** Let (X, T) be a fuzzy topological space. Let λ be any fuzzy set. Then λ is said to be a fuzzy pre open set if $\lambda \leq \text{int}(\text{cl}(\lambda))$. The complement of a fuzzy pre open set is fuzzy pre closed.

Definition 2.7 ([8])**.** Let (X, T) be a fuzzy topological space. Let λ be any fuzzy set. Then λ is said to be a fuzzy semi open set if $\lambda \leq$ cl(int(λ)). The complement of a fuzzy semi open set is fuzzy semi closed.

Notation 2.1. Let (X, T) be a fuzzy topological space. The family of all fuzzy α open(resp. fuzzy semi open, fuzzy pre open, fuzzy β open) sets in (X, T) is denoted by $\alpha(X)$(resp. SO(X), PO(X), $\beta(X)$).

Definition 2.8 ([2])**.** Let E_1 and E_1 be two sets, μ_1 is a fuzzy set in E_1, μ_2 is a fuzzy set in E_2 and f : $E_1 \longrightarrow E_2$ be a map. Then we define the fuzzy sets f(μ_1)(the image of μ_1) and $f^{-1}(\mu_2)$(the preimage of μ_2) by

$$f(\mu_1)(y) = \begin{cases} sup\{\mu_1(x) : x \in f^{-1}(y), & \text{y} \in \text{Range(f)} \\ 1, & \text{otherwise} \end{cases}$$

and $f^{-1}(\mu_2)(x) = \mu_2(f(x))$ fox all x $\in E_1$.

3. Fuzzy $^*G_\delta\beta$ open set and its basic properties

Definition 3.1. Let E be a finite set and let $\mathfrak{F}$ be a family of fuzzy sets of E satisfying the following three conditions:

(i) 0^E, $1^E \in \mathfrak{F}$.

(ii) If μ_1, $\mu_2 \in \mathfrak{F}$, then $\mu_1 \wedge \mu_2 \in \mathfrak{F}$.

(iii) If $\mu \in \mathfrak{F}$ and μ_1, μ_2,......,μ_n are all members of $\mathfrak{F}$ such that $\mu \prec \mu_i$ for all i = 1, 2,...., n, then the fuzzy union of μ_1, μ_2,......,μ_n is equal to 1^E (i.e. $\vee_{i=1}^{n}\mu_i = 1^E$). Then the system FM = (E, $\mathfrak{F}$) is called a fuzzy *matroid and the elements of $\mathfrak{F}$ are fuzzy *flats of FM. The complement of a fuzzy *flat of FM is called a fuzzy *open set of FM.

Definition 3.2. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid and μ be a fuzzy set. Then the fuzzy *closure of μ is

$$\text{cl}^*(\mu) = \bigwedge\{\lambda/\lambda \text{ is a fuzzy }^*\text{flat of FM and } \lambda \geq \mu\}.$$

Definition 3.3. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid and μ be a fuzzy set. Then the fuzzy *interior of μ is

$$\text{int}^*(\mu) = \bigvee\{\lambda/\lambda \text{ is a fuzzy }^*\text{open set of FM and } \lambda \leq \mu\}.$$

Definition 3.4. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid. Let λ be any fuzzy set. Then λ is said to be a fuzzy * G_δ set if $\lambda = \bigwedge_{i=1}^{\infty} \mu_i$, where each μ_i is fuzzy *open set. The complement of a fuzzy $^*G_\delta$ set is fuzzy * F_σ.

Definition 3.5. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid. Let λ be any fuzzy set. Then λ is said to be a fuzzy * β open set if $\lambda \leq \text{cl}^*(\text{int}^*(\text{cl}^*(\lambda)))$. The complement of a fuzzy $^*\beta$ open set is fuzzy * β flat.

Definition 3.6. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid. Let λ be any fuzzy set. Then λ is said to be a fuzzy * $G_\delta\beta$ open set of FM if $\lambda = \mu \wedge \gamma$ where μ is fuzzy $^*G_\delta$ set and γ is fuzzy $^*\beta$ open set. The complement of fuzzy $^*G_\delta\beta$ open set of FM is fuzzy $^*F_\sigma\beta$ flat of FM.

Remark 3.7. (i) The finite intersection of fuzzy $^*F_\sigma$ sets of FM is a fuzzy $^*F_\sigma$ set.

(ii) The finite intersection of fuzzy $^*\beta$ flats of FM is a fuzzy $^*\beta$ flat.

Proposition 3.8. *The finite intersection of fuzzy* $^*F_\sigma\beta$ *flats of FM is a fuzzy* $^*F_\sigma\beta$ *flat.*

Proof. Let $\{\mu_i\}$ be fuzzy $^*\mathrm{F}_\sigma$ sets of FM and $\{\gamma_i\}$ be fuzzy $^*\beta$ flats of FM. Let $\{\lambda_i\}$ be the family of fuzzy $^*\mathrm{F}_\sigma\beta$ flats. Then

$$\begin{aligned} \lambda_i &= \mu_i \vee \gamma_i, \\ \wedge_{i=1}^n \lambda_i &= \wedge_{i=1}^n (\mu_i \vee \gamma_i), \\ &= (\wedge_{i=1}^n \mu_i) \vee (\wedge_{i=1}^n \gamma_i). \end{aligned}$$

Since finite intersection of fuzzy $^*\mathrm{F}_\sigma$ sets is a fuzzy $^*\mathrm{F}_\sigma$ set and finite intersection of fuzzy $^*\beta$ flats is a fuzzy $^*\beta$ flat, $\wedge_{i=1}^n \mu_i$ is a fuzzy $^*\mathrm{F}_\sigma$ set and $\wedge_{i=1}^n \gamma_i$ is a fuzzy $^*\beta$ flat. Thus $(\wedge_{i=1}^n \mu_i) \vee (\wedge_{i=1}^n \gamma_i)$ is fuzzy $^*\mathrm{F}_\sigma\beta$ flat. Hence $\wedge_{i=1}^n \lambda_i$ is fuzzy $^*\mathrm{F}_\sigma\beta$ flat. □

Definition 3.9. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid and λ be a fuzzy set. Then the fuzzy $^*\mathrm{F}_\sigma\beta$ closure of λ denoted by $\mathrm{F}_\sigma\beta$ cl(λ) is defined as

$$\mathrm{F}_\sigma\beta \ \mathrm{cl}(\lambda) = \bigwedge\{\mu/\mu \text{ is a fuzzy } ^*\mathrm{F}_\sigma\beta \text{ flat of FM and } \mu\geq\lambda\}.$$

Proposition 3.10. *Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid and μ, λ be any two fuzzy sets. Then*

(*i*) $F_\sigma\beta$ *cl*$(0^E) = 0^E$.
(*ii*) $F_\sigma\beta$ *cl*(μ) *is a fuzzy* $^*F_\sigma\beta$ *flat.*
(*iii*) $\mu \leq F_\sigma\beta$ *cl*(μ).
(*iv*) *If* $\mu \leq \lambda$, *then* $F_\sigma\beta$ *cl*$(\mu) \leq F_\sigma\beta$ *cl*(λ).
(*v*) $F_\sigma\beta$ *cl*$(F_\sigma\beta$ *cl*$(\mu)) = F_\sigma\beta$ *cl*(μ).
(*vi*) $F_\sigma\beta$ *cl*$(\lambda\vee\mu) = (F_\sigma\beta$ *cl*$(\lambda)) \vee (F_\sigma\beta$ *cl*$(\mu))$.

Definition 3.11. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid and λ be a fuzzy set. Then the fuzzy $^*\mathrm{G}_\delta\beta$ interoir of λ denoted by $\mathrm{G}_\delta\beta$ int(λ) is defined as

$$\mathrm{G}_\delta\beta \ \mathrm{int}(\lambda) = \bigvee\{\mu/\mu \text{ is a fuzzy } ^*\mathrm{G}_\delta\beta \text{ open set of FM and } \mu\leq\lambda\}.$$

Proposition 3.12. *Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid and μ, λ be any two fuzzy sets. Then*

(*i*) $G_\delta\beta$ *int*$(1^E) = 1^E$.
(*ii*) $G_\delta\beta$ *int*(μ) *is a fuzzy* $^*G_\delta\beta$ *open set.*
(*iii*) $G_\delta\beta$ *int*$(\mu) \leq \mu$.
(*iv*) *If* $\mu \leq \lambda$, *then* $G_\delta\beta$ *int*$(\mu) \leq G_\delta\beta$ *int*(λ).
(*v*) $G_\delta\beta$ *int*$(G_\delta\beta$ *int*$(\mu)) = G_\delta\beta$ *int*(μ).
(*vi*) $G_\delta\beta$ *int*$(\lambda\wedge\mu) \leq (G_\delta\beta$ *int*$(\lambda)) \wedge (G_\delta\beta$ *int*$(\mu))$.

Proposition 3.13. *Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid and μ be a fuzzy set. Then μ is a fuzzy $^*F_\sigma\beta$ flat if and only if $F_\sigma\beta$ cl$(\mu) = \mu$.*

Proposition 3.14. *Let FM = (E,$\mathfrak{F}$) be a fuzzy *matroid and λ be a fuzzy set. Then*
(*i*) $F_\sigma\beta$ *cl*$(1^E-\lambda) = 1^E-G_\delta\beta$ *int*(λ).
(*ii*) $G_\delta\beta$ *int*$(1^E-\lambda) = 1^E-F_\sigma\beta$ *cl*(λ).

Definition 3.15. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid and λ be a fuzzy set. Then λ is said to be a fuzzy *regular $\mathrm{G}_\delta\beta$ open set of FM if $\lambda = \mathrm{G}_\delta\beta$ int($\mathrm{F}_\sigma\beta$ cl(λ)).

Definition 3.16. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid and λ be a fuzzy set. Then λ is said to be a fuzzy *regular $F_\sigma\beta$ flat of FM if $\lambda = F_\sigma\beta$ cl($G_\delta\beta$ int(λ)).

Proposition 3.17. *(i) The fuzzy $^*F_\sigma\beta$ closure of a fuzzy $^*G_\delta\beta$ open set is a fuzzy *regular $F_\sigma\beta$ flat.*

(ii) The fuzzy $^*G_\delta\beta$ interior of a fuzzy $^*F_\sigma\beta$ flat is a fuzzy *regular $G_\delta\beta$ open set.

Proof. (i) Let λ be a fuzzy $^*G_\delta\beta$ open set of a fuzzy *matroid. Then

$$\begin{aligned} G_\delta\beta\ int(F_\sigma\beta\ cl(\lambda)) &\leq F_\sigma\beta\ cl(\lambda), \\ F_\sigma\beta\ cl(G_\delta\beta\ int(F_\sigma\beta\ cl(\lambda))) &\leq F_\sigma\beta\ cl(F_\sigma\beta\ cl(\lambda)) \\ &\leq F_\sigma\beta\ cl(\lambda). \end{aligned}$$

Since λ is a fuzzy $^*G_\delta\beta$ open set,

$$\begin{aligned} \lambda &\leq F_\sigma\beta\ cl(\lambda), \\ G_\delta\beta\ int(\lambda) &\leq G_\delta\beta\ int(F_\sigma\beta\ cl(\lambda)), \\ \lambda &\leq G_\delta\beta\ int(F_\sigma\beta\ cl(\lambda)), \\ F_\sigma\beta\ cl(\lambda) &\leq F_\sigma\beta\ cl(G_\delta\beta\ int(F_\sigma\beta\ cl(\lambda))). \end{aligned}$$

Thus $F_\sigma\beta$ cl(λ) = $F_\sigma\beta$ cl($G_\delta\beta$ int($F_\sigma\beta$ cl(λ))).

(ii) Let λ be a fuzzy $^*F_\sigma\beta$ flat of a fuzzy *matroid. Then

$$\begin{aligned} F_\sigma\beta\ cl(G_\delta\beta\ int(\lambda)) &\geq G_\delta\beta\ int(\lambda), \\ G_\delta\beta\ int(F_\sigma\beta\ cl(G_\delta\beta\ int(\lambda))) &\geq G_\delta\beta\ int(G_\delta\beta\ int(\lambda)) \\ &\geq G_\delta\beta\ int(\lambda). \end{aligned}$$

Since λ is a fuzzy $^*F_\sigma\beta$ flat,

$$\begin{aligned} \lambda &\geq G_\delta\beta\ int(\lambda), \\ F_\sigma\beta\ cl(\lambda) &\geq F_\sigma\beta\ cl(G_\delta\beta\ int(\lambda)), \\ \lambda &\geq F_\sigma\beta\ cl(G_\delta\beta\ int(\lambda)), \\ G_\delta\beta\ int(\lambda) &\geq G_\delta\beta\ int(F_\sigma\beta\ cl(G_\delta\beta\ int(\lambda))). \end{aligned}$$

Thus $G_\delta\beta$ int(λ) = $G_\delta\beta$ int($F_\sigma\beta$ cl($G_\delta\beta$ int(λ))). □

Definition 3.18. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid. Let λ be any fuzzy set. Then the fuzzy *kernel of λ denoted by ker$^*(\lambda)$ is defined as

$$\ker^*(\lambda) = \bigwedge\{\mu/\mu \text{ is a fuzzy }^*\text{open set of FM and } \lambda\leq\mu\}.$$

Definition 3.19. Let λ and μ be any two fuzzy sets. A fuzzy set λ is called as a fuzzy *quasicoincident with a fuzzy set μ, denoted by λ q* μ, if $\lambda(x) + \mu(x) > 1^E$, for some $x \in$ E.

Definition 3.20. Let λ and μ be any two fuzzy sets. A fuzzy set λ is called as a fuzzy *non-quasicoincident with a fuzzy set μ, denoted by $\lambda \not{q}^* \mu$, if $\lambda(x) + \mu(x) \leq 1^E$, for all $x \in$ E.

Proposition 3.21. *Let λ and μ be any two fuzzy sets. Then $\lambda \not{q}^* \mu \Leftrightarrow \lambda \leq (1^E - \mu)$.*

Definition 3.22. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid. Let λ be any fuzzy set. Then fuzzy *G$_\delta\beta$ kernel of λ denoted by G$_\delta\beta$ ker$^*(\lambda)$ is defined as

$$\mathrm{G}_\delta\beta\ \mathrm{ker}^*(\lambda) = \bigwedge\{\mu/\mu \text{ is a fuzzy } ^*\mathrm{G}_\delta\beta \text{ open set of FM and } \lambda\leq\mu\}.$$

Proposition 3.23. *Let λ be a fuzzy set in FM. Let μ be a fuzzy $^*F_\sigma\beta$ flat in FM. Then $\mu\ q^*\ G_\delta\beta\ ker^*(\lambda) \Leftrightarrow \mu\ q^*\ \lambda$.*

Proof. Let $\mu\ \not{q}^*\ \lambda$. Then $\lambda \leq (1^E - \mu)$. Now, G$_\delta\beta$ ker$^*(\lambda) \leq$ G$_\delta\beta$ ker$^*(1^E - \mu)$. Since $(1^E - \mu)$ is a fuzzy *G$_\delta\beta$ open set in FM, G$_\delta\beta$ ker$^*(\lambda) \leq (1^E - \mu)$,which implies G$_\delta\beta$ ker$^*(\lambda)\ \not{q}^*\ \mu$.

Conversely, G$_\delta\beta$ ker$^*(\lambda)\ \not{q}^*\ \mu$. This implies that G$_\delta\beta$ ker$^*(\lambda) \leq (1^E - \mu)$. Now, $\lambda \leq$ G$_\delta\beta$ ker$^*(\lambda) \leq (1^E - \mu)$. It follows that $\mu\ \not{q}^*\ \lambda$. □

4. Interrelations of fuzzy *G$_\delta\beta$ open sets with various fuzzy sets

Definition 4.1. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid. Let λ be any fuzzy set. Then λ is said to be a fuzzy * α open set (fuzzy * preopen set) if $\lambda \leq \mathrm{int}^*(\mathrm{cl}^*(\mathrm{int}^*(\lambda)))(\lambda \leq \mathrm{int}^*(\mathrm{cl}^*(\lambda)))$. The complement of a fuzzy $^*\alpha$ open set(fuzzy *pre open set) is fuzzy * α flat (fuzzy * preflat).

Definition 4.2. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid. Let λ be any fuzzy set. Then λ is said to be a fuzzy * semi open set if $\lambda \leq \mathrm{cl}^*(\mathrm{int}^*(\lambda)))$. The complement of a fuzzy *semi open set is fuzzy * semiflat.

Notation 4.1. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid. The family of all fuzzy $^*\alpha$ open(resp. fuzzy *semi open, fuzzy *pre open, fuzzy $^*\beta$ open) sets of FM is denoted by $\alpha(E)$(resp. S(E), P(E), $\beta(E)$).

Definition 4.3. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid. Let λ be any fuzzy set. Then λ is said to be a fuzzy * G_δ^* open set of FM if $\lambda = \mu \wedge \gamma$ where μ is a fuzzy $^*G_\delta$ set and γ is a fuzzy *open set. The complement of a fuzzy $^*G_\delta^*$ open set of FM is a fuzzy *F$_\sigma^*$ flat of FM.

Definition 4.4. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid. Let λ be any fuzzy set. Then λ is said to be a fuzzy * $G_\delta\alpha$ open set of FM if $\lambda = \mu \wedge \gamma$ where μ is a fuzzy $^*G_\delta$ set and γ is a fuzzy $^*\alpha$ open set. The complement of a fuzzy $^*G_\delta\alpha$ open set of FM is a fuzzy *F$_\sigma\alpha$ flat of FM.

Definition 4.5. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid. Let λ be any fuzzy set. Then λ is said to be a fuzzy * G$_\delta$pre open set of FM if $\lambda = \mu \wedge \gamma$ where μ is a fuzzy $^*G_\delta$ set and γ is a fuzzy *pre open set. The complement of a fuzzy $^*G_\delta$pre open set of FM is a fuzzy *F$_\sigma$pre flat of FM.

Definition 4.6. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid. Let λ be any fuzzy set. Then λ is said to be a fuzzy * $G_\delta$semi open set of FM if $\lambda = \mu \wedge \gamma$ where μ is a fuzzy $^*G_\delta$ set and γ is a fuzzy *semi open set. The complement of a fuzzy $^*G_\delta$semi open set of FM is a fuzzy *F$_\sigma$semi flat of FM.

Notation 4.2. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid. The family of all fuzzy $^*G_\delta\alpha$ open(resp. fuzzy $^*G_\delta$semi open, fuzzy $^*G_\delta$pre open, fuzzy $^*G_\delta\beta$ open) sets of FM is denoted by G$_\delta\alpha(E)$(resp. G$_\delta$S(E), G$_\delta$P(E), G$_\delta\beta(E)$).

Remark 4.7. Every fuzzy *open set of FM is a fuzzy $^*\alpha$ open set of FM.

Proposition 4.8. *Every fuzzy* $^*G^*_\delta$ *set of FM is a fuzzy* $^*G_\delta\alpha$ *open set of FM.*

Proof. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid and λ be a fuzzy set. Assume that λ is a fuzzy *G$^*_\delta$ set of FM. That is $\lambda = \mu\wedge\gamma$, where μ is a fuzzy *G$_\delta$ set of FM and γ is a fuzzy *open set of FM. Since every fuzzy *open set of FM is a fuzzy $^*\alpha$ open set of FM, γ is a fuzzy $^*\alpha$ open set of FM. Thus λ is a fuzzy *G$_\delta\alpha$ open set of FM. Hence every fuzzy *G$^*_\delta$ set of FM is a fuzzy *G$_\delta\alpha$ open set of FM. □

Remark 4.9. The converse of the above theorem need not be true as shown in the following example.

Example 4.10. Let E = {a, b} and $\mathfrak{F}$ = { 0^E, 1^E, λ_1, λ_2, λ_3} where λ_i : E $\longrightarrow$ [0, 1] for i=1, 2, 3 is defined as follows λ_1(a) = 0.6, λ_1(b) = 0.5; λ_2(a) = 0.6, λ_2(b) = 1^E; λ_3(a) = 1^E, λ_3(b) = .5. Consider the fuzzy set λ where λ : E$\longrightarrow$[0, 1] and is defined by $\lambda(a) = 0.4$, $\lambda(b) = 0.3$. Then λ is fuzzy $^*\alpha$ open. Consider the fuzzy *G$_\delta$ set $\lambda_1^{'}$. Now, $\lambda_1^{'} \wedge \lambda$ is a fuzzy *G$_\delta\alpha$ open set. But $\lambda_1^{'} \wedge \lambda$ is not a fuzzy *G$^*_\delta$ set since λ is not fuzzy*open. Thus every fuzzy *G$_\delta\alpha$ open set need not be a fuzzy *G$^*_\delta$ set.

Remark 4.11. Every fuzzy $^*\alpha$ open set of FM is a fuzzy*pre open set of FM.

Proposition 4.12. *Every fuzzy* $^*G_\delta\alpha$ *open set of FM is a fuzzy* $^*G_\delta$*pre open set of FM.*

Proof. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid and λ be a fuzzy set. Assume that λ is a fuzzy *G$_\delta\alpha$ open set of FM. That is $\lambda = \mu\wedge\gamma$, where μ is a fuzzy *G$_\delta$ set of FM and γ is a fuzzy $^*\alpha$ open set of FM. Since every fuzzy $^*\alpha$ open set of FM is a fuzzy *pre open set of FM, γ is a fuzzy *pre open set of FM. Hence λ is a fuzzy *G$_\delta$pre open set of FM. Thus every fuzzy *G$_\delta\alpha$ open set of FM is a fuzzy *G$_\delta$pre open set of FM. □

Remark 4.13. The converse of the above theorem need not be true as shown in the following example.

Example 4.14. Let E = {a, b} and $\mathfrak{F}$ = { 0^E, 1^E, λ_1, λ_2, λ_3} where λ_i : E $\longrightarrow$ [0, 1] for i=1, 2, 3 is defined as follows λ_1(a) = 0.6, λ_1(b) = 0.5; λ_2(a) = 0.6, λ_2(b) = 1^E; λ_3(a) = 1^E, λ_3(b) = 0.5. Consider the fuzzy set λ where λ : E$\longrightarrow$[0, 1] and is defined by $\lambda(a) = 0.3$, $\lambda(b) = 0.4$. Then λ is fuzzy *pre open. Consider the fuzzy *G$_\delta$ set $\lambda_1^{'}$. Now, $\lambda_1^{'} \wedge \lambda$ is a fuzzy *G$_\delta$pre open set. But $\lambda_1^{'} \wedge \lambda$ is not a fuzzy *G$_\delta\alpha$ set since λ is not fuzzy $^*\alpha$ open. Thus every fuzzy *G$_\delta$pre open set need not be a fuzzy *G$_\delta\alpha$ set.

Remark 4.15. Every fuzzy *pre open set of FM is a fuzzy $^*\beta$ open set of FM.

Proposition 4.16. *Every fuzzy* $^*G_\delta$*pre open set of FM is a fuzzy* $^*G_\delta\beta$ *open set of FM.*

Proof. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid and λ be a fuzzy set. Assume that λ is a fuzzy *G$_\delta$pre open set of FM. That is $\lambda = \mu\wedge\gamma$, where μ is a fuzzy *G$_\delta$ set of FM and γ is a fuzzy *pre open set of FM. Since every fuzzy *pre open set of FM is a fuzzy $^*\beta$ open set of FM, γ is a fuzzy $^*\beta$ open set of FM. Thus λ is a fuzzy $^*G_\delta\beta$ open set of FM. Hence every fuzzy $^*G_\delta$pre open set of FM is a fuzzy $^*G_\delta\beta$ open set of FM. □

Remark 4.17. The converse of the above theorem need not be true as shown in the following example.

Example 4.18. Let E = {a, b} and $\mathfrak{F}$ = { 0^E, 1^E, λ_1, λ_2, λ_3} where λ_i : E $\longrightarrow$ [0, 1] for i=1, 2, 3 is defined as follows λ_1(a) = 0.4, λ_1(b) = 0.5; λ_2(a) = 0.4, λ_2(b) = 1^E; λ_3(a) = 1^E, λ_3(b) = 0.5. Consider the fuzzy set λ where λ : E$\longrightarrow$[0, 1] and is defined by $\lambda(a) = 0.3$, $\lambda(b) = 0.4$. Then λ is fuzzy$^*\beta$ open. Consider the fuzzy $^*G_\delta$ set $\lambda_1^{'}$. Now, $\lambda_1^{'} \wedge \lambda$ is a fuzzy $^*G_\delta\beta$ open set. But $\lambda_1^{'} \wedge \lambda$ is not a fuzzy $^*G_\delta$pre open set since λ is not fuzzy *pre open. Thus every fuzzy $^*G_\delta\beta$ open set need not be a fuzzy $^*G_\delta$pre open set.

Remark 4.19. Every fuzzy *open set of FM is a fuzzy *semi open set of FM.

Proposition 4.20. *Every fuzzy $^*G_\delta^*$ set of FM is a fuzzy $^*G_\delta$semi open set of FM.*

Proof. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid and λ be a fuzzy set. Assume that λ is a fuzzy $^*G_\delta^*$ set of FM. That is $\lambda = \mu\wedge\gamma$, where μ is a fuzzy $^*G_\delta$ set of FM and γ is a fuzzy *open set of FM. Since every fuzzy *open set of FM is a fuzzy *semi open set of FM, γ is a fuzzy *semi open set of FM. Thus λ is a fuzzy $^*G_\delta$semi open set of FM. Hence every fuzzy $^*G_\delta^*$ set of FM is a fuzzy $^*G_\delta$semi open set of FM. □

Remark 4.21. The converse of the above theorem need not be true as shown in the following example.

Example 4.22. Let E = {a, b} and $\mathfrak{F}$ = { 0^E, 1^E, λ_1, λ_2, λ_3}, where λ_i : E $\longrightarrow$ [0, 1] for i=1, 2, 3 is defined as follows λ_1(a) = 0.4, λ_1(b) = 0.7; λ_2(a) = 0.4, λ_2(b) = 1^E; λ_3(a) = 1^E, λ_3(b) = 0.7. Consider the fuzzy set λ where λ : E$\longrightarrow$[0, 1] and is defined by $\lambda(a) = 0.4$, $\lambda(b) = 0.6$. Then λ is fuzzy *semi open. Consider the fuzzy $^*G_\delta$ set $\lambda_1^{'}$. Now, $\lambda_1^{'} \wedge \lambda$ is a fuzzy $^*G_\delta$semi open set. But $\lambda_1^{'} \wedge \lambda$ is not a fuzzy $^*G_\delta^*$ set since λ is not fuzzy *open. Thus every fuzzy $^*G_\delta$semi open set need not be a fuzzy $^*G_\delta^*$ set.

Remark 4.23. Every fuzzy $^*\alpha$ open set of FM is a fuzzy *semi open set of FM.

Proposition 4.24. *Every fuzzy $^*G_\delta\alpha$ open set of FM is a fuzzy $^*G_\delta$semi open set of FM.*

Proof. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid and λ be a fuzzy set. Assume that λ is a fuzzy $^*G_\delta\alpha$ open set of FM. That is $\lambda = \mu\wedge\gamma$, where μ is a fuzzy $^*G_\delta$ set of FM and γ is a fuzzy $^*\alpha$ open set of FM. Since every fuzzy $^*\alpha$ open set of FM is a fuzzy *semi open set of FM, γ is a fuzzy *semi open set of FM. Thus λ is a fuzzy $^*G_\delta$semi open set of FM. Hence every fuzzy $^*G_\delta\alpha$ open set of FM is a fuzzy $^*G_\delta$semi open set of FM. □

Remark 4.25. The converse of the above theorem need not be true as shown in the following example.

Example 4.26. Let E = {a, b} and $\mathfrak{F}$ = { 0^E, 1^E, λ_1, λ_2, λ_3} where λ_i : E $\longrightarrow$ [0, 1] for i=1, 2, 3 is defined as follows λ_1(a) = 0.4, λ_1(b) = 0.7; λ_2(a) = 0.4, λ_2(b) = 1^E; λ_3(a) = 1^E, λ_3(b) = 0.7. Consider the fuzzy set λ where λ : E$\longrightarrow$[0, 1] and is defined by $\lambda(a) = 0.3$, $\lambda(b) = 0.3$. Then λ is fuzzy *semi open. Consider the fuzzy $^*\mathrm{G}_\delta$ set $\lambda_1^{'}$. Now, $\lambda_1^{'} \wedge \lambda$ is a fuzzy $^*\mathrm{G}_\delta$semi open set. But $\lambda_1^{'} \wedge \lambda$ is not a fuzzy $^*\mathrm{G}_\delta\alpha$ open since λ is not fuzzy $^*\alpha$ open. Thus every fuzzy $^*\mathrm{G}_\delta$semi open set need not be a fuzzy $^*\mathrm{G}_\delta\alpha$ open.

Remark 4.27. Every fuzzy *semi open set of FM is a fuzzy $^*\beta$ open set of FM.

Proposition 4.28. *Every fuzzy* $^*G_\delta$*semi open set of FM is a fuzzy* $^*G_\delta\beta$ *open set of FM.*

Proof. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid and λ be a fuzzy set. Assume that λ is a fuzzy $^*\mathrm{G}_\delta$semi open set of FM. That is $\lambda = \mu\wedge\gamma$, where μ is a fuzzy $^*\mathrm{G}_\delta$ set of FM and γ is a fuzzy *semi open set of FM. Since every fuzzy *semi open set of FM is a fuzzy $^*\beta$ open set of FM, γ is a fuzzy $^*\beta$ open set of FM. Thus λ is a fuzzy $^*\mathrm{G}_\delta\beta$ open set of FM. Hence every fuzzy $^*\mathrm{G}_\delta$semi open set of FM is a fuzzy $^*\mathrm{G}_\delta\beta$ open set of FM. □

Remark 4.29. The converse of the above theorem need not be true as shown in the following example.

Example 4.30. Let E = {a, b} and $\mathfrak{F}$ = { 0^E, 1^E, λ_1, λ_2, λ_3} where λ_i : E $\longrightarrow$ [0, 1] for i=1, 2, 3 is defined as follows λ_1(a) = 0.2, λ_1(b) = 0.8; λ_2(a) = 0.2, λ_2(b) = 1^E; λ_3(a) = 1^E, λ_3(b) = 0.8. Consider the fuzzy set λ where λ : E$\longrightarrow$[0, 1] and is defined by $\lambda(a) = 0.4$, $\lambda(b) = 0.8$. Then λ is fuzzy$^*\beta$ open. Consider the fuzzy $^*\mathrm{G}_\delta$ set $\lambda_1^{'}$. Now, $\lambda_1^{'} \wedge \lambda$ is a fuzzy $^*\mathrm{G}_\delta\beta$ open set. But $\lambda_1^{'} \wedge \lambda$ is not fuzzy $^*\mathrm{G}_\delta$semi open since λ is not a fuzzy *semi open. Thus every fuzzy $^*\mathrm{G}_\delta\beta$ open set need not be a fuzzy $^*\mathrm{G}_\delta$semi open.

Remark 4.31. From the results obtained above, the following implications are obtained.

$$\begin{tikzcd}
 & \boxed{\text{fuzzy } ^*\mathrm{G}_\delta^* \text{ set}} \arrow[dl] \arrow[dr] & \\
\boxed{\text{fuzzy } ^*\mathrm{G}_\delta\alpha \text{ open set}} \arrow[rr] \arrow[d] & & \boxed{\text{fuzzy } ^*\mathrm{G}_\delta\text{semi open set}} \arrow[d] \\
\boxed{\text{fuzzy } ^*\mathrm{G}_\delta\text{pre open set}} \arrow[rr] & & \boxed{\text{fuzzy } ^*\mathrm{G}_\delta\beta \text{ open set}}
\end{tikzcd}$$

5. Fuzzy $^*\mathrm{G}_\delta\beta$ continuous maps and their interrelations with various fuzzy *continuous maps

Definition 5.1. Let $\mathrm{FM}_1 = (E_1, \mathfrak{F}_1)$ and $\mathrm{FM}_2 = (E_2, \mathfrak{F}_2)$ be any two fuzzy *matroids. A map f : $\mathrm{FM}_1 \longrightarrow \mathrm{FM}_2$ is said to be a fuzzy *open map, if the image of every fuzzy *open set in FM_1 is a fuzzy *open set in FM_2.

Definition 5.2. Let $\mathrm{FM}_1 = (E_1, \mathfrak{F}_1)$ and $\mathrm{FM}_2 = (E_2, \mathfrak{F}_2)$ be any two fuzzy *matroids. A map f : $\mathrm{FM}_1 \longrightarrow \mathrm{FM}_2$ is said to be fuzzy *continuous if the inverse image of every fuzzy *open set in FM_2 is fuzzy *open in FM_1.

Definition 5.3. Let $\mathrm{FM}_1 = (E_1, \mathfrak{F}_1)$ and $\mathrm{FM}_2 = (E_2, \mathfrak{F}_2)$ be any two fuzzy *matroids. A map f : $\mathrm{FM}_1 \longrightarrow \mathrm{FM}_2$ is said to be a fuzzy $^*\mathrm{G}_\delta\beta$ open map, if the image of every fuzzy $^*\mathrm{G}_\delta\beta$ open set in FM_1 is a fuzzy $^*\mathrm{G}_\delta\beta$ open set in FM_2.

Definition 5.4. Let $\mathrm{FM}_1 = (E_1, \mathfrak{F}_1)$ and $\mathrm{FM}_2 = (E_2, \mathfrak{F}_2)$ be any two fuzzy *matroids. A map f : $\mathrm{FM}_1 \longrightarrow \mathrm{FM}_2$ is said to be fuzzy $^*\mathrm{G}_\delta\beta$ continuous if the inverse image of every fuzzy *open set in FM_2 is fuzzy $^*\mathrm{G}_\delta\beta$ open in FM_1.

Proposition 5.5. *Let $FM_1 = (E_1, \mathfrak{F}_1)$ and $FM_2 = (E_2, \mathfrak{F}_2)$ be any two fuzzy *matroids. For a map $f : FM_1 \longrightarrow FM_2$, the following are equivalent:*
(i) f is fuzzy $^*G_\delta\beta$ continuous.
(ii) The inverse image of every fuzzy *flat in FM_2 is a fuzzy $^*F_\sigma\beta$ flat in FM_1.

Proposition 5.6. *Let $FM_1 = (E_1, \mathfrak{F}_1)$ and $FM_2 = (E_2, \mathfrak{F}_2)$ be any two fuzzy *matroids. For a map $f : FM_1 \longrightarrow FM_2$, the following are equivalent:*
(i) f is fuzzy $^*G_\delta\beta$ continuous.
(ii) For each $\lambda \in FM_2$, $f^{-1}(int^*(\lambda)) \leq G_\delta\beta\ int(f^{-1}(\lambda))$.
(iii) For each $\lambda \in FM_2$, $F_\sigma\beta\ cl(f^{-1}(\lambda)) \leq f^{-1}(cl^*(\lambda))$.

Proof. (i)⇒(ii) Assume that f is fuzzy $^*\mathrm{G}_\delta\beta$ continuous. Let λ be any fuzzy set in FM_2. Then $\mathrm{int}^*(\lambda)$ is fuzzy *open set in FM_2. Since f is fuzzy $^*\mathrm{G}_\delta\beta$ continuous, $\mathrm{f}^{-1}(\mathrm{int}^*(\lambda))$ is fuzzy $^*\mathrm{G}_\delta\beta$ open in FM_1. Since

$$\begin{aligned} f^{-1}(int^*(\lambda)) &\leq f^{-1}(\lambda), \\ G_\delta\beta\ \ int(f^{-1}(int^*(\lambda))) &\leq G_\delta\beta\ \ int(f^{-1}(\lambda)) \end{aligned}$$

which implies $\mathrm{f}^{-1}(\mathrm{int}^*(\lambda)) \leq \mathrm{G}_\delta\beta\ \mathrm{int}(\mathrm{f}^{-1}(\lambda))$.

(ii)⇒(iii) Assume for each fuzzy set $\lambda \in \mathrm{FM}_2$,

$$\begin{aligned} f^{-1}(int^*(\lambda)) &\leq G_\delta\beta\ \ int(f^{-1}(\lambda)) \\ 1^{E_1} - f^{-1}(int^*(\lambda)) &\geq 1^{E_1} - G_\delta\beta\ \ int(f^{-1}(\lambda)) \\ f^{-1}(1^{E_2}) - f^{-1}(int^*(\lambda)) &\geq F_\sigma\beta\ \ cl(1^{E_2} - f^{-1}(\lambda)) \\ f^{-1}(1^{E_2} - int^*(\lambda)) &\geq F_\sigma\beta\ \ cl(f^{-1}(1^{E_2}) - f^{-1}(\lambda)) \\ f^{-1}(cl^*(1^{E_2} - (\lambda))) &\geq F_\sigma\beta\ \ cl(f^{-1}(1^{E_2}) - \lambda) \end{aligned}$$

for each fuzzy set $(1^{E_2} - \lambda) \in \mathrm{FM}_2$.

(iii)⇒(i) Assume for each fuzzy set $\lambda \in \mathrm{FM}_2$,

$$F_\sigma\beta\ \ cl(f^{-1}(\lambda)) \leq f^{-1}(cl^*(\lambda)). \tag{5.1}$$

Let λ be a fuzzy * flat in FM_2. Then (5.1) becomes

$$F_\sigma\beta\ \ cl(f^{-1}(\lambda)) \leq f^{-1}(\lambda) \tag{5.2}$$

But, from (5.1) and (5.2), $\mathrm{f}^{-1}(\lambda) = \mathrm{F}_\sigma\beta\ \ \mathrm{cl}(\mathrm{f}^{-1}(\lambda))$.
Thus f is fuzzy $^*\mathrm{G}_\delta\beta$ continuous. □

Proposition 5.7. *Let $FM_1 = (E_1, \mathfrak{F}_1)$ and $FM_2 = (E_2, \mathfrak{F}_2)$ be any two fuzzy *matroids. For a bijective map $f : FM_1 \longrightarrow FM_2$, the following are equivalent:*
(i) f is fuzzy $^*G_\delta\beta$ continuous.
(ii) For each $\lambda \in FM_1$, $f(G_\delta\beta\ int(\lambda)) \geq int^*(f(\lambda))$.
(iii) For each $\lambda \in FM_1$, $f(F_\sigma\beta\ cl(\lambda)) \leq cl^*(f(\lambda))$.

Proof. (i)⇒(ii) Assume that f is fuzzy $^*G_\delta\beta$ continuous. Let λ be any fuzzy set in FM_1. Then $f(\lambda)$ is a fuzzy set in FM_2. Now $int^*(f(\lambda))$ is a fuzzy *open set in FM_2. By assumption, $f^{-1}(int^*(f(\lambda)))$ is a fuzzy $^*G_\delta\beta$ open set in FM_1. We know that $int^*(f(\lambda)) \leq f(\lambda)$. Since f is bijective, $f^{-1}(int^*(f(\lambda))) \leq f^{-1}(f(\lambda)) = \lambda$. Thus

$$G_\delta\beta\ int(f^{-1}(int^*(f(\lambda)))) \leq G_\delta\beta\ int(\lambda).$$

By assumption, $f^{-1}(int^*(f(\lambda))) = G_\delta\beta\ int(f^{-1}(int(f(\lambda))))$. So $f^{-1}(int^*(f(\lambda))) \leq G_\delta\beta\ int(\lambda)$. Hence $int^*(f\ (\lambda) \leq f(G_\delta\beta\ int(\lambda))$.

(ii)⇒(iii) Assume for each $\lambda \in FM_1$,

$$\begin{aligned}
f(G_\delta\beta\ \ int(\lambda)) &\geq int^*(f(\lambda)) \\
1^{E_2} - f(G_\delta\beta\ \ int(\lambda)) &\leq 1^{E_2} - int^*(f(\lambda)) \\
f(1^{E_1}) - f(G_\delta\beta\ \ int(\lambda)) &\leq cl^*(1^{E_2} - f(\lambda)) \\
f(1^{E_1}) - f(G_\delta\beta\ \ int(\lambda)) &\leq cl^*(f(1^{E_1}) - f(\lambda)) \\
f(1^{E_1} - G_\delta\beta\ \ int(\lambda)) &\leq cl^*(f(1^{E_1} - \lambda))
\end{aligned}$$

which implies $f(F_\sigma\beta\ cl^*(1^{E_1} - \lambda)) \leq cl^*(f(1^{E_1} - \lambda))$ for each fuzzy set $(1^{E_1} - \lambda) \in FM_1$.

(iii)⇒(i) Assume that for each $\lambda \in FM_1$,

$$f(F_\sigma\beta\ \ cl(\lambda)) \leq cl^*(f(\lambda)). \tag{5.3}$$

Let λ be any fuzzy *flat in FM_2. (5.3) becomes $f(F_\sigma\beta\ cl(\lambda)) \leq f(\lambda)$.

$$F_\sigma\beta\ \ cl(f^{-1}(\lambda)) \leq f^{-1}(\lambda) \tag{5.4}$$

$$But\ \ f^{-1}(\lambda) \leq F_\sigma\beta\ \ cl(f^{-1}(\lambda)) \tag{5.5}$$

Now from equations (5.4) and (5.5), $f^{-1}(\lambda) = F_\sigma\beta\ cl(f^{-1}(\lambda))$.
Thus f is fuzzy $^*G_\delta\beta$ continuous. □

Proposition 5.8. *Let $FM_1 = (E_1, \mathfrak{F}_1)$, $FM_2 = (E_2, \mathfrak{F}_2)$ and $FM_3 = (E_3, \mathfrak{F}_3)$ be any three fuzzy *matroids. A map $f : FM_1 \longrightarrow FM_2$ be fuzzy $^*G_\delta\beta$ continuous and g : $FM_2 \longrightarrow FM_3$ be fuzzy *continuous map. Then g o f : $FM_1 \longrightarrow FM_3$ is fuzzy $^*G_\delta\beta$ continuous.*

Definition 5.9. Let $FM_1 = (E_1, \mathfrak{F}_1)$ and $FM_2 = (E_2, \mathfrak{F}_2)$ be any two fuzzy *matroids. A map $f : FM_1 \longrightarrow FM_2$ is said to be fuzzy $^*G^*_\delta$continuous (fuzzy $^*G_\delta\alpha$ continuous, fuzzy $^*G_\delta$pre continuous, fuzzy $^*G_\delta$semi continuous) if the inverse image of every fuzzy *open set in FM_2 is fuzzy $^*G^*_\delta$ open(fuzzy $^*G_\delta\alpha$ open, fuzzy $^*G_\delta$pre open, fuzzy $^*G_\delta$semi open) in FM_1.

Proposition 5.10. *Every fuzzy $^*G^*_\delta$ continuous map is fuzzy $^*G_\delta\alpha$ continuous.*

Proof. Proof is obvious. □

Remark 5.11. The converse of the above theorem need not be true as shown in the following example.

Example 5.12. Let E = {a, b}, $\mathfrak{F}_1 = \{0^E, 1^E, \lambda_1, \lambda_2, \lambda_3\}$ where λ_i : E ⟶ [0, 1] for i = 1, 2, 3 is defined as follows $\lambda_1(a) = 0.6$, $\lambda_1(b) = 0.4$; $\lambda_2(a) = 0.6$, $\lambda_2(b) = 1^E$; $\lambda_3(a) = 1^E$, $\lambda_3(b) = 0.4$; and $\mathfrak{F}_2 = \{0^E, 1^E, \mu_1, \mu_2, \mu_3\}$ where μ_i : E ⟶ [0, 1] for i = 1, 2, 3 is defined as follows $\mu_1(a) = 0.8$, $\mu_1(b) = 0.4$; $\mu_2(a) = 0.8$, $\mu_2(b)$

= 1^E; μ_3(a) = 1^E, μ_3(b) = 0.4. Let f : $FM_1 \longrightarrow FM_2$ be the identity map. Then f is fuzzy $^*G_\delta\alpha$ continuous but not fuzzy $^*G_\delta^*$ continuous. Consider the fuzzy set $\mu_1^{'}$ in FM_2, $f^{-1}(\mu_1^{'})$ is not fuzzy $^*G_\delta^*$ open in FM_1. Thus every fuzzy $^*G_\delta\alpha$ continuous map need not be fuzzy $^*G_\delta^*$ continuous.

Proposition 5.13. *Every fuzzy $^*G_\delta\alpha$ continuous map is fuzzy $^*G_\delta$pre continuous.*

Proof. Proof is obvious. □

Remark 5.14. The converse of the above theorem need not be true as shown in the following example.

Example 5.15. Let E = {a, b}, $\mathfrak{F}_1 = \{0^E, 1^E, \lambda_1, \lambda_2, \lambda_3\}$ where λ_i : E $\longrightarrow$ [0, 1] for i = 1, 2, 3 is defined as follows λ_1(a) = 0.6, λ_1(b) = 0.5; λ_2(a) = 0.6, λ_2(b) = 1^E; λ_3(a) = 1^E, λ_3(b) = 0.5; and $\mathfrak{F}_2 = \{0^E, 1^E, \mu_1, \mu_2, \mu_3\}$ where μ_i : E $\longrightarrow$ [0, 1] for i = 1, 2, 3 is defined as follows μ_1(a) = 0.7, μ_1(b) = 0.6; μ_2(a) = 0.7, μ_2(b) = 1^E; μ_3(a) = 1^E, μ_3(b) = 0.6. Let f : $FM_1 \longrightarrow FM_2$ be the identity map. Then f is fuzzy $^*G_\delta$pre continuous but not fuzzy $^*G_\delta\alpha$ continuous. Consider the fuzzy set $\mu_1^{'}$ in FM_2, $f^{-1}(\mu_1^{'})$ is not fuzzy $^*G_\delta\alpha$ open in FM_1. Thus every fuzzy $^*G_\delta$pre continuous map need not be fuzzy $^*G_\delta\alpha$ continuous.

Proposition 5.16. *Every fuzzy $^*G_\delta$pre continuous map is fuzzy $^*G_\delta\beta$ continuous.*

Proof. Proof is obvious. □

Remark 5.17. The converse of the above theorem need not be true as shown in the following example.

Example 5.18. Let E = {a, b}, $\mathfrak{F}_1 = \{0^E, 1^E, \lambda_1, \lambda_2, \lambda_3\}$ where λ_i : E $\longrightarrow$ [0, 1] for i = 1, 2, 3 is defined as follows λ_1(a) = 0.4, λ_1(b) = 0.7; λ_2(a) = 0.4, λ_2(b) = 1^E; λ_3(a) = 1^E, λ_3(b) = 0.7; and $\mathfrak{F}_2 = \{0^E, 1^E, \mu_1, \mu_2, \mu_3\}$ where μ_i : E $\longrightarrow$ [0, 1] for i = 1, 2, 3 is defined as follows μ_1(a) = 0.7, μ_1(b) = 0.4; μ_2(a) = 0.7, μ_2(b) = 1^E; μ_3(a) = 1^E, μ_3(b) = 0.4. Let f : $FM_1 \longrightarrow FM_2$ be the identity map. Then f is fuzzy $^*G_\delta\beta$ continuous but not fuzzy $^*G_\delta$pre continuous. Consider the fuzzy set $\mu_1^{'}$ in FM_2, $f^{-1}(\mu_1^{'})$ is not fuzzy $^*G_\delta$pre open in FM_1. Thus every fuzzy $^*G_\delta\beta$ continuous map need not be fuzzy $^*G_\delta$pre continuous.

Proposition 5.19. *Every fuzzy $^*G_\delta^*$ continuous map is fuzzy $^*G_\delta$semi continuous.*

Proof. Proof is obvious. □

Remark 5.20. The converse of the above theorem need not be true as shown in the following example.

Example 5.21. Let E = {a, b}, $\mathfrak{F}_1 = \{0^E, 1^E, \lambda_1, \lambda_2, \lambda_3\}$ where λ_i : E $\longrightarrow$ [0, 1] for i = 1, 2, 3 is defined as follows λ_1(a) = 0.4, λ_1(b) = 0.7; λ_2(a) = 0.4, λ_2(b) = 1^E; λ_3(a) = 1^E, λ_3(b) = 0.7; and $\mathfrak{F}_2 = \{0^E, 1^E, \mu_1, \mu_2, \mu_3\}$ where μ_i : E $\longrightarrow$ [0, 1] for i = 1, 2, 3 is defined as follows μ_1(a) = 0.3, μ_1(b) = 0.7; μ_2(a) = 0.3, μ_2(b) = 1^E; μ_3(a) = 1^E, μ_3(b) = 0.7. Let f : $FM_1 \longrightarrow FM_2$ be the identity map. Then f is fuzzy $^*G_\delta$semi continuous but not fuzzy $^*G_\delta^*$ continuous. Consider the fuzzy set $\mu_1^{'}$ in FM_2, $f^{-1}(\mu_1^{'})$ is not fuzzy $^*G_\delta^*$ open in FM_1. Thus every fuzzy $^*G_\delta$semi continuous map need not be fuzzy $^*G_\delta^*$ continuous.

Proposition 5.22. *Every fuzzy $^*G_\delta\alpha$ continuous map is fuzzy $^*G_\delta$semi continuous.*

Proof. Proof is obvious. □

Remark 5.23. The converse of the above theorem need not be true as shown in the following example.

Example 5.24. Let E = {a, b}, $\mathfrak{F}_1$ = {0^E, 1^E, λ_1, λ_2, λ_3} where λ_i : E $\longrightarrow$ [0, 1] for i = 1, 2, 3 is defined as follows λ_1(a) = 0.4, λ_1(b) = 0.7; λ_2(a) = 0.4, λ_2(b) = 1^E; λ_3(a) = 1^E, λ_3(b) = 0.7; and $\mathfrak{F}_2$ = {0^E, 1^E, μ_1, μ_2, μ_3} where μ_i : E $\longrightarrow$ [0, 1] for i = 1, 2, 3 is defined as follows μ_1(a) = 0.3, μ_1(b) = 0.7; μ_2(a) = 0.3, μ_2(b) = 1^E; μ_3(a) = 1^E, μ_3(b) = 0.7. Let f : $FM_1 \longrightarrow FM_2$ be the identity map. Then f is fuzzy $^*G_\delta$semi continuous but not fuzzy $^*G_\delta\alpha$ continuous. Consider the fuzzy set $\mu_1^{'}$ in FM_2, $f^{-1}(\mu_1^{'})$ is not fuzzy $^*G_\delta\alpha$ open in FM_1. Thus every fuzzy $^*G_\delta$semi continuous map need not be fuzzy $^*G_\delta\alpha$ continuous.

Proposition 5.25. *Every fuzzy $^*G_\delta$semi continuous map is fuzzy $^*G_\delta\beta$ continuous.*

Proof. Proof is obvious. □

Remark 5.26. The converse of the above theorem need not be true as shown in the following example.

Example 5.27. Let E = {a, b}, $\mathfrak{F}_1$ = {0^E, 1^E, λ_1, λ_2, λ_3} where λ_i : E $\longrightarrow$ [0, 1] for i = 1, 2, 3 is defined as follows λ_1(a) = 0.2, λ_1(b) = 0.8; λ_2(a) = 0.2, λ_2(b) = 1^E; λ_3(a) = 1^E, λ_3(b) = 0.8; and $\mathfrak{F}_2$ = {0^E, 1^E, μ_1, μ_2, μ_3} where μ_i : E $\longrightarrow$ [0, 1] for i = 1, 2, 3 is defined as follows μ_1(a) = 0.2, μ_1(b) = 0.1; μ_2(a) = 0.2, μ_2(b) = 1^E; μ_3(a) = 1^E, μ_3(b) = 0.1. Let f : $FM_1 \longrightarrow FM_2$ be the identity map. Then f is fuzzy $^*G_\delta\beta$ continuous but not fuzzy $^*G_\delta$semi continuous. Consider the fuzzy set $\mu_1^{'}$ in FM_2, $f^{-1}(\mu_1^{'})$ is not fuzzy $^*G_\delta$semi open in FM_1. Thus fuzzy $^*G_\delta\beta$ continuous map need not be fuzzy $^*G_\delta$semi continuous.

6. Fuzzy $^*G_\delta\beta$ irresolute map and its properties

Definition 6.1. Let FM_1 = (E_1, $\mathfrak{F}_1$) and FM_2 = (E_2, $\mathfrak{F}_2$) be any two fuzzy *matroids. A map f : $FM_1 \longrightarrow FM_2$ is said to be fuzzy $^*G_\delta\beta$ irresolute if the inverse image of every fuzzy $^*G_\delta\beta$ open set in FM_2 is fuzzy $^*G_\delta\beta$ open in FM_1.

Proposition 6.2. *Let FM_1 = (E_1, $\mathfrak{F}_1$) and FM_2 = (E_2, $\mathfrak{F}_2$) be any two fuzzy *matroids. For a map f : $FM_1 \longrightarrow FM_2$, the following are equivalent:*
(i) f is fuzzy $^*G_\delta\beta$ irresolute.
(ii) The inverse image of fuzzy $^*F_\sigma\beta$ flat in FM_2 is a fuzzy $^*F_\sigma\beta$ flat in FM_1.

Proposition 6.3. *Let FM_1 = (E_1, $\mathfrak{F}_1$) and FM_2 = (E_2, $\mathfrak{F}_2$) be any two fuzzy *matroids. For a bijective map f : $FM_1 \longrightarrow FM_2$, the following are equivalent:*
(i) f is fuzzy $^*G_\delta\beta$ irresolute.
(ii) For each $\lambda \in FM_1$, $f(F_\sigma\beta\ cl(\lambda)) \leq F_\sigma\beta\ cl(f(\lambda))$.
(iii) For each $\mu \in FM_2$, $F_\sigma\beta\ cl(f^{-1}(\mu)) \leq f^{-1}(F_\sigma\beta\ cl(\mu))$.

Proof. (i)⇒(ii) Assume that f is fuzzy $^*G_\delta\beta$ irresolute. Let λ be any fuzzy set in FM_1. Then $F_\sigma\beta$ cl(f(λ)) is a fuzzy $^*F_\sigma\beta$ flat in FM_2. By assumption, $f^{-1}(F_\sigma\beta$

cl(f(λ)) is fuzzy $^*F_\sigma\beta$ flat in FM_1. Thus

$$\begin{aligned} \lambda &\leq f^{-1}(f(\lambda)) \leq f^{-1}(F_\sigma\beta \;\; cl(f(\lambda)), \\ \lambda &\leq f^{-1}(F_\sigma\beta \;\; cl(f(\lambda)), \\ F_\sigma\beta \;\; cl(\lambda) &\leq f^{-1}(F_\sigma\beta \;\; cl(f(\lambda)). \end{aligned}$$

So f($F_\sigma\beta$ cl(λ)) $\leq$($F_\sigma\beta$ cl(f(λ)).

(ii)$\Rightarrow$(iii) Assume for each fuzzy set $\lambda \in FM_1$, f($F_\sigma\beta$ cl(λ)) $\leq$($F_\sigma\beta$ cl(f(λ)). Let μ be a fuzzy set in FM_2. Then $f^{-1}(\mu)$ is a fuzzy set in FM_1. By assumption,

$$\begin{aligned} f(F_\sigma\beta \;\; cl(f^{-1}(\mu))) &\leq F_\sigma\beta \;\; cl(f(f^{-1}(\mu))), \\ f(F_\sigma\beta \;\; cl(f^{-1}(\mu))) &\leq (F_\sigma\beta cl(\mu). \end{aligned}$$

Thus $F_\sigma\beta$ cl($f^{-1}(\mu)$) $\leq f^{-1}$($F_\sigma\beta$ cl(μ)).

(iii)$\Rightarrow$(i) Assume that for each $\mu \in FM_2$, $F_\sigma\beta$ cl($f^{-1}(\mu)$) $\leq f^{-1}$($F_\sigma\beta$ cl(μ)). Let γ be a fuzzy$^*F_\sigma\beta$ flat in FM_2. By assumption, $F_\sigma\beta$ cl($f^{-1}(\gamma)$) $\leq f^{-1}$($F_\sigma\beta$ cl(γ)). Thus $F_\sigma\beta$ cl($f^{-1}(\gamma)$) $\leq f^{-1}(\gamma)$. But $f^{-1}(\gamma) \leq F_\sigma\beta$ cl($f^{-1}(\gamma)$). So $f^{-1}(\gamma) = F_\sigma\beta$ cl($f^{-1}(\gamma)$). Hence f is fuzzy $^*G_\delta\beta$ irresolute. □

Proposition 6.4. *Let FM_1 = (E_1, $\mathfrak{F}_1$), FM_2 = (E_2, $\mathfrak{F}_2$) and FM_3 = (E_3, $\mathfrak{F}_3$) be any three fuzzy *matroids. A map f : $FM_1 \longrightarrow FM_2$ be fuzzy $^*G_\delta\beta$ irresolute and g : $FM_2 \longrightarrow FM_3$ be fuzzy *continuous function. Then g o f : $FM_1 \longrightarrow FM_3$ is fuzzy $^*G_\delta\beta$ continuous.*

Proof. The proof is obvious from the definitions of fuzzy $^*G_\delta\beta$ continuous map and fuzzy $^*G_\delta\beta$ irresolute. □

7. Fuzzy $^*G_\delta\beta$ connected matroid, fuzzy $^*G_\delta\beta$ compact matroid, fuzzy $^*G_\delta\beta$ normal matroid

Definition 7.1. A fuzzy *matroid FM is said to be a fuzzy *connected if it has no proper fuzzy set which is both fuzzy *open and fuzzy *flat of FM.

Definition 7.2. A fuzzy *matroid is said to be a fuzzy $^*G_\delta\beta$ connected if it has no proper fuzzy set which is both fuzzy $^*G_\delta\beta$ open and fuzzy $^*F_\sigma\beta$ flat of FM.(A fuzzy set λ in a fuzzy *matroid is said to be proper if $\lambda \neq 0^E$ and $\lambda \neq 1^E$.)

Proposition 7.3. *A fuzzy *matroid FM is fuzzy $^*G_\delta\beta$ connected if and only if it has no proper fuzzy* $G_\delta\beta$ open sets λ and μ such that $\lambda + \mu = 1^E$.*

Proof. Suppose that fuzzy *matroid FM is fuzzy $^*G_\delta\beta$ connected. Assume that FM has proper fuzzy $^*G_\delta\beta$ open sets λ and μ such that $\lambda + \mu = 1^E$. Then $\lambda + \mu = 1^E$. Thus $\lambda = 1^E - \mu$. So λ is a fuzzy $^*F_\sigma\beta$ flat and fuzzy $^*G_\delta\beta$ open set in FM.
So fuzzy *matroid FM is not fuzzy $^*G_\delta\beta$ connected, which is a contradiction.

Conversely, assume fuzzy *matroid FM has no proper fuzzy $^*G_\delta\beta$ open sets λ and μ such that $\lambda + \mu = 1^E$. Assume that FM is not fuzzy $^*G_\delta\beta$ connected. Then there exists a proper fuzzy set λ which is both fuzzy $^*F_\sigma\beta$ flat and fuzzy $^*G_\delta\beta$ open set in FM. Thus $\mu = 1^E - \lambda$. Since $\lambda \neq 0^E$ and 1^E; $\mu \neq 0^E$ and 1^E.

Thus there exists a proper fuzzy set μ which is both fuzzy $^*F_\sigma\beta$ flat and fuzzy $^*G_\delta\beta$ open set in FM such that $\lambda + \mu = 1^E$ which is a contradiction. □

Proposition 7.4. *The following statements are equivalent for a fuzzy *matroid FM.*
(i) FM = (E, $\mathfrak{F}$) is fuzzy $^*G_\delta\beta$ connected.
(ii) There exists no fuzzy $^*G_\delta\beta$ open sets $\lambda \neq 0^E$ and $\mu \neq 0^E$ such that $\lambda + \mu = 1^E$.
(iii) There exists no fuzzy $^*F_\sigma\beta$ flats $\lambda \neq 1^E$ and $\mu \neq 1^E$ such that $\lambda + \mu = 1^E$.

Proof. (i)⇒(ii) Assume that FM is fuzzy $^*G_\delta\beta$ connected. Then, by the above proposition, it has no proper fuzzy $^*G_\delta\beta$ open sets λ and μ such that $\lambda + \mu = 1^E$.
(ii)⇒(iii) Assume that there exists no fuzzy $^*G_\delta\beta$ open sets $\lambda \neq 0^E$ and $\mu \neq 0^E$ such that $\lambda + \mu = 1^E$. Suppose that there exists fuzzy $^*F_\sigma\beta$ flats $\lambda \neq 1^E$ and $\mu \neq 1^E$ such that $\lambda + \mu = 1^E$. Then $1^E - \lambda \neq 0^E$ is a fuzzy $^*G_\delta\beta$ open set and $1^E - \mu \neq 0^E$ is a fuzzy $^*G_\delta\beta$ flat.

$$\begin{aligned} 1^E - \lambda + 1^E - \mu &= 1^E + 1^E - [\lambda + \mu] \\ &= 1^E + 1^E - 1^E \\ &= 1^E \end{aligned}$$

which is a contradiction.
Thus there exists no fuzzy $^*F_\sigma\beta$ flats $\lambda \neq 1^E$ and $\mu \neq 1^E$ such that $\lambda + \mu = 1^E$.
(iii)⇒(i) Assume that there exists no fuzzy $^*F_\sigma\beta$ flats $\lambda \neq 1^E$ and $\mu \neq 1^E$ such that $\lambda + \mu = 1^E$. Suppose that FM is not fuzzy $^*G_\delta\beta$ connected. There exists a proper fuzzy set λ which is both fuzzy $^*F_\sigma\beta$ flat and fuzzy $^*G_\delta\beta$ open set in FM. Then $1^E - \lambda$ is a proper fuzzy $^*F_\sigma\beta$ flat. Also by assumption, λ is fuzzy $^*F_\sigma\beta$ flat. Now $\lambda + 1^E - \lambda = 1^E$ which is a contradiction. □

Proposition 7.5. *Let FM_1 = (E_1, $\mathfrak{F}_1$) and FM_2 = (E_2, $\mathfrak{F}_2$) be any two fuzzy *matroids. If f : $FM_1 \longrightarrow FM_2$ is fuzzy $^*G_\delta\beta$ continuous surjection and FM_1 is fuzzy $^*G_\delta\beta$ connected, then FM_2 is fuzzy *connected.*

Proposition 7.6. *Let FM_1 = (E_1, $\mathfrak{F}_1$) and FM_2 = (E_2, $\mathfrak{F}_2$) be any two fuzzy *matroids. If f : $FM_1 \longrightarrow FM_2$ is fuzzy $^*G_\delta\beta$ irresolute surjection and FM_1 is fuzzy $^*G_\delta\beta$ connected, then FM_2 is fuzzy $^*G_\delta\beta$ connected.*

Definition 7.7. A fuzzy *matroid FM is said to be fuzzy *compact if whenever $\bigvee_{i\in I}(\lambda_i) = 1^E$, λ_i is fuzzy *open, $i \in$ I, there is a finite subset J of I with $\bigvee_{j\in J}(\lambda_i)$ $= 1^E$.

Definition 7.8. A fuzzy *matroid FM is said to be fuzzy $^*G_\delta\beta$ compact if whenever $\bigvee_{i\in I}(\lambda_i) = 1^E$, λ_i is fuzzy$^*G_\delta\beta$ open, $i \in$ I, there is a finite subset J of I with $\bigvee_{j\in J}(\lambda_i) = 1^E$.

Proposition 7.9. *Let FM_1 = (E_1, $\mathfrak{F}_1$) and FM_2 = (E_2, $\mathfrak{F}_2$) be any two fuzzy *matroids. If f : $FM_1 \longrightarrow FM_2$ is fuzzy $^*G_\delta\beta$ continuous bijection and FM_1 is fuzzy $^*G_\delta\beta$ compact, then FM_2 is fuzzy *compact.*

Proposition 7.10. *Let FM_1 = (E_1, $\mathfrak{F}_1$) and FM_2 = (E_2, $\mathfrak{F}_2$) be any two fuzzy *matroids. If f : $FM_1 \longrightarrow FM_2$ is fuzzy $^*G_\delta\beta$ irresolute bijection and FM_1 is fuzzy $^*G_\delta\beta$ compact, then FM_2 is fuzzy $^*G_\delta\beta$ compact.*

Definition 7.11. Let FM$_1$ = (E_1, $\mathfrak{F}_1$) and FM$_2$ = (E_2, $\mathfrak{F}_2$) be any two fuzzy *matroids. A map f : FM$_1 \longrightarrow$FM$_2$ is said to be fuzzy *M-$G_\delta\beta$ homeomorphism if f is bijective, fuzzy $^*G_\delta\beta$ irresolute and fuzzy $^*G_\delta\beta$ open map.

Definition 7.12. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid. A fuzzy *matroid is said to be fuzzy *normal if for every fuzzy *flat λ and fuzzy *open set μ in FM such that $\lambda \leq \mu$, there exists a fuzzy set γ such that $\lambda \leq \mathrm{int}^*(\gamma) \leq \mathrm{cl}^*(\gamma) \leq \mu$.

Definition 7.13. Let FM = (E, $\mathfrak{F}$) be a fuzzy *matroid. A fuzzy *matroid is said to be fuzzy $^*\mathrm{G}_\delta\beta$ normal if for every fuzzy $^*\mathrm{F}_\sigma\beta$ flat λ and fuzzy $^*\mathrm{G}_\delta\beta$ open set μ in FM such that $\lambda \leq \mu$, there exists a fuzzy set γ such that $\lambda \leq \mathrm{G}_\delta\beta$ int$(\gamma) \leq \mathrm{F}_\sigma\beta$ cl$(\gamma) \leq \mu$.

Proposition 7.14. *Let $FM_1 = (E_1, \mathfrak{F}_1)$ and $FM_2 = (E_2, \mathfrak{F}_2)$ be any two fuzzy*matroids. If $f : FM_1 \longrightarrow FM_2$ is fuzzy *M-$G_\delta\beta$ homeomorphism and FM_2 is fuzzy $^*G_\delta\beta$ normal matroid, then FM_1 is fuzzy $^*G_\delta\beta$ normal matroid.*

Proof. Let λ be any fuzzy $^*\mathrm{F}_\sigma\beta$ flat and μ be any fuzzy $^*\mathrm{G}_\delta\beta$ open set in FM_1 such that $\lambda \leq \mu$. Since f is fuzzy *M-$\mathrm{G}_\delta\beta$ homeomorphism, f(λ) is fuzzy $^*\mathrm{F}_\sigma\beta$ flat in FM_2 and f(μ) is fuzzy $^*\mathrm{G}_\delta\beta$ open in FM_2. Since FM_2 is fuzzy $^*\mathrm{G}_\delta\beta$ normal, there exists a fuzzy set γ in FM_2 such that f$(\lambda) \leq \mathrm{G}_\delta\beta$ int$(\gamma) \leq \mathrm{F}_\sigma\beta$ cl$(\gamma) \leq$ f(μ). Now,

$$\begin{aligned} f^{-1}(f(\lambda)) &\leq f^{-1}(G_\delta\beta \;\; int(\gamma)) \leq f^{-1}(F_\sigma\beta \;\; cl(\gamma)) \leq f^{-1}(f(\mu)), \\ \lambda &\leq f^{-1}(G_\delta\beta \;\; int(\gamma)) \leq f^{-1}(F_\sigma\beta \;\; cl(\gamma)) \leq \mu. \end{aligned}$$

That is, $\lambda \leq \mathrm{G}_\delta\beta$ int(f$^{-1}(\gamma)$) $\leq \mathrm{F}_\sigma\beta$ cl(f$^{-1}(\gamma)$) $\leq \mu$. Theus FM_1 is fuzzy $^*\mathrm{G}_\delta\beta$ normal matroid. □

Proposition 7.15. *Let $f : FM_1 \longrightarrow FM_2$ be a fuzzy *M-$G_\delta\beta$ homeomorphism from a fuzzy $^*G_\delta\beta$ normal matroid FM_1 onto a fuzzy *matroid FM_2. Then FM_2 is fuzzy $^*G_\delta\beta$ normal.*

Proof. Let λ be any fuzzy $^*\mathrm{F}_\sigma\beta$ flat and μ be any fuzzy $^*\mathrm{G}_\delta\beta$ open set in FM_2 such that $\lambda \leq \mu$. Since f is fuzzy $^*\mathrm{G}_\delta\beta$ irresolute, f$^{-1}(\lambda)$ is fuzzy $^*\mathrm{F}_\sigma\beta$ flat and f$^{-1}(\mu)$ is fuzzy $^*\mathrm{G}_\delta\beta$ open in FM_1. Since FM_1 is fuzzy $^*\mathrm{G}_\delta\beta$ normal, there exists a fuzzy set γ in FM_1 such that f$^{-1}(\lambda) \leq \mathrm{G}_\delta\beta$ int$(\gamma) \leq \mathrm{F}_\sigma\beta$ cl$(\gamma) \leq$ f$^{-1}(\mu)$. Now,

$$\begin{aligned} f(f^{-1}(\lambda)) &\leq f(G_\delta\beta \;\; int(\gamma)) \leq f(F_\sigma\beta \;\; cl(\gamma)) \leq f(f(^{-1}\mu)), \\ \lambda &\leq f(G_\delta\beta \;\; int(\gamma)) \leq f(F_\sigma\beta \;\; cl(\gamma)) \leq \mu. \end{aligned}$$

That is, $\lambda \leq \mathrm{G}_\delta\beta$ int(f(γ)) $\leq \mathrm{F}_\sigma\beta$ cl(f(γ)) $\leq \mu$. Thus FM_2 is fuzzy$^*\mathrm{G}_\delta\beta$ normal matroid. □

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