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A note on pairwise fuzzy σ-Baire bitopological spaces

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Abstract. In this paper we further investigate some characterizations of pairwise fuzzy σ-Baire spaces. The conditions under which pairwise fuzzy σ-Baire spaces become pairwise fuzzy Baire spaces, are also investigated.

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1. Introduction

The fundamental concept of a fuzzy set was introduced by L. A. Zadeh [14] in 1965. Subsequently, in 1968, C. L. Chang [4] introduced fuzzy topological spaces based on the concept of fuzzy sets introduced by L. A. Zadeh. Thereafter, the paper of C. L. Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts.

In 1989, A. Kandil [7] introduced and studied fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces. The concept of Baire bitopological spaces have been studied extensively in classical topology in [1, 2, 4, 5, 6]. Pairwise fuzzy σ-nowhere dense sets have been defined and studied in [11]. By using pairwise fuzzy σ-nowhere dense sets, the concept of pairwise fuzzy σ-Baire spaces is defined and studied by authors in [13]. In this paper we further investigate some characterizations of pairwise fuzzy σ-Baire spaces.

2. Preliminaries

Now we give some basic notions and results used in the sequel. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). By a fuzzy bitopological space (Kandil, 1989) we mean an ordered triple (X, T_1, T_2), where T_1 and T_2 are fuzzy topologies on the non-empty set X.

Definition 2.1 ([4]). Let λ and μ be any two fuzzy sets in a fuzzy topological space (X, T). Then we define :

(i) $\lambda \vee \mu$: X → [0,1] as follows : $(\lambda \vee \mu)(x) = \max \{\lambda(x), \mu(x)\}$.
(ii) $\lambda \wedge \mu$: X → [0, 1] as follows : $(\lambda \wedge \mu)(x) = \min \{\lambda(x), \mu(x)\}$.
(iii) $\mu = \lambda^c \Leftrightarrow \mu(x) = 1 - \lambda(x)$.

More generally, for a family of $\{\lambda_i \mid i \in I\}$ of fuzzy sets in X, $\vee_i \lambda_i$ and $\wedge_i \lambda_i$ are defined as $\vee_i \lambda_i = sup_i\{\lambda_i(x) \mid x \in X\}$ and $\wedge_i \lambda_i = inf_i\{\lambda_i(x) \mid x \in X\}$.

Definition 2.2 ([4]). Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). We define the interior and the closure of λ respectively as follows :

(i) $int(\lambda) = \vee\{\mu \mid \mu \leq \lambda, \mu \in T\}$,
(ii) $cl(\lambda) = \wedge\{\mu \mid \lambda \leq \mu, 1 - \mu \in T\}$.

Lemma 2.3 ([3]). *For a fuzzy set λ in a fuzzy topological space X,*

(i) $1 - int(\lambda) = cl(1 - \lambda)$,
(ii) $1 - cl(\lambda) = int(1 - \lambda)$.

Definition 2.4 ([11]). A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy open set if $\lambda \in T_i$ $(i = 1, 2)$. The complement of pairwise fuzzy open set in (X, T_1, T_2) is called a pairwise fuzzy closed set.

Definition 2.5 ([11]). A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy G_δ-set if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are pairwise fuzzy open sets in (X, T_1, T_2).

Definition 2.6 ([11]). A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy F_σ-set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are pairwise fuzzy closed sets in (X, T_1, T_2).

Lemma 2.7 ([3]). *For a family $\mathscr{A} = \{\lambda_\alpha\}$ of fuzzy sets of a fuzzy topological space (X, T), $\vee cl(\lambda_\alpha) \leq cl(\vee \lambda_\alpha)$. In case $\mathscr{A}$ is a finite set, $\vee cl(\lambda_\alpha) = cl(\vee \lambda_\alpha)$. Also $\vee int(\lambda_\alpha) \leq int(\vee \lambda_\alpha)$ in (X, T).*

Definition 2.8 ([8]). A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy dense set if $cl_{T_1} cl_{T_2}(\lambda) = cl_{T_2} cl_{T_1}(\lambda) = 1$.

Definition 2.9 ([9]). A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy nowhere dense set if $int_{T_1} cl_{T_2}(\lambda) = int_{T_2} cl_{T_1}(\lambda) = 0$.

Definition 2.10 ([11]). A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy σ-nowhere dense set if λ is a pairwise fuzzy F_σ-set in (X, T_1, T_2) such that $int_{T_1} int_{T_2}(\lambda) = int_{T_2} int_{T_1}(\lambda) = 0$.

Definition 2.11 ([13]). Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called a pairwise fuzzy σ-first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where

(λ_i)'s are pairwise fuzzy σ-nowhere dense sets in (X, T_1, T_2). Any other fuzzy set in (X, T_1, T_2) is said to be a pairwise fuzzy σ-second category set in (X, T_1, T_2).

Definition 2.12 ([13]). If λ is a pairwise fuzzy σ-first category set in a fuzzy bitopological space (X, T_1, T_2), then the fuzzy set $1 - \lambda$ is called a pairwise fuzzy σ-residual set in (X, T_1, T_2).

Definition 2.13 ([13]). A fuzzy bitopological space (X, T_1, T_2) is called pairwise fuzzy σ-first category if the fuzzy set 1_X is a pairwise fuzzy σ-first category set in (X, T_1, T_2). That is $1_X = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy σ-nowhere dense sets in (X, T_1, T_2). Otherwise (X, T_1, T_2) will be called a pairwise fuzzy σ-second category space.

Definition 2.14 ([9]). A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy Baire space if $int_{T_i}\big(\vee_{k=1}^{\infty}(\lambda_k)\big) = 0$, (i=1,2) where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2).

3. Pairwise fuzzy σ -Baire Spaces

Definition 3.1 ([11]). A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy σ-Baire space if $int_{T_i}\Big(\vee_{k=1}^{\infty}(\lambda_k)\Big) = 0,\ (i = 1, 2)$ where (λ_k)'s are pairwise fuzzy σ-nowhere dense sets in (X, T_1, T_2).

Proposition 3.2. *If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy σ-Baire space, then no non-zero pairwise fuzzy open set is a pairwise fuzzy σ-first category set in (X, T_1, T_2).*

Proof. Let λ be a non-zero pairwise fuzzy open set in a pairwise fuzzy σ-Baire space (X, T_1, T_2). Suppose that $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy σ-nowhere dense sets in (X, T_1, T_2). Then $int_{T_i}(\lambda) = int_{T_i}\big(\vee_{k=1}^{\infty}(\lambda_k)\big)$. Since (X, T_1, T_2) is a pairwise fuzzy σ-Baire space, $int_{T_i}\big(\vee_{k=1}^{\infty}(\lambda_k)\big) = 0$. This implies that $int_{T_i}(\lambda) = 0$. Thus we will have $\lambda = int_{T_i}(\lambda) = 0$, a contradiction to λ being a non-zero pairwise fuzzy open set in (X, T_1, T_2). Hence no non-zero pairwise fuzzy open set is a pairwise fuzzy σ-first category set in (X, T_1, T_2). □

Proposition 3.3. *If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy σ-Baire space and if $\vee_{k=1}^{\infty}(\lambda_k) = 1$, where the fuzzy sets $(\lambda_k)'s(k = 1\ \ to\ \ \infty)$ are pairwise fuzzy F_σ-sets in (X, T_1, T_2), then there exists atleast one F_σ-set λ_k such that $int_{T_1}int_{T_2}(\lambda_k) \neq 0 \neq int_{T_2}int_{T_1}(\lambda_k)$.*

Proof. Suppose that $int_{T_1}int_{T_2}(\lambda_k) = 0$ and $int_{T_2}int_{T_1}(\lambda_k) = 0$, where the fuzzy sets $(\lambda_k)'s\ (k = 1to\infty)$ are pairwise fuzzy F_σ-sets in (X, T_1, T_2). Then the fuzzy sets $(\lambda_k)'s$ are pairwise fuzzy σ-nowhere dense sets in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy σ-Baire space, $int_{T_i}\big(\vee_{k=1}^{\infty}(\lambda_k)\big) = 0 (i = 1, 2)$. Now $\vee_{k=1}^{\infty}(\lambda_k) = 1$ in (X, T_1, T_2), implies that $int_{T_i}\big(\vee_{k=1}^{\infty}(\lambda_k)\big) = int_{T_i}(1) = 1 \neq 0$, a contradiction. Thus $int_{T_1}int_{T_2}(\lambda_k) \neq 0 \neq int_{T_2}int_{T_1}(\lambda_k)$, for atleast one F_σ-set $\lambda_k$in (X, T_1, T_2). □

Theorem 3.4 ([11]). *In a fuzzy bitopological space (X, T_1, T_2), a fuzzy set λ is a pairwise fuzzy σ-nowhere dense set in (X, T_1, T_2) if and only if $(1 - \lambda)$ is a pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X, T_1, T_2).*

Proposition 3.5. *If $\wedge_{k=1}^{\infty}(\lambda_k) \neq 0$, where the fuzzy sets (λ_k)'s are pairwise fuzzy dense and pairwise fuzzy G_δ-sets in a fuzzy bitopological space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy σ-second category space.*

Proof. Let $(\lambda_k)'s(k = 1to\infty)$ be pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X, T_1, T_2). By theorem 3.4, $(1 - \lambda_k)$'s are pairwise fuzzy σ-nowhere dense sets in (X, T_1, T_2). Now $\wedge_{k=1}^{\infty}(\lambda_k) \neq 0$ implies that $1 - \wedge_{k=1}^{\infty}(\lambda_k) \neq 1$. Then $\vee_{k=1}^{\infty}(1 - \lambda_k) \neq 1$ in (X, T_1, T_2). Thus (X, T_1, T_2) is not a pairwise fuzzy σ-first category space. So (X, T_1, T_2) is a pairwise fuzzy σ-second category space. □

Proposition 3.6. *If λ is a pairwise fuzzy σ-first category set in (X, T_1, T_2), then there is a pairwise fuzzy F_σ-set δ in (X, T_1, T_2) such that $\lambda \leq \delta$*

Proof. Let λ be a pairwise fuzzy σ-first category set in (X, T_1, T_2). Then $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$ where (λ_k)'s are pairwise fuzzy σ-nowhere dense sets in (X, T_1, T_2). Now $\big(1 - cl_{T_i}(\lambda_k)\big)$'s are pairwise fuzzy open sets in (X, T_1, T_2). Thus

$$\mu = \wedge_{k=1}^{\infty}(1 - cl_{T_i}(\lambda_k)) \text{ is a pairwise fuzzy } G_\delta\text{-set in } (X, T_1, T_2)$$

and

$$1 - \mu = 1 - \big(\wedge_{k=1}^{\infty}(1 - cl_{T_i}(\lambda_k))\big) = \vee_{k=1}^{\infty}(cl_{T_i}(\lambda_k)).$$

Now $\lambda = \vee_{k=1}^{\infty}(\lambda_k) \leq \vee_{k=1}^{\infty}(cl_{T_i}(\lambda_k)) = 1 - \mu$. That is $\lambda \leq 1 - \mu$ and $(1 - \mu)$is a pairwise fuzzy F_σ-set in (X, T_1, T_2). Let $\delta = 1 - \mu$. Hence, if λ is a pairwise fuzzy σ-first category set in (X, T_1, T_2), then there is a pairwise fuzzy F_σ-set δ in (X, T_1, T_2) such that $\lambda \leq \delta$. □

Theorem 3.7 ([11])**.** *If λ is a pairwise fuzzy dense and pairwise fuzzy G_δ-set in a fuzzy bitopological space (X, T_1, T_2), then $(1 - \lambda)$ is a pairwise fuzzy first category set in (X, T_1, T_2).*

Theorem 3.8 ([13])**.** *If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy σ-Baire space, then $cl_{T_i}\big(\wedge_{k=1}^{\infty}(\lambda_k)\big) = 1, (i = 1, 2)$ where the fuzzy sets (λ_k)'s are pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X, T_1, T_2).*

Proposition 3.9. *If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy σ-Baire space, then $int_{T_i}\big(\vee_{k=1}^{\infty}(1 - \lambda_k)\big) = 0$ where the fuzzy sets $(1 - \lambda_k)$'s are pairwise fuzzy first category sets formed from the pairwise fuzzy dense and pairwise fuzzy G_δ-sets (λ_k) in (X, T_1, T_2).*

Proof. Let the fuzzy bitopological space (X, T_1, T_2) be a pairwise fuzzy σ-Baire space and the fuzzy sets (λ_k)'s $(k = 1 \;\; to \;\; \infty)$ be pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X, T_1, T_2). Then, by Theorem 3.8, $cl_{T_i}\big(\wedge_{k=1}^{\infty}(\lambda_k)\big) = 1, \; (i = 1, 2)$. Thus $1 - cl_{T_i}\big(\wedge_{k=1}^{\infty}(\lambda_k)\big) = 0$. This implies that $int_{T_i}\big(\vee_{k=1}^{\infty}(1 - \lambda_k)\big) = 0$. Also, by Theorem 3.7, $(1 - \lambda_k)$'s are pairwise fuzzy first category sets in (X, T_1, T_2). So $int_{T_i}\big(\vee_{k=1}^{\infty}(1 - \lambda_k)\big) = 0$, where the fuzzy sets $(1 - \lambda_k)$'s are pairwise fuzzy first category sets formed from the pairwise fuzzy dense and pairwise fuzzy G_δ-sets (λ_k) in (X, T_1, T_2). □

Theorem 3.10 ([13])**.** *Let (X, T_1, T_2) be a fuzzy bitopological space. Then the following are equivalent :*

(1) (X, T_1, T_2) *is a pairwise fuzzy* σ*-Baire space.*
(2) $int_{T_i}(\lambda) = 0$, $(i = 1, 2)$ *for every pairwise fuzzy* σ*-first category set* λ *in* (X, T_1, T_2).
(3) $cl_{T_i}(\mu) = 1$, $(i = 1, 2)$ *for every pairwise fuzzy* σ*-residual set* μ *in* (X, T_1, T_2).

Proposition 3.11. *If each pairwise fuzzy* σ*-residual set* δ *in a fuzzy bitopological space* (X, T_1, T_2) *is such that* $\eta \leq \delta$ *where* $cl_{T_i}(\eta) = 1$ *(i=1,2) in* (X, T_1, T_2)*, then* (X, T_1, T_2) *is a pairwise fuzzy* σ*-Baire space.*

Proof. Let δ be a pairwise fuzzy σ-residual set in a fuzzy bitopological space (X, T_1, T_2). Then $1 - \delta$ is a pairwise fuzzy σ-first category set in (X, T_1, T_2). Now, by Proposition 3.6, there is a pairwise fuzzy F_σ-set μ in (X, T_1, T_2) such that $1 - \delta \leq \mu$. This implies that $1 - \mu \leq \delta$. Let $\eta = 1 - \mu$. Then η is a pairwise fuzzy G_δ-set in (X, T_1, T_2) and $\eta \leq \delta$ implies that $cl_{T_i}(\eta) \leq cl_{T_i}(\delta)$. Now $cl_{T_i}(\eta) = 1$, (i=1,2) implies that $cl_{T_i}(\delta) = 1$. Hence, by Theorem 3.10, (X, T_1, T_2) is a pairwise fuzzy σ-Baire space. □

Definition 3.12 ([11])**.** A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy P-space if every non-zero pairwise fuzzy G_δ-set in (X, T_1, T_2) is pairwise fuzzy open in (X, T_1, T_2).

Theorem 3.13 ([9])**.** *Let* (X, T_1, T_2) *be a fuzzy bitopological space. Then the following are equivalent :*

(1) (X, T_1, T_2) *is a pairwise fuzzy Baire space.*
(2) $int_{T_i}(\lambda) = 0$, $(i = 1, 2)$ *for every pairwise fuzzy first category set* λ *in* (X, T_1, T_2).
(3) $cl_{T_i}(\mu) = 1$, $(i = 1, 2)$ *for every pairwise fuzzy residual set* μ *in* (X, T_1, T_2).

Proposition 3.14. *If the fuzzy bitopological space* (X, T_1, T_2) *is a pairwise fuzzy* σ*-Baire space and pairwise fuzzy P-space, then* (X, T_1, T_2) *is a pairwise fuzzy Baire space.*

Proof. Let the fuzzy bitopological space (X, T_1, T_2) be a pairwise fuzzy σ-Baire space. Then, by theorem 3.8, $cl_{T_i}\big(\wedge_{k=1}^{\infty}(\lambda_k)\big) = 1$, where the fuzzy sets (λ_k)'s are pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X, T_1, T_2). Since $cl_{T_i}\big(\wedge_{k=1}^{\infty}(\lambda_k)\big) = 1$, we have $1 - cl_{T_i}\big(\wedge_{k=1}^{\infty}(\lambda_k)\big) = 0$. This implies that $int_{T_i}\big(\vee_{k=1}^{\infty}(1-\lambda_k)\big) = 0$ in (X, T_1, T_2). But $\vee_{k=1}^{\infty} int_{T_i}(1 - \lambda_k) \leq int_{T_i}\big(\vee_{k=1}^{\infty}(1 - \lambda_k)\big) = 0 (i = 1, 2)$ implies that $\vee_{k=1}^{\infty} int_{T_i}(1 - \lambda_k) = 0$. Thus

(A) $$int_{T_i}(1 - \lambda_k) = 0 (k = 1 \ \ to \ \ \infty) \text{ in } (X, T_1, T_2).$$

Since (X, T_1, T_2) is a pairwise fuzzy P-space, the non-zero pairwise fuzzy G_δ-sets (λ_k)'s in (X, T_1, T_2), are pairwise fuzzy open in (X, T_1, T_2). Then $(1 - \lambda_k)$'s are pairwise fuzzy closed sets in (X, T_1, T_2). Now from (A),

$$int_{T_1} cl_{T_2}(1 - \lambda_k) = int_{T_1}(1 - \lambda_k) = 0$$

and

$$int_{T_2} cl_{T_1}(1 - \lambda_k) = int_{T_2}(1 - \lambda_k) = 0.$$

Thus $(1-\lambda_k)$'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). So $int_{T_i}\big(\vee_{k=1}^{\infty}(1-\lambda_k)\big) = 0$, where the fuzzy sets $(1-\lambda_k)$'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Hence (X, T_1, T_2) is a pairwise fuzzy Baire space. □

Definition 3.15 ([10]). A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy almost resolvable space if $\vee_{k=1}^{\infty}(\lambda_k) = 1$, where the fuzzy sets (λ_k)'s in (X, T_1, T_2) are such that $int_{T_1} int_{T_2}(\lambda_k) = 0 = int_{T_2} int_{T_1}(\lambda_k)$. Otherwise (X, T_1, T_2) is called a pairwise fuzzy almost irresolvable space.

Proposition 3.16. *If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy almost irresolvable space, then (X, T_1, T_2) is a pairwise fuzzy σ-second category space.*

Proof. Let $(\lambda_k)'s (k = 1 \ \ to \ \ \infty)$ be pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X, T_1, T_2). Since $cl_{T_1} cl_{T_2}(\lambda_k) = 1 = cl_{T_2} cl_{T_1}(\lambda_k)$ implies that $1 - cl_{T_1} cl_{T_2}(\lambda_k) = 0 = 1 - cl_{T_2} cl_{T_1}(\lambda_k)$, $int_{T_1} int_{T_2}(1-\lambda_k) = 0 = int_{T_2} int_{T_1}(1-\lambda_k)$. Since (X, T_1, T_2) is a pairwise fuzzy almost irresolvable space, $\vee_{k=1}^{\infty}(1-\lambda_k) \neq 1$, where the fuzzy sets $(1-\lambda_k)'s$ in (X, T_1, T_2) are such that $int_{T_1} int_{T_2}(1-\lambda_k) = 0 = int_{T_2} int_{T_1}(1-\lambda_k)$. Now $\vee_{k=1}^{\infty}(1-\lambda_k) \neq 1$ implies that $1 - \wedge_{k=1}^{\infty}(\lambda_k) \neq 1$. Then we have $\wedge_{k=1}^{\infty}(\lambda_k) \neq 0$, where the fuzzy sets $(\lambda_k)'s$ are pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X, T_1, T_2). Thus, by proposition 3.5, (X, T_1, T_2) is a pairwise fuzzy σ-second category space. □

Definition 3.17. [12] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy hyperconnected space if every pairwise fuzzy open set λ is a pairwise fuzzy dense in (X, T_1, T_2). That is $cl_{T_1} cl_{T_2}(\lambda) = 1 = cl_{T_2} cl_{T_1}(\lambda)$ where $\lambda \in T_i$ (i=1,2).

Theorem 3.18 ([13]). *If $cl_{T_i}\big(\wedge_{k=1}^{\infty}(\lambda_k)\big) = 1$, where $(\lambda_k)'s$ are pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy σ-Baire space.*

Proposition 3.19. *If $cl_{T_i}\big(\wedge_{k=1}^{\infty}(\lambda_k)\big) = 1$, where $(\lambda_k)'s$ are pairwise fuzzy G_δ-sets in a pairwise fuzzy hyperconnected and pairwise fuzzy P-space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy σ-Baire space.*

Proof. Let $(\lambda_k)'s (k = 1 \ \ to \ \ \infty)$ be pairwise fuzzy G_δ-sets in (X, T_1, T_2) such that $cl_{T_i}\big(\wedge_{k=1}^{\infty}(\lambda_k)\big) = 1$. Since (X, T_1, T_2) is a pairwise fuzzy P-space, the pairwise fuzzy G_δ-sets $(\lambda_k)'s$ in (X, T_1, T_2), are pairwise fuzzy open sets in (X, T_1, T_2). Also since (X, T_1, T_2) is a pairwise fuzzy hyperconnected, the pairwise fuzzy open sets $(\lambda_k)'s$ in (X, T_1, T_2), are pairwise fuzzy dense sets in (X, T_1, T_2). Then the fuzzy sets $(\lambda_k)'s$ are pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X, T_1, T_2) and $cl_{T_i}\big(\wedge_{k=1}^{\infty}(\lambda_k)\big) = 1$. Thus by theorem 3.18, (X, T_1, T_2) is a pairwise fuzzy σ-Baire space. □

Proposition 3.20. *If the pairwise fuzzy first category sets are formed from the pairwise fuzzy dense and pairwise fuzzy G_δ-sets in a pairwise fuzzy σ-Baire space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy Baire space.*

Proof. Let the fuzzy bitopological space (X, T_1, T_2) be a pairwise fuzzy σ-Baire space. By proposition 3.9, $int_{T_i}\big(\vee_{k=1}^{\infty}(1-\lambda_k)\big) = 0$, where the fuzzy sets $(1-\lambda_k)$'s $(k = \ 1 \ \ to \ \ \infty)$are pairwise fuzzy first category sets formed from the pairwise fuzzy dense and pairwise fuzzy G_δ-sets (λ_k) in (X, T_1, T_2). Now $\vee_{k=1}^{\infty}\big(int_{T_i}(1-\lambda_k)\big) \leq$

$int_{T_i}\big(\vee_{k=1}^{\infty}(1-\lambda_k)\big)$. Then we have $\vee_{k=1}^{\infty}\big(int_{T_i}(1-\lambda_k)\big)=0$. This implies that $int_{T_i}(1-\lambda_k)=0$,where $(1-\lambda_k)'s$ are pairwise fuzzy first category sets in (X,T_1,T_2). Thus (X,T_1,T_2) is a pairwise fuzzy Baire space. □

Definition 3.21 ([10]). A fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy submaximal space if for each fuzzy set λ in (X,T_1,T_2) such that $cl_{T_i}(\lambda)=1$ then $\lambda\in T_i$ in (X,T_1,T_2). That is, each pairwise fuzzy dense set is a pairwise fuzzy open set in (X,T_1,T_2).

Proposition 3.22. *If the fuzzy bitopological space (X,T_1,T_2) is a pairwise fuzzy σ-Baire space and pairwise fuzzy submaximal space, then (X,T_1,T_2) is a pairwise fuzzy Baire space.*

Proof. Let the fuzzy bitopological space (X,T_1,T_2) be a pairwise fuzzy σ-Baire space. Then, by theorem 3.8, $cl_{T_i}\big(\wedge_{k=1}^{\infty}(\lambda_k)\big)=1$ (i=1,2), where (λ_k)'s are pairwise fuzzy dense and pairwise fuzzy G_δ sets in (X,T_1,T_2). Then $1-cl_{T_i}\big(\wedge_{k=1}^{\infty}(\lambda_k)\big)=0$. Thus

$$int_{T_i}(1-\lambda_k)=0(k=1\ \ to\ \ \infty)\text{ in }(X,T_1,T_2). \qquad \text{(B)}$$

But $\vee_{k=1}^{\infty}int_{T_i}(1-\lambda_k)\leq int_{T_i}\big(\vee_{k=1}^{\infty}(1-\lambda_k)\big)$ implies that $\vee_{k=1}^{\infty}int_{T_i}(1-\lambda_k)\leq 0$. Then $\vee_{k=1}^{\infty}int_{T_i}(1-\lambda_k)=0$. This implies that $int_{T_i}(1-\lambda_k)=0$ (i=1,2 and k=1 to ∞). Since $(\lambda_k)'s$ are pairwise fuzzy dense sets in the pairwise fuzzy submaximal space (X,T_1,T_2), $(\lambda_k)'s$ are pairwise fuzzy open sets in (X,T_1,T_2). Thus (1-λ_k)'s are pairwise fuzzy closed sets in (X,T_1,T_2). So $cl_{T_i}(1-\lambda_k)=1-\lambda_k$. Now

$$int_{T_1}cl_{T_2}(1-\lambda_k)=int_{T_1}(1-\lambda_k)=0$$

and

$$int_{T_2}cl_{T_1}(1-\lambda_k)=int_{T_2}(1-\lambda_k)=0.$$

Hence $(1-\lambda_k)$'s are pairwise fuzzy nowhere dense sets in (X,T_1,T_2). From (B), $int_{T_i}\big(\vee_{k=1}^{\infty}(1-\lambda_k)\big)=0$, where $(1-\lambda_k)$'s are pairwise fuzzy nowhere dense sets in (X,T_1,T_2). Therefore (X,T_1,T_2) is a pairwise fuzzy Baire space. □

4. Conclusions

In this paper, some characterizations of pairwise fuzzy σ-Baire bitopological spaces are studied. The conditions for pairwise fuzzy σ-Baire spaces to be pairwise fuzzy Baire spaces are obtained. Also, by making use of the pairwise fuzzy denseness of the countable intersection of pairwise fuzzy G_δ-sets in a pairwise fuzzy hyperconnected and pairwise fuzzy P-space, the pairwise fuzzy σ-Baireness of a fuzzy bitopological space is obtained. It is also established in this paper that each pairwise fuzzy almost irresolvable space is a pairwise fuzzy σ-second category space.

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