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Soft strongly generalized closed sets with respect to soft ideals

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Abstract. The concept of soft ideal was first introduced by Kandil et al. [16]. The notion of generalized closed soft sets in soft topological spaces was introduced by Kannan [23] in 2012, which is generalized in [22, 24, 31]. In this paper, we generalize the notions of soft strongly g-closed sets and soft strongly g-open sets [24] by using the soft ideals notion [16] in soft topological spaces and study their basic properties. Here we used the concept of soft ideals, soft closure, soft interior and soft open sets to define soft strongly $\tilde{I}g$-closed sets. The relationship between soft strongly $\tilde{I}g$-closed sets and other existing soft sets have been investigated. Furthermore, the union and intersection of two soft strongly $\tilde{I}g$-closed (resp. open) sets have been obtained.

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1. Introduction

In real life situation, the problems in economics, engineering, social sciences, medical science etc. do not always involve crisp data. So, we cannot successfully use the traditional classical methods because of various types of uncertainties presented in these problems. To exceed these uncertainties, some kinds of theories were given like theory of fuzzy set, intuitionistic fuzzy set, rough set, bipolar fuzzy set, i.e. which we can use as mathematical tools for dealings with uncertainties. But, all these theories have their inherent difficulties. The reason for these difficulties Molodtsov [30] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties which is free from the above difficulties. In [30, 29], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on. After presentation of the operations of soft sets [28], the properties and applications of soft set theory have been studied increasingly [5, 25, 29]. In [13], Kandil et al. introduced some soft operations such as semi open soft, pre open soft, α-open soft and β-open soft and investigated their properties in detail. Kandil et al. [20] introduced the notion of soft semi separation axioms, which is extended by Abd-El-latif et al. in [3] . Maji et al. [27] initiated the study involving both fuzzy sets and soft sets. In [4], Karal and Ahmed defined the notion of a mapping on classes of fuzzy soft sets, which is a fundamental important in fuzzy soft set theory, to improve this work and they studied properties of fuzzy soft images and fuzzy soft inverse images of fuzzy soft sets. Some fuzzy soft topological properties were introduced in [1, 2, 11, 12, 21].

In 1970, Levine [26] introduced the notion of g-closed sets in topological spaces as a generalization of closed sets. Indeed ideals are very important tools in general topology. It was the works of Newcomb [32], Rancin [33], Samuels [34] and Hamlet Jankovic [8, 9] which motivated the research in applying topological ideals to generalize the most basic properties in general Topology. S. Jafari and N. Rajesh introduced the concept of g-closed sets with respect to an ideal which is a extension of the concept of g-closed sets. The notion of soft ideal was initiated for the first time by Kandil et al. [16]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal $(X, \tau, E, \tilde{I})$. Applications to various fields were further investigated in [10, 14, 15, 17, 18, 19, 22]. Recently, K. Kannan [23] introduced the concept of g-closed soft sets in a soft topological spaces, which is generalized in [22, 24, 31].

In this paper, we introduce and study the concept of soft strongly g-closed sets with respect to an ideal, which is the extension of the concept of soft strongly g-closed set. Also, we study the relationship between soft strongly $\tilde{I}g$-closed sets and other existing soft sets have been investigated.

2. Preliminaries

In this section, we present the basic definitions and results of soft set theory.

Definition 2.1 ([30])**.** Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, A) denoted by F_A is called a soft set over X , where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parametrized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set F_A and if $e \notin A$, then $F(e) = \phi$ i.e
$F_A = \{F(e) : e \in A \subseteq E,\ F : A \to P(X)\}$. The family of all these soft sets denoted by $SS(X)_A$.

Definition 2.2 ([28])**.** Let $F_A,\ G_B \in SS(X)_E$. Then, F_A is soft subset of G_B, denoted by $F_A \tilde{\subseteq} G_B$, if

(1) $A \subseteq B$ and

(2) $F(e) \subseteq G(e), \forall e \in A$.
In this case, F_A is said to be a soft subset of G_B, and G_B is said to be a soft superset of F_A, $G_B \tilde{\supseteq} F_A$.

Definition 2.3 ([28])**.** Two soft subset F_A and G_B over a common universe set X are said to be soft equal if F_A is soft subset of G_B and G_B is soft subset of F_A.

Definition 2.4 ([5])**.** The complement of a soft set F_A, denoted by F_A^c, is defined by $F_A^c = (F^c, A)$, $F^c : A \to P(X)$ is mapping given by $F^c(e) = X - F(e),\ \forall\, e \in A$ and F^c is called the soft complement function of F.
Clearly $(F^c)^c$ is the same as F and $(F_A^c)^c = F_A$.

Definition 2.5 ([35])**.** The difference of two soft sets F_E and G_E over the common universe X, denoted by $F_E - G_E$ is the soft set H_E where for all $e \in E$, $H(e) = F(e) - G(e)$.

Definition 2.6 ([35])**.** Let F_E be a soft set over X and $x \in X$. We say that $x \in F_E$ read as x belongs to the soft set F_E whenever $x \in F(e)$ for all $e \in E$.

Definition 2.7 ([28])**.** A soft set F_A over X is said to be a null soft set denoted by $\tilde{\phi}$ or ϕ_A if for all $e \in A,\ F(e) = \phi$ (null set).

Definition 2.8 ([28])**.** A soft set F_A over X is said to be an absolute soft set denoted by $\tilde{A}$ or X_A if for all $e \in A,\ F(e) = X$. Clearly, we have $X_A^c = \phi_A$ and $\phi_A^c = X_A$.

Definition 2.9 ([28])**.** The union of two soft sets F_A and G_B over the common universe X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e),\ e \in A - B, \\ G(e),\ e \in B - A, \\ F(e) \cup G(e),\ e \in A \cap B \end{cases}.$$

Definition 2.10 ([28])**.** The intersection of two soft sets F_A and G_B over the common universe X is the soft set (H, C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$. Note that, in order to efficiently discuss, we consider only soft sets F_E over a universe X in which all the parameter set E are same. We denote the family of these soft sets by $SS(X)_E$.

Definition 2.11 ([35])**.** Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if
(1) $\tilde{X}, \tilde{\phi} \in \tau$, where $\tilde{\phi}(e) = \phi$ and $\tilde{X}(e) = X,\ \forall e \in E$,
(2) the union of any number of soft sets in τ belongs to τ,
(3) the intersection of any two soft sets in τ belongs to τ.
The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.12 ([35])**.** Let (X, τ, E) be a soft topological space. A soft set F_A over X is said to be closed soft set in X, if its relative complement F_A^c is open soft set.

Definition 2.13 ([35])**.** Let(X, τ, E) be a soft topological space. The members of τ are said to be open soft sets in X. We denote the set of all open soft sets over X by $OS(X, \tau, E)$, or when there can be no confusion by $OS(X)$ and the set of all closed soft sets by $CS(X, \tau, E)$, or $CS(X)$.

Definition 2.14 ([6]). Let (X, τ, E) be a soft topological space and $x, y \in X$ such that $x \neq y$. Then, (X, τ, E) is called a soft Hausdorff space or soft T_2 space if there exist open soft sets F_A and G_B such that $x \in F_A$, $y \in G_B$ and $F_A \tilde{\cap} G_B = \tilde{\phi}$.

Definition 2.15 ([35]). Let (X, τ, E) be a soft topological space, $F_E \in SS(X)_E$ and Y be a non-null subset of X. Then, the sub soft set of F_E over Y denoted by (F_Y, E), is defined as follows:

$$F_Y(e) = Y \cap F(e) \ \forall e \in E.$$

In other words $(F_Y, E) = \tilde{Y} \tilde{\cap} F_E$.

Definition 2.16 ([35]). Let (X, τ, E) be a soft topological space and Y be a non-null subset of X. Then,

$$\tau_Y = \{(F_Y, E) : F_E \in \tau\}$$

is said to be the soft relative topology on Y and (Y, τ_Y, E) is called a soft subspace of (X, τ, E).

Definition 2.17 ([13]). Let (X, τ, E) be a soft topological space and $F_E \in SS(X)_E$. Then F_E is said to be semi open soft set if $F_E \tilde{\subseteq} cl(int F_E)$. The set of all semi open soft sets is denoted by $SOS(X)$ and the set of all semi closed soft sets is denoted by $SCS(X)$.

Definition 2.18 ([36]). Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then,

(1) The function f_{pu} is called continuous soft (cts-soft) if $f_{pu}^{-1}(G, B) \in \tau_1 \ \forall \ (G, B) \in \tau_2$.

(2) The function f_{pu} is called open soft if $f_{pu}(G, A) \in \tau_2 \forall \ (G, A) \in \tau_1$.

(3) The function f_{pu} is called closed soft if $f_{pu}(G, A) \in \tau_2^c \forall \ (G, A) \in \tau_1^c$.

Theorem 2.1 ([4]). *Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets. For the soft function $f_{pu} : SS(X)_A \to SS(Y)_B$, the following statements hold :*

(a) $f_{pu}^{-1}((G, B)^c) = (f_{pu}^{-1}(G, B))^c \forall \ (G, B) \in SS(Y)_B$.

(b) $f_{pu}(f_{pu}^{-1}((G, B))) \tilde{\subseteq} (G, B) \forall \ (G, B) \in SS(Y)_B$. *If f_{pu} is surjective, then the equality holds.*

(c) $(F, A) \tilde{\subseteq} f_{pu}^{-1}(f_{pu}((F, A))) \forall \ (F, A) \in SS(X)_A$. *If f_{pu} is injective, then the equality holds.*

(d) $f_{pu}(\tilde{X}) \tilde{\subseteq} \tilde{Y}$. *If f_{pu} is surjective, then the equality holds.*

(e) $f_{pu}^{-1}(\tilde{Y}) = \tilde{X}$ *and* $f_{pu}(\tilde{\phi}_A) = \tilde{\phi}_B$.

(f) *If* $(F, A) \tilde{\subseteq} (G, A)$, *then* $f_{pu}(F, A) \tilde{\subseteq} f_{pu}(G, A)$.

(g) *If* $(F, B) \tilde{\subseteq} (G, B)$, *then* $f_{pu}^{-1}(F, B) \tilde{\subseteq} f_{pu}^{-1}(G, B) \ \forall \ (F, B), (G, B) \in SS(Y)_B$.

(h) $\forall \ (F, B), (G, B) \in SS(Y)_B$,

$$f_{pu}^{-1}[(F, B) \tilde{\cup} (G, B)] = f_{pu}^{-1}(F, B) \tilde{\cup} f_{pu}^{-1}(G, B)$$

and

$$f_{pu}^{-1}[(F, B) \tilde{\cap} (G, B)] = f_{pu}^{-1}(F, B) \tilde{\cap} f_{pu}^{-1}(G, B).$$

(i) $\forall \ (F, A), (G, A) \in SS(X)_A$,

$$f_{pu}[(F, A) \tilde{\cup} (G, A)] = f_{pu}(F, A) \tilde{\cup} f_{pu}(G, A)$$

and

$$f_{pu}[(F,A)\tilde{\cap}(G,A)]\tilde{\subseteq} f_{pu}(F,A)\tilde{\cap} f_{pu}(G,A).$$

If f_{pu} is injective, then the equality holds.

Definition 2.19 ([23])**.** A soft set $F_E \in SS(X,E)$ is called soft generalized closed in a soft topological space (X,τ,E) if $cl(F_E)\tilde{\subseteq} G_E$ whenever $F_E\tilde{\subseteq} G_E$ and $G_E \in \tau$.

Definition 2.20 ([16])**.** Let $\tilde{I}$ be a non-null collection of soft sets over a universe X with the same set of parameters E. Then, $\tilde{I} \subseteq SS(X)_E$ is called a soft ideal on X with the same set E if

(1) $F_E \in \tilde{I}$ and $G_E \in \tilde{I} \Rightarrow F_E\tilde{\cup} G_E \in \tilde{I}$,

(2) $F_E \in \tilde{I}$ and $G_E\tilde{\subseteq} F_E \Rightarrow G_E \in \tilde{I}$,

i.e., $\tilde{I}$ is closed under finite soft unions and soft subsets.

Definition 2.21 ([31])**.** A soft set $F_E \in SS(X,E)$ is called soft generalized closed set with respect to soft ideal $\tilde{I}$ (soft $\tilde{I}g$-closed set) in a soft topological space (X,τ,E) if $cl(F_E)\setminus G_E \in \tilde{I}$ whenever $F_E\tilde{\subseteq} G_E$ and $G_E \in \tau$.

3. Soft strongly generalized closed sets with respect to soft ideal

Kannan et al. [24] introduced the notion of soft strongly generalized closed sets in soft topological spaces. In this section, we generalize this notion by using the soft ideal notion.

Definition 3.1. A soft set $F_E \in SS(X,E)$ is called soft strongly generalized closed set with respect to soft ideal $\tilde{I}$ (soft strongly $\tilde{I}g$-closed set) in a soft topological space (X,τ,E) if $cl(int(F_E))\setminus G_E \in \tilde{I}$ whenever $F_E\tilde{\subseteq} G_E$ and $G_E \in \tau$.

Example 3.1. Let $X = \{a,b\}$ be the set of two phones under consideration and, $E = \{e_1(costly), e_2(Luxurious)\}$. Let A_E, B_E, C_E be three soft sets representing the attractiveness of the phone which Mr. X, Mr. Y and M. Z are going to buy, where

$A(e_1) = \phi \quad A(e_2) = \{a\}$,
$B(e_1) = \{b\} \quad B(e_2) = \phi$,
$C(e_1) = \{b\} \quad C(e_2) = \{a\}$.

Then A_E, B_E and C_E are soft sets over X and

$$\tau = \{\tilde{X}, \tilde{\phi}, A_E, B_E, C_E\}$$

is the soft topology over X. Let $\tilde{I} = \{\tilde{\phi}, F_E, G_E, H_E\}$ be a soft ideal on X, where

$F(e_1) = \{b\} \quad F(e_2) = \phi$,
$G(e_1) = \{b\} \quad G(e_2) = \{a\}$,
$H(e_1) = \phi \quad H(e_2) = \{a\}$.

The soft sets M_E, N_E, L_E are soft strongly $\tilde{I}g$-closed, where

$M(e_1) = \phi \quad M(e_2) = X$,
$N(e_1) = X \quad N(e_2) = \{a\}$,
$L(e_1) = \{b\} \quad L(e_2) = \{b\}$.

On the other hand, the soft set S_E is not soft strongly $\tilde{I}g$-closed, where
$S(e_1) = \phi \quad S(e_2) = \{a\}$.

Theorem 3.1. *Every soft g-closed set is soft strongly $\tilde{I}g$-closed.*

Proof. Let $F_E \tilde{\subseteq} G_E$ and $G_E \in \tau$. Since F_E is soft g-closed, then $cl(F_E)\tilde{\subseteq} G_E$ and $cl(int(F_E))\tilde{\subseteq} cl(F_E)$. Hence, $cl(int(F_E))\setminus G_E = \tilde{\phi} \in \tilde{I}$. Therefore, F_E is soft strongly $\tilde{I}g$-closed. □

Remark 3.1. The converse of the above theorem is not true in general. The following example supports our claim.

Example 3.2. Suppose that there are two dresses in the universe X given by $X = \{a, b\}$. Let $E = \{e_1(cotton), e_2(woollen)\}$ be the set of parameters showing the material of the dresses.
Let A_E, B_E, C_E, D_E be four soft sets over the common universe X, which describe the composition of the dresses, where
$A(e_1) = \{a\} \quad A(e_2) = X,$
$B(e_1) = \{a\} \quad B(e_2) = \phi,$
$C(e_1) = \{a\} \quad C(e_2) = \{b\},$
$D(e_1) = \{a\} \quad D(e_2) = \{a\}.$
Then, $\tau = \{\phi, \tilde{X}, A_E, B_E, C_E, D_E\}$ is the soft topology over X. Let

$$\tilde{I} = \{\phi, F_E, G_E, H_E\}$$

be a soft ideal on X, where
$F(e_1) = \{a\} \quad F(e_2) = \phi,$
$G(e_1) = \{a\} \quad G(e_2) = \{a\},$
$H(e_1) = \phi \quad H(e_2) = \{a\}.$
The soft set M_E is soft strongly $\tilde{I}g$-closed but not soft g-closed, where
$M(e_1) = \phi \quad M(e_2) = \{b\}.$

Theorem 3.2. *Every closed soft set is soft strongly $\tilde{I}g$-closed.*

Proof. Let $F_E \tilde{\subseteq} G_E$ and $G_E \in \tau$. Since F_E is closed soft, then $cl(int(F_E))\tilde{\subseteq} cl(F_E) = F_E \tilde{\subseteq} G_E$. Hence, $cl(int(F_E)) \setminus G_E = \tilde{\phi} \in \tilde{I}$. Therefore, F_E is soft strongly $\tilde{I}g$-closed. □

Remark 3.2. The converse of the above theorem is not true in general as shall shown in the following example.

Example 3.3. In Example 3.1, the soft set N_E is soft strongly $\tilde{I}g$-closed but not closed soft set, where
$N(e_1) = X$ and $N(e_2) = \{a\}$.

Theorem 3.3. *Every soft $\tilde{I}g$-closed set is soft strongly $\tilde{I}g$-closed.*

Proof. Let $F_E \tilde{\subseteq} H_E$ and $H_E \in \tau$. Then, $cl(int(F_E)) \setminus I_E \tilde{\subseteq} cl(F_E) \setminus I_E \in \tilde{I}$ for some $I_E \in \tilde{I}$. Therefore, F_E is strongly $\tilde{I}g$-closed. □

Remark 3.3. The converse of the above theorem is not true in general, as shown in the following example.

Example 3.4. In Example 3.2, the soft set M_E is soft strongly $\tilde{I}g$-closed but not closed soft, where $M(e_1) = \phi$ and $M(e_2) = \{b\}$.

Theorem 3.4. *If a soft subset F_E of a soft topological space (X, τ, E) is open soft, then it is soft strongly $\tilde{I}g$-closed if and only if it is soft $\tilde{I}g$-closed.*

Proof. It is clear. □

Theorem 3.5. *A soft set A_E is soft strongly $\tilde{I}g$-closed in a soft topological space (X, τ, E) if and only if $F_E \tilde{\subseteq} cl(int(A_E)) \setminus A_E$ and F_E is closed soft implies $F_E \in \tilde{I}$.*

Proof. ($\Rightarrow$) Let $F_E \tilde{\subseteq} cl(int(A_E)) \setminus A_E$ and F_E is closed soft. Then, $A_E \tilde{\subseteq} F_E^c$. By hypothesis, $cl(int(A_E)) \setminus F_E^c \in \tilde{I}$. But, $F_E \tilde{\subseteq} cl(int(A_E)) \tilde{\cap} F_E = cl(int(A_E)) \setminus F_E^c$. Thus, $F_E \in \tilde{I}$ from Definition 2.20.

($\Leftarrow$) Assume that $A_E \tilde{\subseteq} G_E$ and $G_E \in \tau$. Then, $cl(int(A_E)) \setminus G_E = cl(int(A_E)) \tilde{\cap} G_E^c$ is a closed soft set and $cl(int(A_E)) \setminus G_E \tilde{\subseteq} cl(int(A_E)) \setminus G_E$. By assumption, $cl(int(A_E)) \setminus G_E \in \tilde{I}$. So, A_E is soft strongly $\tilde{I}g$-closed. □

Theorem 3.6. *If F_E is soft strongly $\tilde{I}g$-closed in a soft topological space (X, τ, E) and $F_E \tilde{\subseteq} G_E \tilde{\subseteq} cl(int(F_E))$, then G_E is soft strongly $\tilde{I}g$-closed.*

Proof. Let $G_E \tilde{\subseteq} H_E$ and $H_E \in \tau$. Then, $F_E \tilde{\subseteq} H_E$. Since F_E is soft strongly $\tilde{I}g$-closed, then $cl(int(F_E)) \setminus H_E \in \tilde{I}$. Now, $G_E \tilde{\subseteq} cl(int(F_E))$ implies that $cl(G_E) \tilde{\subseteq} cl(int(F_E))$. Thus, $cl(int(G_E)) \setminus H_E \tilde{\subseteq} cl(G_E) \setminus H_E \tilde{\subseteq} cl(int(F_E)) \setminus H_E$. So, $cl(int(G_E)) \setminus H_E \in \tilde{I}$ from Definition 2.20. Hence, G_E is soft strongly $\tilde{I}g$-closed. □

Remark 3.4. The soft intersection of two soft strongly $\tilde{I}g$-closed sets need not be a soft strongly $\tilde{I}g$-closed as shown by the following example.

Example 3.5. In Example 3.1, the soft sets M_E, N_E are soft strongly $\tilde{I}g$-closed. But, $P_E = M_E \tilde{\cap} N_E$ is not soft strongly $\tilde{I}g$-closed, where $P(e_1) = \phi$ and $P(e_2) = \{a\}$.

Theorem 3.7. *If A_E is soft strongly $\tilde{I}g$-closed and F_E is closed soft in a soft topological space (X, τ, E). Then, $A_E \tilde{\cap} F_E$ is soft strongly $\tilde{I}g$-closed.*

Proof. Assume that $A_E \tilde{\cap} F_E \tilde{\subseteq} G_E$ and $G_E \in \tau$. Then, $A_E \tilde{\subseteq} G_E \tilde{\cup} F_E^c$. Since A_E is soft strongly $\tilde{I}g$-closed, so $cl(int(A_E)) \setminus (G_E \tilde{\cup} F_E^c) \in \tilde{I}$. Now,

$$\begin{aligned} cl(int((A_E \tilde{\cap} F_E))) &\tilde{\subseteq} cl(int(A_E)) \tilde{\cap} cl(int(F_E)) \\ &\tilde{\subseteq} cl(int(A_E)) \tilde{\cap} cl(F_E) \\ &= cl(int(A_E)) \tilde{\cap} F_E \\ &= [cl(int(A_E)) \tilde{\cap} F_E] \setminus F_E^c. \end{aligned}$$

Thus,

$$\begin{aligned} cl(int((A_E \tilde{\cap} F_E))) \setminus G_E &\tilde{\subseteq} [cl(int(A_E)) \tilde{\cap} F_E] \setminus [F_E^c \tilde{\cup} G_E] \\ &\tilde{\subseteq} cl(int(A_E)) \setminus [G_E \tilde{\cup} F_E^c] \in \tilde{I}. \end{aligned}$$

So, $A_E \tilde{\cap} F_E$ is soft strongly $\tilde{I}g$-closed. □

Theorem 3.8. *Let (Y, τ_Y, E) be a soft subspace of a soft topological space (X, τ, E), $F_E \tilde{\subseteq} Y_E$ and F_E is strongly $\tilde{I}g$-closed in (X, τ, E). Then, F_E is strongly $\tilde{I}_Y g$-closed in (Y, τ_Y, E).*

Proof. Assume that $F_E \tilde{\subseteq} B_E \tilde{\cap} Y_E$ and $B_E \in \tau$. Then, $B_E \tilde{\cap} Y_E \in \tau_Y$ and $F_E \tilde{\subseteq} B_E$. Since F_E is soft strongly $\tilde{I}g$-closed in (X, τ, E), then $cl(int(F_E)) \setminus B_E \in \tilde{I}$. Now,

$$[cl(int(F_E)) \tilde{\cap} Y_E] \setminus [B_E \tilde{\cap} Y_E] = [cl(int(F_E)) \setminus B_E] \tilde{\cap} Y_E \in \tilde{I}_Y.$$

Thus, F_E is soft strongly $\tilde{I_Y}g$-closed in (Y, τ_Y, E). □

Theorem 3.9. *Let (X_1, τ_1, E), (X_2, τ_2, K) be soft topological spaces. Let $f_{pu} : SS(X_1)_E \to SS(X_2)_K$ be a closed and continuous soft function. If $A_E \in SS(X, E)$ is both a soft strongly $\tilde{I}g$-closed and semi open soft in (X_1, τ_1, E), then $f_{pu}(A_E)$ is a soft strongly $f_{pu}(\tilde{I})g$-closed in (X_2, τ_2, K).*

Proof. Let $A_E \in SS(X)_E$ be a soft strongly $\tilde{I}g$-closed in (X_1, τ_1, E) and $f_{pu}(A_E)\tilde{\subseteq}G_K$ for some $G_K \in \tau_2$. Then, $A_E\tilde{\subseteq}f_{pu}^{-1}(G_K)$. It follow that, $cl_{\tau_1}(int_{\tau_1}(A_E))\setminus f_{pu}^{-1}(G_K) \in \tilde{I}$ from Definition 3.1. Thus, $f_{pu}(cl_{\tau_1}(int_{\tau_1}(A_E))) \setminus G_K \in f_{pu}(\tilde{I})$ from Theorem 2.1. Since f_{pu} is a closed soft function, then $f_{pu}(cl_{\tau_1}(int_{\tau_1}(A_E)))$ is a closed soft in τ_2 from Definition 2.18. Since A_E is semi open soft in (X_1, τ_1, E), then

$$cl_{\tau_2}(int_{\tau_2}(f_{pu}(A_E)))\tilde{\subseteq}cl_{\tau_2}[f_{pu}(cl_{\tau_1}(int_{\tau_1}(A_E)))] = f_{pu}(cl_{\tau_1}(int_{\tau_1}(A_E))).$$

This implies that, $[cl_{\tau_2}(int_{\tau_2}(f_{pu}(A_E)))] \setminus G_K\tilde{\subseteq}[f_{pu}(cl_{\tau_1}(int_{\tau_1}(A_E)))] \setminus G_K \in f_{pu}(\tilde{I})$. So, $f_{pu}(A_E)$ is a soft strongly $f_{pu}(I)g$-closed in (X_2, τ_2, K). □

4. Soft generalized open sets with respect to soft ideal

Definition 4.1. A soft set $F_E \in SS(X, E)$ is called soft strongly generalized open set with respect to soft ideal $\tilde{I}$ (soft strongly $\tilde{I}g$-open)in a soft topological space (X, τ, E) if and only if its relative complement F_E^c is soft strongly $\tilde{I}g$-closed in (X, τ, E).

Example 4.1. In Example 3.1, The soft sets M_E^c, N_E^c, L_E^c are soft strongly $\tilde{I}g$-open where M_E^c, N_E^c, L_E^c are defined by

$M^c(e_1) = X \quad M^c(e_2) = \phi$,
$N^c(e_1) = \phi \quad N^c(e_2) = \{b\}$,
$L^c(e_1) = \{a\} \quad L^c(e_2) = \{a\}$.

Theorem 4.1. *A soft set A_E is soft strongly $\tilde{I}g$-open in a soft topological space (X, τ, E) if and only if $F_E \setminus B_E\tilde{\subseteq}int(cl(A_E))$ for some $B_E \in \tilde{I}$, whenever $F_E\tilde{\subseteq}A_E$ and F_E is closed soft in (X, τ, E).*

Proof. (⇒) Suppose that $F_E\tilde{\subseteq}A_E$ and F_E is closed soft. We have $A_E^c\tilde{\subseteq}F_E^c$, A_E^c is soft strongly $\tilde{I}g$-closed and $F_E^c \in \tau$. By assumption, $cl(int((A_E^c))) \setminus F_E^c \in \tilde{I}$. Then, $cl(int((A_E^c))) \setminus F_E^c = B_E$ for some $B_E \in \tilde{I}$. Thus,

$$cl(int((A_E^c))) \setminus F_E^c = cl(int((A_E^c)))\tilde{\cap}F_E = B_E \in \tilde{I}.$$

So, $[cl(int((A_E^c)))\tilde{\cap}F_E]\tilde{\cup}F_E^c = B_E\tilde{\cup}F_E^c$. This implies that,

$$cl(int((A_E^c)))\tilde{\subseteq}cl(int((A_E^c)))\tilde{\cup}F_E^c = B_E\tilde{\cup}F_E^c.$$

Hence, $cl(int((A_E^c)))\tilde{\subseteq}F_E^c\tilde{\cup}B_E$ for some $B_E \in \tilde{I}$. Furthermore,

$$(F_E^c\tilde{\cup}B_E)^c\tilde{\subseteq}[cl(int((A_E^c)))]^c = int(cl(A_E)).$$

Therefore, $F_E \setminus B_E = F_E\tilde{\cap}B_E^c\tilde{\subseteq}int(cl(A_E))$.

(⇐) We want to prove that A_E is soft strongly $\tilde{I}g$-open. It is sufficient to prove that, A_E^c is soft strongly $\tilde{I}g$-closed.

Let $A_E^c\tilde{\subseteq}G_E$ such that $G_E \in \tau$. Then, $G_E^c\tilde{\subseteq}A_E$. By assumption,

$$G^c_E \setminus \tilde{I}_E \tilde{\subseteq} int(cl(A_E)) = [cl(cl(A_E))^c]^c \text{ for some } \tilde{I}_E \in \tilde{I}.$$

Thus,

$$cl(int(A^c_E)) = cl(cl(A_E))^c \tilde{\subseteq} [G^c_E \setminus \tilde{I}_E]^c = G_E \tilde{\cup} \tilde{I}_E.$$

So,

$$cl(int((A^c_E))) \setminus G_E \tilde{\subseteq} [G_E \tilde{\cup} \tilde{I}_E] \setminus G_E = [G_E \tilde{\cup} \tilde{I}_E] \tilde{\cap} G^c_E = \tilde{I}_E \tilde{\cap} G^c_E \tilde{\subseteq} \tilde{I}_E \in \tilde{I}.$$

This shows that, $cl(int(A^c_E)) \setminus G_E \in \tilde{I}$. Therefore, A^c_E is soft strongly $\tilde{I}g$-closed and hence A_E is soft strongly $\tilde{I}g$-open. This completes the proof. □

Theorem 4.2. *Every open soft set is soft strongly $\tilde{I}g$-open.*

Proof. Immediate from Theorem 3.2. □

Remark 4.1. The converse of the above theorem is not true in general as shall shown in the following example.

Example 4.2. In Example 3.1, the soft set S_E is soft strongly $\tilde{I}g$-open but not open soft set, where $S(e_1) = \phi$ and $S(e_2) = \{b\}$.

Theorem 4.3. *Every soft $\tilde{I}g$-open set is soft strongly $\tilde{I}g$-open.*

Proof. Immediate from Theorem 3.3. □

Remark 4.2. The converse of the above theorem is not true in general, as shown in the following example.

Example 4.3. In Example 3.2, the soft set Z_E is soft strongly $\tilde{I}g$-open but not soft $\tilde{I}g$-open, where $Z(e_1) = X$ and $Z(e_2) = \{a\}$.

Remark 4.3. The soft intersection (resp. union) of two soft strongly $\tilde{I}g$-open sets need not be a soft strongly $\tilde{I}g$-open as shown by the following example.

Example 4.4. In Example 3.1, the soft sets M^c_E, N^c_E, L^c_E are soft strongly $\tilde{I}g$-open. But, $H_E = M^c_E \tilde{\cup} N^c_E$ is not soft strongly $\tilde{I}g$-open, where $H(e_1) = X$ and $H(e_2) = \{b\}$. Also, $K_E = U^c_E \tilde{\cap} V^c_E$ is not soft strongly $\tilde{I}g$-open, where $K(e_1) = \{b\}$ and $K(e_2) = \{a\}$.

Theorem 4.4. *If F_E is soft strongly $\tilde{I}g$-open in a soft topological space (X, τ, E) and $int(cl(F_E)) \tilde{\subseteq} G_E \tilde{\subseteq} F_E$, then G_E is soft strongly $\tilde{I}g$-open.*

Proof. Let $H_E \tilde{\subseteq} G_E$ and $H_E \in \tau^c$. Then, $H_E \tilde{\subseteq} F_E$. Since F_E is soft strongly $\tilde{I}g$-open, then $G_E \setminus int(cl(H_E)) \tilde{\subseteq} F_E \setminus int(cl(H_E)) \in \tilde{I}$. It follows that, $G_E \setminus int(cl(H_E)) \in \tilde{I}$. Thus G_E is soft strongly $\tilde{I}g$-open. □

Theorem 4.5. *If a soft subset F_E of a soft topological space (X, τ, E) is closed soft, then it is soft strongly $\tilde{I}g$-open if and only if it is soft $\tilde{I}g$-open.*

Proof. It is clear. □

Theorem 4.6. *A soft set A_E is soft strongly $\tilde{I}g$-closed in a soft topological space (X, τ, E) if and only if $cl(int(A_E)) \setminus A_E$ is soft strongly $\tilde{I}g$-open.*

Proof. ($\Rightarrow$) Let $F_E\tilde{\subseteq} cl(int(A_E)) \setminus A_E$ and F_E be a closed soft set. Then, $F_E \in \tilde{I}$ from Theorem 3.5. Thus, there exists $I_E \in \tilde{I}$ such that $F_E \setminus I_E = \tilde{\phi}$. So, $F_E \setminus I_E = \tilde{\phi}\tilde{\subseteq} int(cl[cl(int(A_E)) \setminus A_E])$. Hence, $cl(int(A_E)) \setminus A_E$ is a soft strongly $\tilde{I}g$-open from Theorem 4.1.

($\Leftarrow$) Let $A_E\tilde{\subseteq} G_E$ such that $G_E \in \tau$. Then, $cl(int(A_E))\tilde{\cap} G^c_E\tilde{\subseteq} cl(int(A_E))\tilde{\cap} A^c_E = cl(int(A_E)) \setminus A_E$. By hypothesis, $[cl(int(A_E))\tilde{\cap} A^c_E] \setminus I_E\tilde{\subseteq} int(cl[cl(int(A_E)) \setminus A_E]) = \tilde{\phi}$, for some $I_E \in \tilde{I}$ from Theorem 4.1. This implies that, $cl(int(A_E))\tilde{\cap} G^c_E\tilde{\subseteq} I_E \in \tilde{I}$. Thus, $cl(int(A_E)) \setminus G_E \in \tilde{I}$. So, A_E is a soft strongly $\tilde{I}g$-closed. □

5. Conclusion

Therefore, the notions of soft strongly $\tilde{I}g$-closed sets and soft strongly $\tilde{I}g$-open sets have been introduced and investigated. In future, the generalization of these concepts to supra soft topological spaces [7] will be introduced and the future research will be undertaken in this direction.

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References

[1] A. M. Abd El-latif, Characterizations of fuzzy soft pre separation axioms, Journal of New Theory 7 (2015) 47–63.
[2] A. M. Abd El-latif and Rodyna A. Hosny, Fuzzy soft pre-connected properties in fuzzy soft topological spaces, South Asian J. Math. 5 (5) (2015) 202–213.
[3] A. M. Abd El-latif and Rodyna A. Hosny, On soft separation axioms via β-open soft sets, South Asian J. Math. 5 (6) (2015) 252–265.
[4] B. Ahmad and A. Kharal, Mappings on soft classes, New Math. Nat. Comput. 7 (3) (2011) 471–481.
[5] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, Computer and Math. with appl. 57 (2009) 1547–1553.
[6] Banu Pazar Varol and Halis Aygun, On soft Hausdorff spaces, Ann. Fuzzy Math. Inform. 5 (1) (2013) 15–24.
[7] S. A. El-Sheikh and A. M. Abd El-latif, Decompositions of some types of supra soft sets and soft continuity, International Journal of Mathematics Trends and Technology 9 (1) (2014) 37–56.
[8] T. R. Hamlett and D. Jankovic, Compatible extensions of ideals, Boll. Un. Mat. Ita. 7 (1992) 453–456.
[9] D. Jankovic and T. R. Hamlett, New topologies from old via ideals, Amer. Math. Month. 97 (1990) 295–310.
[10] A. C. Guler and G. Kale, Regularity and normality on soft ideal topological spaces, Ann. Fuzzy Math. Inform. 9 (3) (2015) 373–383.
[11] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh, A. M. Abd El-latif and S. El-Sayed, Fuzzy soft connectedness based on fuzzy β-open soft sets, Journal of Mathematics and Computer Applications Research 2 (1) (2015) 37–46 .
[12] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Fuzzy soft semi connected properties in fuzzy soft topological spaces, Math. Sci. Lett. 4 (2) (2015) 171–179.
[13] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, γ-operation and decompositions of some forms of soft continuity in soft topological spaces, Ann. Fuzzy Math. Inform. 7 (2) (2014) 181–196.

[14] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, γ-operation and decompositions of some forms of soft continuity of soft topological spaces via soft ideal, Ann. Fuzzy Math. Inform. 9 (3) (2015) 385–402.
[15] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Soft connectedness via soft ideals, Journal of New Results in Science 4 (2014) 90–108.
[16] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Soft ideal theory, Soft local function and generated soft topological spaces, Appl. Math. Inf. Sci. 8 (4) (2014) 1595–1603.
[17] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Soft regularity and normality based on semi open soft sets and soft ideals, Appl. Math. Inf. Sci. Lett. 3 (2) (2015) 47–55.
[18] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Soft semi compactness via soft ideals , Appl. Math. Inf. Sci. 8 (5) (2014) 2297–2306.
[19] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Soft semi (quasi) Hausdorff spaces via soft ideals, South Asian J. Math. 4 (6) (2014) 265–284.
[20] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Soft semi separation axioms and irresolute soft functions, Ann. Fuzzy Math. Inform. 8 (2) (2014) 305-318.
[21] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Some fuzzy soft topological properties based on fuzzy semi open soft sets, South Asian J. Math. 4 (4) (2014) 154–169.
[22] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Supra generalized closed soft sets with respect to an soft ideal in supra soft topological spaces, Appl. Math. Inf. Sci. 8 (4) (2014) 1731–1740.
[23] K. Kannan, Soft generalized closed sets in soft topological spaces, Journal of Theoretical and Applied Technology 37 (2012) 17–21.
[24] K. Kannan, D. Rajalakshmi and R. Srikanth, Soft strongly generalized closed sets, Indian Journal of Science and Technology 8 (18) (2015) 1–6.
[25] D. V. Kovkov, V. M. Kolbanov and D. A. Molodtsov, Soft sets theory-based optimization, Journal of Computer and Systems Sciences Finternational 46 (6) (2007) 872–880 .
[26] N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo 19 (2) (1970) 89–96.
[27] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, J. Fuzzy Math. 9 (3) (2001) 589–602.
[28] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555–562.
[29] D. Molodtsov, V. Y. Leonov and D. V. Kovkov, Soft sets technique and its application, Nechetkie Sistemy i Myagkie Vychisleniya 1 (1) (2006) 8–39 .
[30] D. Molodtsov, Soft set theory-firs tresults, Comput. Math. Appl. 37 (1999) 19–31.
[31] H. I. Mustafa and F. M. Sleim, Soft generalized closed sets with respect to an ideal in soft topological spaces, Appl. Math. Inf. Sci. 8 (2) (2014) 665–671.
[32] R. L. Newcomb, Topologies which are compact modulo an ideal. Ph. D, Dissertation, Univ. Cal. at Santa Barbara 1967.
[33] D. V. Rancin, Compactness modul an ideal, Soviet Math. Dokl. 13 (1972) 193–197.
[34] P. Samuels, A topology from a given topology topology and ideals, J. London Math. Soc. 10 (4) (1975) 409–416.
[35] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl. 61 (2011) 1786–1799.
[36] I. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, Remarks on soft topological spaces, Ann. Fuzzy Math. Inform. 3 (2) (2012) 171–185.

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