Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 11, No. 4, (April 2016), pp. 661–668
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
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Min-weighted and max-weighted power automata

Pamy Sebastian

Received 31 March 2015; Revised 20 Jult 2015; Accepted 11 October 2015

Abstract. In this paper we define min-weighted and max-weighted power automata for every fuzzy automaton and prove some properties related to it.We prove that transition monoids of a fuzzy automaton and its fuzzy power automaton are same.

2010 AMS Classification: 18B20

Keywords: Fuzzy automaton, Transition monoid, Inverse fuzzy automaton.

Corresponding Author: Pamy Sebastian (pamyseb@gmail.com)

1. Introduction

Corresponding to every fuzzy automaton M, we define min-weighted and max-weighted power automata $\mathcal{P}(M)^{\wedge}$ and $\mathcal{P}(M)^{\vee}$ such that the state set is the power set $\mathcal{P}(Q)$ of the state set Q of M and study some algebraic properties of it. The membership value of the transition function between two states of $\mathcal{P}(Q)$ is the minimum(maximum) of the membership values of the transition function between the elements of the state set if the image is the set of all elements with non zero membership value of the transition function from the domain and zero otherwise. If M is a deterministic connected inverse fuzzy automaton then the transition monoid is isomorphic to a subinverse monoid of the inverse monoid of all fuzzy matrices with each row and column containing excatly one non zero entry. A fuzzy automaton is commutative if its transition monoid is commutative. In this paper we prove that a fuzzy automaton and its min-weighted and max-weighted power automata have the same transition monoids as in the case of power automaton [3] and so if the fuzzy automaton is inverse and commutative, then the corresponding fuzzy power automaton has a transition monoid which is inverse and commutative.

2. Preliminaries

Definition 2.1 ([4]). A fuzzy recognizer on an alphabet X is a 5-tuple $M = (Q, X, \mu, i, \tau)$ where Q is a finite set of states , X is a finite set of input symbols and μ is a fuzzy subset of $Q \times X \times Q$ representing the transition mapping, i is a fuzzy subset of Q called initial state, τ is a fuzzy subset of Q called final state. (Q, X, μ) is called a fuzzy finite state machine. A fuzzy automaton can also be represented as a five tuple $(Q, X, \{T_u|u \in X\}, i, \tau)$ where $\{T_u|u \in X\}$ is the set of fuzzy transition matrices, $i = [i_1\ i_2\ ....i_n], i_k \in [0, 1], \tau = [j_1\ j_2\ ....j_n]^T, j_k \in [0, 1]$, for $k = 1, 2.....n$

μ can be extended to the set $Q \times X^* \times Q$ by

$$\mu(q, \Lambda, p) = \begin{cases} 1, q = p \\ 0, q \neq p \end{cases}$$

$$\mu(q, u, p) = \begin{cases} \bigvee_{q_i \in Q} \{\mu(q, x_1, q_1) \wedge \mu(q_1, x_2, q_2) \wedge .... \wedge \mu(q_{k-1}, x_k, p) \\ \qquad |x_1 x_2 ....x_k = u\}. \end{cases}$$

Definition 2.2. For a fuzzy automaton $A = (Q, X, \mu, i, \tau)$ define a congruence θ_A on X^* by $u\theta_A v$ if and only if $\mu(q, u, p) = \mu(q, v, p)$, for all $p, q \in Q$. Then the transition monoid $T(A)$ of A is equal to X^*/θ_A.

Definition 2.3 ([1]). Let $M_1 = (Q_1, X_1, \mu_1)$ and $M_2 = (Q_2, X_2, \mu_2)$ be fuzzy finite state machines. A pair (α, β) of mappings $\alpha : Q_1 \longrightarrow Q_2$ and $\beta : X_1 \longrightarrow X_2$ is called a homomorphism, written $(\alpha, \beta) : M_1 \longrightarrow M_2$ if $\mu_1(q, x, p) \leq \mu_2(\alpha(q), \beta(x), \alpha(p))$ $\forall q, p \in Q$ and $\forall x \in X_1$. (α, β) is called a strong homomorphism if $\mu_2(\alpha(q), \beta(x), \alpha(p)) = \bigvee\{\mu_1(q, x, t)|t \in Q_1, \alpha(t) = \alpha(p)\} \forall p, q \in Q_1$ and $\forall x \in X_1$. A homomorphism is said to be an isomorphism if α and β are both one one and onto.

Definition 2.4. A fuzzy automaton $M = (Q, X, \mu)$ is said to be commutative if $\mu(p, ab, q) = \mu(p, ba, q)\ \forall\ a, b \in X^*, p, q \in Q$. If the fuzzy automaton is commutative then the transition monoid will also be commutative.[1]

Definition 2.5 ([2]). M is said to be an inverse fuzzy automaton if $\forall\ x \in \tilde{X}^*$, $\mu(q, xx^{-1}x, p) = \mu(q, x, p)$ and $\mu(q, x^{-1}xx^{-1}, p) = \mu(q, x^{-1}, p)\ \forall\ p, q \in Q$.

Definition 2.6 ([1]). Let $M_1 = (Q_1, X_1, \mu_1), M_2 = (Q_2, X_2, \mu_2)$ be fuzzy finite state machines such that $Q_1 \cap Q_2 = \phi$ and $X_1 \cap X_2 = \phi$. Then the direct sum is defined as $M_1 \oplus M_2 = (Q_1 \cup Q_2, X_1 \cup X_2, \mu_1 \oplus \mu_2)$ where

$$\mu_1 \oplus \mu_2(p, a, q) = \begin{cases} \mu_1(p, a, q) & \text{if } p, q \in Q_1, a \in X_1 \\ \mu_2(p, a, q) & \text{if } p, q \in Q_2, a \in X_2 \\ 1 & \text{if either } (p, a) \in Q_1 \times X_1, q \in Q_2 \\ & \text{or } (p, a) \in Q_2 \times X_2, q \in Q_1 \\ 0 & \text{otherwise} \end{cases}$$

and

the cartesian composition is defined as $M_1.M_2 = (Q_1 \times Q_2, X_1 \cup X_2, \mu_1.\mu_2)$ where

$$(\mu_1.\mu_2)((p_1,p_2),a,(q_1,q_2)) = \begin{cases} \mu_1(p_1,a,q_1) & \text{if } a \in X_1 \text{ and } p_2 = q_2 \\ \mu_2(p_2,a,q_2) & \text{if } a \in X_2 \text{ and } p_1 = q_1 \\ 0 & \text{otherwise.} \end{cases}$$

3. Min-weighted power automata

Definition 3.1. Let $M = (Q, X, \mu)$ be a fuzzy automaton. Let $\mathcal{P}(Q)$ be the power set of Q. Define $\mu^\wedge : \mathcal{P}(Q) \times X \times \mathcal{P}(Q) \longrightarrow [0,1]$ as

$$\mu^\wedge(A,a,B) = \begin{cases} \bigwedge_{q\in A} \bigwedge_{p\in B} \mu(q,a,p) & \text{if } B = \{p : \mu(q,a,p) > 0\} \\ 0 & \text{otherwise} \end{cases}$$

for all$A \neq \phi, B \neq \phi, A, B \in \mathcal{P}(Q)$,

$\mu^\wedge(\phi,a,\phi) = 1$ and $\mu^\wedge(A,a,B) = 0$ if $A = \phi$ or $B = \phi$, for all $a \in X$. Then $\mathcal{P}^\wedge(M) = (\mathcal{P}(Q), X, \mu^\wedge)$ is called the min-weighted power automaton. We can extend $\mu^\wedge$ to $\mathcal{P}(Q) \times X^* \times \mathcal{P}(Q)$ as

$$\mu^\wedge(A,xa,B) = \bigvee_{C\in\mathcal{P}(Q)} \mu^\wedge(A,x,C) \wedge \mu^\wedge(C,a,B) \text{ where } A,B,C \in \mathcal{P}(Q).$$

M can be embedded in $\mathcal{P}^\wedge(M)$ with the isomorphism $p \longrightarrow \{p\}$.

Theorem 3.2. *Every mapping (α, β) of a fuzzy automaton $M_1 = (Q_1, X_1, \mu_1)$ into a fuzzy automaton $M_2 = (Q_2, X_2, \mu_2)$ can be extended to a mapping from $\mathcal{P}^\wedge(M_1)$ into $\mathcal{P}^\wedge(M_2)$ such that (α, β) is an isomorphism if and only if the extended map is an isomorphism.*

Proof. Consider the extension $\hat{\alpha}$ of $\alpha : Q_1 \longrightarrow Q_2$ to $\mathcal{P}(Q_1) \longrightarrow \mathcal{P}(Q_2)$ such that for $A \in \mathcal{P}(Q)$ define $\hat{\alpha}(A) = \{\alpha(q), q \in A)\}$.
Suppose (α, β) is a homomorphism and let $a \in X_1$,. Then

$$\begin{aligned} \mu_1{}^\wedge(A,a,B) &= \begin{cases} \bigwedge_{q\in A} \bigwedge_{p\in B} \mu_1(q,a,p) & \text{if } B = \{p : \mu_1(q,a,p) > 0\} \\ 0 & \text{otherwise} \end{cases} \\ &\leq \begin{cases} \bigwedge_{q\in A} \bigwedge_{p\in B} \mu_2(\alpha(q),\beta(a),\alpha(p)) & \text{if } B = \{p : \mu_1(q,a,p) > 0\} \\ 0 & \text{otherwise} \end{cases} \\ &= \mu_2{}^\wedge(\hat{\alpha}(A), \beta(a), \hat{\alpha}(B). \end{aligned}$$

Thus $(\hat{\alpha}, \beta)$ is a homomorphism.

Suppose (α, β) is one-one and let $\hat{\alpha}(A) = \hat{\alpha}(B)$ for $A, B \in \mathcal{P}(Q)$. Then $\{\alpha(q) : q \in A\} = \{\alpha(q) : q \in B\}$. For $q \in A$, $\alpha(q) \in \hat{\alpha}(A) = \hat{\alpha}(B)$. Thus $\alpha(q) = \alpha(q')$ for some $q' \in B$. Since α is one-one, q=q'. So $q \in B$ and thus $A \subseteq B$.

Similarly, we can prove that $B \subseteq A$. Hence $A = B$. Therefore $\hat{\alpha}$ is one-one and so $(\hat{\alpha}, \beta)$ is one-one. Similarly $\hat{\alpha}$ is onto since α is onto. Converse is clear since α is the restriction of $\hat{\alpha}$ to Q. □

4. Max-weighted power automaton

As in the case of min-weighted power automaton, we can define max-weighted power automaton for a fuzzy automaton $M = (Q, X, \mu)$.

For $A, B \in \mathcal{P}(Q)$, define $\mu^{\vee}(A, a, B)$ as follows.

$$\mu^{\vee}(A,a,B) = \begin{cases} \bigvee_{q\in A}\bigvee_{p\in B} \mu(q,a,p) & \text{if } B = \{p : \mu(q,a,p) > 0\} \\ 0 & \text{otherwise.} \end{cases}$$

for all $A \neq \phi, B \neq \phi$ in $\mathcal{P}(Q)$,

$\mu^{\vee}(\phi, a, \phi) = 1$ and $\mu^{\vee}(A, a, B) = 0$ if $A = \phi$ or $B = \phi$, for all $a \in X$. Then $\mathcal{P}^{\vee}(M) = (\mathcal{P}(Q), X, \mu^{\vee})$ is called the max-weighted power automaton. We can extend $\mu^{\vee}$ to $\mathcal{P}(Q) \times X^* \times \mathcal{P}(Q)$ as

$$\mu^{\vee}(A, xa, B) = \bigvee_{C\in\mathcal{P}(Q)} \mu^{\vee}(A, x, C) \wedge \mu^{\vee}(C, a, B),$$

where $A, B, C \in \mathcal{P}(Q)$.
M can be embedded in $\mathcal{P}^{\vee}(M)$ with the isomorphism $p \longrightarrow \{p\}$.

5. Some algebraic properties of fuzzy power automata

Theorem 5.1. *If M_1 and M_2 are fuzzy automata, then* $\mathcal{P}^{\wedge}(M_1 \oplus M_2) \cong \mathcal{P}(M_1).\mathcal{P}(M_2)^{\wedge}$.

Proof. $\mathcal{P}^{\wedge}(M_1 \bigoplus M_2) = (\mathcal{P}(Q_1 \cup Q_2), X_1 \cup X_2, (\mu_1 \oplus \mu_2)^{\wedge})$
and

$$\mathcal{P}(M_1).\mathcal{P}(M_2)^{\wedge} = (\mathcal{P}(Q_1) \times \mathcal{P}(Q_2), X_1 \cup X_2, \mu_1.\mu_2^{\wedge}).$$

Define a mapping $\alpha : \mathcal{P}(Q_1) \times \mathcal{P}(Q_2) \longrightarrow \mathcal{P}(Q_1 \cup Q_2)$ as $\alpha(A, B) = A \cup B$ where $A \in \mathcal{P}(Q_1)$ and $B \in \mathcal{P}(Q_2)$ and β is the identity map on $X_1 \cup X_2$.
We claim that (α, β) is an isomorphism from $\mathcal{P}(M_1).\mathcal{P}(M_2)^{\wedge} \longrightarrow \mathcal{P}^{\wedge}(M_1 \oplus M_2)$.
We have (α, β) is a isomorphism iff α and β are one-one onto and
$\mu_1.\mu_2{}^{\wedge}((A_1, B_1), a, (A_2, B_2)) \leq (\mu_1 \oplus \mu_2)^{\wedge}(\alpha(A_1, B_1), a, \alpha(A_2, B_2))$
$\forall\ (A_1, B_1), (A_2, B_2) \in \mathcal{P}(Q_1) \times \mathcal{P}(Q_2)$.
Clearly α and β are one-one onto.
Let $(A_1, B_1), (A_2, B_2) \in \mathcal{P}(Q_1) \times \mathcal{P}(Q_2)$ and $a \in X_1 \cup X_2$.
Then $\alpha(A_1, B_1) = A_1 \cup B_1$, $\alpha(A_2, B_2) = A_2 \cup B_2$ and

$$\mu_1 \oplus \mu_2{}^{\wedge}(\alpha(A_1, B_1), a, \alpha(A_2, B_2)) = \mu_1 \oplus \mu_2{}^{\wedge}(A_1 \cup B_1, a, A_2 \cup B_2)$$

$$= \begin{cases} \bigwedge_{q\in A_1\cup B_1}\bigwedge_{p\in A_2\cup B_2} \mu_1 \oplus \mu_2(q,a,p) & \text{if } A_2 \cup B_2 = \{p : \mu_1 \oplus \mu_2(q,a,p) > 0\} \\ 0 & \text{otherwise} \end{cases}$$

$$(5.1) \qquad = \begin{cases} \bigwedge_{q\in A_1}\bigwedge_{p\in A_2} \mu_1(q,a,p) & \text{if } a \in X_1, A_2 = \{p : \mu_1(q,a,p) > 0\} \\ \bigwedge_{q\in B_1}\bigwedge_{p\in B_2} \mu_2(q,a,p) & \text{if } a \in X_2, B_2 = \{p : \mu_2(q,a,p) > 0\} \\ 1 & \begin{array}{l}\text{if either } q \in A_1, a \in X_1, p \in B_2 \\ \text{or } q \in B_1, a \in X_2, p \in A_2\end{array} \\ 0 & \text{otherwise.} \end{cases}$$

Now,
$\mu_1.\mu_2{}^{\wedge}(A_1, B_1), a, (A_2, B_2)$

$$= \begin{cases} \bigwedge_{(p_1,q_1)\in(A_1,B_1)} \bigwedge_{(p_2,q_2)\in(A_2,B_2)} \mu_1.\mu_2(p_1,q_1),a,(p_2,q_2) \\ \qquad \text{if}(A_2,B_2)=\{(p_2,q_2):\mu_1\mu_2((p_1,q_1),a,(p_2,q_2))>0\} \\ 0 \qquad \text{otherwise} \end{cases}$$

$$(5.2) \qquad = \begin{cases} \bigwedge_{p_1\in A_1} \bigwedge_{p_2\in A_2} \mu_1(p_1,a,p_2) & \text{if } a\in X_1, A_2=\{p_2:\mu_1(p_1,a,p_2)>0\} \\ \bigwedge_{q_1\in B_1} \bigwedge_{q_2\in B_2} \mu_2(q_1,a,q_2) & \text{if } a\in X_2, B_2=\{q_2:\mu_2(q_1,a,q_2)>0\} \\ 0 & \text{otherwise} \end{cases}$$

From equations (5.1) and (5.2), we get

$$\mu_1.\mu_2{}^\wedge((A_1,B_1),a,(A_2,B_2)) \leq \mu_1 \oplus \mu_2{}^\wedge(\alpha(A_1,B_1),\beta(a),\alpha(A_2,B_2))$$

$\forall (A_1,B_1),(A_2,B_2) \in \mathcal{P}(Q_1)\times\mathcal{P}(Q_2), a\in X_1$.
Thus (α,β) is an isomorphism from $\mathcal{P}^\wedge(M_1\oplus M_2)\longrightarrow\mathcal{P}(M_1).\mathcal{P}(M_2)^\wedge$. □

Theorem 5.2. *A fuzzy automaton M and its min-weighed power automaton $\mathcal{P}^\wedge(M)$ have the same transition monoids.*

Proof. The transition monoid of the fuzzy automaton is X^*/μ_M where μ_M is the congruence defined on X^* by $a\mu_M b$ if and only if $\mu(q,a,p)=\mu(q,b,p)\ \forall\ q,p\in Q$.
$\mu_{\mathcal{P}^\wedge(M)}$ is defined on X^* by $a\mu_{\mathcal{P}^\wedge(M)}b$ if and only if $\mu^\wedge(A,a,B)=\mu^\wedge(A,b,B)$ $\forall\ A,B\in\mathcal{P}(Q)$ and the transition monoid of $\mathcal{P}^\wedge(M)$ is $X^*/\mu_{\mathcal{P}^\wedge(M)}$.
Let $[a]_{\mu_M}\in X^*/\mu_M$ and $[a]_{\mu_{\mathcal{P}^\wedge(M)}}\in X^*/\mu_{\mathcal{P}^\wedge(M)}$.
First suppose $a\in X$ and let $b\in[a]_{\mu_M}$. Then $\mu(q,a,p)=\mu(q,b,p)\ \forall\ q,p\in Q$.
Let $A,B\neq\phi\in\mathcal{P}(Q)$. Then

$$\mu^\wedge(A,a,B) = \begin{cases} \bigwedge_{q\in A}\bigwedge_{p\in B}\mu(q,a,p) & \text{if } B=\{p:\mu(q,a,p)>0\} \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} \bigwedge_{q\in A}\bigwedge_{p\in B}\mu(q,b,p) & \text{if } B=\{p:\mu(q,b,p)>0\} \\ 0 & \text{otherwise} \end{cases}$$

$$=\mu^\wedge(A,b,B).$$

Thus $b\in[a]_{\mu^\wedge_{\mathcal{P}(M)}}$.

If A or B or both equal to ϕ, then clearly $\mu^\wedge(A,a,B)=\mu^\wedge(A,b,B))$.

Thus $[a]_{\mu_M}\subseteq[a]_{\mu^\wedge_{\mathcal{P}(M)}}$.

Now let $a=a_1a_2$ where $a_1,a_2\in X$. Then

$$\mu^\wedge(A,a,B) = \bigvee_{C\in\mathcal{P}(Q)} \mu^\wedge(A,a_1,C)\wedge\mu^\wedge(C,a_2,B).$$

If $b \in [a]_{\mu_M}, b \in [a_1a_2]_{\mu_M} = [a_1]_{\mu_M}[a_2]_{\mu_M}$. Then there exist $b_1, b_2 \in X$ such that $b = b_1b_2$ and $[b_1] \in [a_1]_{\mu_M}, [b_2] \in [a_2]_{\mu_M}$. This implies $\mu(p, a_1, q) = \mu(p, b_1, q)$ and $\mu(p, a_2, q) = \mu(p, b_2, q)$ for all $p, q \in Q$. Thus

$$\begin{aligned}\mu^\wedge(A, a, B) &= \bigvee_{C \in \mathcal{P}(Q)} \mu^\wedge(A, a_1, C) \wedge \mu^\wedge(C, a_2, B)\\ &= \bigvee_{C \in \mathcal{P}(Q)} \mu^\wedge(A, b_1, C) \wedge \mu^\wedge(C, b_2, B)\\ &= \mu^\wedge(A, b_1b_2, B) = \mu^\wedge(A, b, B).\end{aligned}$$

Thus $b \in [a]_{\mu_{\mathcal{P}^\wedge(M)}}$ and so $[a]_{\mu_M} \subseteq [a]_{\mu_{\mathcal{P}^\wedge(M)}}$ for every $a \in X^*$.

Conversely, suppose $b \in [a]_{\mu^\wedge(\mathcal{P}(M)}$. Then $\mu^\wedge(A, a, B) = \mu^\wedge(A, b, B)\ \forall\ A, B \in \mathcal{P}(Q)$. Take $A = \{p\}$ and $B = \{q\}$.
Then $\mu(\{p\}, a, \{q\}) = \mu(\{p\}, b, \{q\})\ \forall\ p, q \in Q$. Thus $b \in [a]_{\mu_M}$. So $[a]_{\mu_{\mathcal{P}^\wedge(M)}} \subseteq [a]_{\mu_M}$. Hence $[a]_{\mu_{\mathcal{P}^\wedge(M)}} = [a]_{\mu_M}$. □

Example 5.3. Consider the example of a fuzzy automaton M=$(Q, \tilde{X}, \mu)$, where $Q = \{q_0, q_1, q_2\}$, $\tilde{X} = \{a, b\}$ and $\mu : Q \times \tilde{X} \times Q \longrightarrow [0, 1]$ as defined below
$\mu(q_0, a, q_1) = 0.7$, $\mu(q_1, a, q_2) = 0.4$, $\mu(q_2, a, q_0) = 0.3$, $\mu(q_1, b, q_0) = 0.7$, $\mu(q_0, b, q_2) = 0.3$, $\mu(q_2, b, q_1) = 0.4$ and $= 0$ for all other elements of $Q \times X \times Q$. The transition semigroup of this fuzzy automaton is the semigroup generated by T_a and T_b where $T_a = T_{aba}$ and $T_b = T_{bab}$.

$\mathcal{P}(Q)$ has elements, say $\phi,\ A_1 = \{q_0\},\ A_2 = \{q_1\},\ A_3 = \{q_2\},\ A_4 = \{q_0, q_1\},\ A_5 = \{q_1, q_2\},\ A_6 = \{q_0, q_2\},\ A_7 = \{q_0, q_1, q_2\}$.

Now, $\mu^\wedge(\phi, a, \phi) = \mu^\wedge(\phi, b, \phi) = 1,\ \mu^\wedge(\phi, a, A_i) = \mu^\wedge(A_i, a, \phi) = \mu^\wedge(\phi, b, A_i) = \mu^\wedge(A_i, b, \phi) = 0$ for $i = 1, 2, \ldots, 7$. The other values of $\mu^\wedge(A_i, a, A_j)$ and $\mu^\wedge(A_i, b, A_j)$ can be calculated by the formula

$$\begin{cases} \bigwedge\limits_{q \in A_i} \bigwedge\limits_{p \in A_j} \mu(q, a, p) & \text{if } A_j = \{p : \mu(q, a, p) > 0\} \\ 0 & \text{otherwise.} \end{cases}$$

The transition monoid of the min-weighted power automaton
$\mathcal{P}^\wedge(M) = (\mathcal{P}(Q), X, \mu^\wedge)$ is the semigroup generated by

$$T_a = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.3 \end{bmatrix}$$

and

$$T_b = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.3 \end{bmatrix}$$

We can verify that $T_{aba} = T_a$ and $T_{bab} = T_b$.
Also,
$T(\mathcal{P}^{\wedge}(M))$ is $\{T_a, T_{a^2}, T_{a^3}, T_{a^4}, T_{a^5}, T_b, T_{b^2}, T_{ab}, T_{ba}, T_{ab^2}, T_{b^2a}, T_{a^2b}, T_{ba^2}, T_{a^2b^2}, T_{b^2a^2}, T_{ab^2a}\}$ which is same as $T(M)$

Theorem 5.4. *A fuzzy automaton M and its max-weighed power automaton $\mathcal{P}^{\vee}(M)$ have same transition monoids.*

Proof similar as in the case of min-weighted power automaton

Theorem 5.5. *If M is a commutative fuzzy automaton then the min-weighted (max-weighted) power automaton is commutative.*

Proof. Let $M = (Q, X, \mu, i, \tau)$ be a commutative fuzzy automaton.
Then $\mu(p, xy, q) = \mu(p, yx, q)$ for all $x, y \in X^*, p, q \in Q$. Thus $[xy]_{\mu_M} = [yx]_{\mu_M}$ for all $x, y \in X^*$. By Theorem 5.2, we get $[xy]_{\mu^{\wedge}_{\mathcal{P}}(M)} = [yx]_{\mu^{\wedge}_{\mathcal{P}}(M)}$. So $\mu^{\wedge}(A, xy, B) = \mu^{\wedge}(A, yx, B)$.

Similarly, we get $\mu^{\vee}(A, xy, B) = \mu^{\vee}(A, yx, B)$. □

Theorem 5.6. *If $M = (Q, X, \mu)$ is a commutative and inverse fuzzy automaton then the min-weighted (max-weighted) power automaton is also commutative and inverse.*

Proof. From theorems 5.4 and 5.5, $\mathcal{P}^{\wedge}(M)(\mathcal{P}^{\vee}(M))$ is a commutative and inverse fuzzy automaton. □

6. Conclusions

In this paper we defined min-weigted and max-weighted power automaton and studied some of its properties. We also proved that a fuzzy automaton and its min-weighted and max-weighted power automata have the same transition monoids and so if the fuzzy automaton is inverse and commutative, then the corresponding fuzzy power automaton has a transition monoid which is inverse and commutative.

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PAMY SEBASTIAN (pamyseb@gmail.com)
Department of Mathematics, Mary Matha Arts & Science College, Mananthavady