Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 11, No. 4, (April 2016), pp. 621–631
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Interval valued fuzzy quasi-ideals of near-rings

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Received 22 July 2015; Revised 13 September 2015; Accepted 28 September 2015

Abstract. In this paper, we introduce the notion of interval valued fuzzy quasi-ideals of near-rings. Some examples and characterizations of interval valued fuzzy quasi-ideals of near-rings are discussed here.

2010 AMS Classification: 18B40, 03E72, 16D25

Keywords: Near-rings, Quasi-ideals, Interval valued fuzzy quasi-ideals.

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1. Introduction

The concept of fuzzy set was first initiated by Zadeh[15] in 1965. After ten years, Zadeh[16] introduced a new notion of fuzzy subsets viz., interval valued fuzzy subset (in short i-v fuzzy subsets) where the values of the membership functions are closed intervals of numbers instead of a number. Interval valued fuzzy sets have many application in several areas. In [12], Rosenfeld defined fuzzy subgroup and gave some of its properties. In 1991, Abou Said[1] introduced the idea of fuzzy subnear-rings and fuzzy ideals in near-rings. Jun and Kim[5] and Davvaz[2, 3] applied a few concepts of interval valued fuzzy subsets in near-rings. Deena and Coumaressane[4] discussed some concepts of generalized fuzzy ideals in near-ring. Narayanan et al.[10, 11] introduced the concept of generalized fuzzy quasi-ideals of near-rings. Manikantan[7] defined and discussed fuzzy bi-ideals of near-rings. Recently, Muhammad Shabir et al.[8, 9] introduced and discussed some characterizations of fuzzy h-ideals of hemirings with interval valued fuzzy set. In this paper we introduce the notion of i-v fuzzy quasi-ideals of near-rings. We investigate some of their properties. We give examples which are i-v fuzzy quasi-ideal and i-v fuzzy quasi-ideal but not i-v fuzzy ideal of near-rings.

2. Preliminaries

In this section, we list some basic concepts and well known results of interval valued fuzzy set theory. Throughout this paper, R will denote a left near-ring.

Definition 2.1 ([6]). A non-empty set R with two binary operations $+$ and $\cdot$ is called a near-ring if

(1) $(R, +)$ is a group,
(2) $(R, \cdot)$ is a semigroup,
(3) $x \cdot (y + z) = x \cdot y + x \cdot z$, for all $x, y, z \in R$.

We use the word 'near-ring' to mean 'left near-ring'. We denote xy instead of $x \cdot y$. Note that $x0 = 0$ and $x(-y) = -xy$ but in general $0x \neq 0$ for some $x \in R$.

Definition 2.2 ([6]). An ideal I of a near-ring R is a subset of R such that

(4) $(I, +)$ is a normal subgroup of $(R, +)$,
(5) $RI \subseteq I$,
(6) $((x + i)y - xy) \in I$ for any $i \in I$ and $x, y \in R$.

Note that I is a left ideal of R if I satisfies (4) and (5), and I is a right ideal of R if I satisfies (4) and (6).

Definition 2.3 ([5]). A two sided R-subgroup of a near-ring R is a subset H of R such that

(i) $(H, +)$ is a subgroup of $(R, +)$,
(ii) $RH \subset H$, (iii) $HR \subset H$.

If H satisfies (i) and (ii) then it is called a left R-subgroup of R. If H satisfies (i) and (iii) then it is called a right R-subgroup of R.

Definition 2.4 ([4]). Let R be a near-ring. Given two subsets A and B of R, the product $AB = \{ab | a \in A, b \in B\}$ and $A * B = \{(a' + b)a - a'a | a,\ a' \in A,\ b \in B\}$.

Definition 2.5 ([10]). A subgroup Q of $(R, +)$ is said to be a quasi-ideal of R if $QR \cap RQ \cap Q * R \subseteq Q$.

Notation 2.6 ([13, 3]). By an interval number $\widetilde{a}$, we mean an interval $[a^-, a^+]$ such that $0 \leq a^- \leq a^+ \leq 1$ where a^- and a^+ are the lower and upper limits of $\widetilde{a}$ respectively. The set of all closed subintervals of $[0, 1]$ is denoted by $D[0, 1]$. We also identify the interval $[a, a]$ by the number $a \in [0, 1]$. For any interval numbers $\widetilde{a}_i = [a_i^-, a_i^+], \widetilde{b}_i = [b_i^-, b_i^+] \in D[0, 1], i \in I$ we define

$$\max{}^i\{\widetilde{a}_i, \widetilde{b}_i\} = [\max\{a_i^-, b_i^-\}, \max\{a_i^+, b_i^+\}],$$
$$\min{}^i\{\widetilde{a}_i, \widetilde{b}_i\} = [\min\{a_i^-, b_i^-\}, \min\{a_i^+, b_i^+\}],$$
$$\inf{}^i \widetilde{a}_i = \left[\bigcap_{i \in I} a_i^-, \bigcap_{i \in I} a_i^+\right], \sup{}^i \widetilde{a}_i = \left[\bigcup_{i \in I} a_i^-, \bigcup_{i \in I} a_i^+\right]$$

and let

(1) $\widetilde{a} \leq \widetilde{b} \Longleftrightarrow a^- \leq b^-$ and $a^+ \leq b^+$,
(2) $\widetilde{a} = \widetilde{b} \Longleftrightarrow a^- = b^-$ and $a^+ = b^+$,
(3) $\widetilde{a} < \widetilde{b} \Longleftrightarrow \widetilde{a} \leq \widetilde{b}$ and $\widetilde{a} \neq \widetilde{b}$,
(4) $k\widetilde{a} = [ka^-, ka^+]$, whenever $0 \leq k \leq 1$.

Definition 2.7 ([13]). Let X be a non-empty set. A mapping $\widetilde{\mu} : X \to D[0, 1]$ is called an i-v fuzzy subset of X. For any $x \in X$, $\widetilde{\mu}(x) = [\mu^-(x), \mu^+(x)]$, where μ^- and μ^+ are fuzzy subsets of X such that $\mu^-(x) \leq \mu^+(x)$. Thus $\widetilde{\mu}(x)$ is an interval(a closed subset of $[0, 1]$) and not a number from the interval $[0, 1]$ as in the case of a fuzzy set.

Let $\widetilde{\mu}, \widetilde{\nu}$ be i-v fuzzy subsets of X. The following are defined by
(1) $\widetilde{\mu} \leq \widetilde{\nu} \Leftrightarrow \widetilde{\mu}(x) \leq \widetilde{\nu}(x)$.
(2) $\widetilde{\mu} = \widetilde{\nu} \Leftrightarrow \widetilde{\mu}(x) = \widetilde{\nu}(x)$.
(3) $(\widetilde{\mu} \cup \widetilde{\nu})(x) = \max^i\{\widetilde{\mu}(x), \widetilde{\nu}(x)\}$.
(4) $(\widetilde{\mu} \cap \widetilde{\nu})(x) = \min^i\{\widetilde{\mu}(x), \widetilde{\nu}(x)\}$.

Definition 2.8 ([13])**.** Let $\widetilde{\mu}$ be an i-v fuzzy subset of X and $[t_1, t_2] \in D[0,1]$. Then the set $\widetilde{U}(\widetilde{\mu} : [t_1, t_2]) = \{x \in X \mid \widetilde{\mu}(x) \geq [t_1, t_2]\}$ is called the upper level set of $\widetilde{\mu}$.

Definition 2.9 ([14])**.** An i-v fuzzy subset $\widetilde{\mu}$ of a near-ring R is called an i-v fuzzy subnear-ring of R if
(1) $\widetilde{\mu}(x - y) \geq \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}$,
(2) $\widetilde{\mu}(xy) \geq \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}$,
for all $x, y \in R$.

An i-v fuzzy subset $\widetilde{\mu}$ of a near-ring R is called an i-v fuzzy ideal of R if $\widetilde{\mu}$ is an i-v fuzzy subnear-ring of R and
(3) $\widetilde{\mu}(x) = \widetilde{\mu}(y + x - y)$,
(4) $\widetilde{\mu}(xy) \geq \widetilde{\mu}(y)$,
(5) $\widetilde{\mu}((x + i)y - xy) \geq \widetilde{\mu}(i)$,
for any $x, y, i \in R$.

Note that $\widetilde{\mu}$ is an i-v fuzzy left ideal of R if it satisfies (1), (3) and (4), and $\widetilde{\mu}$ is an i-v fuzzy right ideal of R if it satisfies (1), (2), (3) and (5).

Definition 2.10 ([5])**.** An i-v fuzzy subset $\widetilde{\mu}$ of a near-ring R is called an i-v fuzzy R-subgroup of R if for all $x, y \in R$,
(1) $\widetilde{\mu}(x - y) \geq \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}$,
(2) $\widetilde{\mu}(xy) \geq \widetilde{\mu}(y)$,
(3) $\widetilde{\mu}(xy) \geq \widetilde{\mu}(x)$.

Note that $\widetilde{\mu}$ is an i-v fuzzy left ideal of R if it satisfies (1) and (2), and $\widetilde{\mu}$ is an i-v fuzzy right ideal of R if it satisfies (1) and (3).

3. Interval valued fuzzy quasi-ideal of near-ring

In this section, we introduce the notion of i-v fuzzy quasi-ideal of R. We characterize i-v fuzzy quasi-ideal of R. Throughout this paper, $\overline{f}_I$ is an i-v fuzzy characteristic function of a subset I of R and the i-v fuzzy characteristic function of R is denoted by $\mathbf{R}$, that means, $\mathbf{R} : R \to D[0,1]$ mapping every element of R to $[1,1]$.

Definition 3.1. An i-v fuzzy subset $\widetilde{\mu}$ of R is said to be an i-v fuzzy subgroup of R if $x,\ y \in R$ implies $\widetilde{\mu}(x - y) \geq \min^i\{\widetilde{\mu}(x),\ \widetilde{\mu}(y)\}$.

Definition 3.2. An i-v fuzzy subgroup $\widetilde{\mu}$ of R is called an i-v fuzzy quasi-ideal of R if $(\widetilde{\mu}\mathbf{R}) \cap (\mathbf{R}\widetilde{\mu}) \cap (\widetilde{\mu} * \mathbf{R}) \subseteq \widetilde{\mu}$.

Definition 3.3. Let $\widetilde{f}$ and $\widetilde{g}$ be any two i-v fuzzy subsets of R. Then $\widetilde{f}\cap\widetilde{g}$, $\widetilde{f}\cup\widetilde{g}$, $\widetilde{f}+\widetilde{g}$, $\widetilde{f}\widetilde{g}$ and $\widetilde{f}*\widetilde{g}$ are i-v fuzzy subsets of R defined by:

$$(\widetilde{f}\cap\widetilde{g})(x)=\min{}^i\{\widetilde{f}(x),\ \widetilde{g}(x)\}.$$

$$(\widetilde{f}\cup\widetilde{g})(x)=\max{}^i\{\widetilde{f}(x),\ \widetilde{g}(x)\}.$$

$$(\widetilde{f}+\widetilde{g})(x)=\begin{cases}\sup^i_{x=y+z}\min^i\{\widetilde{f}(y),\ \widetilde{g}(z)\} & \text{if } x \text{ can be expressed as } x=y+z\\ 0 & \text{otherwise.}\end{cases}$$

$$(\widetilde{f}\widetilde{g})(x)=\begin{cases}\sup^i_{x=yz}\min^i\{\widetilde{f}(y),\ \widetilde{g}(z)\} & \text{if } x \text{ can be expressed as } x=yz\\ 0 & \text{otherwise.}\end{cases}$$

$$(\widetilde{f}*\widetilde{g})(x)=\begin{cases}\sup^i_{x=(a+c)b-ab}\min^i\{\widetilde{f}(c),\ \widetilde{g}(b)\} & \text{if } x \text{ can be expressed as}\\ & x=(a+c)b-ab.\\ 0 & \text{otherwise.}\end{cases}$$

Example 3.4. Let $R=\{a,\ b,\ c,\ d\}$ be a set with two binary operations is defined as follows:

$+$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

$\cdot$	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	a	b	b

Then clearly $(R,+,\cdot)$ is a left near-ring. Let $\widetilde{\mu}:R\to D[0,1]$ be an i-v fuzzy subset of R such that $\widetilde{\mu}(a)=[0.5,0.6]$, $\widetilde{\mu}(b)=[0.3,0.4]$ and $\widetilde{\mu}(c)=[0.2,0.3]=\widetilde{\mu}(d)$. Clearly $\widetilde{\mu}$ is an i-v fuzzy subgroup of R. $(\widetilde{\mu}\mathbf{R})(a)=[0.5,0.6]=(\mathbf{R}\widetilde{\mu})(a)$, similarly,$(\widetilde{\mu}*\mathbf{R})(a)=[0.5,0.6]$ and $\min^i\{(\mathbf{R}\widetilde{\mu})(a),\ (\widetilde{\mu}\mathbf{R})(a),\ (\widetilde{\mu}*\mathbf{R})(a)\}=[0.5,0.6]=\widetilde{\mu}(a)$. Similarly, $(\widetilde{\mu}\mathbf{R})(b)=(\mathbf{R}\widetilde{\mu})(b)=(\widetilde{\mu}*\mathbf{R})(b)=[0.2,0.3]$, $(\widetilde{\mu}\mathbf{R})(c)=(\mathbf{R}\widetilde{\mu})(c)=(\widetilde{\mu}*\mathbf{R})(c)=[0,0]$ and $(\widetilde{\mu}\mathbf{R})(d)=(\mathbf{R}\widetilde{\mu})(d)=(\widetilde{\mu}*\mathbf{R})(d)=[0,0]$. Thus $\widetilde{\mu}$ is an i-v fuzzy quasi-ideal of R.

Lemma 3.5. *Let $\widetilde{\mu}$ be an i-v fuzzy subset of R. If $\widetilde{\mu}$ is an i-v fuzzy right ideal of R, then $\widetilde{\mu}$ is an i-v fuzzy quasi-ideal of R.*

Proof. Let $x'\in R$ and $x'=ab=(x+z)y-xy$, where a,b,x,y and z are in R. Then

$$\begin{aligned}&((\widetilde{\mu}\mathbf{R})\cap(\mathbf{R}\widetilde{\mu})\cap(\widetilde{\mu}*\mathbf{R}))(x')\\ &=\min{}^i\{(\widetilde{\mu}\mathbf{R})(x'),(\mathbf{R}\widetilde{\mu})(x'),(\widetilde{\mu}*\mathbf{R})(x')\}\\ &=\min{}^i\{\sup{}^i_{x'=ab}\min{}^i\{\widetilde{\mu}(a),\mathbf{R}(b)\},\sup{}^i_{x'=ab}\min{}^i\{\mathbf{R}(a),\widetilde{\mu}(b)\},\\ &\qquad\qquad\sup{}^i_{x'=(x+z)y-xy}\min{}^i\{\widetilde{\mu}(z),\mathbf{R}(y)\}\}\\ &=\min{}^i\left\{\sup{}^i\{\widetilde{\mu}(a)\},\sup{}^i\{\widetilde{\mu}(b)\},\sup{}^i\{\widetilde{\mu}(z)\}\right\}\\ &\qquad\qquad\text{Since } \widetilde{\mu} \text{ is an i-v fuzzy right ideal,} \widetilde{\mu}((x+z)y-xy)\geq\widetilde{\mu}(z).\\ &\leq\min{}^i\left\{\widetilde{1},\widetilde{1},\widetilde{\mu}((x+z)y-xy)\right\}=\widetilde{\mu}((x+z)y-xy)=\widetilde{\mu}(x').\end{aligned}$$

If x' is not expressed as $x'=ab=(x+z)y-xy$, then $(\widetilde{\mu}\mathbf{R}\cap\mathbf{R}\widetilde{\mu}\cap\widetilde{\mu}*\mathbf{R})(x')=0\leq\widetilde{\mu}(x')$. Thus $\widetilde{\mu}\mathbf{R}\cap\mathbf{R}\widetilde{\mu}\cap\widetilde{\mu}*\mathbf{R}\subseteq\widetilde{\mu}$. Hence $\widetilde{\mu}$ is an i-v fuzzy quasi-ideal of R. □

Lemma 3.6. *Let $\widetilde{\mu}$ be an i-v fuzzy subset of R. If $\widetilde{\mu}$ is an i-v fuzzy left ideal of R, then $\widetilde{\mu}$ is an i-v fuzzy quasi-ideal of R.*

Proof. Let $x' \in R$ and $x' = ab = (x+z)y - xy$, where a, b, x, y and z are in R. Consider,

$$
\begin{aligned}
&((\widetilde{\mu}\mathbf{R}) \cap (\mathbf{R}\widetilde{\mu}) \cap (\widetilde{\mu} * \mathbf{R}))(x') \\
&= \min^i\{(\widetilde{\mu}\mathbf{R})(x'), (\mathbf{R}\widetilde{\mu})(x'), (\widetilde{\mu} * \mathbf{R})(x')\} \\
&= \min^i\{\sup^i_{x'=ab} \min^i\{\widetilde{\mu}(a), \mathbf{R}(b)\}, \sup^i_{x'=ab} \min^i\{\mathbf{R}(a), \widetilde{\mu}(b)\}, \\
&\qquad\qquad \sup^i_{x'=(x+z)y-xy} \min^i\{\widetilde{\mu}(z), \mathbf{R}(y)\}\} \\
&= \min^i \left\{\sup^i\{\widetilde{\mu}(a)\}, \sup^i\{\widetilde{\mu}(b)\}, \sup^i\{\widetilde{\mu}(z)\}\right\} \\
&\qquad \text{Since } \widetilde{\mu} \text{ is an i-v fuzzy left ideal,} \widetilde{\mu}(ab) \geq \widetilde{\mu}(b). \\
&\leq \min^i \left\{\widetilde{1}, \widetilde{\mu}(ab), \widetilde{1}\right\} = \widetilde{\mu}(ab) = \widetilde{\mu}(x').
\end{aligned}
$$

If x' is not expressed as $x' = ab = (x+z)y - xy$, then $(\widetilde{\mu}\mathbf{R} \cap \mathbf{R}\widetilde{\mu} \cap \widetilde{\mu} * \mathbf{R})(x') = 0 \leq \widetilde{\mu}(x')$. Thus $\widetilde{\mu}\mathbf{R} \cap \mathbf{R}\widetilde{\mu} \cap \widetilde{\mu} * \mathbf{R} \subseteq \widetilde{\mu}$. Hence $\widetilde{\mu}$ is an i-v fuzzy quasi-ideal of R. □

Theorem 3.7. *Let $\widetilde{\mu}$ be an i-v fuzzy subset of R. If $\widetilde{\mu}$ is an i-v fuzzy ideal of R, then $\widetilde{\mu}$ is an i-v fuzzy quasi-ideal of R.*

However the converse of the Theorem 3.7 is not true in general which is demonstrated by the following Example.

Example 3.8. Let $R = \{a, b, c, d\}$ be a set with two binary operations defined as follows:

$+$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

$\cdot$	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	b	c	d

Then clearly $(R, +, \cdot)$ is a left near-ring. Define an i-v fuzzy subset $\widetilde{\mu} : R \to D[0,1]$ by $\widetilde{\mu}(a) = [0.8, 0.9], \widetilde{\mu}(b) = [0.6, 0.7], \widetilde{\mu}(c) = [0.3, 0.4] = \widetilde{\mu}(d)$. Then $(\widetilde{\mu}\mathbf{R})(a) = [0.8, 0.9], (\mathbf{R}\widetilde{\mu})(a) = [0.8, 0.9], (\widetilde{\mu} * \mathbf{R})(a) = [0.8, 0.9]$. $\min^i\{(\widetilde{\mu}\mathbf{R})(a), (\mathbf{R}\widetilde{\mu})(a), (\widetilde{\mu} * \mathbf{R})(a)\} = \min^i\{[0.8, 0.9], [0.8, 0.9], [0.8, 0.9]\} = [0.8, 0.9] = \widetilde{\mu}(a)$. Thus $\widetilde{\mu}$ is an i-v fuzzy quasi ideal of R and $\widetilde{\mu}$ is not an i-v fuzzy right ideal of R , since $\widetilde{\mu}((c+b)d - cd) = \widetilde{\mu}(d) < \widetilde{\mu}(b)$. Thus $\widetilde{\mu}$ is not an i-v fuzzy ideal of R.

Lemma 3.9. *Every i-v fuzzy quasi-ideals in a zero-symmetric near-ring R is an i-v fuzzy subnear-ring of R.*

Proof. Let μ be an i-v fuzzy quasi-ideals of a zero-symmetric near-ring R. Choose $a, b, c, x, y, z \in R$ such that $a = bc = (x+z)y - xy$. Then

$$\begin{aligned}\widetilde{\mu}(bc) = \widetilde{\mu}(a) &\geq \min^i\{(\widetilde{\mu}\mathbf{R})(a), (\mathbf{R}\mu)(a)(\mu * \mathbf{R})(a)\} \\ &= \min^i\{\sup^i_{a=bc}\min^i\{\widetilde{\mu}(b), \mathbf{R}(c)\}, \sup^i_{a=bc}\min^i\{\mathbf{R}(b), \widetilde{\mu}(c)\}, \\ &\qquad \sup^i_{a=(x+z)y-xy}\min^i\{\widetilde{\mu}(z), \mathbf{R}(y)\}\} \\ &\geq \min^i\{\sup^i_{a=bc}\min^i\{\widetilde{\mu}(b), \mathbf{R}(c)\}, \sup^i_{a=bc}\min^i\{\widetilde{\mu}(c), \mathbf{R}(b)\}, \\ &\qquad \sup^i_{a=(0+c)b-0b}\min^i\{\widetilde{\mu}(c)\}\} \\ &= \min^i\{\widetilde{\mu}(b), \widetilde{\mu}(c), \widetilde{\mu}(c)\} \\ &= \min^i\{\widetilde{\mu}(b), \widetilde{\mu}(c)\}.\end{aligned}$$

Therefore $\widetilde{\mu}(bc) \geq \min^i\{\widetilde{\mu}(b), \widetilde{\mu}(c)\}$ and since $\widetilde{\mu}$ is an i-v fuzzy quasi-ideal of a zero-symmetric near-ring R, then $\widetilde{\mu}(b-c) \geq \min^i\{\widetilde{\mu}(b), \widetilde{\mu}(c)\}$ for all $b, c \in R$. Thus μ is an i-v fuzzy subnear-ring of R. □

Theorem 3.10. *Let $\widetilde{\mu}$ be an i-v fuzzy subset of R. Then $\widetilde{\mu}$ is an i-v fuzzy quasi-ideal of R if and only if upper level subsets $\widetilde{U}(\widetilde{\mu} : [t_1, t_2])$ is a quasi-ideal of R, for all $[t_1, t_2] \in D[0, 1]$ with $[t_1, t_2] \neq [0, 0]$.*

Proof. Assume that $\widetilde{\mu}$ is an i-v fuzzy quasi-ideal of R. Let $[t_1, t_2] \in D[0, 1]$ with $[t_1, t_2] \neq [0, 0]$. Let $x, y \in \widetilde{U}(\widetilde{\mu} : [t_1, t_2])$. Then $\widetilde{\mu}(x) \geq [t_1, t_2]$ and $\widetilde{\mu}(y) \geq [t_1, t_2]$. Since $\widetilde{\mu}$ is an i-v fuzzy quasi-ideal of R, we have $\widetilde{\mu}(x-y) \geq \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \geq [t_1, t_2]$. It follows that $x - y \in \widetilde{U}(\widetilde{\mu} : [t_1, t_2])$. Let $x' \in R$ and $x' \in \widetilde{U}(\widetilde{\mu} : [t_1, t_2])\mathbf{R} \cap \mathbf{R}\widetilde{U}(\widetilde{\mu} : [t_1, t_2]) \cap \widetilde{U}(\widetilde{\mu} : [t_1, t_2]) * \mathbf{R}$. If there exist $a, b_1, z \in \widetilde{U}(\widetilde{\mu} : [t_1, t_2])$ and $a_1, b, x, y \in R$ such that $x' = ab = a_1 b_1 = (x+z)y - xy$. Then $\widetilde{\mu}(a) \geq [t_1, t_2], \widetilde{\mu}(b_1) \geq [t_1, t_2]$ and $\widetilde{\mu}(z) \geq [t_1, t_2]$. Thus

$$\begin{aligned}\widetilde{\mu}(x') &\geq (\widetilde{\mu}\mathbf{R} \cap \mathbf{R}\widetilde{\mu} \cap \widetilde{\mu} * \mathbf{R})(x') \\ &= \min^i\{(\widetilde{\mu}\mathbf{R})(x'), (\mathbf{R}\widetilde{\mu})(x'), (\widetilde{\mu} * \mathbf{R})(x')\} \\ &= \min^i\{\sup^i_{x'=ab}\min^i\{\widetilde{\mu}(a), \mathbf{R}(b)\}, \sup^i_{x'=a_1b_1}\min^i\{\mathbf{R}(a_1), \widetilde{\mu}(b_1)\}, \\ &\qquad \sup^i_{x'=(x+z)y-xy)}\min^i\{\widetilde{\mu}(z), \mathbf{R}(y)\}\} \\ &= \min^i\{\sup^i_{x'=ab}\{\widetilde{\mu}(a)\}, \sup^i_{x'=a_1b_1}\{\widetilde{\mu}(b_1)\}, \sup^i_{x'=(x+z)y-xy)}\{\widetilde{\mu}(z)\}\}.\end{aligned}$$

This implies that $\widetilde{\mu}(x') \geq [t_1, t_2]$ and so $x' \in \widetilde{U}(\widetilde{\mu} : [t_1, t_2])$, that is, $\widetilde{U}(\widetilde{\mu} : [t_1, t_2])\mathbf{R} \cap \mathbf{R}\widetilde{U}(\widetilde{\mu} : [t_1, t_2]) \cap \widetilde{U}(\widetilde{\mu} : [t_1, t_2]) * \mathbf{R} \subseteq \widetilde{U}(\widetilde{\mu} : [t_1, t_2])$ and hence $\widetilde{U}(\widetilde{\mu} : [t_1, t_2])$ is a quasi-ideal of R.

Conversely, assume that $\widetilde{U}(\widetilde{\mu} : [t_1, t_2]), [t_1, t_2] \in D[0, 1]$ with $[t_1, t_2] \neq [0, 0]$, is a quasi-ideal of R. Let $x' \in R$. Suppose that $(\widetilde{\mu}\mathbf{R} \cap \mathbf{R}\widetilde{\mu} \cap \widetilde{\mu} * \mathbf{R})(x') > \widetilde{\mu}(x')$. Choose $[0, 0] < [t_1, t_2] \leq [1, 1]$ such that $(\widetilde{\mu}\mathbf{R} \cap \mathbf{R}\widetilde{\mu} \cap \widetilde{\mu} * \mathbf{R})(x') > [t_1, t_2] > \widetilde{\mu}(x')$. This implies that $(\widetilde{\mu}\mathbf{R})(x') \geq [t_1, t_2], (\mathbf{R}\widetilde{\mu})(x') \geq [t_1, t_2]$ and $(\widetilde{\mu} * \mathbf{R})(x') \geq [t_1, t_2]$. So,

$(\widetilde{\mu}\mathbf{R})(x') = \sup^i_{x'=ab}\min^i\{\widetilde{\mu}(a), \mathbf{R}(b)\} = \sup^i_{x'=ab}\{\widetilde{\mu}(a)\} \geq [t_1, t_2]$ and

$(\mathbf{R}\widetilde{\mu})(x') = \sup^i_{x'=a_1b_1}\min^i\{\mathbf{R}(a_1), \widetilde{\mu}(b_1)\} = \sup^i_{x'=a_1b_1}\{\widetilde{\mu}(b_1)\} \geq [t_1, t_2]$ and $(\widetilde{\mu} * \mathbf{R})(x') = \sup^i_{x'=(x+z)y-xy}\min^i\{\widetilde{\mu}(z), \mathbf{R}(y)\} = \sup^i_{x'=(x+z)y-xy}\{\widetilde{\mu}(z)\} \geq [t_1, t_2]$. Then $a, b_1, z \in \widetilde{U}(\widetilde{\mu} : [t_1, t_2])$. Since $\widetilde{U}(\widetilde{\mu} : [t_1, t_2])$ is a quasi-ideal of R, then

$x' = ab \in \widetilde{U}(\widetilde{\mu} : [t_1, t_2])\mathbf{R}, x' = a_1b_1 \in \mathbf{R}\widetilde{U}(\widetilde{\mu} : [t_1, t_2])$ and $x' = (x+z)y - xy \in \widetilde{U}(\widetilde{\mu} : [t_1, t_2]) * \mathbf{R}$. Thus $x' \in \widetilde{U}(\widetilde{\mu} : [t_1, t_2])\mathbf{R} \cap \mathbf{R}\widetilde{U}(\widetilde{\mu} : [t_1, t_2]) \cap \widetilde{U}(\widetilde{\mu} : [t_1, t_2]) * \mathbf{R}$, that is, $x' \in \widetilde{U}(\widetilde{\mu} : [t_1, t_2])$, because $\widetilde{U}(\widetilde{\mu} : [t_1, t_2])$ is a quasi-ideal of R. Thus $\widetilde{\mu}(x') \geq [t_1, t_2]$, which is a contradiction. Therefore, $\widetilde{\mu}\mathbf{R} \cap \mathbf{R}\widetilde{\mu} \cap \widetilde{\mu} * \mathbf{R} \subseteq \widetilde{\mu}$ and hence $\widetilde{\mu}$ is an i-v fuzzy quasi-ideal of R. □

Lemma 3.11. *Let A and B be two nonempty subsets of R. Then the following properties hold:*

(1) $\widetilde{f}_A \cap \widetilde{f}_B = \widetilde{f}_{A\cap B}$.
(2) $\widetilde{f}_A \cup \widetilde{f}_B = \widetilde{f}_{A\cup B}$.
(3) $\widetilde{f}_A \widetilde{f}_B = \widetilde{f}_{AB}$.
(4) $\widetilde{f}_A * \widetilde{f}_B = \widetilde{f}_{A*B}$.

Lemma 3.12. *Let Q be a subgroup of R. Then Q is a quasi-ideal of R if and only if $\widetilde{f}_Q$ is an i-v fuzzy quasi-ideal of R.*

Proof. Assume that Q is a quasi-ideal of R. Then $\widetilde{f}_Q$ is an i-v fuzzy subgroup of R.

$$\begin{aligned}(\widetilde{f}_Q\mathbf{R}) \cap (\mathbf{R}\widetilde{f}_Q) \cap (\widetilde{f}_Q * \mathbf{R}) &= (\widetilde{f}_Q\widetilde{f}_R) \cap (\widetilde{f}_R\widetilde{f}_Q) \cap (\widetilde{f}_Q * \widetilde{f}_R) \\ &= \widetilde{f}_{QR} \cap \widetilde{f}_{RQ} \cap \widetilde{f}_{Q*R} \\ &= \widetilde{f}_{QR\cap RQ\cap Q*R} \subseteq \widetilde{f}_Q.\end{aligned}$$

This means that $\widetilde{f}_Q$ is an i-v fuzzy quasi-ideal of R.

Conversely, let us assume that $\widetilde{f}_Q$ is an i-v fuzzy quasi-ideal of R. Let x be any element of $QR \cap RQ \cap Q * R$. Then, we have

$$\begin{aligned}\widetilde{f}_Q(x) &\geq (\widetilde{f}_Q\mathbf{R} \cap \mathbf{R}\ \widetilde{f}_Q \cap \widetilde{f}_Q * \mathbf{R})(x) \\ &= \min{}^i \left\{(\widetilde{f}_Q\mathbf{R})(x), (\mathbf{R}\ \widetilde{f}_Q)(x), (\widetilde{f}_Q * \mathbf{R})(x)\right\} \\ &= \min{}^i \left\{(\widetilde{f}_Q\widetilde{f}_R)(x), (\widetilde{f}_R\widetilde{f}_Q)(x), (\widetilde{f}_Q * \widetilde{f}_R)(x)\right\} \\ &= \min{}^i \left\{\widetilde{f}_{QR}(x), \widetilde{f}_{RQ}(x), \widetilde{f}_{Q*R}(x)\right\} \\ &= \widetilde{f}_{QR\cap RQ\cap Q*R}(x) = \widetilde{1}.\end{aligned}$$

This implies that $x \in Q$ and so $QR \cap RQ \cap Q * R \subseteq Q$. This means that Q is a quasi ideal of R.

□

Theorem 3.13. *Every i-v fuzzy right R-subgroup of R is an i-v fuzzy quasi-ideal.*

Proof. Assume that $\widetilde{\mu}$ is an i-v fuzzy right R-subgroup of R. Let $a, b, x, y, z \in R$ be such that $x' = ab = (x+z)y - xy$. Then

$$((\widetilde{\mu}\mathbf{R}) \cap (\mathbf{R}\widetilde{\mu}) \cap (\widetilde{\mu} * \mathbf{R}))(x')$$
$$= \min^i\{(\widetilde{\mu}\mathbf{R})(x'), (\mathbf{R}\widetilde{\mu})(x'), (\widetilde{\mu} * \mathbf{R})(x')\}$$
$$= \min^i\{\sup^i_{x'=ab} \min^i\{\widetilde{\mu}(a), \mathbf{R}(b)\}, \sup^i_{x'=ab} \min^i\{\mathbf{R}(a), \widetilde{\mu}(b)\}, (\widetilde{\mu} * \mathbf{R})(x')\}\}$$
$$= \min^i \left\{\sup^i\{\widetilde{\mu}(a)\}, \sup^i\{\widetilde{\mu}(b)\}, (\widetilde{\mu} * \mathbf{R})(x')\right\}$$

Since $\widetilde{\mu}$ is an i-v fuzzy right R-subgroup of R,$\widetilde{\mu}(ab) \geq \widetilde{\mu}(a)$.

$$\leq \min^i\{\widetilde{\mu}(ab), \mathbf{R}(b), \mathbf{R}((x+z)y - xy)\}$$
$$= \min^i\{\widetilde{\mu}(ab), \widetilde{1}, \widetilde{1}\} = \widetilde{\mu}(ab) = \widetilde{\mu}(x').$$

Therefore $\widetilde{\mu}$ is an i-v fuzzy quasi-ideal of R. □

Theorem 3.14. *Every i-v fuzzy left R-subgroup of R is an i-v fuzzy quasi-ideal.*

Proof. Assume that $\widetilde{\mu}$ is an i-v fuzzy left R-subgroup of R. Let $a, b, x, y, z \in R$ be such that $x' = ab = (x+z)y - xy$. Then

$$((\widetilde{\mu}\mathbf{R}) \cap (\mathbf{R}\widetilde{\mu}) \cap (\widetilde{\mu} * \mathbf{R}))(x')$$
$$= \min^i\{(\widetilde{\mu}\mathbf{R})(x'), (\mathbf{R}\widetilde{\mu})(x'), (\widetilde{\mu} * \mathbf{R})(x')\}$$
$$= \min^i\{\sup^i_{x'=ab} \min^i\{\widetilde{\mu}(a), \mathbf{R}(b)\}, \sup^i_{x'=ab} \min^i\{\mathbf{R}(a), \widetilde{\mu}(b)\}, (\widetilde{\mu} * \mathbf{R})(x')\}\}$$
$$= \min^i \left\{\sup^i\{\widetilde{\mu}(a)\}, \sup^i\{\widetilde{\mu}(b)\}, (\widetilde{\mu} * \mathbf{R})(x')\right\}$$

Since $\widetilde{\mu}$ is an i-v fuzzy right R-subgroup of R,$\widetilde{\mu}(ab) \geq \widetilde{\mu}(b)$.

$$\leq \min^i\{\mathbf{R}(a), \widetilde{\mu}(ab), \mathbf{R}((x+z)y - xy)\} = \min^i\left\{\widetilde{1}, \widetilde{\mu}(ab), \widetilde{1}\right\} = \widetilde{\mu}(ab) = \widetilde{\mu}(x').$$

Therefore $\widetilde{\mu}$ is an i-v fuzzy quasi-ideal of R. □

Theorem 3.15. *Every i-v fuzzy R-subgroup of R is an i-v fuzzy quasi-ideal.*

Proof. The proof is straightforward from Theorem 3.13 and Theorem 3.14. □

The converse of the Theorem 3.15 is not true in general as shown in following Example.

Example 3.16. Let $R = \{0, a, b, c\}$ be a set with two binary operations $+$ and $\cdot$ is defined as follws:

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

·	0	a	b	c
0	0	a	0	a
a	0	a	0	a
b	0	a	b	c
c	0	a	b	c

Then clearly $(R, +, \cdot)$ is a left near-ring. Let $\widetilde{\mu} : R \to D[0,1]$ be an i-v fuzzy subset defined by $\widetilde{\mu}(0) = [0.7, 0.8], \widetilde{\mu}(a) = [0.2, 0.3] = \widetilde{\mu}(b)$ and $\widetilde{\mu}(d) = [0.4, 0.6]$.
Thus, $(\widetilde{\mu}\mathbf{R})(0) = [0.7, 0.8], (\mathbf{R}\widetilde{\mu})(0) = [0.7, 0.8], (\widetilde{\mu} * \mathbf{R})(0) = [0.7, 0.8]$,
$(\widetilde{\mu}\mathbf{R})(a) = [0.7, 0.8], (\mathbf{R}\widetilde{\mu})(a) = [0.4, 0.6], (\widetilde{\mu} * \mathbf{R})(a) = \overline{0}$,
$(\widetilde{\mu}\mathbf{R})(b) = [0.4, 0.6], (\mathbf{R}\widetilde{\mu})(b) = [0.4, 0.6], (\widetilde{\mu} * \mathbf{R})(b) = [0.4, 0.6]$, and
$(\widetilde{\mu}\mathbf{R})(c) = [0.4, 0.6], (\mathbf{R}\widetilde{\mu})(c) = [0.4, 0.6], (\widetilde{\mu} * \mathbf{R}(c) = \overline{0}$,

Hence $\widetilde{\mu}$ is an i-v fuzzy quasi-ideal of R. But $\widetilde{\mu}$ is not an i-v fuzzy R-subgroups of R, because $\widetilde{\mu}(0c) = \widetilde{\mu}(a) = [0.2, 0.3] < [0.4, 0.6] = \widetilde{\mu}(c)$ and $\widetilde{\mu}(ca) = \widetilde{\mu}(a) = [0.2, 0.3] < [0.4, 0.6] = \widetilde{\mu}(c)$.

Theorem 3.17. *Let $\widetilde{\mu}$ be an i-v fuzzy subset of R. Then $\widetilde{\mu} = [\mu^-, \mu^+]$ is an i-v fuzzy quasi-ideal of R if and only if μ^-, μ^+ are fuzzy quasi-ideals of R.*

Proof. Assume that $\widetilde{\mu}$ is an i-v fuzzy quasi-ideal of R. For any $x, y \in R$, we have

$$\begin{aligned}[\mu^-(x-y), \mu^+(x-y)] = \widetilde{\mu}(x-y) &\geq \min{}^i\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \\ &= \min{}^i\{[\mu^-(x), \mu^+(x)], [\mu^-(y), \mu^+(y)]\} \\ &= [\min\{\mu^-(x), \mu^-(y)\}, \min\{\mu^+(x), \mu^+(y)\}].\end{aligned}$$

It follows that $\mu^-(x-y) \geq \min\{\mu^-(x), \mu^-(y)\}$ and $\mu^+(x-y) \geq \min\{\mu^+(x), \mu^+(y)\}$. Thus $\widetilde{\mu}$ is an additive subgroup of R. Next,

$$\begin{aligned}[((\mu^-\mathbf{R}^-) \cap (\mathbf{R}^-\mu^-) \cap (\mu^- * \mathbf{R}^-))(x), ((\mu^+\mathbf{R}^+) \cap (\mathbf{R}^+\mu^+) \cap (\mu^+ * \mathbf{R}^+))(x)] \\ = ((\widetilde{\mu}\mathbf{R}) \cap (\mathbf{R}\widetilde{\mu}) \cap (\widetilde{\mu} * \mathbf{R}))(x) \\ \leq \widetilde{\mu}(x) = [\mu^-(x), \mu^+(x)].\end{aligned}$$

It follows that $((\mu^-\mathbf{R}^-) \cap (\mathbf{R}^-\mu^-) \cap (\mu^- * \mathbf{R}))(x) \leq \mu^-(x)$ and
$((\mu^+\mathbf{R}^+) \cap (\mathbf{R}^+\mu^+) \cap (\mu^+ * \mathbf{R}^+))(x) \leq \mu^+(x)$.
Therefore μ^- and μ^+ are fuzzy quasi-ideals of R.
Conversely, assume that μ^- and μ^+ are fuzzy quasi-ideals of R and $x \in R$
(i.e,)$(\mu^-\mathbf{R}^-) \cap (\mathbf{R}^-\mu^-) \cap (\mu^- * \mathbf{R}^-)(x) \leq \mu^-(x)$.
$((\mu^+\mathbf{R}^+) \cap (\mathbf{R}^+\mu^+) \cap (\mu^+ * \mathbf{R}^+))(x) \leq \mu^+(x)$.

$$\begin{aligned}&((\widetilde{\mu}\mathbf{R}) \cap (\mathbf{R}\widetilde{\mu}) \cap (\widetilde{\mu} * \mathbf{R}))(x) \\ &\quad = [(\mu^-\mathbf{R}^- \cap \mathbf{R}^-\mu^- \cap \mu^- * \mathbf{R}^-)(x), (\mu^+\mathbf{R}^+ \cap \mathbf{R}^+\mu^+ \cap \mu^+ * \mathbf{R}^+)(x)] \\ &\quad \leq [\mu^-(x), \mu^+(x)] = \widetilde{\mu}(x).\end{aligned}$$

Therefore $\widetilde{\mu}$ is an i-v fuzzy quasi-ideal of R. □

Theorem 3.18. *Let $\{\widetilde{\mu}_i : i \in \Omega\}$ be any family of i-v fuzzy quasi-ideals of R. Then $\mu = \bigcap_{i\in\Omega} \widetilde{\mu}_i$ is also an i-v fuzzy quasi ideal of R, where Ω is an index set.*

Proof. Let $\{\widetilde{\mu}_i : i \in \Omega\}$ be i-v fuzzy quasi-ideals of R. Let $x, y \in R$. By Theorem 3.1 of [14], μ is an i-v fuzzy subgroup of R. Since $\widetilde{\mu} = \bigcap_{i\in\Omega} \widetilde{\mu}_i \subseteq \widetilde{\mu}_i$, for every $i \in \Omega$. Let $x \in R$. Then

$$\begin{aligned}(\widetilde{\mu}\mathbf{R} \cap \mathbf{R}\widetilde{\mu} \cap \widetilde{\mu} * \mathbf{R})(x) &\leq (\widetilde{\mu}_i\mathbf{R}_i \cap \mathbf{R}_i\widetilde{\mu}_i \cap \widetilde{\mu}_i * \mathbf{R}_i)(x) \\ &\qquad \text{since } \widetilde{\mu}_i \text{ is an i-v fuzzy quasi-ideals of } R. \\ &\leq \widetilde{\mu}_i(x), \text{for every } i \in \Omega.\end{aligned}$$

This implies that
$(\widetilde{\mu}\mathbf{R} \cap \mathbf{R}\widetilde{\mu} \cap \widetilde{\mu} * \mathbf{R})(x) \leq \inf{}^i\{\widetilde{\mu}_i(x) : i \in \Omega\} = \left(\bigcap_{i\in\Omega} \widetilde{\mu}_i\right)(x) = \widetilde{\mu}(x)$.
Thus, $(\widetilde{\mu}\mathbf{R} \cap \mathbf{R}\widetilde{\mu} \cap \widetilde{\mu} * \mathbf{R}) \subseteq \widetilde{\mu}$. Hence $\mu = \bigcap_{i\in\Omega} \widetilde{\mu}_i$ is an i-v fuzzy quasi-ideal of R. □

Theorem 3.19. *Let R be a zero-symmetric near-ring and $\widetilde{\mu}$ be an i-v fuzzy subgroup of R. Then the following conditions are equivalent:*
(i) $\widetilde{\mu}$ is an i-v fuzzy quasi-ideal of R.
(ii) $\widetilde{\mu}\mathbf{R} \cap \mathbf{R}\widetilde{\mu} \subseteq \widetilde{\mu}$.

Proof. Let R be a zero-symmetric near-ring and $\widetilde{\mu}$ be an i-v fuzzy subgroup of R.

(i)⇒(ii): Assume that $\widetilde{\mu}$ be an i-v fuzzy quasi-ideal of R. This implies that $\widetilde{\mu}\mathbf{R}\cap\mathbf{R}\widetilde{\mu}\cap\widetilde{\mu}*\mathbf{R} \subseteq \widetilde{\mu}$. Since $\widetilde{\mu}$ is an i-v fuzzy subgroup of R, then we have $\widetilde{\mu}(0) \geq \widetilde{\mu}(x)$, for all $x \in R$. Clearly, $\mathbf{R}(x) = \widetilde{1}$, for all $x \in R$. Then $(\widetilde{\mu}\mathbf{R})(0) \geq (\widetilde{\mu}\mathbf{R})(x)$, for all $x \in R$. By our assumption R is a zero-symmetric near-ring, then $\widetilde{\mu}\mathbf{R} \cap \mathbf{R}\widetilde{\mu} \subseteq \widetilde{\mu} * \mathbf{R}$. It is clear that $\widetilde{\mu}\mathbf{R} \cap \mathbf{R}\widetilde{\mu} \subseteq \widetilde{\mu}$.

(ii)⇒(i): Let $x \in R$. Then

$$\begin{aligned}(\widetilde{\mu}\mathbf{R} \cap \mathbf{R}\widetilde{\mu} \cap \widetilde{\mu} * \mathbf{R})(x) &= \min{}^i\{(\widetilde{\mu}\mathbf{R})(x), (\mathbf{R}\widetilde{\mu})(x), (\widetilde{\mu} * \mathbf{R})(x)\} \\ &\leq \min{}^i\{(\widetilde{\mu}\mathbf{R})(x), (\mathbf{R}\widetilde{\mu})(x)\} \\ &\leq (\widetilde{\mu}\mathbf{R} \cap \mathbf{R}\widetilde{\mu})(x) \leq \widetilde{\mu}(x).\end{aligned}$$

Hence $\widetilde{\mu}\mathbf{R} \cap \mathbf{R}\widetilde{\mu} \cap \widetilde{\mu} * \mathbf{R} \subseteq \widetilde{\mu}$ and $\widetilde{\mu}$ is an i-v fuzzy quasi-ideal of R. □

4. Aknowledgement

The authors are highly grateful to referees for their valuable comments and suggestions for improving the paper. The second author was supported in part by UGC-BSR Grant # F4-1/2006(BSR)/7-254/2009(BSR) in India.

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