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Intuitionistic fuzzy parameterized intuitionistic fuzzy soft sets with applications in decision making

Faruk Karaaslan

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Abstract. In this work, we present definition of intuitionistic fuzzy parameterized (ifp) intuitionistic fuzzy soft set and its operations. Then we define ifp-aggregation operator to form ifp-intuitionistic fuzzy soft decision making method which allows constructing decision processes.

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Corresponding Author: Faruk Karaaslan (`fkaraaslan@karatekin.edu.tr`)

1. Introduction

In the real world, we face many problems involving uncertainty and vagueness related to areas such as economics, engineering, environment, social sciences, medical sciences and business management. Uncertain data in these applications could be caused by complexities and difficulties in classical mathematical modeling. Therefore, researchers need new approaches for mathematical modeling. Some of these new approaches are fuzzy set theory introduced by Zadeh [44], intuitionistic fuzzy set theory introduced by Atanassov [4] as a generalized of fuzzy set theory, soft set theory [38], intuitionistic fuzzy soft set theory [4], fuzzy soft set theory [13, 34], neutrosophic soft set theory [33] and so on. Recently, the properties and applications on the neutrosophic soft set theory have been studied extensively (see [15, 17, 18, 19, 20]).

The idea of the soft set which is completely a new approach for modeling fuzzy, not clearly defined objects, was introduced by Molodtsov [38], in 1999. Then, Maji et al.[36] defined several operation on soft sets and made a theoretical study on the theory of soft set. Soft set theory has a rich potential for application in solving practical problems in economics, social science, medical science etc. Many interesting results of soft set theory have been studied by embedding the ideas of fuzzy sets and intuitionistic fuzzy set. Some of them are fuzzy soft sets [7, 6, 8, 13, 21, 26, 34, 37, 39] and intuitionistic fuzzy soft sets [14, 23, 24, 31, 35, 40, 41]. The algebraic structures of soft sets were first studied by Aktaş and Çağman [2]. Aktaş and Çağman introduced the basic properties of soft sets to the related concept of fuzzy sets as well as rough sets. Then, they gave a definition of soft group and derived the basic properties. The algebraic structure of soft set theory has also been studied in detail [1, 3, 5, 22, 25, 26, 27, 28, 30, 32, 42, 43, 45, 46, 47, 48]. In 2011, Çağman et al. defined concepts of fuzzy parameterized fuzzy soft set [8] and fuzzy parameterized soft set [9] by assigning fuzzy values to the elements of parameter set, respectively. In 2012, Çağman and Deli [10] defined t-norm and t-conorm products of fuzzy parameterized soft sets (FP-soft sets) and proposed two decision making methods called $AND-FP-$soft decision making method and $OR-FP-$soft decision making method. Also they defined means of FP-soft sets [11]. Intuitionistic fuzzy parameterized soft set was defined by Çağman and Deli [16] and intuitionistic fuzzy parameterized fuzzy soft set and its operations were introduced by Karaaslan [29].

In this study, we take parameter set of an intuitionistic fuzzy soft as an intuitionistic fuzzy set and so we get a generalization of the soft set [38], fuzzy soft set [34], intuitionistic fuzzy soft set [35], fuzzy parameterized soft set [13], fuzzy parameterized fuzzy soft set [8], intuitionistic fuzzy soft set [14], intuitionistic fuzzy parameterized soft set [16] and intuitionistic fuzzy parameterized fuzzy soft set [29].

From the discussion above, initially we present some basic definitions required in next sections of the paper in section 2. Then we define concept of intuitionistic fuzzy parameterized (ifp) intuitionistic fuzzy soft set. After we define set theoretical operations of the $(ifp)-$intuitionistic fuzzy soft sets, we investigate some properties of these operations.We also define ifp-aggregation operator to form ifp-intuitionistic fuzzy soft decision making method that allows constructing more efficient decision processes. We finally present an example which shows that the method can be successfully applied to many problems that contain uncertainties.

2. Preliminaries

In this section, we present definitions and some results of soft set, fuzzy set, fuzzy soft set, intuitionistic fuzzy set and intuitionistic fuzzy soft set theory that can be found details [4, 8, 12, 24, 34, 36, 38, 44].

Throughout this subsection U refers to an initial universe, E be the set of parameters, $P(U)$ is the power set of U.

Definition 2.1 ([12])**.** Let U be an initial universe, $P(U)$ be the power set of U, E be the set of all parameters and $A \subseteq E$. Then, a soft set F_A on the universe U is defined by a function f_A representing a mapping

$$f_A : E \to P(U) \text{ such that } f_A(x) = \varnothing \text{ if } x \notin A$$

Here, f_A is called approximate function of soft set F_A, and the value $f_A(x)$ is a set called x-element of the soft set for all $x \in E$. It is worth nothing that the set $f_A(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. Thus, a soft set F_A over U can be represented by the set of ordered pairs

$$F_A = \{(x, f_A) : x \in E, f_A(x) \in P(U)\}$$

Note that the set of all soft sets over U will be denoted by $S(U)$.

Definition 2.2 ([44]). Let U be an initial universal set. A fuzzy set X over U is a set defined by a function μ_X representing a mapping

$$\mu_X : U \to [0, 1]$$

Here, μ_X called membership function of X and the value $\mu_X(u)$ is called the grade of membership of $u \in U$. The value represents the degree of u belonging to fuzzy set X. Thus, a fuzzy set X over U can be represented as follows,

$$X = \{(u, \mu_X(u)) : u \in U, \mu_X(u) \in [0, 1]\}$$

Note that the set of all the fuzzy sets over U will be denoted by $F(U)$.

Definition 2.3 ([4]). An intuitionistic fuzzy set (IFS) X over U is defined as an object of the following form

$$X = \{(x, \mu_X(u), \nu_X(u)) : u \in U\},$$

where the functions $\mu_X : U \to [0, 1]$ and $\nu_X : U \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $u \in U$, respectively, and for every $u \in U$,

$$0 \le \mu_X(u) + \nu_X(u) \le 1.$$

In addition for all $u \in U$, $U = \{(u, 1, 0) : u \in U\}$, $\varnothing = \{(u, 0, 1) : u \in U\}$ are intuitionistic fuzzy universal and intuitionistic fuzzy empty set, respectively.

Theorem 2.4 ([4]). *Let X and Y be two intuitionistic fuzzy sets. Then*

(1) $X \subseteq Y \Leftrightarrow \forall u \in U, \mu_X(u) \le \mu_Y(u), \nu_X(u) \ge \nu_Y(u)$.
(2) $X \cap Y = \{(u, min\{\mu_X(u), \mu_Y(u)\}, max\{\nu_X(u), \nu_Y(u)\}) : u \in U\}$.
(3) $X \cup Y = \{(u, max\{\mu_X(u), \mu_Y(u)\}, min\{\nu_X(u), \nu_Y(u)\}) : u \in U\}$.
(4) $X^c = \{(u, \nu_X(u), \mu_X(u)) : u \in U\}$.

Note that the set of all the intuitionistic fuzzy sets over U will be denoted by $\mathcal{IF}(U)$.

Definition 2.5 ([14]). Let U be an initial universe, $\mathcal{IF}(U)$ be the set of all intuitionistic fuzzy sets over U, E be a set of all parameters and $A \subseteq E$. Then, an intuitionistic fuzzy soft set (IFS-set) γ_A over U is a function from E into $\mathcal{IF}(U)$.

Where, the value $\gamma_A(x)$ is an intuitionistic fuzzy set over U. That is, $\gamma_A(x) = \{(u, \overline{\gamma}_{A(x)}(u), \underline{\gamma}_{A(x)}(u)) : x \in E, u \in U)\}$, where $\overline{\gamma}_{A(x)}(u)$ and $\underline{\gamma}_{A(x)}(u)$ are the membership and non-membership degrees of u to the parameter x, respectively.

Note that, the set of all intuitionistic fuzzy soft sets over U is denoted by $\mathcal{IFS}(U)$.

Definition 2.6 ([14]). Let $A, B \subseteq E$, γ_A and γ_B be two IFS-sets. Then, γ_A is said to be an intuitionistic fuzzy soft subset of γ_B if

(1) $A \subseteq B$ and
(2) $\gamma_A(x)$ is an intuitionistic fuzzy subset of $\gamma_B(x)$ $\forall x \in A$.

This relationship is denoted by $\gamma_A \tilde{\subseteq} \gamma_B$. Similarly, γ_A is said to be an intuitionistic fuzzy soft superset of γ_B, if γ_B is an intuitionistic fuzzy soft subset of γ_A and denoted by $\gamma_A \tilde{\supseteq} \gamma_B$.

Definition 2.7 ([14]). Let γ_A and γ_B be two intuitionistic fuzzy soft sets over $\mathcal{IF}(U)$. Then, γ_A and γ_B are said to be intuitionistic fuzzy soft equal if and only if γ_A is an intuitionistic fuzzy soft subset of γ_B and γ_B is an intuitionistic fuzzy soft subset of γ_A, and written by $\gamma_A \tilde{=} \gamma_B$.

Definition 2.8 ([14]). Let γ_A be an IFS-set over $\mathcal{IF}(U)$. If $\gamma_A(x) = \varnothing$ for all $x \in E$, then γ_A is called empty IFS-set and denoted by γ_ϕ.

Definition 2.9 ([14]). Let γ_A be an IFS-set over $\mathcal{IF}(U)$. If $\gamma_A(x) = \{(u, 1, 0) : \forall u \in U\}$ for all $x \in A$, then γ_A is called A-universal IFS-set and denoted by $\gamma_{\hat{A}}$.

If A=E, then the A-universal IFS-set is called universal IFS-set and denoted by $\gamma_{\hat{E}}$.

Definition 2.10 ([14]). Let γ_A and γ_B be two IFS-sets over $\mathcal{IF}(U)$. Union of γ_A and γ_B, denoted by $\gamma_A \tilde{\cup} \gamma_B$, and is defined by

$$\gamma_A \tilde{\cup} \gamma_B = \{(x, \gamma_{A \tilde{\cup} B}(x)) : x \in E\}$$

where

$$\gamma_{A \tilde{\cup} B}(x) = \{(u, max\{\overline{\gamma}_{A(x)}(u), \overline{\gamma}_{B(x)}(u)\}, min\{\underline{\gamma}_{A(x)}(u), \underline{\gamma}_{B(x)}(u)\}) : u \in U\}.$$

Definition 2.11 ([14]). Let γ_A and γ_B be two IFS-sets over $\mathcal{IF}(U)$. Intersection of γ_A and γ_B, denoted by $\gamma_A \tilde{\cap} \gamma_B$, and is defined by

$$\gamma_A \tilde{\cap} \gamma_B = \{(x, \gamma_{A \tilde{\cap} B}(x)) : x \in E\}$$

where

$$\gamma_{A \tilde{\cap} B}(x) = \{(u, min\{\overline{\gamma}_{A(x)}(u), \overline{\gamma}_{B(x)}(u)\}, max\{\underline{\gamma}_{A(x)}(u), \underline{\gamma}_{B(x)}(u)\}) : u \in U\}.$$

Definition 2.12 ([14]). Let γ_A be an IFS-set over $\mathcal{IF}(U)$. Complement of γ_A, denoted by γ_A^c, and is defined by

$$\gamma_A^c = \{(x, \gamma_{A^c}(x)) : x \in E\}$$

where $\gamma_{A^c}(x) = \gamma_A^c(x)$ is the complement of intuitionistic fuzzy set $\gamma_A(x)$, defined by

$$\gamma_A^c(x) = \{(u, \underline{\gamma}_{A(x)}(u), \overline{\gamma}_{A(x)}(u)) : u \in U\}$$

for all $x \in E$.

Definition 2.13 ([14]). Let γ_A and γ_B be two IFS-sets over $\mathcal{IF}(U)$. $\wedge-$product of γ_A and γ_B, denoted by $\gamma_A \wedge \gamma_B$, and is defined by

$$\gamma_A \wedge \gamma_B = \{((x, y), \gamma_{A \wedge B}(x, y)) : (x, y) \in E \times E\}$$

where

$$\gamma_{A \wedge B}(x, y)) = \{(u, min(\mu_{\gamma_{A(x)}}(u), \mu_{\gamma_{B(x)}}(u))), max(\nu_{\gamma_{A(x)}}(u), \nu_{\gamma_{B(x)}}(u))) : u \in U\}$$

for all $x, y \in E$.

Definition 2.14 ([14]). Let γ_A and γ_B be two IFS-sets over $\mathcal{IF}(U)$. $\vee-$product of γ_A and γ_B, denoted by $\gamma_A \vee \gamma_B$, and is defined by

$$\gamma_A \vee \gamma_B = \{((x, y), \gamma_{A \vee B}(x, y)) : (x, y) \in E \times E\}$$

where

$$\gamma_{A\vee B}(x,y)) = \{(u, max(\mu_{\gamma_{A(x)}}(u), \mu_{\gamma_{B(x)}}(u))), min(\nu_{\gamma_{A(x)}}(u), \nu_{\gamma_{B(x)}}(u))) : u \in U\}$$

for all $x, y \in E$.

Definition 2.15 ([8])**.** Let U be an initial universe, E be the set of all parameters and X be a fuzzy set over E with the membership function $\mu_X : E \to [0,1]$ and $\gamma_X(x)$ be a fuzzy set over U for all $x \in E$. Then, and $fpfs-$set Γ_X over U is a set defined by a function $\gamma_X(x)$ representing a mapping

$$\gamma_X : E \to F(U) \text{ such that } \gamma_X(x) = \varnothing \text{ if } \mu_X(x) = 0$$

Here, γ_X is called fuzzy approximate function of ($fpfs$-set) Γ_X, and the value $\gamma_X(x)$ is a fuzzy set called $x-$element of the $fpfs-$set for all $x \in E$. Thus, an $fpfs-$set Γ_X over U can be represented by the set of ordered pairs

$$\Gamma_X = \{(\mu_X(x)/x, \gamma_X(x)) : x \in E, \gamma_X(x) \in F(U), \mu_X(x) \in [0,1]\},$$

It must be noted that the sets of all $fpfs$-sets over U will be denoted by $FPFS(U)$.

Definition 2.16 ([16])**.** Let U be an initial universe, $P(U)$ be the power set of U, E is the set of all parameters and X be a intuitionistic fuzzy set over E with the membership function $\mu_X : E \to [0,1]$ and non-membership function $\nu_X : E \to [0,1]$. Then, an $ifps-$set F_X over U is a set defined by a function f_X representing a mapping

$$f_X : E \to P(U) \text{ such that } f_X(x) = \varnothing \text{ if } \mu_X(x) = 0, \nu_X(x) = 1$$

Here, f_X is called approximate function of the $ifps$-set F_X, and the value $f_X(x)$ is a set called $x-$element of $ifps$-set for all $x \in E$. Thus, $ifps-$set F_X over U can be represented by the set of ordered pairs

$$F_X = \{((\mu_X(x), \nu_X(x))/x, f_X(x)) : x \in E, f_X(x) \in P(U), \mu_X(x), \nu_X(x) \in [0,1]\}$$

3. Intuitionistic fuzzy parameterized (Ifp) intuitionistic fuzzy soft sets

In this section, we define intuitionistic fuzzy parameterized intuitionistic fuzzy soft sets and their operations with examples.

Throughout this work, we use $\Omega_X, \Omega_Y, \Omega_Z, ...$,etc. for ifp-intuitionistic fuzzy soft sets and $\omega_X, \omega_Y, \omega_Z, ...$,etc. for their intuitionistic fuzzy approximate functions, respectively.

Definition 3.1. Let U be an initial universe, E be the set of all parameters and X be an intuitionistic fuzzy set over E with the membership function $\mu_X : E \to [0,1]$ and non-membership function $\nu_X : E \to [0,1]$. An ifp-intuitionistic fuzzy soft set Ω_X over $\mathcal{IF}(U)$ is defined as follows:

$$\Omega_X = \{\big((\mu_X(x), \nu_X(x))/x, \omega_X(x)\big) : x \in E, \omega_X(x) \in \mathcal{IF}(U)\},$$

where $\omega_X : E \to \mathcal{IF}(U)$ and $\omega_X(x)$ is called intuitionistic fuzzy approximation of ifp-intuitionistic fuzzy soft set Ω_X.

Note that, If $\mu_X(x) = 0, \nu_X(x) = 1$ and $\omega_X(x) = \varnothing$, we don't display such elements in the ifp-intuitionistic fuzzy soft set. Also, it must be noted that the sets of all ifp-intuitionistic fuzzy soft set Ω_X over $\mathcal{IF}(U)$ will be denoted by $\Omega(U)$.

From now on, to be more comprehensible we will use $\omega_X((\mu(x), \nu(x))/x)$ instead of the notation $\omega_X(x)$ for all $x \in E$.

Example 3.2. Assume that $U = \{u_1, u_2\, u_3, u_4, u_5\}$ is a universal set and $E = \{x_1, x_2, x_3, x_4\}$ a set of parameters. If
$X = \{(0.5, 0.2)/x_1, (0.6, 0.3)/x_3, (1.0, 0.0)/x_4\}$,
$\omega_X((0.5, 0.2)/x_1) = \{(0.7, 0.2)/u_1, (0.5, 0.4)/u_4\}$,
$\omega_X((0.0, 1.0)/x_2) = \varnothing$,
$\omega_X((0.6, 0.3)/x_3) = \{(0.4, 0.3)/u_2, (0.8, 0.1)/u_3, (0.6, 0.3)/u_5\}$,
$\omega_X((1.0, 0.0)/x_4) = U$,
then the Ω_X is written as follow

$$\begin{array}{rcl}\Omega_X &=& \{((0.5, 0.2)/x_1, \{(0.7, 0.2)/u_1, (0.5, 0.4)/u_4\}),\\ && ((0.6, 0.3)/x_3, \{(0.4, 0.3)/u_2, (0.8, 0.1)/u_3, (0.6, 0.3)/u_5\}),\\ && ((1.0, 0.0)/x_4, U)\}.\end{array}$$

Definition 3.3. Let $\Omega_X \in \Omega(\mathrm{U})$. If $\omega_X((\mu(x), \nu(x))/x) = \varnothing$ for all $(\mu(x), \nu(x))/x \in X$, then Ω_X is called an X-empty ifp-intuitionistic fuzzy soft set, denoted by $\Omega_{\varnothing_X}$. If $X = \varnothing$, then the $X-$empty ifp-intuitionistic fuzzy soft set ($\Omega_{\varnothing_X}$) is called empty ifp-intuitionistic fuzzy soft set, denoted by $\Omega_{\varnothing}$. Here, $\varnothing$ mean that intuitionistic fuzzy empty set.

Definition 3.4. Let $\Omega_X \in \Omega(\mathrm{U})$. If $\mu_X(x) = 1$, $\nu_X(x) = 0$ and $\omega_X((\mu(x), \nu(x))/x) = U$ for all $(\mu(x), \nu(x))/x \in X$, then Ω_X is called X-universal ifp-intuitionistic fuzzy soft set, denoted by $\Omega_{\tilde{X}}$.

If X is equal to intuitionistic fuzzy universal set over E, then the X-universal ifp-intuitionistic fuzzy soft set is called universal ifp-intuitionistic fuzzy soft set, denoted by $\Omega_{\tilde{E}}$. Here, U mean that intuitionistic fuzzy universal set.

Example 3.5. Let $U = \{u_1, u_2, u_3, u_4\}$ be a universal set and $E = \{x_1, x_2, x_3, x_4\}$ be a set of parameters. If

$$X = \{(0.2, 0.5)/x_2, (0.5, 0.3)/x_3, (1.0, 0.0)/x_4\} \text{ and}$$

$$\begin{array}{rcl}\omega_X((0.0, 1.0)/x_1) &=& \varnothing,\\ \omega_X((0.2, 0.5)/x_2) &=& \{(0.5, 0.4)/u_1, (0.7, 0.3)/u_5\},\\ \omega_X((0.5, 0.3)/x_3) &=& \varnothing,\\ \omega_X((1.0, 0.0)/x_4) &=& U,\end{array}$$

then the ifp-intuitionistic fuzzy soft set Ω_X is written by

$$\begin{array}{rcl}\Omega_X &=& \{((0.2, 0, 5)/x_2, \{(0.5, 0.4)/u_1, (0, 1.0)/u_2, (0, 1.0)/u_3,\\ && (0.7, 0.3)/u_4\}), ((0, 5.0, 3)/x_3, \varnothing), ((1.0, 0)/x_4, U)\}.\end{array}$$

If $Y = \{(1.0, 0)/x_1, (0.7, 0.2)/x_4\}$ and $\omega_Y((1.0, 0)/x_1) = \varnothing$, $\omega_Y((0.7, 0.2)/x_4) = \varnothing$ then the ifp-intuitionistic fuzzy soft set Ω_Y is an Y-empty ifp-intuitionistic fuzzy soft set, i.e., $\Omega_Y = \Omega_{\Phi_Y}$.

If $Z = \{(1.0, 0)/x_1, (1.0, 0)/x_2\}$, $\omega_Z((1.0, 0)/x_1) = U$, and $\omega_Z((1.0, 0)/x_2) = U$, then the ifp-intuitionistic fuzzy soft set Ω_Z is Z-universal ifp-intuitionistic fuzzy soft sets, i.e., $\Omega_Z = \Omega_{\tilde{Z}}$.

If $X = \gamma_E$ and $\omega_X((\mu(x_i), \nu(x_i))/x_i) = U$ for all $x_i \in \gamma_E$, where $i = 1, 2, 3, 4$, then the ifp-intuitionistic fuzzy soft set Ω_X is a universal ifp-intuitionistic fuzzy soft set, i.e., $\Omega_X = \Omega_{\tilde{E}}$.

Definition 3.6. Let $\Omega_X, \Omega_Y \in \Omega(\mathrm{U})$. Then, Ω_X is an ifp-intuitionistic fuzzy soft subset of Ω_Y, denoted by $\Omega_X \tilde{\subseteq} \Omega_Y$, if $\mu_X(x) \leq \mu_Y(x), \nu_X(x) \geq \nu_Y(x)$ and $\omega_X((\mu(x), \nu(x))/x) \subseteq \omega_Y((\mu(x), \nu(x))/x)$ for all $x \in E$.

Proposition 3.7. *Let $\Omega_X, \Omega_Y \in \Omega(U)$. Then*

i. $\Omega_X \tilde{\subseteq} \Omega_{\tilde{E}}$.
ii. $\Omega_{\Phi_X} \tilde{\subseteq} \Omega_X$.
iii. $\Omega_\Phi \tilde{\subseteq} \Omega_X$.
iv. $\Omega_X \tilde{\subseteq} \Omega_X$.
v. $\Omega_X \tilde{\subseteq} \Omega_Y$ *and* $\Omega_Y \tilde{\subseteq} \Omega_Z \Rightarrow \Omega_X \tilde{\subseteq} \Omega_Z$.

Proof. The proofs can be proved easily by using the intuitionistic fuzzy approximate and membership functions of the ifp-intuitionistic fuzzy soft sets. □

Definition 3.8. Let $\Omega_X, \Omega_Y \in \Omega(U)$. Then, Ω_X and Ω_Y are ifp intuitionistic fuzzy soft equal, written as $\Omega_X = \Omega_Y$, if and only if $\mu_X(x) = \mu_Y(x)$, $\nu_X(x) = \nu_Y(x)$ and $\omega_X((\mu(x), \nu(x))/x) = \omega_Y((\mu(x), \nu(x))/x)$ for all $x \in E$.

Proposition 3.9. *Let $\Omega_X, \Omega_Y, \Omega_Z \in \Omega(U)$. Then*

i. $\Omega_X = \Omega_Y$ *and* $\Omega_Y = \Omega_Z \Leftrightarrow \Omega_X = \Omega_Z$.
ii. $\Omega_X \tilde{\subseteq} \Omega_Y$ *and* $\Omega_Y \tilde{\subseteq} \Omega_X \Leftrightarrow \Omega_X = \Omega_Y$.

Proof. The proofs are trivial. □

Definition 3.10. Let $\Omega_X \in \Omega(\mathrm{U})$. Then, the complement of Ω_X, denoted by $\Omega_X^{\tilde{c}}$, is defined by

$$\Omega_X^{\tilde{c}} = \{((\nu_X(x), \mu_X(x))/x, \omega_X^c((\mu(x), \nu(x))/x)) : x \in E, \omega_X^c((\mu(x), \nu(x))/x) \in \mathcal{IF}(U)\}$$

where $\omega_X^c(x)$ is complement of the intuitionistic fuzzy set $\omega_X(x)$, that is, $\omega_X^c((\mu(x), \nu(x))/x) = \omega_{X^c}((\mu(x), \nu(x))/x)$ for every $x \in E$.

Proposition 3.11. *Let $\Omega_X \in \Omega(U)$. Then*

i. $(\Omega_X^{\tilde{c}})^{\tilde{c}} = \Omega_X$.
ii. $\Omega_\Phi^{\tilde{c}} = \Omega_{\tilde{E}}$.

Proof. i. Let $\Omega_X \in \Omega(U)$. Then, from Definition 3.10, we know that

$$\Omega_X^{\tilde{c}} = \{((\nu_X(x), \mu_X(x))/x, \omega_X^c((\mu(x), \nu(x))/x)) : x \in E, \omega_X^c((\mu(x), \nu(x))/x) \in \mathcal{IF}(U)\}.$$

Since $(\omega_X^c)^c(x) = \omega_X((\mu(x), \nu(x))/x)$,

$$\begin{aligned} (\Omega_X^{\tilde{c}})^{\tilde{c}} &= \{((\mu_X(x), \nu_X(x))/x, (\omega_X^c)^c((\mu(x), \nu(x))/x)) : x \in E, \\ &\quad (\omega_X^c)^c((\mu(x), \nu(x))/x) \in \mathcal{IF}(U)\} \\ &= \Omega_X. \end{aligned}$$

ii. From Definition 3.3, $\Omega_\Phi = \{((0,1)/x, \emptyset) : x \in E, \emptyset \in \mathcal{IF}(U)\}$. By Definition 3.10, $\Omega_\Phi^{\tilde{c}} = \{((1,0)/x, U) : x \in E, U \in \mathcal{IF}(U)\}$. We have that $\Omega_\Phi^{\tilde{c}} = \Omega_{\tilde{E}}$. □

Definition 3.12. Let $\Omega_X, \Omega_Y \in \Omega(U)$. Then, union of Ω_X and Ω_Y, denoted by $\Omega_X \tilde{\cup} \Omega_Y$, is defined by

$$\Omega_X \tilde{\cup} \Omega_Y = \{((\mu_{X\tilde{\cup}Y}(x), \nu_{X\tilde{\cap}Y}(x)/x, \omega_{X\tilde{\cup}Y}((\mu(x), \nu(x))/x) : x \in E\}.$$

Here,

$$\mu_{X\tilde{\cup}Y}(x) = \max\{\mu_X(x), \mu_Y(x)\}, \nu_{X\tilde{\cup}Y}(x) = \min\{\nu_X(x), \nu_Y(x)\} \text{ and}$$

$$\omega_{X\tilde{\cup}Y}((\mu(x), \nu(x))/x) = \omega_X((\mu(x), \nu(x))/x) \cup \omega_Y((\mu(x), \nu(x))/x), \text{ for all } x \in E.$$

Proposition 3.13. *Let $\Omega_X, \Omega_Y, \Omega_Z \in \Omega(U)$. Then*

i. $\Omega_X \tilde{\cup} \Omega_X = \Omega_X$.
ii. $\Omega_X \tilde{\cup} \Omega_\Phi = \Omega_X$.
iii. $\Omega_X \tilde{\cup} \Omega_{\tilde{E}} = \Omega_{\tilde{E}}$.
iv. $\Omega_X \tilde{\cup} \Omega_Y = \Omega_Y \tilde{\cup} \Omega_X$.
v. $(\Omega_X \tilde{\cup} \Omega_Y) \tilde{\cup} \Omega_Z = \Omega_X \tilde{\cup} (\Omega_Y \tilde{\cup} \Omega_Z)$.

Proof. The proofs can be easily obtained from Definition 3.12. □

Definition 3.14. Let $\Omega_X, \Omega_Y \in \Omega(U)$. Then, intersection of Ω_X and Ω_Y, denoted by $\Omega_X \tilde{\cap} \Omega_Y$, is defined by

$$\Omega_X \tilde{\cap} \Omega_Y = \{((\mu_{X\tilde{\cap}Y}(x), \nu_{X\tilde{\cup}Y}(x))/x, \omega_{X\tilde{\cap}Y}((\mu(x), \nu(x))/x) : x \in E\}.$$

Here,

$$\mu_{X\tilde{\cap}Y}(x) = \min\{\mu_X(x), \mu_Y(x)\}, \nu_{X\tilde{\cap}Y}(x) = \max\{\nu_X(x), \nu_Y(x)\}$$

and

$$\omega_{X\tilde{\cap}Y}((\mu(x), \nu(x))/x) = \omega_X((\mu(x), \nu(x))/x) \cap \omega_Y((\mu(x), \nu(x))/x) \text{ for all } x \in E.$$

Note that here $\omega_X((\mu(x), \nu(x))/x)$ and $\omega_Y((\mu(x), \nu(x))/x)$ are intuitionistic fuzzy set. Thus, in operations of between $\omega_X((\mu(x), \nu(x))/x)$ and $\omega_Y((\mu(x), \nu(x))/x)$, we use the operations of intuitionistic fuzzy sets.

Proposition 3.15. *Let $\Omega_X, \Omega_Y, \Omega_Z \in \Omega(U)$. Then*

i. $\Omega_X \tilde{\cap} \Omega_X = \Omega_X$.
ii. $\Omega_X \tilde{\cap} \Omega_\Phi = \Omega_\Phi$.
iii. $\Omega_X \tilde{\cap} \Omega_{\tilde{E}} = \Omega_X$.
iv. $\Omega_X \tilde{\cap} \Omega_Y = \Omega_Y \tilde{\cap} \Omega_X$.
v. $(\Omega_X \tilde{\cap} \Omega_Y) \tilde{\cap} \Omega_Z = \Omega_X \tilde{\cap} (\Omega_Y \tilde{\cap} \Omega_Z)$.

Proof. The proofs can be easily obtained from Definition 3.14. □

Remark 3.16. Let $\Omega_X \in \Omega(U)$. If $\Omega_X \neq \Omega_\Phi$ or $\Omega_X \neq \Omega_{\tilde{E}}$, then $\Omega_X \tilde{\cup} \Omega_X^{\tilde{c}} \neq \Omega_{\tilde{E}}$ and $\Omega_X \tilde{\cap} \Omega_X^{\tilde{c}} \neq \Omega_\Phi$.

Proposition 3.17. *Let $\Omega_X, \Omega_Y \in \Omega(U)$. Then De Morgan's laws are valid*

i. $(\Omega_X \tilde{\cup} \Omega_Y)^{\tilde{c}} = \Omega_X^{\tilde{c}} \tilde{\cap} \Omega_Y^{\tilde{c}}$.
ii. $(\Omega_X \tilde{\cap} \Omega_Y)^{\tilde{c}} = \Omega_X^{\tilde{c}} \tilde{\cup} \Omega_Y^{\tilde{c}}$.

Proof. i. Firstly, for all $x \in E$,

$$\begin{aligned}
\omega_{(X\widetilde{\cup}Y)^{\tilde{c}}}((\mu(x),\nu(x))/x) &= \omega^c_{X\widetilde{\cup}Y}((\mu(x),\nu(x))/x) \\
&= (\omega_X((\mu(x),\nu(x))/x) \cup \omega_Y((\mu(x),\nu(x))/x))^c \\
&= (\omega_X((\mu(x),\nu(x))/x))^c \cap (\omega_Y((\mu(x),\nu(x))/x))^c \\
&= \omega^c_X((\mu(x),\nu(x))/x) \cap \omega^c_Y((\mu(x),\nu(x))/x) \\
&= \omega_{X^{\tilde{c}}}((\mu(x),\nu(x))/x) \cap \omega_{Y^{\tilde{c}}}((\mu(x),\nu(x))/x) \\
&= \omega_{X^{\tilde{c}}\widetilde{\cap}Y^{\tilde{c}}}((\mu(x),\nu(x))/x),
\end{aligned}$$

$$\begin{aligned}
\Omega_Y = \{(\max\{\mu_X(x),\mu_Y(x)\}, \min\{\nu_X(x),\nu_Y(x)\}))/x, \\
\omega_{X\widetilde{\cup}Y}((\mu(x),\nu(x))/x) : x \in E\}
\end{aligned}$$

and

$$\begin{aligned}
(\Omega_X\widetilde{\cup}\Omega_Y)^{\tilde{c}} &= \{((\min\{\nu_X(x),\nu_Y(x)\}, \max\{\mu_X(x),\mu_Y(x)\})/x, \\
&\qquad \omega_{(X\widetilde{\cup}Y)^c}((\mu(x),\nu(x))/x)) : x \in E\} \\
&= \{((\min\{\nu_X(x),\nu_Y(x)\}, \max\{\mu_X(x),\mu_Y(x)\})/x, \\
&\qquad \omega_{X^c\widetilde{\cap}Y^c}((\mu(x),\nu(x))/x)) : x \in E\} \\
&= \{((\nu_X(x),\mu_X(x))/x, \omega_{X^c}((\mu(x),\nu(x))/x)) : x \in E\} \\
&\quad \cap \{((\nu_Y(x),\mu_Y(x)))/x, \omega_{Y^c}((\mu(x),\nu(x))/x)) : x \in E\} \\
&= \Omega^{\tilde{c}}_X \widetilde{\cap} \Omega^{\tilde{c}}_Y.
\end{aligned}$$

The proof of (ii) can be made similarly. □

Proposition 3.18. *Let* $\Omega_X, \Omega_Y, \Omega_Z \in \Omega(U)$. *Then*

i. $\Omega_X\widetilde{\cup}(\Omega_Y\widetilde{\cap}\Omega_Z) = (\Omega_X\widetilde{\cup}\Omega_Y)\widetilde{\cap}(\Omega_X\widetilde{\cup}\Omega_Z)$.
ii. $\Omega_X\widetilde{\cap}(\Omega_Y\widetilde{\cup}\Omega_Z) = (\Omega_X\widetilde{\cap}\Omega_Y)\widetilde{\cup}(\Omega_X\widetilde{\cap}\Omega_Z)$.

Proof. i. For all $x \in E$,

$$\begin{aligned}
\mu_{X\widetilde{\cup}(Y\widetilde{\cap}Z)}(x) &= \max\{\mu_X(x), \mu_{Y\widetilde{\cap}Z}(x)\} \\
&= \max\{\mu_X(x), \min\{\mu_Y(x),\mu_Z(x)\}\} \\
&= \min\{\max\{\mu_X(x),\mu_Y(x)\}, \max\{\mu_X(x),\mu_Z(x)\}\} \\
&= \min\{\mu_{X\widetilde{\cup}Y}(x), \mu_{X\widetilde{\cup}Z}(x)\} \\
&= \mu_{(X\widetilde{\cup}Y)\widetilde{\cap}(X\widetilde{\cup}Z)}(x),
\end{aligned}$$

$$\begin{aligned}
\nu_{X\widetilde{\cup}(Y\widetilde{\cap}Z)}(x) &= \min\{\nu_X(x), \nu_{Y\widetilde{\cap}Z}(x)\} \\
&= \min\{\nu_X(x), \max\{\nu_Y(x),\nu_Z(x)\}\} \\
&= \min\{\max\{\nu_X(x),\nu_Y(x)\}, \max\{\nu_X(x),\nu_Z(x)\}\} \\
&= \min\{\nu_{X\widetilde{\cup}Y}(x), \nu_{X\widetilde{\cup}Z}(x)\} \\
&= \nu_{(X\widetilde{\cup}Y)\widetilde{\cap}(X\widetilde{\cup}Z)}(x)
\end{aligned}$$

and

$$\begin{aligned}
\omega_{X\tilde{\cup}(Y\tilde{\cap}Z)}((\mu(x),\nu(x))/x) &= \omega_X((\mu(x),\nu(x))/x) \cup \omega_{Y\tilde{\cap}Z}((\mu(x),\nu(x))/x) \\
&= \omega_X((\mu(x),\nu(x))/x) \\
&\quad \cup (\omega_Y((\mu(x),\nu(x))/x) \cap \omega_Z((\mu(x),\nu(x))/x)) \\
&= (\omega_X((\mu(x),\nu(x))/x) \cup \omega_Y((\mu(x),\nu(x))/x)) \\
&\quad \cap (\omega_X((\mu(x),\nu(x))/x) \cup \omega_Z((\mu(x),\nu(x))/x)) \\
&= \omega_{X\tilde{\cup}Y}((\mu(x),\nu(x))/x) \cap \omega_{X\tilde{\cup}Z}((\mu(x),\nu(x))/x) \\
&= \omega_{(X\tilde{\cup}Y)\tilde{\cap}(X\tilde{\cup}Z)}((\mu(x),\nu(x))/x)
\end{aligned}$$

The proof of (ii) can be made similarly. □

4. Ω-aggregation operator

In this section, we define an aggregate intuitionistic fuzzy set of an ifp-intuitionistic fuzzy soft set and Ω-aggregation operator that produce an aggregate intuitionistic fuzzy set from an ifp-intuitionistic fuzzy soft set and its intuitionistic fuzzy parameter set.

Definition 4.1. Let $\Omega_X \in \Omega(U)$. Then Ω-aggregation operator, denoted by Ω_{agg}, is defined by

$$\Omega_{agg} : \mathcal{IF}(E) \times \Omega(U) \to \mathcal{IF}(U), \quad \Omega_{agg}(X, \Omega_X) = \Omega^*_X$$

where

$$\Omega^*_X = \{(\mu_{\Omega^*_X}(u), \nu_{\Omega^*_X}(u))/u : u \in U\}$$

which is an intuitionistic fuzzy set over U. The value Ω^*_X is called aggregate intuitionistic fuzzy set of the Ω_X. Here, the membership degree $\mu_{\Omega^*_X}(u)$ and nonmembership degree $\nu_{\Omega^*_X}(u)$of u is defined as follows

$$\mu_{\Omega^*_X}(u) = \frac{1}{|E|}\sum_{x\in E} \mu_X(x)\mu_{\omega_X((\mu(x),\nu(x))/x)}(u)$$

and

$$\nu_{\Omega^*_X}(u) = \frac{1}{|E|}\sum_{x\in E} \nu_X(x)\nu_{\omega_X((\mu(x),\nu(x))/x)}(u)$$

where $|E|$ is the cardinality of E and $\mu_{\omega_X((\mu(x),\nu(x))/x)}(u)$ is membership degree of $u \in U$ and $\nu_{\omega_X((\mu(x),\nu(x))/x)}(u)$ non-membership degree of $u \in U$ in the intuitionistic fuzzy set $\omega_X((\mu(x),\nu(x))/x)$.

5. Decision making method

In this section, we give to choose optimum element a decision making method using intuitionistic fuzzy set obtained using operation Ω_{agg} defined as above. We can construct this decision making method by the following algorithm.

***Step 1*:** Construct an ifp-intuitionistic fuzzy soft set Ω_X over U,

***Step 2*:** Find the aggregate intuitionistic fuzzy set Ω^*_X of ifp-intuitionistic fuzzy soft set Ω_X,

***Step 3*:** Find
$max(u) = max\{\mu_{\Omega^*_X}(u) : u \in U\}$ and $min(v) = min\{\nu_{\Omega^*_X}(v) : v \in U\}$

***Step 4*:** Find $\alpha \in [0,1]$ such that $(max(u), \alpha)/u \in \Omega^*_X$ and $\beta \in [0,1]$ such that $(\beta, min(v))/v \in \Omega^*_X$

***Step 5*:** Find $\frac{max(u)}{max(u)+\alpha} = \alpha'$ and $\frac{\beta}{min(v)+\beta} = \beta'$

***Step 6*:** Opportune element of U is denoted by $Opp(u)$ and it is chosen as follow

$$Opp(u)=\begin{cases} u, if\ \alpha' \geq \beta' \\ v, if\ \alpha' < \beta' \end{cases}$$

Example 5.1. In this example, we present an application for the ifp-intuitionistic fuzzy soft-decision making method.

Let us assume that a company wants to fill a position. There are five candidates who form the set of alternatives, $U = \{u_1, u_2, u_3, u_4, u_5\}$. The choosing committee consider a set of parameters, $E = \{x_1, x_2, x_3, x_4\}$. For $i = 1, 2, 3, 4, 5$, the parameters x_i stand for "experience", "computer knowledge", "young age" and "good speaking", respectively.

After a serious discussion each candidate is evaluated from point of view of the goals and the constraint according to a chosen subset
$X = \{(0.7, 0.2)/x_2, (0.8, 0, 2)/x_3, (0.6, 0.3)/x_4\}$ of E. Finally, the committee constructs the following ifp-intuitionistic fuzzy soft set over U.

Step 1: Let the constructed ifp-intuitionistic fuzzy soft set Ω_X, be as follows,

$$\Omega_X = \Big\{(0.7, 0.2)/x_2, \{(0.4, 0.3)/u_1, (0.7, 0.3)/u_2, (0.6, 0.2)/u_3, (0.1, 0.5)/u_4, (0.9, 0.1)/u_5\}), ((0.8, 0.2)/x_3, \{0.8, 0.1)/u_1, (0.8, 0.2)/u_2, (0.5, 0.3)/u_3, (0.7, 0.3)/u_4\}), ((0.6, 0.3)/x_4, \{(0.5, 0.5)/u_1, (0.6, 0.1)/u_3, (0.3, 0, 6)/u_5\})\Big\}.$$

Step 2: The aggregate intuitionistic fuzzy set can be found as,

$$\Omega^*_X = \{(0.318, 0.057)/u_1, (0.248, 0.100)/u_2, (0.295, 0.033)/u_3, (0.158, 0.115)/u_4, (0.203, 0.100)/u_5\}$$

Step 3: $max(u) = 0.318$ and $min(v) = 0.033$

Step 4: $(0.318, 0.057)/u_1 \in \Omega^*_X$ and $(0.295, 0.033)/u_3 \in \Omega^*_X$

Step 5: $\alpha' = \frac{0.318}{0.318+0.057} = 0.848$ and $\beta' = \frac{0.295}{0.295+0.033} = 0.899$

Step 6: Since $\alpha' < \beta'$, $Opp(u) = u_3$.

Note that, although membership degree of u_1 is bigger than u_3, opportune element of U is u_3. This example show how the effect on decision making of non-membership degrees of elements.

6. Conclusion

In this paper, firstly we have defined ifp-intuitionistic fuzzy soft sets and their operations. Then we have presented a decision making method on the ifp-intuitionistic fuzzy soft set theory. Finally, we have provided an example that demonstrating that this method can successfully work. It can be applied to problems of many fields that contain uncertainty. In the future, methods of similarity measure and computing correlation coefficient under ifp-intuitionistic fuzzy soft environment can be improved by researchers.

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Faruk Karaaslan (`fkaraaslan@karatekin.edu.tr`)
Department of Mathematics, Faculty of Science, University of Çankırı Karatekin, Postal code 18100, Çankırı, Turkey