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Intuitionistic fuzzy lexical analyzer

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Abstract. This paper tries to establish a minimum state intuitionistic fuzzy finite automaton with unique membership transition on an input symbol(IFAUM) for a given intuitionistic fuzzy regular expression(IFRE). It is considered that IFAUM is a suitable model for implementing intuitionistic fuzzy lexical analyzers. An intuitionistic fuzzy lexical analyzer generator is also proposed.

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1. Introduction

In a conventional fuzzy set, a membership function assigned to each element of the universe of discourse, a number from the unit interval indicates the degree of belongingness to the set under consideration. Zadeh [16] was the first to consider the theory of fuzzy sets which had been used in dealing with problems of imprecision and uncertainty. The concept of fuzzy automata was presented by Santos [13], Wee and Fu [15], in which there were more than one fuzzy transition from a state on an input symbol for a membership value. This development was followed by the postulation called deterministic fuzzy finite state automaton(DFFSA) as in Malik and Mordeson [8], in which there can be atleast one transition on an input symbol. An equivalent DFFSA can be constructed from a fuzzy finite state automaton. However it only acts as a deterministic fuzzy recognizer, the fuzzy regular languages accepted by the fuzzy finite state automaton and deterministic fuzzy finite state automaton need not necessarily be equal (i.e. the degrees of a string in these systems need not be the same). Rajaretnam and Ayyaswamy [10] introduced fuzzy finite state automaton with unique membership transition on an input symbol. One kind of determinism of a fuzzy finite state automaton, is that the membership value of any recognized string in both the systems are the same. Jun proposed intuitionistic fuzzy finite state machines in [6, 7] . The two IF topologies (also called, intuitionistic fuzzy topologies) can be associated with the state sets of IF-fuzzy automata whose level topologies have relationships with the topologies introduced by Srivastava and Tiwari [14] for fuzzy automata. Using the notion of intuitionistic fuzzy sets by Atanassov [1, 2, 3, 4], it is possible to obtain intuitionistic fuzzy language [11], by introducing nonmembership value to the strings of fuzzy language. This is a natural generalisation of a fuzzy language characterised by two functions expressing the degree of belongingness and nonbelongingness. Zhang and Li [17] introduced the notions of intuitionistic fuzzy recognizer and intuitionistic fuzzy automaton. Ravi and Alka Choubey [12] stated the finite automaton (NDFA and DFA) with intuitionistic fuzzy transitions. Mateescu and et al. [9] proposed a model for fuzzy lexical analysis.

In the following, we first review the basic concepts of intuitionistic fuzzy finite automata with unique membership transition on an input symbol(IFAUM) and design three algorithms namely EPSILON, IFAUM, MINIFAUM to find the minimum state IFAUM for a given IFRE. To implement intuitionistic fuzzy lexical analyzer, we construct an intuitionistic fuzzy finite state automaton recognizing all tokens of the programming language being considered, using the above mentioned algorithms. For two different tokens we may have the same membership values(degrees), so there is likely a conflict to resolve it. In order to avoid this, there is a linear order of priorities associated with token names. Finally, we propose a model to design intuitionistic fuzzy lexical analyzer to recognize tokens.

2. Basic definitions

Definition 2.1 ([6]). Given a nonempty set Σ. Intuitionistic fuzzy sets(*ifs*) in Σ is an object having the form

$$A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in \Sigma\}$$

where the functions $\mu_A : \Sigma \to [0, 1]$ and $\nu_A : \Sigma \to [0, 1]$ denote the degree of membership and the degree of non-membership of each element $x \in \Sigma$ to the set A respectively, and $0 \leq \mu_A(x)+\nu_A(x) \leq 1$ for each $x \in \Sigma$. For the sake of simplicity, we shall use the notation $A = (\mu_A, \nu_A)$ instead of $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in \Sigma\}$.

Now we define the following.

Definition 2.2. An intuitionistic fuzzy finite automaton with unique membership transition on an input symbol is an ordered 5-tuple(*IFAUM*) $\mathscr{A} = (Q, \Sigma, A, i, f)$, where

(i) Q is a finite non-empty set of states.
(ii) Σ is a finite non-empty set of input symbols.
(iii) $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subset of $Q \times \Sigma \times Q$. The fuzzy subset $\mu_A : Q \times \Sigma \times Q \to [0, 1]$ denotes the degree of membership value such that

$\mu_A(p,a,q) = \mu_A(p,a,q')$ for some $q,q' \in Q$ then $q = q'$ and
$\nu_A : Q \times \Sigma \times Q \to [0,1]$ denotes the degree of non-membership value of every element in $Q \times \Sigma \times Q$.

(iv) $i = (i_{\mu_A}, i_{\nu_A})$ is an intuitionistic fuzzy subset of Q, i.e. $i_{\mu_A} : Q \to [0,1]$ and $i_{\nu_A} : Q \to [0,1]$ called the intuitionistic fuzzy initial state.

(v) $f = (f_{\mu_A}, f_{\nu_A})$ is an intuitionistic fuzzy subset of Q, i.e. $f_{\mu_A} : Q \to [0,1]$ and $f_{\nu_A} : Q \to [0,1]$ called the intuitionistic fuzzy subset of final states.

Definition 2.3. Let $\mathscr{A} = (Q, \Sigma, A, i, f)$ be an *IFAUM*. Define an *ifs* $A^* = (\mu_A^*, \nu_A^*)$ in $Q \times \Sigma^* \times Q$ as follows: $\forall p,q \in Q, x \in \Sigma^*, a \in \Sigma$.

$$\mu_A^*(q,\epsilon,p) = \begin{cases} 1, & \text{if } p = q \\ 0, & \text{if } p \neq q \end{cases} \quad , \quad \nu_A^*(q,\epsilon,p) = \begin{cases} 0, & \text{if } p = q \\ 1, & \text{if } p \neq q \end{cases}$$

$$\mu_A^*(q,xa,p) = \bigvee\{\mu_A^*(q,x,r) \wedge \mu_A(r,a,p) | r \in Q\}$$

$$\nu_A^*(q,xa,p) = \bigwedge\{\nu_A^*(q,x,r) \vee \nu_A(r,a,p) | r \in Q\}$$

Definition 2.4. Let $\mathscr{A} = (Q, \Sigma, A, i, f)$ be an *IFAUM* and $x \in \Sigma^*$. Then x is recognized by $\mathscr{A}$, if $L_{\mu_{\mathscr{A}}}(x) > 0$ and $L_{\nu_{\mathscr{A}}}(x) < 1$

Definition 2.5. Let $\mathscr{A} = (Q, \Sigma, A, i, f)$ be an *IFAUM*. The behavior of an *IFAUM* is $L_{\mathscr{A}} = (L_{\mu_{\mathscr{A}}}, L_{\nu_{\mathscr{A}}})$

3. Construction of an IFAUM from a given IFRE

Definition 3.1 ([12])**.** Let Σ be a finite alphabet set and (μ_A, ν_A) be finite sets of real numbers in $[0,1]$.

1. Let e be a regular expression over Σ. Then, we call $\overline{e} = \dfrac{e}{m/n}$, where $m \in \mu_A$ and $n \in \nu_A$ an intuitionistic fuzzy regular expression (IFRE).
2. Let $\overline{e}_1$ and $\overline{e}_2$ be two intuitionistic fuzzy regular expressions, then $\overline{e}_1 + \overline{e}_2$, $(\overline{e}_1\overline{e}_2)$ and $(\overline{e}_1)^*$ are all IFRE's.
3. An intuitionistic fuzzy regular expression is formed by applying 1 and 2, a finite number of times.

Definition 3.2 ([12])**.** Let $\overline{e}$ be an IFRE, then the corresponding behavior, i.e. intuitionistic fuzzy regular behavior(IFRB) $\overline{L}(\overline{e})$ is defined by

(i) if $\overline{e} = \dfrac{e}{m/n}$, where e is a regular expression, then $\overline{L}(\overline{e}) = \{(x,m,n) | x \in \overline{L}(\overline{e})\}$.

(ii) if $\overline{e} = \overline{e}_1 + \overline{e}_2$, $\overline{e} = (\overline{e}_1\overline{e}_2)$ or $\overline{e} = (\overline{e}_1)^*$, then $\overline{L}(\overline{e}) = \overline{L}(\overline{e}_1) \cup L(\overline{e}_2)$, $\overline{L}(\overline{e}) = \overline{L}(\overline{e}_1)\overline{L}(\overline{e}_2)$ or $\overline{L}(\overline{e}) = \left(\overline{L}(\overline{e}_1)\right)^*$ respectively.

Note that if $m = 1$ and $n = 0$ then $\dfrac{e}{m/n}$ is written as e.

3.1. **Algorithm EPSILON ($\overline{e}$).** This algorithm finds an intuitionistic fuzzy finite state automaton [17] (IFFSA) $\mathscr{A}$ with ϵ-transitions from a given IFRE $\overline{e}$ accepting an intuitionsitic fuzzy regular behavior $\overline{L}$ denoted by $\overline{e}$.

Input: An IFRE $\overline{e}$ over an alphabet Σ denoting an intuitionistic fuzzy regular behavior $\overline{L}$.

Output: An IFFSA $\mathscr{A} = (Q, \Sigma \cup \{\epsilon\}, A, i, f)$ such that an intuitionistic fuzzy regular behavior of $\mathscr{A}$ is $\overline{L}$.

Method: Decompose $\overline{e}$ into its primitive components. For each component, we construct an IFAUM inductively as follows:-

Case (i): $\overline{e} = \dfrac{e}{m/n}$

First, we find an NFA for the regular expression e as in [5].

Let it be $M = (Q, \Sigma \cup \{\epsilon\}, \delta, q_0, F)$ with ϵ-transitions such that $L(M) = L(e) = L$.

Now, define an IFFSA $\mathscr{A} = (Q, \Sigma \cup \{\epsilon\}, A, i, f)$ with ϵ-transitions,

where $\mu_A, \nu_A : Q \times \Sigma \cup \{\epsilon\} \times Q \to [0,1]$ by

(i) $\forall\ p, q \in Q\ ,\ a \in \Sigma$

$$\mu_A(p,a,q) = \begin{cases} m, & \text{if } q \in \delta(p,a) \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad \nu_A(p,a,q) = \begin{cases} n, & \text{if } q \in \delta(p,a) \\ 0, & \text{otherwise} \end{cases}$$

(ii) $\forall\ p, q \in Q\ ,\ \epsilon \in \Sigma$

$$\mu_A(p,\epsilon,q) = \begin{cases} 1, & \text{if } q \in \delta(p,\epsilon) \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad \nu_A(p,\epsilon,q) = \begin{cases} 0, & \text{if } q \in \delta(p,\epsilon) \\ 1, & \text{otherwise} \end{cases}$$

(iii) $i : Q \to [0,1]$ is defined by,

$$i_{\mu_A}(p) = \begin{cases} 1, & \text{if } p = q_0 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad i_{\nu_A}(p) = \begin{cases} 0, & \text{if } p = q_0 \\ 1, & \text{otherwise} \end{cases}$$

(iv) $f : Q \to [0,1]$ is defined by,

$$f_{\mu_A}(p) = \begin{cases} 1, & \text{if } p \in F \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad f_{\nu_A}(p) = \begin{cases} 0, & \text{if } p \in F \\ 1, & \text{otherwise} \end{cases}$$

Let $L_{\mu_{\mathscr{A}}}$ and $L_{\nu_{\mathscr{A}}}$ be an intuitionistic fuzzy behavior of $\mathscr{A}$. For all $x \in \Sigma^*$

$$L_{\mu_{\mathscr{A}}}(x) = \bigvee \big\{\{i_{\mu_A}(p) \wedge \mu_A^*(p,x,q) \wedge f_{\mu_A}(q) | q \in Q\} | p \in Q\big\}$$

and

$$L_{\nu_{\mathscr{A}}}(x) = \bigwedge \big\{\{i_{\nu_A}(p) \vee \nu_A^*(p,x,q) \vee f_{\nu_A}(q) | q \in Q\} | p \in Q\big\}$$

Now,

$$\mu^*_A(p,x,q) = \begin{cases} m, & \text{if } x \in L(e) \,\&\, q \in F \\ 0, & \text{otherwise} \end{cases}$$

$$\text{and} \quad \nu^*_A(p,x,q) = \begin{cases} n, & \text{if } x \in L(e) \,\&\, q \in F \\ 0, & \text{otherwise} \end{cases}$$

$$\therefore L_{\mu_{\mathscr{A}}}(x) = \begin{cases} 1 \wedge m \wedge 1 = m, & \text{if } x \in L(e) \\ 0, & \text{otherwise} \end{cases}$$

$$\text{and} \quad L_{\nu_{\mathscr{A}}}(x) = \begin{cases} 0 \vee n \vee 0 = n, & \text{if } x \in L(e) \\ 0, & \text{otherwise} \end{cases}$$

Thus, $L_{\mathscr{A}} = \overline{L}$.

Having constructed an IFFSA for $\overline{e}_1$ and $\overline{e}_2$, we now proceed to combine them in ways that correspond to an IFRE's. Suppose $\mathscr{A}_1$ and $\mathscr{A}_2$ are the IFFSA's with ϵ-transitions for $\overline{e}_1$ and $\overline{e}_2$ with an IFRB's $\overline{L}_{\mathscr{A}_1}$ and $\overline{L}_{\mathscr{A}_2}$ respectively. Let $\mathscr{A}_1 = (Q_1, \Sigma \cup \{\epsilon\}, A_1, i_1, f_1)$ and $\mathscr{A}_2 = (Q_2, \Sigma \cup \{\epsilon\}, A_2, i_2, f_2)$.

Suppose $i_{\mu_{A_1}}(p_1) = 1, f_{\mu_{A_1}}(q_1) = 1$ and $i_{\nu_{A_1}}(p_1) = 0, f_{\nu_{A_1}}(q_1) = 0$, $i_{\mu_{A_2}}(p_2) = 1$, $f_{\mu_{A_2}}(q_2) = 1$ and $i_{\nu_{A_2}}(p_2) = 0, f_{\nu_{A_2}}(q_2) = 0$ where $p_1, q_1 \in Q_1; p_2, q_2 \in Q_2$.

Case (ii): $\overline{e} = \overline{e}_1 + \overline{e}_2$

Constructing an IFFSA for $\overline{e}$. Let q_i and q_f be two new initial and final states, $Q = Q_1 \cup Q_2 \cup \{q_i, q_f\}$. Define an IFFSA $\mathscr{A} = (Q, \Sigma, A, i, f)$ such that

(i) $\forall\ q \in Q_1$ with $i_{\mu_{A_1}}(q) = 1$ and $i_{\nu_{A_1}}(q) = 0$
include $\mu_A(q_i, \epsilon, q) = 1$ and $\nu_A(q_i, \epsilon, q) = 0$.

(ii) $\forall\ q \in Q_2$ with $i_{\mu_{A_2}}(q) = 1$ and $i_{\nu_{A_2}}(q) = 0$
include $\mu_A(q_i, \epsilon, q) = 1$ and $\nu_A(q_i, \epsilon, q) = 0$.

(iii) $\forall\ p, q \in Q_1$, $a \in \Sigma \cup \{\epsilon\}$ include $\mu_A(p, a, q) = \mu_{A_1}(p, a, q)$
and $\nu_A(p, a, q) = \nu_{A_1}(p, a, q)$.

(iv) $\forall\ p, q \in Q_2$, $a \in \Sigma \cup \{\epsilon\}$ include $\mu_A(p, a, q) = \mu_{A_2}(p, a, q)$
and $\nu_A(p, a, q) = \nu_{A_2}(p, a, q)$.

(v) $\forall\ p \in Q_1$ with $f_{\mu_{A_1}}(p) = 1$ include $\mu_A(p, \epsilon, q_f) = 1$
and $f_{\nu_{A_1}}(p) = 0$ include $\nu_A(p, \epsilon, q_f) = 0$.

(vi) $\forall\ p \in Q_2$ with $f_{\mu_{A_2}}(p) = 1$ include $\mu_A(p, \epsilon, q_f) = 1$
and $f_{\nu_{A_2}}(p) = 0$ include $\nu_A(p, \epsilon, q_f) = 0$.

(vii) $\mu_A(p, a, q) = 0$ and $\nu_A(p, a, q) = 1$, for all other possibilities.
$i : Q \to [0, 1]$ is defined by

$$i_{\mu_A}(p) = \begin{cases} 1, & \text{if } p = q_i \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad i_{\nu_A}(p) = \begin{cases} 0, & \text{if } p = q_f \\ 1, & \text{otherwise} \end{cases}$$

$f : Q \to [0, 1]$ is defined by

$$f_{\mu_A}(p) = \begin{cases} 1, & \text{if } p = q_f \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad f_{\nu_A}(p) = \begin{cases} 0, & \text{if } p = q_f \\ 1, & \text{otherwise} \end{cases}$$

Thus, $L_{\mu_{\mathscr{A}}}(x) = \overline{L}_{\mu_{\mathscr{A}_1}}(x) \vee \overline{L}_{\mu_{\mathscr{A}_2}}(x)$ and $L_{\nu_{\mathscr{A}}}(x) = \overline{L}_{\nu_{\mathscr{A}_1}}(x) \wedge \overline{L}_{\nu_{\mathscr{A}_2}}(x) \forall x \in \Sigma^*$
Case (iii): $\overline{e} = \overline{e}_1\overline{e}_2$
Let $Q = Q_1 \cup Q_2$. Define an IFFSA $\mathscr{A} = (Q, \Sigma \cup \{\epsilon\}, A, i, f)$ such that

(i) $\forall\ p \in Q_1$ with $f_{\mu_{A_1}}(p) = 1, f_{\nu_{A_1}}(p) = 0$ and $\forall\ q \in Q_2$ with $i_{\mu_{A_2}}(q) = 1$, $i_{\nu_{A_2}}(q) = 0$ include $\mu_A(p, \epsilon, q) = 1$ and $\nu_A(p, \epsilon, q) = 0$.
(ii) $\forall\ p, q \in Q_1$, $a \in \Sigma \cup \{\epsilon\}$,
include $\mu_A(p, a, q) = \mu_{A_1}(p, a, q)$ and $\nu_A(p, a, q) = \nu_{A_1}(p, a, q)$.
(iii) $\forall\ p, q \in Q_2$, $a \in \Sigma \cup \{\epsilon\}$,
include $\mu_A(p, a, q) = \mu_{A_2}(p, a, q)$ and $\nu_A(p, a, q) = \nu_{A_2}(p, a, q)$.

$i : Q \to [0, 1]$ is defined by

$$i_{\mu_A}(p) = \begin{cases} 1, & \text{if } i_{\mu_{A_1}}(p) = 1, p \in Q_1 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad i_{\nu_A}(p) = \begin{cases} 0, & \text{if } i_{\nu_{A_1}}(p) = 0, p \in Q_1 \\ 1, & \text{otherwise} \end{cases}$$

$f : Q \to [0, 1]$ is defined by

$$f_{\mu_A}(p) = \begin{cases} 1, & \text{if } f_{\mu_{A_2}}(p) = 1, p \in Q_2 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad f_{\nu_A}(p) = \begin{cases} 0, & \text{if } f_{\nu_{A_2}}(p) = 0, p \in Q_2 \\ 1, & \text{otherwise} \end{cases}$$

$$\text{Clearly,} L_{\mu_{\mathscr{A}}}(x) = \bigvee\{\overline{L}_{\mu_{\mathscr{A}_1}}(y) \wedge \overline{L}_{\mu_{\mathscr{A}_2}}(z)\,|x = yz, x, y \in \Sigma^*\}$$
$$\text{and} \quad L_{\nu_{\mathscr{A}}}(x) = \bigwedge\{\overline{L}_{\nu_{\mathscr{A}_1}}(y) \vee \overline{L}_{\nu_{\mathscr{A}_2}}(z)\,|x = yz, x, y \in \Sigma^*\}$$

Therefore,$L_{\mathscr{A}} = \overline{L}_{\mathscr{A}_1}\overline{L}_{\mathscr{A}_2}$
Case (iv): $\overline{e} = (\overline{e}_1)^*$
Let $Q = Q_1 \cup \{p_i, q_f\}$, where p_i and q_f are new initial and final states which are not in Q_1. Define an IFFSA $\mathscr{A} = (Q, \Sigma \cup \{\epsilon\}, A, i, f)$ such that

(i) $\forall\ p \in Q_1$ with $i_{\mu_{A_1}}(p) = 1$, include $\mu_A(p_i, \epsilon, p) = 1$
and $i_{\nu_{A_1}}(p) = 0$, include $\nu_A(p_i, \epsilon, p) = 0$.
(ii) $\mu_A(p_i, \epsilon, q_f) = 1$ and $\nu_A(p_i, \epsilon, q_f) = 0$.
(iii) $\forall\ q \in Q_1$ with $f_{\mu_{A_1}}(q) = 1$, include $\mu_A(q, \epsilon, q_f) = 1$
and $f_{\nu_{A_1}}(q) = 0$, include $\nu_A(q, \epsilon, q_f) = 0$.
(iv) $\forall\ p, q \in Q_1$ with $f_{\mu_{A_1}}(q) = 1$, $i_{\mu_{A_1}}(p) = 1$, include $\mu_A(q, \epsilon, p) = 1$
and $f_{\nu_{A_1}}(q) = 0$, $i_{\nu_{A_1}}(p) = 0$, include $\nu_A(q, \epsilon, p) = 0$.
(v) $\forall\ p, q \in Q_1$, $a \in \Sigma \cup \{\epsilon\}$, include $\mu_A(p, a, q) = \mu_{A_1}(p, a, q)$
and $\nu_A(p, a, q) = \nu_{A_1}(p, a, q)$.

For all $x \in \Sigma^*$, $L_{\mu_{\mathscr{A}}}(x) > 0$ and $L_{\nu_{\mathscr{A}}}(x) < 1$ if and only if $x = x_1x_2\cdots x_n$, $\overline{L}_{\mu_{\mathscr{A}_1}}(x_i) \geq 0$ and $\overline{L}_{\nu_{\mathscr{A}_1}}(x_i) \leq 1$, $i = 1, 2, \ldots, n$.
Moreover,

$$L_{\mu_{\mathscr{A}}}(x) = \overline{L}_{\mu_{\mathscr{A}_1}}(x_1) \wedge \overline{L}_{\mu_{\mathscr{A}_1}}(x_2) \wedge \cdots \wedge \overline{L}_{\mu_{\mathscr{A}_1}}(x_n)$$
$$\text{and}\, L_{\nu_{\mathscr{A}}}(x) = \overline{L}_{\nu_{\mathscr{A}_1}}(x_1) \vee \overline{L}_{\nu_{\mathscr{A}_1}}(x_2) \vee \cdots \vee \overline{L}_{\nu_{\mathscr{A}_1}}(x_n)$$
$$\text{Clearly,}\, L_{\mu_{\mathscr{A}}}(x) = \bigvee \{\overline{L}_{\mu_{\mathscr{A}_1}}(x_1) \wedge \overline{L}_{\mu_{\mathscr{A}_1}}(x_2) \wedge \cdots \wedge \overline{L}_{\mu_{\mathscr{A}_1}}(x_n)$$

$$|x = x_1 x_2 \cdots x_n,\ x_i \in \Sigma^*,\ i = 1, 2, \ldots, n\}$$

$$\text{and } L_{\nu_{\mathscr{A}}}(x) = \bigwedge \{\overline{L}_{\nu_{\mathscr{A}_1}}(x_1) \vee \overline{L}_{\nu_{\mathscr{A}_1}}(x_2) \vee \cdots \vee \overline{L}_{\nu_{\mathscr{A}_1}}(x_n)$$
$$|x = x_1 x_2 \cdots x_n,\ x_i \in \Sigma^*,\ i = 1, 2, \ldots, n\}$$

Hence $L_{\mathscr{A}} = \overline{L}^*_{\mathscr{A}_1}$.

Example 3.3. Consider an IFRE $\overline{e} = \dfrac{ba^*}{0.7/0.2} + \Big(\dfrac{a}{0.3/0.5} + \dfrac{ab}{0.6/0.1}\Big)^*$.

Following the Algorithm EPSILON ($\overline{e}$),an IFFSA $\mathscr{A} = (Q, \Sigma \cup \{\epsilon\}, A, i, f)$, where $Q = \{q_0, q_1, q_2, q_3, ..., q_{15}\}, \Sigma = \{a, b\},\ i_{\mu_A}(q_0) = 1$ and $i_{\nu_A}(q_0) = 0$, $f_{\mu_A}(q_{15}) = 1,\ f_{\nu_A}(q_{15}) = 0$ and A is shown below:

$\mu_A(q_0, \epsilon, q_1) = 1.0$	$\nu_A(q_0, \epsilon, q_1) = 0.0$	$\mu_A(q_1, b, q_2) = 0.7$	$\nu_A(q_1, b, q_2) = 0.2$
$\mu_A(q_2, \epsilon, q_3) = 1.0$	$\nu_A(q_2, \epsilon, q_3) = 0.0$	$\mu_A(q_3, a, q_4) = 0.7$	$\nu_A(q_3, a, q_4) = 0.2$
$\mu_A(q_4, \epsilon, q_3) = 1.0$	$\nu_A(q_4, \epsilon, q_3) = 0.0$	$\mu_A(q_4, \epsilon, q_5) = 1.0$	$\nu_A(q_4, \epsilon, q_5) = 0.0$
$\mu_A(q_2, \epsilon, q_5) = 1.0$	$\nu_A(q_2, \epsilon, q_5) = 0.0$	$\mu_A(q_5, \epsilon, q_{15}) = 1.0$	$\nu_A(q_5, \epsilon, q_{15}) = 0.0$
$\mu_A(q_0, \epsilon, q_6) = 1.0$	$\nu_A(q_0, \epsilon, q_6) = 0.0$	$\mu_A(q_6, \epsilon, q_7) = 1.0$	$\nu_A(q_6, \epsilon, q_7) = 0.0$
$\mu_A(q_6, \epsilon, q_{14}) = 1.0$	$\nu_A(q_6, \epsilon, q_{14}) = 0.0$	$\mu_A(q_7, \epsilon, q_8) = 1.0$	$\nu_A(q_7, \epsilon, q_8) = 0.0$
$\mu_A(q_7, \epsilon, q_{10}) = 1.0$	$\nu_A(q_7, \epsilon, q_{10}) = 0.0$	$\mu_A(q_8, a, q_9) = 0.3$	$\nu_A(q_8, a, q_9) = 0.5$
$\mu_A(q_{10}, a, q_{11}) = 0.6$	$\nu_A(q_{10}, a, q_{11}) = 0.1$	$\mu_A(q_{11}, b, q_{12}) = 0.6$	$\nu_A(q_{11}, b, q_{12}) = 0.1$
$\mu_A(q_9), \epsilon, q_{13}) = 1.0$	$\nu_A(q_9, \epsilon, q_{13}) = 0.0$	$\mu_A(q_{12}, \epsilon, q_{13}) = 1.0$	$\nu_A(q_{12}, \epsilon, q_{13}) = 0.0$
$\mu_A(q_{13}, \epsilon, q_{14}) = 1.0$	$\nu_A(q_{13}, \epsilon, q_{14}) = 0.0$	$\mu_A(q_{13}, \epsilon, q_7) = 1.0$	$\nu_A(q_{13}, \epsilon, q_7) = 0.0$
$\mu_A(q_{14}, \epsilon, q_{15}) = 1.0$	$\nu_A(q_{14}, \epsilon, q_{15}) = 0.0$		

Clearly, an intuitionistic fuzzy behavior of $\mathscr{A}$ and an intuitionistic fuzzy regular behavior denoted by an IFRE $\overline{e}$ are the same.

3.2. **Algorithm IFAUM ($\mathscr{A}$).** This algorithm constructs an IFAUM $\mathscr{A}'$ from an IFFSA $\mathscr{A}$ which is obtained using Algorithm EPSILON ($\overline{e}$).
Input: Let $\mathscr{A} = (Q, \Sigma \cup \{\epsilon\}, A, i, f)$ be an IFFSA with ϵ-transitions, which is the output of Algorithm EPSILON ($\overline{e}$) i.e. $\mathscr{A}$:= EPSILON ($\overline{e}$). In an IFFSA $\mathscr{A}$ there is only one state with non-zero initial and final intuitionistic fuzzy value. Let $q_0, q_f \in Q$ such that $i_{\mu_A}(q_0) = 1$, $i_{\nu_A}(q_0) = 0$, $f_{\mu_A}(q_f) = 1$ and $f_{\nu_A}(q_f) = 0$.
Output: IFAUM $\mathscr{A}'$ with $L_{\mathscr{A}'} = L_{\mathscr{A}}$.
Method: For $s \in Q$, we define

$$s^\epsilon = \{q \in Q \,|\, \mu^*_A(s, \epsilon, q) = 1 \text{ and } \nu^*_A(s, \epsilon, q) = 0\}$$

i.e. the set of all reachable states from s by means of ϵ-transitions with membership value 1 and non-membership 0. Clearly, s is a member of s^ϵ, $\forall\, s \in Q$. If T is a set of states, then $T^\epsilon = \cup\{s^\epsilon \,|\, s \in T\}$. We present the procedure in the following Algorithm 1 ,to obtain T^ϵ, using data structure stack called STACK.

Algorithm 1. EPSILON $\overline{e}$

```
begin
   push all states in T into STACK;
   T^ϵ := T;
   while STACK ≠ ∅ do
     begin
        pop s; the top element of STACK;
        for each t ∈ Q with μ_A(s,ϵ,t) = 1, ν_A(s,ϵ,t) = 0 do
        if t is not in T^ϵ then
            begin
               add t to T^ϵ;
               push t onto STACK;
            end
     end
end
```

Construct $\mathscr{A}' = (Q', \Sigma, A', i', f')$. Each state of $\mathscr{A}'$ is a set of states of $\mathscr{A}$. Q', the states of $\mathscr{A}'$ and their intuitionistic fuzzy transitions on input symbols are defined as follows:

Step 1: Let $s_0 = q_0^\epsilon$. We assume that $s_0 \in Q'$ and is initially unmarked.

Algorithm 2. IFAUM

```
begin
   while there is an unmarked state s = {s_1, s_2, ..., s_n} do
     begin
        mark s;
        for each input symbol a and for each membership value m
           and non-membership value n do
          begin
           T := {r_j | μ_A(s_i,a,r_j) = m and ν_A(s_i,a,r_j) = n, s_i ∈ s};
           y := T^ϵ;
           if y ∉ Q' then include y an unmarked state of Q';
           set μ_A'(s,a,y) := m and ν_A'(s,a,y) := n;
          end
     end
end
```

Step 2: Do the procedure given in Algorithm 2.

Thus Q' , $\mu_{A'} : Q' \times \Sigma \times Q' \to [0, 1]$ and $\nu_{A'} : Q' \times \Sigma \times Q' \to [0, 1]$ are defined.

Step 3: Define $i' : Q' \to [0, 1]$ by

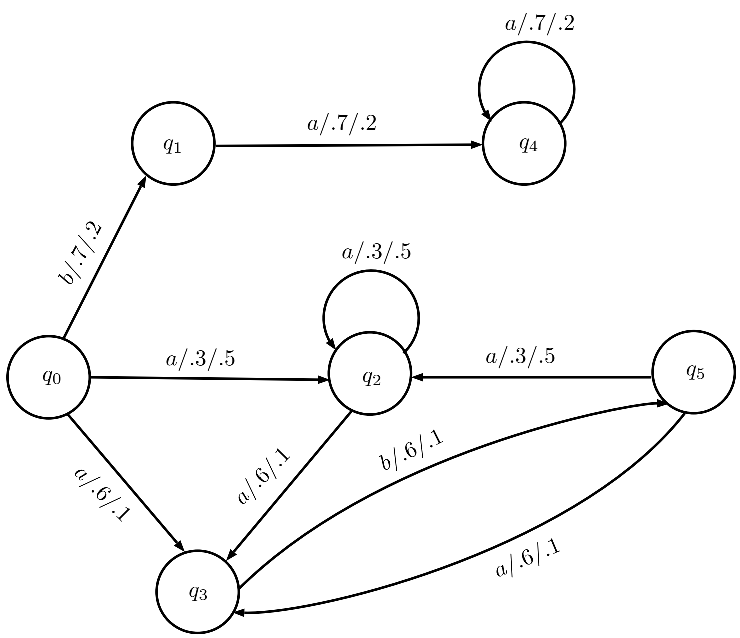
$$i_{\mu_{A'}}(p) = \begin{cases} 1, \text{if } p = s_0 \\ 0, \text{otherwise} \end{cases} \quad \text{and } i_{\nu_{A'}}(p) = \begin{cases} 0, \text{if } p = s_0 \\ 1, \text{otherwise} \end{cases}$$

and $f' : Q' \to [0,1]$ by

$$f_{\mu_{A'}}(p) = \begin{cases} 1, \text{if } q_f \in p \\ 0, \text{otherwise} \end{cases} \quad \text{and } f_{\nu_{A'}}(p) = \begin{cases} 0, \text{if } q_f \in p \\ 1, \text{otherwise} \end{cases}$$

Clearly, $\mathscr{A}'$ is an IFAUM without ϵ-transitions with $L_{\mathscr{A}'} = L_{\mathscr{A}}$.

Example 3.4. Consider an IFFSA $\mathscr{A} = (Q, \Sigma \cup \{\epsilon\}, A, i, f)$ which is obtained in Example 3.3. Systematically applying the Algorithms 1 and 2, we obtain $\mathscr{A}' = (Q', \Sigma, A', i', f')$, where $Q' = \{q_0, q_1, q_2, q_3, q_4, q_5\}, i_{\mu_{A'}}(q_0) = 1$, $i_{\nu_{A'}}(q_0) = 0, f_{\mu_{A'}}(q_i) = 1, f_{\nu_{A'}}(q_i) = 0, i = 0, 1, 2, 4, 5$ and intuitionistic fuzzy transition is shown in the diagram below.



3.3. **Algorithm MINIFAUM** $(\mathscr{A}')$**.** The number of states in an IFAUM $\mathscr{A}'$ which is constructed using Algorithm IFAUM $\mathscr{A}$ is not the smallest possible. This algorithm gives a more general way of reducing the number of states of $\mathscr{A}'$ as few as possible.

Input: An IFAUM $\mathscr{A}' = (Q', \Sigma, A', i', f')$ which is the output of Algorithm IFAUM $(\mathscr{A})$ i.e. $\mathscr{A}' := IFAUM(\mathscr{A})$.

Output: An IFAUM $\mathscr{A}'' = (Q'', \Sigma, A'', i'', f'')$ such that $L_{\mathscr{A}''} = L_{\mathscr{A}'}$ and having as few states as possible.

Method:

Step 1: We construct a partition Π of the set states of Q'. Initially, Π consists of three groups, Q'_1, Q'_2, Q'_3 such that

$$Q'_1 = \{q \in Q' \mid i_{\mu_{A'}}(q) = 1 \text{ and } i_{\nu_{A'}}(q) = 0\},$$
$$Q'_2 = \{q \in Q' \mid f_{\mu_{A'}}(q) = 1 \text{ and } f_{\nu_{A'}}(q) = 0\} - Q'_1,$$
$$Q'_3 = Q'_1 - Q'_1 \cup Q'_2.$$

Then we construct a new partition Π_{new} using the procedure given in Algorithm 3 and followed by the procedure given in Algorithm 4.

Step 2: From the final partition Π, which is the output of the procedure given in Algorithm 4, pick a representative for each group. The representatives will be the states of the reduced IFAUM $\mathscr{A}''$. Let s be a representative state and $\mu_{A'}(s, a, t) := m$ and $\nu_{A'}(s, a, t) := n$ in $\mathscr{A}'$, then the set $\mu_{A''}(s, a, r) := m$ and $\nu_{A''}(s, a, r) := n$, if r is the representative of t's group (r may be t).

Algorithm 3. MINIFAUM $\mathscr{A}'$

```
begin
  Π_new := Π;
  repeat
    begin
      Π := Π_new;
      make all groups of Π are unmarked;
      while there is an unmarked group G in Π do
        begin
          mark G;
          new := ∅;
          for each input a ∈ Σ do
            begin
              if ((μ_A'(p,a,r) = m1, ν_A'(p,a,r) = n1 and
              μ_A'(p',a,r') = m2, ν_A'(p',a,r') = n2), p,p' ∈ G,
               m1 ≠ m2, m2 > 0 or n1 ≠ n2, n2 < 1 and r,r' are in the same
              group G1 of Π (G1 may be G)) then
              new := new ∪{r};
              /* thus r and r' are placed in different groups */
            end
        end
      place new as a group in Π_new;
    end
  until (Π := Π_new);
end
```

Step 3: Formally, we define the initial intuitionistic fuzzy values $(i_{\mu_{A''}}, i_{\nu_{A''}})$ and final intuitionistic fuzzy values $(f_{\mu_{A''}}\, f_{\nu_{A''}})$, using the procedure in Algorithm 5.

Algorithm 4. MINIFAUM $\mathscr{A}'$

```
begin
  repeat
    Π := Π_new;
    for each group G of Π do
      begin
        partition G into subgroups such that two states s and
        t of G are in the same group if and only if for each
        input symbol a and each m > 0 and n < 1 such that
        μ_A'(s,a,s') = μ_A'(t,a,t') = m, & ν_A'(s,a,s') = ν_A'(t,a,t') = n
        and s',t' are in the same group of Π
      end
    place all subgroups so formed in Π_new;
  until (Π := Π_new);
end
```

Step 4: If $\mathscr{A}''$ has a dead state d i.e. there exists no $d' \in Q''$ such that $\mu_{A''}(d, a, d') > 0$ and $\nu_{A''}(d, a, d') < 1$, for each $a \in \Sigma$ and $f_{\mu_{A''}}(d) = 0$ and $f_{\mu_{A''}}(d) = 1$, remove d from Q'' and set $\mu_{A''}(p, a, d) := 0$ and $\nu_{A''}(p, a, d) := 1$ $\forall\, p \in Q'', a \in \Sigma$.

Step 5: If $s \in Q''$ and for any string $x \in \Sigma^*$, $\mu^*_{A''}(q, x, s) = 0$ and $\nu^*_{A''}(q, x, s) = 1$ for $q \in Q'', i_{\mu_{A''}}(q) = 1$ and $i_{\nu_{A''}}(q) = 0$, then s is not reachable, remove s from Q'' and set $\mu_{A''}(p, a, s) := 0$ and $\nu_{A''}(p, a, s) := 1$, $p \in Q'', a \in \Sigma$.

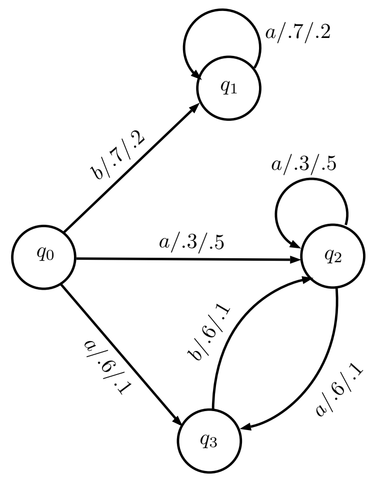
Algorithm 5. MINIFAUM $\mathscr{A}'$

```
begin
  for each group G in Π do
    begin
      let s be the representative of group G;
      if (i_μA'(r) = 1 and i_νA'(r) = 0) for each r ∈ G then
      set i_μA''(s) := 1 and i_νA''(s) := 0;
      else set i_μA''(s) := 0 and i_νA''(s) := 1;
      if (f_μA'(r) = 1 and f_νA'(r) = 0) for each r ∈ G then
      set f_μA''(s) := 1 and f_νA''(s) := 0;
      else set f_μA''(s) := 0 and f_νA''(s) := 1;
      /* thus i'' and f'' are defined */
    end
end
```

Example 3.5. Consider an IFAUM $\mathscr{A}'$ obtained in Example 3.4, Using the procedure given in the Algorithm 3, 4, 5 we get

$$\mathscr{A}'' = (Q'', \Sigma, A'', i'', f'')$$

where $Q'' = \{q_0, q_1, q_2, q_3\}, \Sigma = \{a, b\}, i_{\mu_{A''}}(q_0) = 1, i_{\nu_{A''}}(q_0) = 0$, $f_{\mu_{A''}}(q_i) = 1, f_{\nu_{A''}}(q_i) = 0, i = 0, 1, 2.$



Clearly, the behavior of $\mathscr{A}''$ is same as the intuitionistic fuzzy regular behavior denoted by the given IFRE in Example 3.3.

4. Intuitionistic Fuzzy Lexical Analyzer

The function of the lexical analyzer is to read the source program, one character at a time, and to translate it into a sequence of primitive units called tokens. Keywords, identifiers, constants, and operators are examples of tokens.We propose the intuitionistic fuzzy lexical analyzer to recognize strings with degrees in [0,1]. According to the degree of the input string, any of the four actions may be taken which is described in this section with an example.

Definition 4.1. Let $L_{\mathscr{A}}$ be an intuitionistic fuzzy behavior, T is a finite set of names with a linear order $<$, and $\eta : \Sigma^* \to T$ a function. Then we call the set $\{(x, L_{\mathscr{A}}(x), \eta(x)) \mid x \in \Sigma^*\}$ a marked intuitionistic fuzzy behavior, denoted $(L_{\mathscr{A}}, \eta)$, where η is the marking function of the behavior.

Definition 4.2. Let $(L_{\mathscr{A}}, \eta)$ be a marked intuitionistic fuzzy behavior then $(L_{\mathscr{A}}, \eta)$ is called a marked intuitionistic fuzzy regular behavior, if $L_{\mathscr{A}}$ is an intuitionistic fuzzy regular behavior.

Definition 4.3. Let $(L_{\mathscr{A}_1}, \eta_1)$ and $(L_{\mathscr{A}_2}, \eta_2)$ be two marked intuitionistic fuzzy behavior. A marked intuitionistic behavior $(L_{\mathscr{A}}, \eta)$ is called the marked union of $(L_{\mathscr{A}_1}, \eta_1)$ and $(L_{\mathscr{A}_2}, \eta_2)$, if $L_{\mathscr{A}} = L_{\mathscr{A}_1} \cup L_{\mathscr{A}_2}$ and $\eta : \Sigma^* \to T$ is defined by

$$\eta(x) = \begin{cases} \eta_1(x), & \text{if } \left(L_{\mu_{\mathscr{A}_1}}(x) > L_{\mu_{\mathscr{A}_2}}(x)\, and\, L_{\nu_{\mathscr{A}_1}}(x) < L_{\nu_{\mathscr{A}_2}}(x)\right) \\ & \text{or } \left(L_{\mathscr{A}_1} = L_{\mathscr{A}_2}\, and\, \eta_2(x) < \eta_1(x)\right) \\ \eta_2(x), & \text{otherwise} \end{cases}$$

Note that if $L_{\mu_{\mathscr{A}}}(x) = 0$ and $L_{\nu_{\mathscr{A}}}(x) = 1$, for some $x \in \Sigma^*$, then the value of $\eta(x)$ is unimprotant.

Example 4.4. Consider the token "identifier" which is described by the following IFRE. Let $\Sigma_l = \{a, b, c \ldots, z\}$, $\Sigma_d = \{0, 1, 2, \ldots, 9\}$, $\Sigma_o = \Sigma_l \cup \Sigma_d$, where l denotes any letter belonging to Σ_l and d denotes any digit belonging to Σ_d.

$$\Sigma_l\Sigma_o^* + \frac{\Sigma_l((+))\Sigma_o^*}{0.9/0.1} + \frac{\Sigma_d\Sigma_l\Sigma_o^*}{0.7/0.2} + \frac{\Sigma_d\Sigma_d\Sigma_o^*}{0.3/0.5}$$

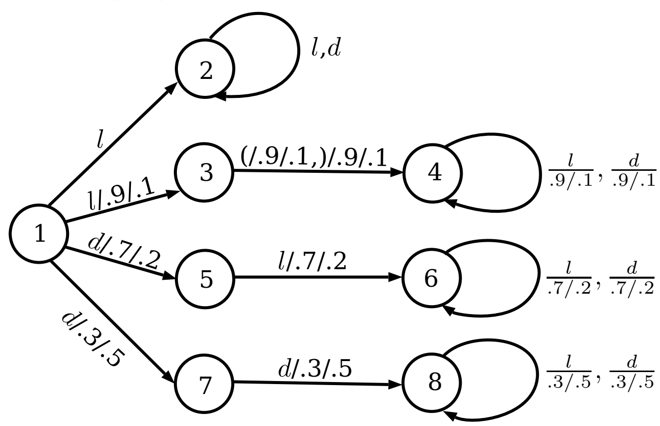
Following the three algorithms, EPSILON, IFAUM, MINIFAUM systematically, we obtain an IFAUM corresponding to the IFRE is shown below: $\mathscr{A} = (Q, \Sigma, A, i, f)$ with intuitionistic fuzzy regular behavior $L_{\mathscr{A}}$, where $Q = \{j \mid 1 \leq j \leq 8\}, \Sigma$ is the character set of the programming language.
$i : Q \to [0, 1]$ is defined by

$$i_{\mu_A}(j) = \begin{cases} 1, \text{if } j = 1 \\ 0, \text{otherwise} \end{cases} \quad \text{and} \quad i_{\nu_A}(j) = \begin{cases} 0, \text{if } j = 1 \\ 1, \text{otherwise} \end{cases}$$

$f : Q \to [0, 1]$ is defined by

$$f_{\mu_A}(j) = \begin{cases} 1, \text{if } j = 2, 4, 6, 8 \\ 0, \text{otherwise} \end{cases} \quad \text{and} \quad f_{\nu_A}(j) = \begin{cases} 0, \text{if } j = 2, 4, 6, 8 \\ 1, \text{otherwise} \end{cases}$$

$\mu_A, \nu_A : Q \times \Sigma \times Q \to [0, 1]$ is shown in the following transition diagram,



We assume the following actions for different ranges of degrees:

$$(0.9, 1] : \{accept\}$$
$$[0.8, 0.9] : \{warning; accept\}$$
$$[0.7, 0.8) : \{question\}$$
$$[0, 0.7) : \{reject\}$$

The classification of strings with some examples, to be an identifier is illustrated below.

(i) Let $x = max$, then $L_{\mu_{\mathscr{A}}}(x) = 1$, $L_{\nu_{\mathscr{A}}}(x) = 0$
$\therefore max$ is accepted as the token identifier.

(ii) Let $x = m(ax,$ then $L_{\mu_{\mathscr{A}}}(x) = 0.9,\ L_{\nu_{\mathscr{A}}}(x) = 0.1$
$\therefore m(ax$ is accepted as the token identifier with warning.

(iii) Let $x = 1y,$ then $L_{\mu_{\mathscr{A}}}(x) = 0.7,\ L_{\nu_{\mathscr{A}}}(x) = 0.2$
$\therefore 1y$ is accepted if the user wants to accept it as an identifier, by answering "yes", otherwise $1y$ will be rejected as non-identifier.

(iv) Let $x = 123x,$ then $L_{\mu_{\mathscr{A}}}(x) = 0.3,\ L_{\nu_{\mathscr{A}}}(x) = 0.5 \therefore 123x$ is rejected.

4.1. **Algorithm MIFRB** $(t_1, t_2, \ldots, t_n)$**.** The algorithm uses the input alphabet Σ, the character set of the programming language and $\{t_1, t_2, \ldots, t_n\}$, $n > 0$, the set of all tokens with linear order of priorities of their names η_i, $i = 1, 2, 3, \ldots, n$. It computes IFAUM for each token and then marked intuitionistic fuzzy regular behaviors are generated. It returns the marked intuitionistic fuzzy behavior for each token t_i, $i = 1, 2, 3, \ldots, n$.

Algorithm 6. MIFRB

```
begin
   for i := 1  to  n do
     begin
        Let ē_i be the IFRE that denotes t_i;
        /* Now find an IFFSA for the IFRE ē  */
        𝒜_i:= EPSILON (ē);
        /* Now find an IFAUM for the IFFSA 𝒜_1  */
        𝒜'_i:= IFAUM (𝒜_i);
        /* Find the minimum state IFAUM for the IFFSA 𝒜'_i  */
        𝒜''_i:= MINIFAUM (𝒜'_i);
        /* We consider the final IFAUM for ē as 𝒜_i  */
        Let 𝒜_i := 𝒜''_i and L_𝒜_i be an intuitionistic fuzzy regular behavior;
     end
   for i := 1  to  n do
        Let (L_𝒜_i, η_i) be the marked intuitionistic fuzzy regular behavior
        for token t_i;
end
```

4.2. **Algorithm IFLEX.** The algorithm uses the marked intuitionistic fuzzy regular behaviors $(L_{\mathscr{A}_i}, \eta_i)$ for each token t_i, $i = 1, 2, 3 \ldots, n$. These are generated by calling the Algorithm MIFRB. It determines the degree of the input string $x \in \Sigma^*$ and decides the action to be taken.

Algorithm 7. IFLEX

```
begin
   Let L_μ_𝒜_k(x) := ∨{L_μ_𝒜_i(x) | i = 1,2,...,n}, for some k, 1 ≤ k ≤ n;
     /* thus the degree of x is defined */
       if(L_μ_𝒜_k(x) ≠ L_μ_𝒜_j(x), j ≠ k) then
            begin
              /* no conflict, maximum is unique */
              L_𝒜(x) := (L_μ_𝒜_k(x), L_ν_𝒜_k(x));
```

```
            η(x) := η_k(x);
          end
      else if (L_{μ_{𝒜_k}}(x) = L_{μ_{𝒜_j}}(x)) then
          begin
            if (L_{ν_{𝒜_k}}(x) < L_{ν_{𝒜_j}}(x)) then
              begin
                η(x) := η_k(x);
              end
            else if (L_{ν_{𝒜_k}}(x) > L_{ν_{𝒜_j}}(x)) then
              begin
                η(x) := η_j(x);
              end
            else
              begin
                if(η_{𝒜_k}(x) ≤ η_{𝒜_j}(x)) then
                η(x) := η_j(x);
                else
                η(x) := η_k(x);
              end
          end
    /* deciding the action to be taken */
      if(0.9 < L_𝒜(x) ≤ 1) then
        begin
          write (“accepted”);
          write (“Token name=”,η(x));
        end
      else if(0.8 < L_𝒜(x) ≤ 0.9) then
        begin
          write (“warning; accepted”);
          write (“Token name=”, η(x));
        end
      else if(0.7 < L_𝒜(x) ≤ 0.8) then
        begin
          write (“degree is low”);
          write (“Do you want to accept?  yes/ no”);
          if ( yes ) then
              begin
              write (“accepted”);
              write (“Token name=”, η(x));
              end
          else
              write (“rejected”);
        end
      else
          write (“ERROR, the string is rejected”);
end
```

5. Conclusions

Authors have investigated by designing three algorithms namely EPSILON, IFAUM, MINIFAUM in order to find minimum state IFAUM for a given intuitionistic fuzzy regular expression. They are thoroughly discussed by implementing the notion of an intuitionistic fuzzy lexical analyzer.

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