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Pairwise open (closed) soft sets in soft bitopological spaces

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Abstract. The aim of this paper is to introduce and study the notions of pairwise open (closed) soft sets, pairwise soft interior (respectively closure, kernel) operator in a soft bitopological spaces. Moreover, some important results related to these notions are obtained. The important of these notions is, it is consider to be a generalization of the notions of open soft sets, closed soft sets, soft interior, soft closure in soft topological spaces. Furthermore, the concept of pairwise Λ-soft sets is presented and it is proved that the family of all pairwise Λ-soft sets is an Alexandroff soft topology. In addition, we introduce and characterize a new type of soft sets in a soft bitopological spaces namely pairwise λ-closed soft set and investigate some of its basic properties. Finally, comparisons between these soft sets are obtained.

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Keywords: Soft sets, Soft topology, Soft bitopology, Soft bitopological spaces, Pairwise open soft sets, Pairwise closed soft sets, Pairwise soft closure, Pairwise soft interior, Pairwise soft kernel, Pairwise Λ-soft sets, and Pairwise λ-closed soft sets.

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1. Introduction

Molodtsov [12] introduced soft sets as a mathematical tool for dealing with uncertainties. Recently, topological structures of soft set have been studied. In 2011, Shabir and Naz [18] and N. Cagman et al. [4] initiated the study of soft topology and soft topological spaces independently. Shabir and Naz [18] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. They introduced the definitions of open soft sets, closed soft sets, soft interior, soft closure, soft subspace and soft separation axioms. N. Cagman et al. [4] defined the soft topology on a soft set and presented its related properties and foundations of the theory of soft topological spaces. Moreover, the other properties of topological structures of soft topological space has been discussed in [2, 6, 13, 14, 15, 20, 21].

As a generalized of soft topological spaces, El-Sheikh and Abd El-latif [5] introduced the notion of supra soft topological spaces by dropping only the soft intersection condition.

Ittanagi [3] introduced the notion of soft bitopological space which is defined over an initial universal set X with fixed set of parameters E, also he introduced some types of soft separation axioms. A study of soft bitopological spaces is a generalization of the study of soft topological spaces as every soft bitopological space (X, η_1, η_2, E) can be regarded as a soft topological space (X, η, E) if $\eta_1 = \eta_2 = \eta$.

In 2014, G. Senel and N. Cagman [17] defined closed soft sets, α-closed soft sets, semi-closed soft sets, pre-closed soft sets, regular closed soft sets, g-closed soft sets and sg-closed soft sets on soft bitopological spaces. They also gave related properties of these soft sets and compared their properties with each other.

The main purpose of the present paper is to generalized the notions of soft topological spaces such as open soft sets, closed soft sets, soft interior, soft closure in a soft bitopological spaces. So, we introduce and study the notions of pairwise open (closed) soft sets, pairwise soft interior (respectively, closure, kernel) operator in a soft bitopological space (X, η_1, η_2, E). The properties of these notions and some important results related to it are obtained. We show that the family of all pairwise open soft sets is a supra soft topology η_{12} which is containing η_1, η_2 but it is not soft topology in general. For this reason, we introduce the concept of pairwise Λ-soft sets as soft sets that coincide with their pairwise soft kernel. Moreover, we conclude that the family of all pairwise Λ-soft sets is an Alexandroff soft topology $\eta_{P\Lambda}$ which is finer than η_1, η_2 and containing η_{12}. In addition, we introduce and characterize a new type of soft sets in a soft bitopological spaces namely pairwise λ-closed soft set and investigate some of its basic properties. Relationships between these soft sets are obtained.

2. Preliminaries

In this section, we collect some needed definitions and theorems of the material used in this paper.

Definition 2.1 ([12])**.** Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F, A) and if $e \notin A$, then $F(e) = \phi$ i.e.,
$(F, A) = \{(e, F(e)) : e \in A \subseteq E,\ F : A \to P(X)\}$. The family of all these soft sets denoted by $SS(X)_E$.

According to [14], any soft set (F, A) can be extended to a soft set of type (F, E) as follows: Let E be the set of parameters and $A \subseteq E$. Then, for each soft set (F, A) over X, a soft set (H, E) is constructed over X; where $\forall e \in E$,

$$H(e) = \begin{cases} F(e) & if\ \ e \in A, \\ \phi & if\ \ e \in E \setminus A. \end{cases}$$

Thus the soft sets (F, A) and (H, E) are equivalent to each other and the usual set operations of the soft sets $(F_i; A_i)$; $i \in \Delta$ is the same as those of the soft sets (H_i, E); $i \in \Delta$. For this reason, in this paper, we have considered our soft sets over the same parameter set E. Thus, a more general definition can be given as follows.

Definition 2.2 ([14])**.** Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X. A pair (F, E) is called a soft set over X, where F is a mapping given by $F : E \to P(X)$. A soft set can also be defined by the set of ordered pairs

$$(F, E) = \{(e, F(e)) : e \in E, \ \ F : E \to P(X)\}.$$

From now on, $SS(X)_E$ denotes the family of all soft sets over X with a fixed set of parameters E.

Definition 2.3 ([10])**.** Let (F, E), $(G, E) \in SS(X)_E$. Then

(1) (F, E) is a soft subset of (G, E), denoted by $(F, E)\tilde{\subseteq}(G, E)$, if $F(e) \subseteq G(e)$, $\forall e \in E$. In this case, (F, E) is said to be a soft subset of (G, E) and (G, E) is said to be a soft superset of (F, E).

(2) Two soft sets $(F, E)$and (G, E) over a common universe set X are said to be equal, denoted by $(F, E) = (G, E)$, if $F(e) = G(e)$, $\forall\, e \in E$.

(3) The union of two soft sets (F, E) and (G, E) over the common universe X, denoted by $(F, E)\tilde{\cup}(G, E)$, is the soft set (H, E), where $H(e) = F(e) \cup G(e)$, for all $e \in E$.

(4) The intersection of two soft sets (F, E) and (G, E) over the common universe X, denoted by $(F, E)\tilde{\cap}(G, E)$, is the soft set (M, E), where $M(e) = F(e) \cap G(e)$, for all $e \in E$.

Definition 2.4 ([1])**.** The complement of a soft set (F, E), denoted by $(F, E)^c$, is defined by $(F, E)^c = (F^c, E)$, where $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X \setminus F(e)$, $\forall\, e \in E$ and F^c is called the soft complement function of F.

Clearly, $(F^c)^c$ is the same as F and $((F, E)^c)^c = (F, E)$.

Definition 2.5 ([10])**.** Let X be an initial universe and E be a set of parameters. Then

(1) A soft set (F, E) over X is said to be a NULL soft set, denoted by $(\tilde{\phi}, E)$ or $\tilde{\phi}$, if $F(e) = \phi$ for all $e \in E$.

(2) A soft set (F, E) over X is said to be an absolute soft set, denoted by $(\tilde{X}, E)$ or $\tilde{X}$, if $F(e) = X$ for all $e \in E$.

Clearly we have $(\tilde{\phi}, E)^c = (\tilde{X}, E)$ and $(\tilde{X}, E)^c = (\tilde{\phi}, E)$.

Definition 2.6 ([21])**.** Let I be an arbitrary indexed set and $L = \{(F_i, E), i \in I\}$ be a subfamily of $SS(X)_E$.

(1) The union of L is the soft set (H, E), where $H(e) = \bigcup_{i \in I} F_i(e)$ for each $e \in E$. We write $\tilde{\bigcup}_{i \in I}(F_i, E) = (H, E)$.

(2) The intersection of L is the soft set (M, E), where $M(e) = \bigcap_{i \in I} F_i(e)$ for each $e \in E$. We write $\tilde{\bigcap}_{i \in I}(F_i, E) = (M, E)$.

Proposition 2.7 ([6]). *Let I be an arbitrary set and $\{(F_i, E) : i \in I\} \subseteq SS(X)_E$. Then the following statements are true.*

(1) $(F_i, E)\tilde{\subseteq}\tilde{\bigcup}\{(F_i, E) : i \in I\}$, *for every* $i \in I$.

(2) $\tilde{\bigcap}\{(F_i, E) : i \in I\}\tilde{\subseteq}(F_i, E)$, *for every* $i \in I$.

(3) $[\tilde{\bigcup}\{(F_i, E) : i \in I\}]^c = \tilde{\bigcap}\{(F_i, E)^c : i \in I\}$.

(4) $[\tilde{\bigcap}\{(F_i, E) : i \in I\}]^c = \tilde{\bigcup}\{(F_i, E)^c : i \in I\}$.

For more details about the properties of the union, the intersection and the complement of soft sets can you see in [6, 9, 16, 19, 21].

Definition 2.8 ([13, 19]). The soft set $(F, E) \in SS(X)_E$ is called a soft point in $(\tilde{X}, E)$ if there exist $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \phi$ for each $e' \in E \setminus \{e\}$, and the soft point (F, E) is denoted by (x_e, E) or x_e, i.e.,
$x_e : E \to P(X)$ is a mapping defined by

$$x_e(a) = \begin{cases} \{x\} & if \;\; e = a, \\ \phi & if \;\; e \neq a \end{cases} \qquad \text{for all } a \in E.$$

The set of all soft points in $(\tilde{X}, E)$ is denoted by $\xi(X)_E$.

Definition 2.9 ([19]). The soft point x_e is said to be belonging to the soft set (G, E), denoted by $x_e\tilde{\in}(G, E)$, if $x_e(e) \subseteq G(e)$, i.e., $\{x\} \subseteq G(e)$.

Proposition 2.10 ([19]). *The union of any collection of soft points can be considered as a soft set and every soft set can be expressed as a union of all soft points belonging to it, i.e.,* $(G, E) = \tilde{\bigcup}\{x_e : x_e\tilde{\in}(G, E)\}$.

Proposition 2.11 ([19]). *Let (G, E), (H, E) be two soft sets over X. Then*

(1) $x_e\tilde{\in}(G, E) \Leftrightarrow x_e\tilde{\notin}(G, E)^c$.

(2) $x_e\tilde{\in}(G, E)\tilde{\cup}(H, E) \Leftrightarrow x_e\tilde{\in}(G, E)$ *or* $x_e\tilde{\in}(H, E)$.

(3) $x_e\tilde{\in}(G, E)\tilde{\cap}(H, E) \Leftrightarrow x_e\tilde{\in}(G, E)$ *and* $x_e\tilde{\in}(H, E)$.

(4) $(G, E)\tilde{\subseteq}(H, E) \Leftrightarrow x_e\tilde{\in}(G, E) \Rightarrow x_e\tilde{\in}(H, E)$.

Definition 2.12 ([18]). Let η be a collection of soft sets over a universe X with a fixed set of parameters E. Then, $\eta \subseteq SS(X)_E$ is called a soft topology on X if it satisfies the following axioms:

(1) $(\tilde{X}, E), (\tilde{\phi}, E) \in \eta$, where $\tilde{\phi}(e) = \phi$ and $\tilde{X}(e) = X$, $\forall e \in E$,

(2) the union of any number of soft sets in η belongs to η,

(3) the intersection of any two soft sets in η belongs to η.

The triplet (X, η, E) is called a soft topological space.

Definition 2.13 ([8]). Let (X, η, E) be a soft topological space. Then

(1) Any member of η is said to be an open soft set in (X, η, E).

(2) A soft set (F, E) over X is said to be a closed soft set in (X, η, E), if its complement $(F, E)^c$ is an open soft set in (X, η, E).

Proposition 2.14 ([18]). *Let (X, η, E) be a soft topological space over X. Then*

(1) $(\tilde{X}, E), (\tilde{\phi}, E)$ *are closed soft sets in* (X, η, E).

(2) *The intersection of any number of closed soft sets is a closed soft set.*

(3) *The union of any two closed soft sets is a closed soft set.*

We denote the set of all closed soft sets by $CS(X, \eta, E)$, or $CS_\eta(X)$. For more details about the properties of the soft topological space can you see in [2, 6, 7, 8, 11, 13, 14, 15, 18, 20, 21].

Theorem 2.15 ([20])**.** *Let (X, η, E) be a soft topological space. Then, for any soft point x_e, $x_e \tilde{\in} scl_\eta(F, E)$ if and only if each soft neighborhood of x_e intersects (F, E), i.e.,*

$$x_e \tilde{\in} scl_\eta(G, E) \;\Leftrightarrow\; (O_{x_e}, E)\tilde{\cap}(G, E) \neq (\tilde{\phi}, E) \;\; \forall\, (O_{x_e}, E) \in \eta(x_e),$$

where $\eta(x_e)$ denote the family of all η-open soft sets which contains x_e.

Definition 2.16 ([5])**.** Let μ be a collection of soft sets over X [i.e., $\mu \subseteq SS(X)_E$]. Then, μ is said to be a supra soft topology on X if it satisfies the following conditions:
(1) $(\tilde{X}, E)$, $(\tilde{\phi}, E) \in \mu$,
(2) the union of any number of soft sets in μ belongs to μ.

Definition 2.17 ([3])**.** A quadrable system (X, η_1, η_2, E) is called a soft bitopological space, where η_1, η_2 are arbitrary soft topologies on X and E be a set of parameters.

3. Some properties of soft bitopological spaces

In this section, we introduce and study the notions of pairwise open (closed) soft sets, pairwise soft interior (respectively, closure, kernel) operator, pairwise Λ-soft sets and pairwise λ-closed soft sets in a soft bitopological space (X, η_1, η_2, E). Moreover, the basic properties of these notions are presented. Finally, relationships between these soft sets are obtained.

Definition 3.1. Let (X, η_1, η_2, E) be a soft bitopological space. A soft set (G, E) over X is said to be a pairwise open soft set in (X, η_1, η_2, E) (p-open soft set, for short) if there exist an open soft set (G_1, E) in η_1 and an open soft set (G_2, E) in η_2 such that $(G, E) = (G_1, E)\tilde{\cup}(G_2, E)$.

Definition 3.2. Let (X, η_1, η_2, E) be a soft bitopological space. A soft set (G, E) over X is said to be a pairwise closed soft set in (X, η_1, η_2, E) (p-closed soft set, for short) if its complement is a p-open soft set in (X, η_1, η_2, E). Clearly, a soft set (F, E) over X is a p-closed soft set in (X, η_1, η_2, E) if there exist a closed soft set (F_1, E) in η_1^c and a closed soft set (F_2, E) in η_2^c such that $(F, E) = (F_1, E)\tilde{\cap}(F_2, E)$, where

$$\eta_i^c = \{(G, E)^c \in SS(X)_E : (G, E) \in \eta_i\},\; i = 1, 2.$$

The family of all p-open (closed) soft sets in (X, η_1, η_2, E) is denoted by $POS(X, \eta_1, \eta_2)_E$ $(PCS(X, \eta_1, \eta_2)_E)$, respectively.

Example 3.3. Let $X = \{x, y, z\}$ and $E = \{\alpha, \beta\}$. We consider the following soft sets over X.

$$(G_1, E) = \{(\alpha, \{x\}), (\beta, \{x, z\})\},$$
$$(G_2, E) = \{(\alpha, \{x, y\}), (\beta, X)\},$$
$$(G_3, E) = \{(\alpha, \{y\}), (\beta, \{y\})\},$$
$$(H_1, E) = \{(\alpha, \{y, z\}), (\beta, \{x\})\},$$
$$(H_2, E) = \{(\alpha, \phi), (\beta, \{y\})\},$$

$(H_3, E) = \{(\alpha, \{y, z\}), (\beta, \{x, y\})\}$.
Then, (X, η_1, η_2, E) is a soft bitopological space,
where

$$\eta_1 = \{(\tilde{\phi}, E), (\tilde{X}, E), (G_1, E), (G_2, E), (G_3, E)\}$$

and

$$\eta_2 = \{(\tilde{\phi}, E), (\tilde{X}, E), (H_1, E), (H_2, E), (H_3, E)\}.$$

It is clear that

$$\eta_{12} = \eta_1 \cup \eta_2 \cup \{(G_1, E)\tilde{\cup}(H_1, E), (G_1, E)\tilde{\cup}(H_2, E)\},$$

because

$$\begin{aligned}
&(G_1, E)\tilde{\cup}(H_3, E) = (\tilde{X}, E),\\
&(G_2, E)\tilde{\cup}(H_1, E) = (G_2, E)\tilde{\cup}(H_3, E) = (\tilde{X}, E),\\
&(G_2, E)\tilde{\cup}(H_2, E) = (G_2, E),\\
&(G_3, E)\tilde{\cup}(H_1, E) = (H_3, E),\\
&(G_3, E)\tilde{\cup}(H_2, E) = (G_3, E),\\
&(G_3, E)\tilde{\cup}(H_3, E) = (H_3, E).
\end{aligned}$$

While $(G_1, E)\tilde{\cup}(H_1, E) = \{(\alpha, X), (\beta, \{x, z\})\}$ which not belongs to either η_1 nor η_2, also $(G_1, E)\tilde{\cup}(H_2, E) = \{(\alpha, \{x\}), (\beta, X)\}$ which not belongs to either η_1 nor η_2. It is easy to find the family of all p-closed soft sets.

The following theorem studies the main properties of p-open (closed) soft sets.

Theorem 3.4. *Let (X, η_1, η_2, E) be a soft bitopological space. Then*
(1) *$(\tilde{\phi}, E)$, $(\tilde{X}, E)$ are p-open soft sets and p-closed soft sets.*
(2) *An arbitrary union of p-open soft sets is a p-open soft set.*
(3) *An arbitrary intersection of p-closed soft sets is a p-closed soft set.*
(4) *If $(G, E) \in \eta_1 \cap \eta_2$ and $(H, E) \in POS(X, \eta_1, \eta_2)_E$, then $(G, E)\tilde{\cap}(H, E) \in POS(X, \eta_1, \eta_2)_E$.*

Proof. (1) Since $(\tilde{\phi}, E) \in \eta_1, \eta_2$, $(\tilde{\phi}, E) = (\tilde{\phi}, E)\tilde{\cup}(\tilde{\phi}, E)$, $(\tilde{\phi}, E)$ is a p-open soft set. Similarly, $(\tilde{X}, E)$ is a p-open soft set.

(2) Let $\{(F_i, E) : i \in \Delta\} \subseteq POS(X, \eta_1, \eta_2)_E$. Then (F_i, E) is a p-open soft set for all $i \in \Delta$. Thus there exist $(F_i^1, E) \in \eta_1$ and $(F_i^2, E) \in \eta_2$ such that $(F_i, E) = (F_i^1, E)\tilde{\cup}(F_i^2, E)$ for all $i \in \Delta$ which implies that

$$\tilde{\bigcup}_{i\in\Delta}(F_i, E) = \tilde{\bigcup}_{i\in\Delta}[(F_i^1, E)\tilde{\cup}(F_i^2, E)] = [\tilde{\bigcup}_{i\in\Delta}(F_i^1, E)]\tilde{\cup}[\tilde{\bigcup}_{i\in\Delta}(F_i^2, E)].$$

Now, since η_1 and η_2 are soft topologies, $[\tilde{\bigcup}_{i\in\Delta}(F_i^1, E)] \in \eta_1$ and $[\tilde{\bigcup}_{i\in\Delta}(F_i^2, E)] \in \eta_2$. Consequently, $\tilde{\bigcup}_{i\in\Delta}(F_i, E)$ is a p-open soft set.

(3) It is immediate from the Definition 3.2 , Propositions 2.7 and 2.14.

(4) It is obvious. □

Corollary 3.5. *Let (X, η_1, η_2, E) be a soft bitopological space. Then, the family of all p-open soft sets is a supra soft topology on X. This supra soft topology we denoted by η_{12}, i.e.,*

$$\eta_{12} = POS(X, \eta_1, \eta_2)_E = \{(G, E) = (G_1, E)\tilde{\cup}(G_2, E) : (G_i, E) \in \eta_i, i = 1, 2\},$$

and the triple (X, η_{12}, E) is the supra soft topological space associated to the soft bitopological space (X, η_1, η_2, E).

The following example show that:
(1) η_{12} is not soft topology in general.
(2) The finite intersection of p-open soft sets need not be a p-open soft set.
(3) The arbitrary union of p-closed soft sets need not be a p-closed soft set.

Example 3.6. Let $X = \mathbb{N}$, $E = \{0, 1\}$ and let (G_n, E) be a soft set over $\mathbb{N}$ defined as

$$G_n : E \to P(\mathbb{N})$$

such that

$$G_n(1) = \{n,\ n+1,\ n+2,\ ...\} \text{ and } G_n(0) = \phi \text{ for all } n \in \mathbb{N} \text{ and let}$$
$$\eta_1 = \{(\tilde{\phi}, E), (\tilde{\mathbb{N}}, E)\} \cup \{(G_n, E) :\ n = 1, 2, 3, ...\}.$$

It is clear that η_1 is a soft topology on $\mathbb{N}$.

Now, let (H_m, E) be a soft set over $\mathbb{N}$ defined as

$$H_m : E \to P(\mathbb{N})$$

such that

$$H_m(1) = \{1,\ 2,\ 3,\ ...,\ m\} \text{ and } H_m(0) = \phi, \text{ for all } m \in \mathbb{N} \text{ and let}$$
$$\eta_2 = \{(\tilde{\phi}, E), (\tilde{\mathbb{N}}, E)\} \cup \{(H_m, E) :\ m = 1, 2, 3, ...\}.$$

Then η_2 is a soft topology on $\mathbb{N}$. Consequently, $(\mathbb{N}, \eta_1, \eta_2, E)$ is a soft bitopological space. Now, $(G_3, E) \in \eta_1$ and $(H_3, E) \in \eta_2$ are p-open soft sets (for $\eta_1, \eta_2 \subseteq \eta_{12}$). But, $(G_3, E)\tilde{\cap}(H_3, E)$ is not p-open soft set, for $(G_3, E)\tilde{\cap}(H_3, E) = (F, E)$ such that $F(1) = G_3(1) \cap H_3(1) = \{3, 4, 5, ...\} \cap \{1, 2, 3\} = \{3\}$, $F(0) = G_3(0) \cap H_3(0) = \phi$. It is clear that (F, E) cannot be expressed as a union of two soft sets one belongs to η_1 and the other belongs to η_2, i.e., (F, E) is not p-open soft set. Hence, the finite intersection of p-open soft sets need not be a p-open soft set. Consequently, η_{12} is not soft topology in general.

On the other hand, since (G_3, E) and (H_3, E) are p-open soft sets, $(G_3, E)^c$ and $(H_3, E)^c$ are p-closed soft sets, but $(G_3, E)^c\tilde{\cup}(H_3, E)^c$ is not p-closed soft set, because $(G_3, E)^c\tilde{\cup}(H_3, E)^c = (M, E)$ such that $M(1) = G_3^c(1) \cup H_3^c(1) = \mathbb{N} \setminus \{3, 4, 5, ...\} \cup \mathbb{N} \setminus \{1, 2, 3\} = \mathbb{N} \setminus \{3\}$, $M(0) = G_3^c(0) \cup H_3^c(0) = \mathbb{N}$. It is clear that (M, E) cannot be expressed as an intersection of two soft sets one belongs to η_1^c and the other belongs to η_2^c, i.e., (M, E) is not p-closed soft set. Therefore, the arbitrary union of p-closed soft sets need not be a p-closed soft set.

Theorem 3.7. *Let (X, η_1, η_2, E) be a soft bitopological space. Then*
(1) *every η_i - open soft set is a p-open soft set, $i = 1, 2$, i.e., $\eta_1 \cup \eta_2 \subseteq \eta_{12}$.*
(2) *every η_i - closed soft set is a p-closed soft set, $i = 1, 2$, i.e., $\eta_1^c \cup \eta_2^c \subseteq \eta_{12}^c (= PCS(X, \eta_1, \eta_2)_E)$.*
(3) *if $\eta_1 \subseteq \eta_2$, then $\eta_{12} = \eta_2$ and $\eta_{12}^c = \eta_2^c$.*

Proof. Straightforward. □

Lemma 3.8. *Let (X, η_1, η_2, E) be a soft bitopological space. A soft set (G, E) over X is a p-open soft set if and only if for all $x_e \tilde{\in} (G, E)$ there exists $(O^1_{x_e}, E) \in \eta_1$ such that $(O^1_{x_e}, E) \tilde{\subseteq} (G, E)$ or there exists $(O^2_{x_e}, E) \in \eta_2$ such that $(O^2_{x_e}, E) \tilde{\subseteq} (G, E)$, where $(O^i_{x_e}, E)$ denote a η_i-open soft set containing x_e, $i = 1, 2$.*

Proof. Let (G, E) be a p-open soft set in (X, η_1, η_2, E). Then, there exist $(G_1, E) \in \eta_1$ and $(G_2, E) \in \eta_2$ such that $(G, E) = (G_1, E)\tilde{\cup}(G_2, E)$. Now, let $x_e\tilde{\in}(G, E)$. Then, $x_e\tilde{\in}(G_1, E)\tilde{\cup}(G_2, E)$ which implies that $x_e\tilde{\in}(G_1, E)$ or $x_e\tilde{\in}(G_2, E)$ [by Proposition 2.11], we get $x_e\tilde{\in}(G_1, E)\tilde{\subseteq}(G, E)$ or $x_e\tilde{\in}(G_2, E)\tilde{\subseteq}(G, E)$. Hence, for every $x_e\tilde{\in}(G, E)$ there exists $(O^1_{x_e}, E) = (G_1, E) \in \eta_1$ such that $(O^1_{x_e}, E)\tilde{\subseteq}(G, E)$ or there exists $(O^2_{x_e}, E) = (G_2, E) \in \eta_2$ such that $(O^2_{x_e}, E)\tilde{\subseteq}(G, E)$.

Conversely, suppose that for every $x_e\tilde{\in}(G, E)$ there exists $(O^1_{x_e}, E) \in \eta_1$ such that $(O^1_{x_e}, E)\tilde{\subseteq}(G, E)$ or there exists $(O^2_{x_e}, E) \in \eta_2$ such that $(O^2_{x_e}, E)\tilde{\subseteq}(G, E)$. We shall prove that $(G, E) \in \eta_{12}$. From hypothesis, we set $\Gamma_1 = \tilde{\bigcup}_{x_e\tilde{\in}(G,E)}\{(O^1_{x_e}, E)\tilde{\subseteq}(G, E) : (O^1_{x_e}, E) \in \eta_1\}$ and $\Gamma_2 = \tilde{\bigcup}_{x_e\tilde{\in}(G,E)}\{(O^2_{x_e}, E)\tilde{\subseteq}(G, E) : (O^2_{x_e}, E) \in \eta_2\}$. Then, by Proposition 2.10, we have $\Gamma_1\tilde{\cup}\Gamma_2 = (G, E)$. Since, $\Gamma_1 \in \eta_1$ and $\Gamma_2 \in \eta_2$, then $(G, E) \in \eta_{12}$. □

Definition 3.9. Let (X, η_1, η_2, E) be a soft bitopological space and let $(G, E) \in SS(X)_E$. The pairwise soft closure of (G, E), denoted by $scl_p(G, E)$, is the intersection of all p-closed soft super sets of (G, E), i.e.,

$$scl_p(G, E) = \tilde{\bigcap}\{(F, E) \in \eta^c_{12} : (G, E)\tilde{\subseteq}(F, E)\}.$$

Clearly, $scl_p(G, E)$ is the smallest p-closed soft set containing (G, E).

Example 3.10. Let $X = \{x, y, z\}$, $E = \{\alpha, \beta\}$ and let

$\eta_1 = \{(\tilde{\phi}, E), (\tilde{X}, E), (G_1, E), (G_2, E), (G_3, E)\}$,

$\eta_2 = \{(\tilde{\phi}, E), (\tilde{X}, E), (F_1, E), (F_2, E)\}$,

where

$(G_1, E) = \{(\alpha, \{x\}), (\beta, \{y\})\}$,
$(G_2, E) = \{(\alpha, \{y\}), (\beta, \{z\})\}$,
$(G_3, E) = \{(\alpha, \{x, y\}), (\beta, \{y, z\})\}$,
$(F_1, E) = \{(\alpha, \{x, z\}), (\beta, \{y, z\})\}$,
$(F_2, E) = \{(\alpha, \{x\}), (\beta, \{z\})\}$.

Then (X, η_1, η_2, E) is a soft bitopological space. It is clear that

$\eta_{12} = \{(\tilde{\phi}, E), (\tilde{X}, E), (G_1, E), (G_2, E), (G_3, E), (F_1, E), (F_2, E), (P_1, E), (P_2, E), (P_3, E)\}$,

where

$(P_1, E) = (G_1, E)\tilde{\cup}(F_2, E) = \{(\alpha, \{x\}), (\beta, \{y, z\})\}$,
$(P_2, E) = (G_2, E)\tilde{\cup}\ (F_1, E) = \{(\alpha, X), (\beta, \{y, z\})\}$,
$(P_3, E) = (G_2, E)\tilde{\cup}(F_2, E) = \{(\alpha, \{x, y\}), (\beta, \{z\})\}$.

Consequently,

$\eta^c_{12} = \{(\tilde{\phi}, E), (\tilde{X}, E), (G_1, E)^c, (G_2, E)^c, (G_3, E)^c, (F_1, E)^c, (F_2, E)^c, (P_1, E)^c, (P_2, E)^c, (P_3, E)^c\}$,

where

$(G_1, E)^c = \{(\alpha, \{y, z\}), (\beta, \{x, z\})\}$,
$(G_2, E)^c = \{(\alpha, \{x, z\}), (\beta, \{x, y\})\}$,
$(G_3, E)^c = \{(\alpha, \{z\}), (\beta, \{x\})\}$,
$(F_1, E)^c = \{(\alpha, \{y\}), (\beta, \{x\})\}$,
$(F_2, E)^c = \{(\alpha, \{y, z\}), (\beta, \{x, y\})\}$,
$(P_1, E)^c = \{(\alpha, \{y, z\}), (\beta, \{x\})\}$,

$(P_2, E)^c = \{(\alpha, \phi), (\beta, \{x\})\}$,
$(P_3, E)^c = \{(\alpha, \{z\}), (\beta, \{x, y\})\}$.

Now, let $(M, E) = \{(\alpha, \{z\}), (\beta, \{x, z\})\}$ and $(F, E) = \{(\alpha, \{y, z\}), (\beta, \{x\})\}$. Then, the p-closed soft sets which contains (M, E) are $(G_1, E)^c$ and $(\tilde{X}, E)$ it follows that $scl_p(M, E) = (G_1, E)^c \tilde{\cap} (\tilde{X}, E)$. Thus $scl_p(M, E) = \{(\alpha, \{y, z\}), (\beta, \{x, z\})\}$. Also, the p-closed soft sets which contains (F, E) are $(G_1, E)^c$, $(F_2, E)^c$, $(P_1, E)^c$ and $(\tilde{X}, E)$. Then $scl_p(F, E) = (G_1, E)^c \tilde{\cap} (F_2, E)^c \tilde{\cap} (P_1, E)^c \tilde{\cap} (\tilde{X}, E)$. Hence

$$scl_p(F, E) = \{(\alpha, \{y, z\}), (\beta, \{x\})\} = (F, E).$$

Theorem 3.11. *Let (X, η_1, η_2, E) be a soft bitopological space and (G, E), $(H, E) \in SS(X)_E$. Then*

(1) $scl_p(\tilde{\phi}, E) = (\tilde{\phi}, E)$ *and* $scl_p(\tilde{X}, E) = (\tilde{X}, E)$.
(2) $(G, E) \tilde{\subseteq} scl_p(G, E)$.
(3) (G, E) *is a p-closed soft set if and only if* $scl_p(G, E) = (G, E)$.
(4) $(G, E) \tilde{\subseteq} (H, E) \Rightarrow scl_p(G, E) \tilde{\subseteq} scl_p(H, E)$.
(5) $scl_p(G, E) \tilde{\cup} scl_p(H, E) \tilde{\subseteq} scl_p[(G, E) \tilde{\cup} (H, E)]$.
(6) $scl_p[scl_p(G, E)] = scl_p(G, E)$, *i.e,* $scl_p(G, E)$ *is a p-closed soft set.*

Proof. Straightforward. □

Theorem 3.12. *Let (X, η_1, η_2, E) be a soft bitopological space and $(G, E) \in SS(X)_E$. Then*

$$x_e \tilde{\in} scl_p(G, E) \Leftrightarrow (O_{x_e}, E) \tilde{\cap} (G, E) \neq (\tilde{\phi}, E), \ \forall \ (O_{x_e}, E) \in \eta_{12}(x_e),$$

where (O_{x_e}, E) is any p-open soft set contains x_e and $\eta_{12}(x_e)$ is the family of all p-open soft sets contains x_e.

Proof. Let $x_e \tilde{\in} scl_p(G, E)$ and assume that there exists $(O_{x_e}, E) \in \eta_{12}(x_e)$ such that $(O_{x_e}, E) \tilde{\cap} (G, E) = (\tilde{\phi}, E)$. Then $(G, E) \tilde{\subseteq} (O_{x_e}, E)^c$. Thus $scl_p(G, E) \tilde{\subseteq} scl_p(O_{x_e}, E)^c$ $= (O_{x_e}, E)^c$ which implies $scl_p(G, E) \tilde{\cap} (O_{x_e}, E) = (\tilde{\phi}, E)$, a contradiction.

Conversely, assume that $x_e \tilde{\notin} scl_p(G, E)$. Then $x_e \tilde{\in} [scl_p(G, E)]^c$. Thus $[scl_p(G, E)]^c$ $\in \eta_{12}(x_e)$. So, by hypothesis, $[scl_p(G, E)]^c \tilde{\cap} (G, E) \neq (\tilde{\phi}, E)$, a contradiction. □

Example 3.13. Let (X, η_1, η_2, E) be the same as in Example 3.10.

Now, we find $scl_p(M, E)$, where $(M, E) = \{(\alpha, \{z\}), (\beta, \{x, z\})\}$, by using Theorem 3.12 as follows:

$\xi(X)_E = \{x_\alpha, x_\beta, y_\alpha, y_\beta, z_\alpha, z_\beta\}$.

It is clear that

$\eta_{12}(x_\alpha) = \{(\tilde{X}, E), (G_1, E), (G_3, E), (F_1, E), (F_2, E), (P_1, E), (P_2, E), (P_3, E)\}$,
$\eta_{12}(x_\beta) = \{(\tilde{X}, E)\}$,
$\eta_{12}(y_\alpha) = \{(\tilde{X}, E), (G_2, E), (G_3, E), (P_2, E), (P_3, E)\}$,
$\eta_{12}(y_\beta) = \{(\tilde{X}, E), (G_1, E), (G_3, E), (F_1, E), (P_1, E), (P_2, E)\}$,
$\eta_{12}(z_\alpha) = \{(\tilde{X}, E), (F_1, E), (P_2, E)\}$,
$\eta_{12}(z_\beta) = \{(\tilde{X}, E), (G_2, E), (G_3, E), (F_1, E), (F_2, E), (P_1, E), (P_2, E), (P_3, E)\}$.

Thus we have $x_\alpha \tilde{\notin} scl_p(M, E)$, for $(G_1, E) \in \eta_{12}(x_\alpha)$ and $(G_1, E) \tilde{\cap} (M, E) = \{(\alpha, \{x\}),$ $(\beta, \{y\})\} \tilde{\cap} \{(\alpha, \{z\}), (\beta, \{x, z\})\} = (\tilde{\phi}, E)$, but $x_\beta \tilde{\in} scl_p(M, E)$, for $(\tilde{X}, E)$ is the only p-open soft set contains x_β and $(\tilde{X}, E)\tilde{\cap}(M, E) \neq (\tilde{\phi}, E)$. Also, $y_\alpha \tilde{\in}\ scl_p(M, E)$ because $(O_{y_\alpha}, E)\tilde{\cap}(M, E) \neq (\tilde{\phi}, E)\ \ \forall\ \ (O_{y_\alpha}, E) \in \eta_{12}(y_\alpha)$ but $y_\beta \tilde{\notin}\ scl_p(M, E)$ because $(G_1, E) \in \eta_{12}(y_\beta)$ and $(G_1, E)\tilde{\cap}(M, E) = (\tilde{\phi}, E)$.

Similarly, we have that $z_\alpha \tilde{\in}\ scl_p(M, E)$ because $(O_{z_\alpha}, E)\tilde{\cap}(M, E) \neq (\tilde{\phi}, E)$ $\forall\ \ (O_{z_\alpha}, E) \in \eta_{12}(z_\alpha)$, and $z_\beta \tilde{\in}\ scl_p(M, E)$ because $(O_{z_\beta}, E)\tilde{\cap}\ (M, E) \neq (\tilde{\phi}, E)$ $\forall\ \ (O_{z_\beta}, E) \in \eta_{12}(z_\beta)$. Consequently,

$$\begin{aligned} scl_p(M, E) &= \tilde{\bigcup}\{x_\beta, y_\alpha, z_\alpha, z_\beta\} \\ &= \{(\alpha, \phi), (\beta, \{x\})\}\tilde{\cup}\{(\alpha, \{y\}), (\beta, \phi)\}\tilde{\cup}\{(\alpha, \{z\}), (\beta, \phi)\}\tilde{\cup}\{(\alpha, \phi), (\beta, \{z\})\} \\ &= \{(\alpha, \{y, z\}), (\beta, \{x, z\})\}. \end{aligned}$$

Hence $scl_p(M, E) = \{(\alpha, \{y, z\}), (\beta, \{x, z\})\}$.

Theorem 3.14. *Let (X, η_1, η_2, E) be a soft bitopological space. A soft set (F, E) over X is a p-closed soft set if and only if $(F, E) = scl_{\eta_1}(F, E)\ \tilde{\cap}\ scl_{\eta_2}(F, E)$.*

Proof. Let (F, E) be a p-closed soft set and let $x_e \tilde{\notin} (F, E)$. Then $x_e \tilde{\notin} scl_p(F, E)$ [for $scl_p(F, E) = (F, E)$]. Thus, by Theorem 3.12, there exists $(O_{x_e}, E) \in \eta_{12}(x_e)$ such that $(O_{x_e}, E)\tilde{\cap}(F, E) = (\tilde{\phi}, E)$. Since $(O_{x_e}, E) \in \eta_{12}(x_e)$, there exists $(G_1, E) \in \eta_1$ and $(G_2, E) \in \eta_2$ such that $(O_{x_e}, E) = (G_1, E)\tilde{\cup}(G_2, E)$. So $[(G_1, E)\tilde{\cup}(G_2, E)]\tilde{\cap}(F, E) = (\tilde{\phi}, E)$. It follows that $(G_1, E)\tilde{\cap}(F, E) = (\tilde{\phi}, E)$ and $(G_2, E)\tilde{\cap}(F, E) = (\tilde{\phi}, E)$. Since $x_e \tilde{\in} (O_{x_e}, E)$, $x_e \tilde{\in} (G_1, E)$ or $x_e \tilde{\in} (G_2, E)$ and thus by Theorem 2.15, $x_e \tilde{\notin} scl_{\eta_1}(F, E)$ or $x_e \tilde{\notin} scl_{\eta_2}(F, E)$. Hence $x_e \tilde{\notin} scl_{\eta_1}(F, E)\tilde{\cap}scl_{\eta_2}(F, E)$. Therefore $scl_{\eta_1}(F, E)\tilde{\cap}scl_{\eta_2}(F, E)\tilde{\subseteq}(F, E)$.

On the other hand, we have $(F, E)\tilde{\subseteq}scl_{\eta_1}(F, E)\tilde{\cap}scl_{\eta_2}(F, E)$. Thus $(F, E) = scl_{\eta_1}(F, E)\ \tilde{\cap}\ scl_{\eta_2}(F, E)$.

Conversely, assume that $(F, E) = scl_{\eta_1}(F, E)\ \tilde{\cap}\ scl_{\eta_2}(F, E)$. Since $scl_{\eta_1}(F, E)$ is a closed soft set in (X, η_1, E) and $scl_{\eta_2}(F, E)$ is a closed soft set in (X, η_2, E), by Definition 3.2, $scl_{\eta_1}(F, E)\ \tilde{\cap}\ scl_{\eta_2}(F, E)$ is a p-closed soft set in (X, η_1, η_2, E). Hence (F, E) is a p-closed soft set. □

Corollary 3.15. *Let (X, η_1, η_2, E) be a soft bitopological space. Then,*

$$scl_p(G, E) = scl_{\eta_1}(G, E)\ \tilde{\cap}\ scl_{\eta_2}(G, E),\ \forall\ (G, E) \in SS(X)_E.$$

Definition 3.16. Let (X, η_1, η_2, E) be a soft bitopological space and let $(G, E) \in SS(X)_E$. The pairwise soft interior of (G, E), denoted by $sint_p(G, E)$, is the union of all p-open soft subsets of (G, E), i.e.,

$$sint_p(G, E) = \tilde{\bigcup}\{(H, E) \in \eta_{12} : (H, E)\tilde{\subseteq}(G, E)\}.$$

Clearly, $sint_p(G, E)$ is the largest p-open soft set contained in (G, E).

Example 3.17. Let (X, η_1, η_2, E) be the same as in Example 3.10. Now, let $(M, E) = \{(\alpha, \{y, z\}), (\beta, \{x, z\})\}$, $(F, E) = \{(\alpha, \{y, z\}), (\beta, \{x\})\}$, then the p-open soft sets which containing in (M, E) are (G_2, E), $(\tilde{\phi}, E)$. Therefore, $sint_p(M, E) = (G_2, E) = \{(\alpha, \{y\}), (\beta, \{z\})\}$. Also, the only p-open soft set which containing in (F, E) is $(\tilde{\phi}, E)$. Hence, $sint_p(F, E) = (\tilde{\phi}, E)$.

Theorem 3.18. *Let (X, η_1, η_2, E) be a soft bitopological space and (G, E), $(H, E) \in SS(X)_E$. Then*

(1) $sint_p(\tilde{\phi}, E) = (\tilde{\phi}, E)$ *and* $sint_p(\tilde{X}, E) = (\tilde{X}, E)$.

(2) $sint_p(G,E)\tilde{\subseteq}(G,E)$.
(3) (G,E) *is a p-open soft set if and only if* $sint_p(G,E)=(G,E)$.
(4) $(G,E)\tilde{\subseteq}(H,E) \Rightarrow sint_p(G,E)\tilde{\subseteq}sint_p(H,E)$.
(5) $sint_p[(G,E)\tilde{\cap}(H,E)]\tilde{\subseteq}sint_p(G,E)\tilde{\cap}sint_p(H,E)$.
(6) $sint_p[sint_p(G,E)]=sint_p(G,E)$, *i.e.,* $sint_p(G,E)$ *is a p-open soft set.*

Proof. Straightforward. □

Theorem 3.19. *Let* (X,η_1,η_2,E) *be a soft bitopological space and* $(G,E)\in SS(X)_E$. *Then,*

$$x_e\tilde{\in}sint_p(G,E) \;\Leftrightarrow\; \exists(O_{x_e},E)\in\eta_{12}(x_e) \text{ such that } (O_{x_e},E)\tilde{\subseteq}(G,E).$$

Proof. Straightforward. □

Theorem 3.20. *Let* (X,η_1,η_2,E) *be a soft bitopological space. A soft set* (G,E) *over* X *is a p-open soft set if and only if* $(G,E)=sint_{\eta_1}(G,E)\;\tilde{\cup}\;sint_{\eta_2}(G,E)$.

Proof. Let (G,E) be a p-open soft set. Since, $sint_{\eta_i}(G,E)\tilde{\subseteq}(G,E)$, $i=1,2$, then $sint_{\eta_1}(G,E)\tilde{\cup}sint_{\eta_2}(G,E)\tilde{\subseteq}(G,E)$. Now, let $x_e\tilde{\in}(G,E)$. Then, by Lemma 3.8, there exists $(O^1_{x_e},E)\in\eta_1$ such that $(O^1_{x_e},E)\tilde{\subseteq}(G,E)$ or there exists $(O^2_{x_e},E)\in\eta_2$ such that $(O^2_{x_e},E)\tilde{\subseteq}(G,E)$. thus $x_e\tilde{\in}sint_{\eta_1}(G,E)$ or $x_e\tilde{\in}sint_{\eta_2}(G,E)$. So $x_e\tilde{\in}sint_{\eta_1}(G,E)\;\tilde{\cup}\;sint_{\eta_2}(G,E)$. Consequently, $(G,E)=sint_{\eta_1}(G,E)\;\tilde{\cup}\;sint_{\eta_2}(G,E)$.

Conversely, since $sint_{\eta_1}(G,E)$ is an open soft set in (X,η_1,E) and $sint_{\eta_2}(G,E)$ is an open soft set in (X,η_2,E), by Definition 3.1, $sint_{\eta_1}(G,E)\;\tilde{\cup}\;sint_{\eta_2}(G,E)$ is a p-open soft set in (X,η_1,η_2,E). Thus (G,E) is a p-open soft set. □

Corollary 3.21. *Let* (X,η_1,η_2,E) *be a soft bitopological space. Then*

$$sint_p(G,E)=sint_{\eta_1}(G,E)\;\tilde{\cup}\;sint_{\eta_2}(G,E).$$

Remark 3.22. Let (X,η_1,η_2,E) be a soft bitopological space. Then
(1) $scl_p[(G,E)\tilde{\cup}(H,E)]\neq scl_p(G,E)\tilde{\cup}scl_p(H,E)$, in general.
(2) $scl_p(F,E)=(F,E)\not\Rightarrow(F,E)\in\eta_1^c\cup\eta_2^c$, in general.
(3) $sint_p[(G,E)\tilde{\cap}(H,E)]\neq sint_p(G,E)\tilde{\cap}sint_p(H,E)$, in general.
(4) $sint_p(G,E)=(G,E)\not\Rightarrow(G,E)\in\eta_1\cup\eta_2$, in general.

The following example proved the previous remark.

Example 3.23. Let (X,η_1,η_2,E) be the same soft bitopological space in Example 3.10. Then

(1) Let $(G,E)=\{(\alpha,\{z\}),(\beta,\{x,z\})\}$ and let $(H,E)=\{(\alpha,\{z\}),(\beta,\{y\})\}$. Then

$$scl_p(G,E)=\{(\alpha,\{y,z\}),(\beta,\{x,z\})\}$$

and

$$scl_p(H,E)=\{(\alpha,\{z\}),(\beta,\{x,y\})\}.$$

Thus

$$scl_p(G,E)\tilde{\cup}scl_p(H,E)=\{(\alpha,\{y,z\}),(\beta,X)\}.$$

Now, $(G,E)\tilde{\cup}(H,E)=\{(\alpha,\{z\}),(\beta,X)\}$, but $scl_p[(G,E)\tilde{\cup}(H,E)]=(\tilde{X},E)$. It is clear that $scl_p[(G,E)\tilde{\cup}(H,E)]\neq scl_p(G,E)\tilde{\cup}scl_p(H,E)$, in general.

(2) Let $(F, E) = \{(\alpha, \{y, z\}), (\beta, \{x\})\}$. Then $scl_p(F, E) = (F, E)$ but (F, E) not belongs to either η_1^c nor η_2^c, that is means $\eta_1^c \cup \eta_1^c \neq \eta_{12}^c$, in general.

(3) Let $(G, E) = \{(\alpha, \{y\}), (\beta, \{z\})\}$ and let $(H, E) = \{(\alpha, \{x\}), (\beta, \{y, z\})\}$. Then $sint_p(G, E) = (G, E)$ and $sint_p(H, E) = (H, E)$. Thus $sint_p(G, E)\tilde{\cap} sint_p(H, E) = \{(\alpha, \phi), (\beta, \{z\})\}$.

On the other hand, since $(G, E)\tilde{\cap}(H, E) = \{(\alpha, \phi), (\beta, \{z\})\}$, $sint_p[(G, E)\tilde{\cap} (H, E)] = (\tilde{\phi}, E)$ because $(\tilde{\phi}, E)$ is the only p-open soft set containing in $(G, E)\tilde{\cap}(H, E)$. It is clear that $sint_p[(G, E)\tilde{\cap}(H, E)] \neq sint_p(G, E)\tilde{\cap} sint_p(H, E)$, in general.

(4) Let $(G, E) = \{(\alpha, \{x\}), (\beta, \{y, z\})\}$. Then $sint_p(G, E) = (G, E)$, but (G, E) not belongs to either η_1 nor η_2, that is means $\eta_1 \cup \eta_1 \neq \eta_{12}$, in general.

Definition 3.24. An operator $\Psi : SS(X)_E \to SS(X)_E$ is called a supra soft closure operator if it satisfies the following conditions for all (G, E), $(H, E) \in SS(X)_E$.

Ψ_1**:** : $\Psi((\tilde{\phi}, E)) = (\tilde{\phi}, E)$,
Ψ_2**:** : $(G, E)\tilde{\subseteq}\Psi((G, E))$,
Ψ_3**:** : $\Psi((G, E))\tilde{\cup}\Psi((H, E))\tilde{\subseteq}\Psi((G, E)\tilde{\cup}(H, E))$ and
Ψ_4**:** : $\Psi(\Psi((G, E))) = \Psi((G, E))$.

Theorem 3.25. *Let (X, η_1, η_2, E) be a soft bitopological space. Then, the operator $scl_p : SS(X)_E \to SS(X)_E$ which defined by*

$$scl_p(G, E) = scl_{\eta_1}(G, E) \tilde{\cap} scl_{\eta_2}(G, E)$$

is a supra soft closure operator and it is induced, in usual manner, a unique supra soft topology given by $\{(F, E) \in SS(X)_E : scl_p(F, E)^c = (F, E)^c\}$ which is precisely η_{12}.

Proof. Straightforward. □

Definition 3.26. An operator $I : SS(X)_E \to SS(X)_E$ is a supra soft interior operator if it satisfies the following conditions for all (G, E), $(H, E) \in SS(X)_E$.

I_1**:** : $I(\tilde{\phi}, E) = (\tilde{\phi}, E)$,
I_2**:** : $I(G, E)\tilde{\subseteq}(G, E)$,
I_3**:** : $I[(G, E)\tilde{\cap}(H, E)]\tilde{\subseteq} I(G, E)\tilde{\cap} I(H, E)$ and
I_4**:** : $I[I(G, E)] = I(G, E)$.

Theorem 3.27. *Let (X, η_1, η_2, E) be a soft bitopological space. Then, the operator $sint_p : SS(X)_E \to SS(X)_E$ which defined by*

$$sint_p(G, E) = sint_{\eta_1}(G, E) \tilde{\cup} sint_{\eta_2}(G, E)$$

is a supra soft interior operator and it is induced a unique supra soft topology given by $\{(G, E) \in SS(X)_E : sint_p(G, E) = (G, E)\}(= \eta_{12})$.

Proof. Straightforward. □

Theorem 3.28. *Let (X, η_1, η_2, E) be a soft bitopological space and $(G, E) \in SS(X)_E$. Then*

(1) $sint_p(G, E) = [scl_p(G, E)^c]^c$.
(2) $scl_p(G, E) = [sint_p(G, E)^c]^c$.

Proof. Straightforward. □

Definition 3.29. Let (X, η_1, η_2, E) be a soft bitopological space and let $(G, E) \in SS(X)_E$. The pairwise soft kernel of (G, E) [briefly, $sker_p(G, E)$], is the intersection of all p-open soft supersets of (G, E), i.e.,

$$sker_p(G, E) = \tilde{\bigcap}\{(H, E) \in \eta_{12} : (G, E)\tilde{\subseteq}(H, E)\}.$$

Remark 3.30. The pairwise soft kernel of (G, E) is not p-open soft set in general which is shown in the following example.

Example 3.31. Let $X = \{a, b\}$, $E = \{e_1, e_2\}$ and let

$$\eta_1 = \{(\tilde{\phi}, E), (\tilde{X}, E), (G_1, E), (G_2, E)\},$$
$$\eta_2 = \{(\tilde{\phi}, E), (\tilde{X}, E), (F_1, E), (F_2, E)\},$$

where

$$(G_1, E) = \{(e_1, \{a\}), (e_2, X)\},$$
$$(G_2, E) = \{(e_1, \phi), (e_2, \{a\})\},$$
$$(F_1, E) = \{(e_1, \{b\}), (e_2, \phi)\},$$
$$(F_2, E) = \{(e_1, \{b\}), (e_2, X)\}.$$

Then (X, η_1, η_2, E) is a soft bitopological space.
It is clear that

$$\eta_{12} = \{(\tilde{\phi}, E), (\tilde{X}, E), (G_1, E), (G_2, E), (F_1, E), (F_2, E), (P, E)\},$$

where

$(P, E) = (G_2, E)\tilde{\cup}(F_1, E) = \{(e_1, \{b\}), (e_2, \{a\})\}$, while
$(G_1, E)\tilde{\cup}(F_1, E) = (G_1, E)\tilde{\cup}(F_2, E) = (\tilde{X}, E)$ and $(G_2, E)\tilde{\cup}(F_2, E) = (F_2, E)$.

Now, let $(G, E) = \{(e_1, \{a\}), (e_2, \phi)\}$. Then the p-open soft sets which contains (G, E) are (G_1, E), $(\tilde{X}, E)$. Thus $sker_p(G, E) = (G_1, E)\tilde{\cap}(\tilde{X}, E) = (G_1, E)$.

If $(M, E) = \{(e_1, \phi), (e_2, \{b\})\}$, then the p-open soft sets which contains (M, E) are (G_1, E), (F_2, E). thus $sker_p(M, E) = (G_1, E)\tilde{\cap}(F_2, E) = \{(e_1, \phi), (e_2, X)\}$. It is obvious that $sker_p(M, E)$ is not a p-open soft set.

Let $(H, E) = (F_1, E)$. Then the p-open soft sets which contains (H, E) are (F_1, E), (F_2, E),(P, E), and $(\tilde{X}, E)$. Thus

$$sker_p(H, E) = (G_1, E)\tilde{\cap}(F_2, E)\tilde{\cap}(P, E)\tilde{\cap}(\tilde{X}, E) = (H, E).$$

The following theorem studies the main properties of pairwise soft kernel.

Theorem 3.32. *Let (X, η_1, η_2, E) be a soft bitopological space and let $(G, E), (H, E) \in SS(X)_E$. Then*

(1) $sker_p(\tilde{X}, E) = (\tilde{X}, E)$ *and* $sker_p(\tilde{\phi}, E) = (\tilde{\phi}, E)$.
(2) $(G, E)\tilde{\subseteq} sker_p(G, E)$.
(3) $(G, E)\tilde{\subseteq}(H, E) \Rightarrow sker_p(G, E)\tilde{\subseteq} sker_p(H, E)$.
(4) *If* $(G, E) \in \eta_{12}$, *then* $sker_p(G, E) = (G, E)$.
(5) $sker_p[sker_p(G, E)] = sker_p(G, E)$.
(6) $sker_p[\tilde{\bigcap}\{(H_i, E) : i \in \Delta\}]\tilde{\subseteq}\tilde{\bigcap}\{sker_p(H_i, E) : i \in \Delta\}$.
(7) $sker_p[\tilde{\cup}\{(G_i, E) : i \in \Delta\}] = \tilde{\cup}\{sker_p(G_i, E) : i \in \Delta\}$.

Proof. (1),(2),(3) and (4) are obvious.

(5) Since $(G, E)\tilde{\subseteq} sker_p(G, E)$, t $sker_p(G, E)\tilde{\subseteq} sker_p[sker_p(G, E)]$. Now, since $sker_p(G, E) = \tilde{\bigcap}\{(H, E) \in \eta_{12} : (G, E)\tilde{\subseteq}(H, E)\}$, $sker_p(G, E)\tilde{\subseteq}(H, E)$ for all $(H, E) \in \eta_{12}, (G, E)\tilde{\subseteq}(H, E)$. Then $sker_p[sker_p(G, E)]\tilde{\subseteq} sker_p(H, E)$

for all $(H,E) \in \eta_{12}, (G,E)\tilde{\subseteq}(H,E)$. Thus $sker_p[sker_p(G,E)]\tilde{\subseteq}(H,E)$ [by (4)] for all $(H,E) \in \eta_{12}, (G,E)\tilde{\subseteq}(H,E)$. So $sker_p[sker_p(G,E)]\tilde{\subseteq}\tilde{\bigcap}\{(H,E) \in \eta_{12}, (G,E)\tilde{\subseteq}(H,E)\} = sker_p(G,E)$. Hence (5) holds.

(6) Since $\tilde{\bigcap}\{(H_i,E) : i \in \Delta\}\tilde{\subseteq}(H_i,E)\ \forall\ i \in \Delta$, by (3), $sker_p[\tilde{\bigcap}\{(H_i,E) : i \in \Delta\}]\tilde{\subseteq}sker_p(H_i,E)\ \forall\ i \in \Delta$. Thus $sker_p[\tilde{\bigcap}\{(H_i,E) : i \in \Delta\}]\tilde{\subseteq}\tilde{\bigcap}\{sker_p(H_i,E) : i \in \Delta\}$.

(7) Since $(G_i,E)\tilde{\subseteq}\tilde{\cup}\{(G_i,E) : i \in \Delta\}$, $sker_p(G_i,E)\tilde{\subseteq}sker_p[\tilde{\cup}\{(G_i,E) : i \in \Delta\}]$. Thus $\tilde{\cup}\{sker_p(G_i,E) : i \in \Delta\}\tilde{\subseteq}sker_p[\tilde{\cup}\{(G_i,E) : i \in \Delta\}]$. To prove the inverse inclusion, let $x_e\tilde{\notin}\tilde{\cup}\{sker_p(G_i,E) : i \in \Delta\}$. Then $x_e\tilde{\notin}sker_p(G_i,E)\ \forall\ i \in \Delta$. Thus for all $i \in \Delta$ there exists $(H_i,E) \in \eta_{12}$ such that $(G_i,E)\tilde{\subseteq}(H_i,E)$ and $x_e\tilde{\notin}(H_i,E)$. We set $(H,E) = \tilde{\cup}\{(H_i,E) : i \in \Delta\}$. So $(H,E) \in \eta_{12}$ and $x_e\tilde{\notin}(H,E)$. Since $\tilde{\cup}\{(G_i,E) : i \in \Delta\}\tilde{\subseteq}\tilde{\cup}\{(H_i,E) : i \in \Delta\}$, $\tilde{\cup}\{(G_i,E) : i \in \Delta\}\tilde{\subseteq}(H,E)$, $(H,E) \in \eta_{12}$ and $x_e\tilde{\notin}(H,E)$. Hence $x_e\tilde{\notin}sker_p[\tilde{\cup}\{(G_i,E) : i \in \Delta\}]$. Therefore (7) holds. □

Theorem 3.33. *Let (X,η_1,η_2,E) be a soft bitopological space and $(G,E) \in SS(X)_E$. Then,*

$$x_e\tilde{\in}sker_p(G,E) \Leftrightarrow (C_{x_e},E)\tilde{\cap}(G,E) \neq (\tilde{\phi},E)\ \ \forall\ (C_{x_e},E) \in \eta_{12}^c(x_e),$$

where (C_{x_e},E) is any p-closed soft set contains x_e and $\eta_{12}^c(x_e)$ is the family of all p-closed soft sets contains x_e.

Proof. Let $x_e\tilde{\in}sker_p(G,E)$ and assume that there exists $(C_{x_e},E) \in \eta_{12}^c(x_e)$ such that $(C_{x_e},E)\tilde{\cap}(G,E) = (\tilde{\phi},E)$. Then $(G,E)\tilde{\subseteq}(C_{x_e},E)^c$ implies $sker_p(G,E)\tilde{\subseteq}sker_p(C_{x_e},E)^c = (C_{x_e},E)^c$ which implies $sker_p(G,E)\tilde{\cap}(C_{x_e},E) = (\tilde{\phi},E)$, a contradiction.

Conversely, suppose that $x_e\tilde{\notin}sker_p(G,E)$. Then there exists $(H,E) \in \eta_{12}$ such that $(G,E)\tilde{\subseteq}(H,E)$ and $x_e\tilde{\notin}(H,E)$. Thus $x_e\tilde{\in}(H,E)^c$, but $(H,E)^c \in \eta_{12}^c(x_e)$. So, by hypothesis, $(H,E)^c\tilde{\cap}(G,E) \neq (\tilde{\phi},E)$ which contradicts with $(G,E)\tilde{\subseteq}(H,E)$. Hence $x_e\tilde{\in}sker_p(G,E)$. □

Definition 3.34. A soft set (G,E) is said to be a pairwise Λ- soft set in a soft bitopological space (X,η_1,η_2,E) [briefly, PΛ-soft set] if $sker_p(G,E) = (G,E)$. The family of all PΛ-soft sets we denoted by $P\Lambda S(X,\eta_1,\eta_2)_E$.

In Example 3.31, it is clear that (H,E) is a PΛ-soft set.

Theorem 3.35. *Let (X,η_1,η_2,E) be a soft bitopological space. If (G,E) is a p-open soft set, then it is a PΛ-soft set, i.e.,*

$$\eta_{12} \subseteq P\Lambda S(X,\eta_1,\eta_2)_E.$$

Proof. Immediate from the Theorem 3.32 and Definition 3.34. □

The Example 3.31 show that the converse of Theorem 3.35 is not true in general. For $(G,E) = \{(e_1,\phi),(e_2,X)\}$, then $sker_p(G,E) = (G_1,E)\tilde{\cap}(F_2,E)\tilde{\cap}(\tilde{X},E)$ implies $sker_p(G,E) = (G,E)$. Therefore, (G,E) is a PΛ-soft set, but it is not a p-open soft set.

Theorem 3.36. *Let (X,η_1,η_2,E) be a soft bitopological space. Then, the class $P\Lambda S(X,\eta_1,\eta_2)_E$ is a soft topology on X. This soft topology, we denoted by $\eta_{P\Lambda}$. The triple $(X,\eta_{P\Lambda},E)$ is the soft topological space associated to the soft bitopological space (X,η_1,η_2,E).*

Proof. Since $sker_p(\tilde{X}, E) = (\tilde{X}, E)$ and $sker_p(\tilde{\phi}, E) = (\tilde{\phi}, E)$, $(\tilde{X}, E)$, $(\tilde{\phi}, E) \in P\Lambda S(X, \eta_1, \eta_2)_E$. Let $(G, E), (H, E) \in P\Lambda S(X, \eta_1, \eta_2)_E$. Then $sker_p(G, E) = (G, E)$ and $sker_p(H, E) = (H, E)$. Thus, by Theorem 3.32 (6), we have $sker_p[(G, E)\tilde{\cap}(H, E)]\tilde{\subseteq}(G, E)\tilde{\cap}(H, E)$, but $(G, E)\tilde{\cap}(H, E)\tilde{\subseteq}sker_p[(G, E)\tilde{\cap}(H, E)]$. So $sker_p[(G, E)\tilde{\cap}(H, E)] = (G, E)\tilde{\cap}(H, E)$. Hence $(G, E)\tilde{\cap}(H, E) \in P\Lambda S(X, \eta_1, \eta_2)_E$.

Finally, let $\{(G_i, E) : i \in \Delta\} \subseteq P\Lambda S(X, \eta_1, \eta_2)_E$. Then $sker_p(G_i, E) = (G_i, E)$ for all $i \in \Delta$, it follows that $sker_p[\tilde{\bigcup}\{(G_i, E) : i \in \Delta\}] = \tilde{\bigcup}\{sker_p(G_i, E) : i \in \Delta\} = \tilde{\bigcup}\{(G_i, E) : i \in \Delta\}$. Thus $\tilde{\bigcup}\{(G_i, E) : i \in \Delta\} \in P\Lambda S(X, \eta_1, \eta_2)_E$. Consequently, the class $P\Lambda S(X, \eta_1, \eta_2)_E$ is a soft topology on X. □

Definition 3.37. Members of $\eta_{P\Lambda}$ are called PΛ-open soft sets. A soft set (G, E) in a soft bitopological space (X, η_1, η_2, E) is called a PΛ-closed soft set if its complement is a PΛ-open soft set. We denote the family of all PΛ-closed soft set by $\eta^c_{P\Lambda}$.

Corollary 3.38. *Let (X, η_1, η_2, E) be a soft bitopological space. Then,*

$$\eta_i \subseteq \eta_{12} \subseteq \eta_{P\Lambda} \subseteq SS(X)_E,\ i = 1, 2.$$

Remark 3.39. The following example shows that $\eta_{12} \neq \eta_{P\Lambda} \neq SS(X)_E$ in general.

Example 3.40. Let $X = \{a, b\}$, $E = \{e_1, e_2\}$ and let

$$\eta_1 = \{(\tilde{\phi}, E), (\tilde{X}, E), (G_1, E), (G_2, E)\},$$
$$\eta_2 = \{(\tilde{\phi}, E), (\tilde{X}, E), (F_1, E), (F_2, E)\},$$

where

$$(G_1, E) = \{(e_1, \{a\}), (e_2, X)\},$$
$$(G_2, E) = \{(e_1, \phi), (e_2, \{a\})\},$$
$$(F_1, E) = \{(e_1, \{b\}), (e_2, \phi)\},$$
$$(F_2, E) = \{(e_1, \{b\}), (e_2, X)\}.$$

Then (X, η_1, η_2, E) is a soft bitopological space.
It is clear that

$$\eta_{12} = \{(\tilde{\phi}, E), (\tilde{X}, E), (G_1, E), (G_2, E), (F_1, E), (F_2, E), (P, E)\},$$

where

$$(P, E) = (G_2, E)\tilde{\cup}(F_1, E) = \{(e_1, \{b\}), (e_2, \{a\})\}.$$ And

$$\eta_{P\Lambda} = \{(\tilde{\phi}, E), (\tilde{X}, E), (G_1, E), (G_2, E), (F_1, E), (F_2, E), (P, E), (K, E)\},$$

where $(K, E) = \{(e_1, \phi), (e_2, X)\}$.
It is clear that $\eta_{12} \neq \eta_{P\Lambda} \neq SS(X)_E$.

Lemma 3.41. *An arbitrary intersection of PΛ-soft sets is a PΛ-soft set.*

Proof. Let $\{(G_i, E) : i \in \Delta\} \subseteq \eta_{P\Lambda}$. Then, $sker_p(G_i, E) = (G_i, E)\ \forall\ i \in \Delta$. From Theorem 3.32 (6), we have $sker_p[\tilde{\bigcap}\{(G_i, E) : i \in \Delta\}]\tilde{\subseteq}\tilde{\bigcap}\{sker_p(G_i, E) : i \in \Delta\}$. Thus $sker_p[\tilde{\bigcap}\{(G_i, E) : i \in \Delta\}]\tilde{\subseteq}\tilde{\bigcap}\{(G_i, E) : i \in \Delta\}$.

On the other hand, from Theorem 3.32 (2), we have

$$\tilde{\bigcap}\{(G_i, E) : i \in \Delta\}\tilde{\subseteq}sker_p[\tilde{\bigcap}\{(G_i, E) : i \in \Delta\}].$$

So $sker_p[\tilde{\bigcap}\{(G_i, E) : i \in \Delta\}] = \tilde{\bigcap}\{(G_i, E) : i \in \Delta\}$. Hence $\tilde{\bigcap}\{(G_i, E) : i \in \Delta\} \in \eta_{P\Lambda}$. □ □

Definition 3.42. A soft topological space (X, η, E) is said to be an Alexandroff soft space if the arbitrary intersection of open soft sets is an open soft set.

Theorem 3.43. *Let (X, η_1, η_2, E) be a soft bitopological space. Then, the soft topological space $(X, \eta_{P\Lambda}, E)$ associated of (X, η_1, η_2, E) is an Alexandroff soft space.*

Proof. Immediate from the Theorem 3.36 and Lemma 3.41. □

Theorem 3.44. *Let (X, η_1, η_2, E) be a soft bitopological space. Then*
(1) *$(\tilde{\phi}, E)$, $(\tilde{X}, E)$ are PΛ-closed soft sets.*
(2) *an arbitrary intersection of a PΛ-closed soft sets is a PΛ-closed soft set.*
(3) *an arbitrary union of a PΛ-closed soft sets is a PΛ-closed soft set.*

Proof. Straightforward. □

Definition 3.45. A soft set (G, E) is said to be a pairwise λ-closed soft set in a soft bitopological space (X, η_1, η_2, E) [briefly, Pλ-closed soft set] if $(G, E) = (F, E)\tilde{\cap}(H, E)$, where (F, E) is a p-closed soft set and (H, E) is a PΛ-open soft set. The family of all Pλ-closed soft sets we denoted by $P\lambda CS(X, \eta_1, \eta_2)_E$.

Theorem 3.46. *Let (X, η_1, η_2, E) be a soft bitopological space. Then*
(1) *every p-closed soft set is a Pλ-closed soft set.*
(2) *every PΛ-open soft set is a Pλ-closed soft set.*

Proof. Straightforward. □

Corollary 3.47. *Let (X, η_1, η_2, E) be a soft bitopological space. Then, the following diagram is hold*

$$POS \Rightarrow P\Lambda S \Rightarrow P\lambda CS \Leftarrow PCS$$

Remark 3.48. The converse of Theorem 3.46 is not true in general which is shown in the following example.

Example 3.49. Let (X, η_1, η_2, E) is the same soft bitopological space in Example 3.31 and let

$$(G, E) = \{(e_1, \phi), (e_2, \{b\})\}.$$

Then

(1) (G, E) is a Pλ-closed soft set, because $(G, E) = (P, E)^c\tilde{\cap}(F_2, E)$, where $(P, E)^c \in \eta_{12}^c$ and $(F_2, E) \in \eta_{P\Lambda}$ [for $sker_p(F_2, E) = (F_2, E)$]. But, (G, E) is not p-closed soft set.

(2) It obvious that (G, E) is a Pλ-closed soft set, but it is not PΛ-open soft set, because $sker_p(G, E) = \{(e_1, \phi), (e_2, X)\} \neq (G, E)$.

Theorem 3.50. *Let (X, η_1, η_2, E) be a soft bitopological space and $(G, E) \in SS(X)_E$. Then, the following statements are equivalent:*
(1) *(G, E) is a Pλ-closed soft set.*
(2) *$(G, E) = (H, E)\tilde{\cap}scl_p(G, E)$, $(H, E) \in \eta_{P\Lambda}$.*
(3) *$(G, E) = sker_p(G, E)\tilde{\cap}scl_p(G, E)$.*

Proof. (1) ⇒ (2): Let (G, E) be a Pλ-closed soft set. Then there exist $(M, E) \in \eta_{12}^c$, $(H, E) \in \eta_{P\Lambda}$ such that $(G, E) = (M, E)\tilde{\cap}(H, E)$. Thus, by Proposition 2.7 (2),

$(G,E)\tilde{\subseteq}(M,E)$. So $scl_p(G,E)\tilde{\subseteq}scl_p(M,E)=(M,E)$. Hence $(H,E)\tilde{\cap}scl_p(G,E)\tilde{\subseteq}(H,E)\tilde{\cap}(M,E)$ and thus $(H,E)\tilde{\cap}scl_p(G,E)\tilde{\subseteq}(G,E)$.

Now, we shall prove the inverse inclusion. Since $(G,E)\tilde{\subseteq}scl_p(G,E)$, $(H,E)\tilde{\cap}(G,E)\tilde{\subseteq}(H,E)\tilde{\cap}scl_p(G,E)$, but $(G,E)\tilde{\cap}(H,E)=(G,E)$. Then $(G,E)\tilde{\subseteq}(H,E)\tilde{\cap}scl_p(G,E)$. Thus $(G,E)=(H,E)\tilde{\cap}scl_p(G,E)$, $(H,E)\in\eta_{P\Lambda}$. So (2) holds.

(2) $\Rightarrow$ (3): Since $(G,E)\tilde{\subseteq}scl_p(G,E)$ and $(G,E)\tilde{\subseteq}sker_p(G,E)$, $(G,E)\tilde{\subseteq}scl_p(G,E)\tilde{\cap}sker_p(G,E)$. Now, by (2), there exists $(H,E)\in\eta_{P\Lambda}$ such that $(G,E)=(H,E)\tilde{\cap}scl_p(G,E)$. Then $(G,E)\tilde{\subseteq}(H,E)$. Thus, by Theorem 3.32 (3), $sker_p(G,E)\tilde{\subseteq}sker_p(H,E)$. So $sker_p(G,E)\tilde{\subseteq}(H,E)$, for $(H,E)\in\eta_{P\Lambda}$. Hence $sker_p(G,E)\tilde{\cap}scl_p(G,E)\tilde{\subseteq}(H,E)\tilde{\cap}scl_p(G,E)$ and thus $sker_p(G,E)\tilde{\cap}scl_p(G,E)\tilde{\subseteq}(G,E)$. Consequently, $(G,E)=sker_p(G,E)\tilde{\cap}scl_p(G,E)$. Therefore (3) holds.

(3) $\Rightarrow$ (1): Since $scl_p(G,E)\in\eta_{12}^c$ and $sker_p(G,E)\in\eta_{P\Lambda}$ [by Theorem 3.32 (5)], by (3) and Definition 3.45, we have that (G,E) is a Pλ-closed soft set. □

4. Conclusion

Generalized open sets play a very important role in general and soft topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in general, soft topology and real analysis concerns the variously modified forms of continuity, separation axioms etc. by utilizing generalized open sets. The concept of a soft bitopological space was introduced by Ittanagi [3]. In this paper, we introduced and studied the notions of pairwise open (closed) soft sets, pairwise soft interior (respectively, closure, kernel) operator in a soft bitopological spaces as a generalization of the notions of open soft sets, closed soft sets, soft closure and soft interior in soft topological spaces. In subsequent endeavors, we introduce some new types of pairwise soft sets and the concept of low pairwise soft separation axioms in a soft bitopological spaces.

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