Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 11, No. 4, (April 2016), pp. 5111–525
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Separation axioms on soft topological spaces

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Received 10 April 2015; Revised 13 July 2015; Accepted 4 October 2015

Abstract. In this paper we introduce the separation axioms soft T_i $(i = 0, 1, 2, 3, 4, 5)$ by using the concept of soft points and we study some of their properties. We observe that (Examples 3.7, 3.8), in a soft T_1-space, the soft point x_e may be not closed soft set, so we have spaces which are soft T_i but not soft T_{i-1} $(i = 3, 4, 5)$. In order to overcome this problem, we presented the necessary condition for a soft space to be soft T_1-space. Also, we show that the soft T_i in the sense of [5] and the current soft T_i are equivalent $(i = 0, 1, 2, 3)$.Finally, we discuss the hereditary and some soft topological properties for such spaces.

2010 AMS Classification: 54A40, 03E72, 20N25, 22A99, 06D72

Keywords: Soft set, Soft topology, Separation axioms, Soft point, Soft continuous.

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1. Introduction

Several set theories can be considered as tools for dealing with uncertainties. In (1999) D. Molodtsov [9] intiated the concept of soft set theory as a new mathematical tool for modeling uncertainties. After the introduction of the notion of soft sets, several researchers improved this concept. Maji et al [10, 11] presented an application of soft sets in decision making problems that based on the reduction of parameters to keep the optimal choice objects. Pei and Miao [13] showed that soft sets are a class of special information systems. Topological structure of soft sets also was studied by Sabir and Naz [14]. They defined the soft topological spaces which are defined over an intial universe with a fixed set of parameter and studied the concepts of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms are also introduced and their basic properties are investigated by them. Separation axioms studied in some researches (see, for example, [5, 6, 8, 14, 15]). In this paper, we introduce separation axioms T_i $(i = 0, 1, 2, 3, 4, 5)$ on a soft topological space (X, τ, E) and study some of their properties. We show that these axioms are soft topological properties under certain soft mapping. Also, we show that the axioms T_i are hereditary properties for $(i = 0, 1, 2, 3, 5)$ and the axioms T_4 is hereditary with respect to soft closed subspaces. In addition, we show that the separation axioms T_i $(i = 0, 1, 2, 3)$ in the sense of [5] are equivalent to our separations (see Lemma 3.1). Finally, some counterexamples have obtained.

2. Preliminaries

Throughout this paper, let X be an universe set, $P(X)$ is the set of all subset of X and E be a set of parameters.

Definition 2.1 ([9]). A pair (F, E) denoted by F_E is called a soft set over X, where F is a mapping given by $F : E \to P(X)$. We denote the family of all soft sets over X by $SS(X, E)$.

Definition 2.2 ([9, 11]). For any two soft sets (F, E) and (G, E) over a common universe X, we say that: (F, E) is a soft subset of (G, E) if $F(e) \subseteq G(e)$ for every $e \in E$ and we can write $(F, E)\tilde{\subseteq}(G, E)$. Also, we say that the pairs (F, E) and (G, E) are soft equal if $(F, E)\tilde{\subseteq}(G, E)$ and $(G, E)\tilde{\subseteq}(F, E)$ and we can write $(F, E) \cong (G, E)$.

Definition 2.3 ([3, 9, 16]). Let I be an arbitrary indexed set and $\{(F_i, E) : i \in I\} \subseteq SS(X, E)$. The soft intersection of these soft sets is the soft set $(F, E) \in SS(X, E)$, where F is a mapping from E into $P(X)$ which defined by $F(p) = \cap\{F_i(p) : i \in I\}$ for every $p \in E$ and we can write $(F, E) = \tilde{\cap}\{(F_i, E) : i \in I\}$.

Definition 2.4 ([3, 9, 16]). Let I be an arbitrary indexed set and $\{(F_i, E) : i \in I\} \subseteq SS(X, E)$. The soft union of these soft sets is the soft set $(F, E) \in SS(X, E)$, where F is a mapping from E into $P(X)$ which defined by $F(p) = \cup\{F_i(p) : i \in I\}$ for every $p \in E$ and we can write $(F, E) = \tilde{\cup}\{(F_i, E) : i \in I\}$.

Definition 2.5 ([9, 11]). A soft set (F, E) over X is called a null soft set and denoted by $\tilde{\varnothing}$, if $F(e) = \varnothing$ for every $e \in E$.

Definition 2.6 ([9, 11]). A soft set (F, E) over X is called an absolute soft set and denoted by $\tilde{X}$, if $F(e) = X$ for every $e \in E$.

Definition 2.7 ([14, 16]). Let τ be a collection of soft sets over X. Then τ is called a soft topology on X, if τ satisfies the following axioms:

(1) $\tilde{\varnothing}, \tilde{X} \in \tau$.
(2) The soft intersection of any two soft sets in τ is in τ, i.e. if $(G, E), (H, E) \in \tau$, then $(G, E)\tilde{\cap}(H, E) \in \tau$.
(3) The soft union of any number of soft sets in τ is in τ, i.e. if $(G_i, E) \in \tau$, for every $i \in I$, then $\tilde{\cup}\{(G_i, E) : i \in I\} \in \tau$.
The triplet (X, τ, E) is called a soft topological space over X. The members of τ are said to be open soft sets in X.

Definition 2.8 ([1, 16]). For a soft set (F, E) over X, the relative complement of (F, E), denoted by $(F, E)^c$ and defined by $(F, E)^c = (F^c, E)$ where $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X - F(e)$ for every $e \in E$.

Definition 2.9 ([14]). A soft set (F, E) over X is said to be closed soft set in X, if its relative complement $(F, E)^c$ belongs to τ.

Proposition 2.10 ([2]). *Let (F,E) be a soft set in $SS(X,E)$. Then the following are hold:*

(1) $(F,E)\tilde{\cap}\tilde{\varnothing}=\tilde{\varnothing}$.
(2) $(F,E)\tilde{\cap}\tilde{X}=(F,E)$.
(3) $(F,E)\tilde{\cup}\tilde{\varnothing}=(F,E)$.
(4) $(F,E)\tilde{\cup}\tilde{X}=\tilde{X}$.

Proposition 2.11 ([16]). *Let (F,E) and (G,E) be two soft sets in $SS(X,E)$. Then the following are true:*

(1) $(F,E)\tilde{\subseteq}(G,E)$ *if and only if* $(F,E)\tilde{\cap}(G,E)=(F,E)$.
(2) $(F,E)\tilde{\subseteq}(G,E)$ *if and only if* $(F,E)\tilde{\cup}(G,E)=(G,E)$.

Proposition 2.12 ([16]). *Let $(F,E),(G,E),(K,E),(H,E)\in SS(X,E)$. Then the following are true:*

(1) *If* $(F,E)\tilde{\cap}(G,E)=\tilde{\varnothing}$, *then* $(F,E)\tilde{\subseteq}(G,E)^c$.
(2) $(F,E)\tilde{\cup}(F,E)^c=\tilde{X}$ *[[2]].*
(3) *If* $(F,E)\tilde{\subseteq}(G,E)$ *and* $(G,E)\tilde{\subseteq}(H,E)$, *then* $(F,E)\tilde{\subseteq}(H,E)$.
(4) *If* $(F,E)\tilde{\subseteq}(G,E)$ *and* $(H,E)\tilde{\subseteq}(K,E)$, *then* $(F,E)\tilde{\cap}(H,E)\tilde{\subseteq}(G,E)\tilde{\cap}(K,E)$.
(5) $(F,E)\tilde{\subseteq}(G,E)$ *if and only if* $(G,E)^c\tilde{\subseteq}(F,E)^c$.

Proposition 2.13 ([1]). *If (F,E) and (G,E) are two soft sets over X, then*

(1) $((F,E)\tilde{\cup}(G,E))^c=(F,E)^c\tilde{\cap}(G,E)^c$.
(2) $((F,E)\tilde{\cap}(G,E))^c=(F,E)^c\tilde{\cup}(G,E)^c$.

Definition 2.14 ([14]). Let (X,τ,E) be a soft topological space and (G,E) be a soft set over X. Then the soft closure of (G,E), denoted by $cl(G,E)$ or $\overline{(G,E)}$ and defined as: $\overline{(G,E)}=\tilde{\cap}\{(S,E):(S,E)\in\tau^c,(S,E)\tilde{\supseteq}(G,E)\}$.

Proposition 2.15 ([16]). *Let (X,τ,E) be a soft topological space and $(F,E),(G,E)\in SS(X,E)$. Then*

(1) $(F,E)\in\tau^c$ *if and only if* $cl(F,E)=(F,E)$.
(2) *If* $(F,E)\tilde{\subseteq}(G,E)$, *then* $cl(F,E)\tilde{\subseteq}cl(G,E)$.

Definition 2.16 ([14]). Let (X,τ,E) be a soft topological space and M be a non-empty subset of X. The family, $\tau_M=\{\tilde{E_M}\tilde{\cap}F_A:F_A\in\tau\}$ is called the soft relative topology on M and (M,τ_M) is called soft subspace of (X,τ), where $\tilde{E_M}:E\to P(M)$ such that $\tilde{E_M}(e)=M$ for every $e\in E$.

Proposition 2.17 ([14]). *Let (M,τ_M) be a soft subspace of (X,τ) and F_A be a soft set over M. Then, F_A is open soft set in M if and only if $F_A=\tilde{E_M}\tilde{\cap}G_B$, for some $G_B\in\tau$.*

Definition 2.18 ([14]). Let $(F,E)\in SS(X,E)$ and $x\in X$. It is said that x belongs to F_E ($x\tilde{\in}F_E$) if $x\in F_E(e)$ for every $e\in E$. It is also said that x does not belong to F_E ($x\tilde{\notin}F_E$) if $x\notin F_E(e)$ for some $e\in E$.

Definition 2.19 ([16]). The soft set $(F,E)\in SS(X,E)$ is called a soft point in X, denoted by x_e, if for the element $e\in E$, $F(e)=\{x\}$ and $F(e^{'})=\varnothing$ for every $e^{'}\in E-\{e\}$.

Definition 2.20 ([16])**.** The soft point x_e is said to be in the soft set (G,E), denoted by $x_e\tilde{\in}(G,E)$, if for the element $e \in E$ we have $F(e)=\{x\}\subseteq G(e)$.

Definition 2.21 ([16])**.** A soft set (G,E) in a soft topological space (X,τ,E) is called a soft neighborhood (briefly, nbd) of the soft set (F,E) if there exists $(H,E)\in\tau$ such that $(F,E)\tilde{\subseteq}(H,E)\tilde{\subseteq}(G,E)$.

Theorem 2.22 ([16])**.** *A soft set (G,E) is open soft set if and only if for each soft set (F,E) contained in (G,E), (G,E) is a soft neighborhood of (F,E).*

Definition 2.23 ([7, 16])**.** Let $SS(X,E)$ and $SS(Y,K)$ be the families of all soft sets over X and Y, respectively. Then

(1) A mapping $f=(\phi,\psi)$ is called a soft mapping from $SS(X,E)$ into $SS(Y,K)$, where $\phi: X\to Y$ and $\psi: E\to K$ are two mappings.

(2) If $F_A\in SS(X,E)$, then the image of F_A under the soft mapping (ϕ,ψ) is a soft set over Y denoted by $(\phi,\psi)(F_A)$ and defined by:

$$(\phi,\psi)(F_A)(k)=\begin{cases}\cup_{e\in\psi^{-1}(k)\cap A}\phi(F_A(e)) & \text{if } \psi^{-1}(k)\cap A\neq\varnothing,\\ \varnothing & \textit{otherwise}.\end{cases}$$

(3) If $G_B\in SS(Y,K)$, then the preimage of G_B under the soft mapping (ϕ,ψ) is a soft set over X denoted by $(\phi,\psi)^{-1}(G_B)$ and defined by:

$$(\phi,\psi)^{-1}(G_B)(e)=\begin{cases}\phi^{-1}(G_B(\psi(e))) & \text{if } \psi(e)\in B,\\ \varnothing & \textit{otherwise}.\end{cases}$$

The soft mapping (ϕ,ψ) is called injective, if ϕ and ψ are injective. The soft mapping (ϕ,ψ) is called surjective, if ϕ and ψ are surjective.

Proposition 2.24 ([4, 7, 16])**.** *Let $(F,A),(F_1,A)\in SS(X,A),(G,B),(G_1,B)\in SS(Y,B)$. The following statements are true:*

(1) $If(F,A)\tilde{\subseteq}(F_1,A), then(\phi,\psi)(F,A)\tilde{\subseteq}(\phi,\psi)(F_1,A)$.
(2) $If(G,B)\tilde{\subseteq}(G_1,B), then(\phi,\psi)^{-1}(G,B)\tilde{\subseteq}(\phi,\psi)^{-1}(G_1,B)$.
(3) $(F,A)\tilde{\subseteq}(\phi,\psi)^{-1}((\phi,\psi)(F,A))$.
(4) $(\phi,\psi)((\phi,\psi)^{-1}(G,B))\tilde{\subseteq}(G,B)$.
(5) $(\phi,\psi)^{-1}((G,B)^c)=((\phi,\psi)^{-1}(G,B))^c$.
(6) $(\phi,\psi)((F,A)\tilde{\cup}(F_1,A))=(\phi,\psi)(F,A)\tilde{\cup}(\phi,\psi)(F_1,A)$.
(7) $(\phi,\psi)((F,A)\tilde{\cap}(F_1,A))\tilde{\subseteq}(\phi,\psi)(F,A)\tilde{\cap}(\phi,\psi)(F_1,A)$.
(8) $(\phi,\psi)^{-1}((G,B)\tilde{\cup}(G_1,B))=(\phi,\psi)^{-1}(G,B)\tilde{\cup}(\phi,\psi)^{-1}(G_1,B)$.
(9) $(\phi,\psi)^{-1}((G,B)\tilde{\cap}(G_1,B))=(\phi,\psi)^{-1}(G,B)\tilde{\cap}(\phi,\psi)^{-1}(G_1,B)$.
(10) $(\phi,\psi)(\tilde{\varnothing}_X)=\tilde{\varnothing}_Y$.
(11) $(\phi,\psi)^{-1}(\tilde{\varnothing}_Y)=\tilde{\varnothing}_X$.

Theorem 2.25 ([16])**.** *Let (X,τ,E) and (Y,τ^*,K) be soft topological spaces and let $(\phi,\psi): SS(X,E)\to SS(Y,K)$ be a soft mapping. Then the following statements are equivalent:*

(1) (ϕ, ψ) *is soft continuous.*
(2) *For each* $(H, B) \in \tau^*$, $(\phi, \psi)^{-1}(H, B) \in \tau$.
(3) *For each soft closed set* (F, B) *over* Y, $(\phi, \psi)^{-1}(F, B)$ *is soft closed set over* X.

Definition 2.26 ([12])**.** Let (X, τ, E) and (Y, τ^*, K) be two soft topological spaces. A mapping $f = (\phi, \psi) : (X, \tau, E) \to (Y, \tau^*, K)$ is said to be a soft homeomorphism if

(1) $f : X \to Y$ is a bijective mapping.
(2) $f : (X, \tau, E) \to (Y, \tau^*, K)$ and $f^{-1} : (Y, \tau^*, K) \to (X, \tau, E)$ are soft continuous.

Definition 2.27 ([12])**.** Let (X, τ, E) and (Y, τ^*, K) be two soft topological spaces. A mapping $f = (\phi, \psi) : (X, \tau, E) \to (Y, \tau^*, K)$ is said to be soft open mapping if $(G, E) \in \tau$, then $f(G, E) \in \tau^*$.

Proposition 2.28 ([12])**.** *Let* (X, τ, E) *and* (Y, τ^*, K) *be two soft topological spaces. For a bijection mapping* $f : (X, \tau, E) \to (Y, \tau^*, K)$, *the following statements are equivalent:*

(1) $f : (X, \tau, E) \to (Y, \tau^*, K)$ *is soft open mapping.*
(2) $f^{-1} : (Y, \tau^*, K) \to (X, \tau, E)$ *is soft continuous mapping.*

Remark 2.29 ([5])**.** Let $(F, E) \in SS(X, E), a \in E$ and $x \in X$. In whats follows we write $x \in_a (F, E)$ (respectively, $x \notin_a (F, E)$) if and only if $x \in F(a)$ (respectively, $x \notin F(a)$).

Definition 2.30 ([5])**.** A soft topological space (X, τ, E) is called a soft T_0-space if for every distinct points x, y of X and for every $a \in E$, there exists an open soft set (G, E) such that $x \in_a (G, E)$ and $y \notin_a (G, E)$ or $x \notin_a (G, E)$and $y \in_a (G, E)$.

Definition 2.31 ([5])**.** A soft topological space (X, τ, E) is called a soft T_1-space if for every distinct points x, y of X and for every $a \in E$, there exists an open soft set (G, E) such that $x \in_a (G, E)$ and $y \notin_a (G, E)$.

Definition 2.32 ([5])**.** A soft topological space (X, τ, E) is called a soft T_2-space if for every distinct points x, y of X and for every $a \in E$, there exists two open soft sets (G, E) and (H, E) such that $x \in_a (G, E), y \in_a (H, E)$ and $G(a) \cap H(a) = \varnothing$.

Definition 2.33 ([5])**.** A soft topological space (X, τ, E) is called a soft T_3-space if for every point $x \in X$, for every $a \in E$ and for every closed soft set (Q, E) such that $x \notin_a (Q, E)$, there exists two open soft sets (G, E) and (H, E) such that $x \in_a (G, E), Q(a) \subseteq H(a)$ and $G(a) \cap H(a) = \varnothing$.

3. Soft separation axioms

Definition 3.1. A soft topological space (X, τ, E) is said to be a soft T_0-space if for every two soft points x_e, y_e such that $x \neq y$ there exists $G_E \in \tau$ such that $x_e \tilde{\in} G_E$, $y_e \tilde{\notin} G_E$ or there exists $H_E \in \tau$ such that $y_e \tilde{\in} H_E$, $x_e \tilde{\notin} H_E$.

Example 3.2. Let $X = \{x, y\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{X}, \tilde{\varnothing}, (F_1, E), (F_2, E), (F_3, E)\}$ where

$$F_1(e) = \begin{cases} X & \text{if } e = e_1, \\ \{y\} & \text{if } e = e_2; \end{cases}$$

$$F_2(e) = \begin{cases} \{x\} & \text{if } e = e_1, \\ X & \text{if } e = e_2; \end{cases}$$

and

$$F_3(e) = \begin{cases} \{x\} & \text{if } e = e_1, \\ \{y\} & \text{if } e = e_2. \end{cases}$$

Then (X, τ, E) is a soft T_0-space because for the soft points x_{e_1}, y_{e_1} there exists $(F_2, E) \in \tau$ such that $x_{e_1} \tilde{\in} (F_2, E)$ but $y_{e_1} \tilde{\notin} (F_2, E)$ and for the soft points x_{e_2}, y_{e_2} there exists $(F_1, E) \in \tau$ such that $y_{e_2} \tilde{\in} (F_1, E)$ but $x_{e_2} \tilde{\notin} (F_1, E)$.

Definition 3.3. A soft topological space (X, τ, E) is said to be a soft T_1-space if for every two soft points x_e, y_e such that $x \neq y$ there exists $G_E, H_E \in \tau$ such that $x_e \tilde{\in} G_E$, $y_e \tilde{\notin} G_E$ and $y_e \tilde{\in} H_E$, $x_e \tilde{\notin} H_E$.

Proposition 3.4. *Every soft T_1-space is a soft T_0-space.*

Proof. Straightforward. □

The following example shows that the soft T_0-space may not be a soft T_1-space.

Example 3.5. Let $X = \{x, y\}, E = \{e\}$ *and* $\tau = \{\tilde{X}, \tilde{\varnothing}, F_E\}$ where $F_E(e) = \{x\}$. Then (X, τ, E) is a soft T_0-space but not soft T_1-space. Since x_e, y_e are two soft points $(x \neq y)$ and the only open soft set which containing y_e is $\tilde{X}$ also contains x_e. Hence (X, τ, E) is not a soft T_1-space. On the other hand it is a soft T_0-space since for each two soft points x_e, y_e, $x \neq y$ and open soft set F_E ($x_e \tilde{\in} F_E$ but $y_e \tilde{\notin} F_E$).

Theorem 3.6. *If the soft point x_e is a closed soft set $\forall e \in E$, then (X, τ, E) is a soft T_1-space.*

Proof. Let x_e, y_e be two soft points over X such that $x \neq y$. From hypothesis, x_e, y_e are closed soft sets $(x \neq y)$. So, $x_e \tilde{\cap} y_e = \tilde{\varnothing}$. Then, $x_e \tilde{\subseteq} y_e^c$ and $y_e \tilde{\subseteq} x_e^c$. Since x_e, y_e are two closed soft sets, then, x_e^c, y_e^c are two open soft sets. Now, $x_e \tilde{\subseteq} y_e^c$, $y_e \tilde{\nsubseteq} y_e^c$ and $y_e \tilde{\subseteq} x_e^c$, $x_e \tilde{\nsubseteq} x_e^c$. Hence, (X, τ, E) is soft T_1-space. □

Example 3.7. Let $X = \{x, y\}, E = \{e_1, e_2\}$ and $\tau = \{\tilde{X}, \tilde{\varnothing}, F_E, G_E\}$ where $F_E, G_E : E \to P(X)$ such that $F_E(e_1) = \{x\}, F_E(e_2) = \{y\}$ and $G_E(e_1) = \{y\}, G_E(e_2) = \{x\}$.
Now, x_{e_1}, y_{e_1}, x_{e_2} and y_{e_2} are soft points.
For the soft points x_{e_1},y_{e_1} we have $x_{e_1} \tilde{\in} F_E$, $y_{e_1} \tilde{\notin} F_E$ and $y_{e_1} \tilde{\in} G_E$, $x_{e_1} \tilde{\notin} G_E$. For the soft points x_{e_2},y_{e_2} we have $x_{e_2} \tilde{\in} G_E$, $y_{e_2} \tilde{\notin} G_E$ and $y_{e_2} \tilde{\in} F_E$, $x_{e_2} \tilde{\notin} F_E$.
Then (X, τ, E) is a soft T_1-space, but x_{e_2} is not closed soft set.

Example 3.8. Let $X = \{x, y\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{X}, \tilde{\varnothing}, F_E, G_E, H_E, W_E\}$ where

$$F_E(e) = x_{e_1}(e) = \begin{cases} \{x\} & \text{if } e = e_1, \\ \varnothing & \text{if } e = e_2; \end{cases}$$

$$G_E(e) = x_{e_1}\tilde{\cup} y_{e_2}(e) = \begin{cases} \{x\} & \text{if } e = e_1, \\ \{y\} & \text{if } e = e_2; \end{cases}$$

$$H_E(e) = x_{e_2}\tilde{\cup} y_{e_1}(e) = \begin{cases} \{y\} & \text{if } e = e_1, \\ \{x\} & \text{if } e = e_2; \end{cases}$$

and

$$W_E(e) = x_{e_1}\tilde{\cup} x_{e_2}\tilde{\cup} y_{e_1}(e) = \begin{cases} X & \text{if } e = e_1, \\ \{x\} & \text{if } e = e_2. \end{cases}$$

For the soft points x_{e_1}, y_{e_1} there exists $G_E, H_E \in \tau$ such that $x_{e_1}\tilde{\in} G_E$ but $y_{e_1}\tilde{\notin} G_E$ and $y_{e_1}\tilde{\in} H_E$ but $x_{e_1}\tilde{\notin} H_E$. For the soft points x_{e_2}, y_{e_2} there exists $G_E, H_E \in \tau$ such that $y_{e_2}\tilde{\in} G_E$ but $x_{e_2}\tilde{\notin} G_E$ and $x_{e_2}\tilde{\in} H_E$ but $y_{e_2}\tilde{\notin} H_E$. Then (X, τ, E) is a soft T_1-space, but x_{e_1} is not a closed soft set.

Theorem 3.9. *A soft topological space (X, τ, E) is a soft T_1-space if and only if $\{x\} = \cap\{G_E(e) : G_E \in \tau, x_e\tilde{\in} G_E\}$ for every soft point x_e in X.*

Proof. "⇒" Let (X, τ, E) be a Soft T_1-space and let x_e be a soft point. Then,
$\{x\} \subseteq \cap\{G_E(e) : G_E \in \tau, x_e\tilde{\in} G_E\}$.
We prove that $\cap\{G_E(e) : G_E \in \tau, x_e\tilde{\in} G_E\} \subseteq \{x\}$. Indeed, let $y \in \cap\{G_E(e) : G_E \in \tau, x_e\tilde{\in} G_E\}$ and $y \neq x$. Since (X, τ, E) is a soft T_1-space, there exists $H_E \in \tau$ such that $x_e\tilde{\in} H_E$ and $y_e\tilde{\notin} H_E$. Hence, $y \notin H(e)$ and, therefore, $y \notin \cap\{G_E(e) : G_E \in \tau, x_e\tilde{\in} G_E\}$ which is a contradiction.

"⇐" Conversely, let x_e and y_e be two soft points such that $x \neq y$. Suppose that for every $G_E \in \tau$ with $x_e\tilde{\in} G_E$ we have $y_e\tilde{\in} G_E$. Then, $y \in \cap\{G_E(e) : G_E \in \tau, x_e\tilde{\in} G_E\} = \{x\}$ which is a contradiction. Therefore, there exists $G_E \in \tau$ such that $x_e\tilde{\in} G_E$ and $y_e\tilde{\notin} G_E$. □

Theorem 3.10. *If (X, τ, E) is soft T_1-space, $\tau \leqslant \tau^*$ (τ coarser than τ^*), then (X, τ^*, E) is a soft T_1-space.*

Proof. Straightforward. □

Definition 3.11. A soft topological space (X, τ, E) is said to be a soft T_2-space if for every two soft points x_e, y_e such that $x \neq y$ there exists $G_E, H_E \in \tau$ such that $x_e\tilde{\in} G_E$, $y_e\tilde{\in} H_E$ and $G_E\tilde{\cap} H_E = \tilde{\varnothing}$.

Theorem 3.12. *A soft topological space (X, τ, E) is soft T_2-space if and only if $\{x\} = \tilde{\cap}\{F_E(e) : F_E$ is a closed soft nbd of $x_e\}$.*

Proof. "⇒" Let (X, τ, E) be a soft T_2-space and let x_e be a soft point. Then, $\{x\} \subseteq \tilde{\cap}\{F_E(e) : F_E$ is a closed soft nbd of $x_e\}$.
We prove that $\tilde{\cap}\{F_E(e) : F_E$ is a closed soft nbd of $x_e\} \subseteq \{x\}$. Indeed, let $y \in \tilde{\cap}\{F_E(e) : F_E$ is a closed soft nbd of $x_e\}$ and $y \neq x$. Since (X, τ, E) is a soft T_2-space, there exists $G_E, H_E \in \tau$ such that $x_e \tilde{\in} G_E$, $y_e \tilde{\in} H_E$ and $G_E \tilde{\cap} H_E = \tilde{\varnothing}$. Then, there exists $H_E^c \in \tau^c$ such that $x_e \tilde{\in} H_E^c$ and $y_e \tilde{\notin} H_E^c$. Hence, $y \notin H_E^c(e)$ and, therefore, $y \notin \tilde{\cap}\{F_E(e) : F_E$ is a closed soft nbd of $x_e\}$ which is a contradiction.

"⇐" Let x_e, y_e be two soft points such that $x \neq y$. Then $y \notin \{x\} = \tilde{\cap}\{F_E(e) : F_E$ is a closed soft nbd of $x_e\}$. It follows that, there exists a closed soft set F_E^* such that $y \notin F_E^*(e)$, $x_e \in F_E^*$.

$$Also, y_e \tilde{\in} F_E^{*^c}. \tag{3.1}$$

Since F_E is a closed nbd of x_e, from the Definition 2.21,

$$\exists G_E \in \tau\ such that\ x_e \tilde{\in} G_E \tilde{\subseteq} F_E. \tag{3.2}$$

From (3.1)and (3.2), we get $G_E, F_E^{*^c} \in \tau$ such that $x_e \tilde{\in} G_E, y_e \tilde{\in} F_E^{*^c}$ and $G_E \tilde{\cap} F_E^{*^c} = \tilde{\Phi}$. (for $G_E \tilde{\subseteq} F_E$). Hence, (X, τ, E) is a soft T_2-space. □

Remark 3.13. If (X, τ, E) is a soft T_2-space, then x_e may be not a closed soft set for every $e \in E$ as the following example show.

Example 3.14. Let $X = \{x, y\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{X}, \tilde{\varnothing}, (F_1, E), (F_2, E)\}$, where

$$F_1(e) = \begin{cases} \{x\} & \text{if } e = e_1, \\ \{y\} & \text{if } e = e_2; \end{cases}$$

and

$$F_2(e) = \begin{cases} \{y\} & \text{if } e = e_1, \\ \{x\} & \text{if } e = e_2. \end{cases}$$

Then, (X, τ, E) is a soft T_2-space, but x_{e_1} is not a closed soft set.

Theorem 3.15. *If (X, τ, E) is a soft T_2-space, $\tau \leqslant \tau^*$, then (X, τ^*, E) is a soft T_2-space.*

Proof. Straightforward. □

Theorem 3.16. *Every soft T_2-space is a soft T_1-space.*

Proof. Immediately follows from the definitions. □

The following example shows that the soft T_1-space may not be a soft T_2-space.

Example 3.17. Let X be a non-empty infinite set and let $\tau_\infty^s = \{\tilde{\varnothing}\} \cup \{F_E \in SS(X, E) : (F_E(e))^c$is finite subset of X for every $e \in E\}$. We try to show that τ_∞^s is a topology on X, so, we satisfies the conditions of Definition 2.7.

(1) Obviously $\tilde{\varnothing}, \tilde{X} \in \tau_\infty^s$.
(2) Let $F_E, G_E \in \tau_\infty^s$. Then $(F_E(e))^c, (G_E(e))^c$ are two finite subsets of X. Since $(F_E(e))^c \cup (G_E(e))^c = (F_E(e) \cap G_E(e))^c = ((F_E \tilde{\cap} G_E)(e))^c$ is a finite subset of X. Hence, $F_E \tilde{\cap} G_E \in \tau_\infty^s$.

(3) Let $F_{i_E} \in \tau^s_\infty \ \forall\ i \in I$. Then $(F_{i_E}(e))^c$ is a finite subset of $X\ \forall i \in I$. Since $\cap_{i\in I}(F_{i_E}(e))^c = (\cup_{i\in I}F_{i_E}(e))^c = ((\tilde{\cup}_{i\in I}F_{i_E})(e))^c$ is a finite subset of X. Hence, $\tilde{\cup}_{i\in I}F_{i_E} \in \tau^s_\infty$ and τ^s_∞ is a soft topology on X.

It follows that (X, τ^s_∞, E) is a soft topological space over X and we call it soft Co-finite topology. Next, we show that (X, τ^s_∞) is a soft T_1-space but not a soft T_2-space. So, let x_e, y_e be two soft points such that $x \neq y$. Since

$$x_e(\epsilon) = \begin{cases} \{x\} & \text{if } \epsilon = e, \\ \varnothing & \text{if } \epsilon \neq e; \end{cases}$$

and

$$y_e(\epsilon) = \begin{cases} \{y\} & \text{if } \epsilon = e, \\ \varnothing & \text{if } \epsilon \neq e. \end{cases}$$

x^c_e and y^c_e are two open soft sets in τ^s_∞ such that $y_e\tilde{\in}x^c_e, x_e\tilde{\notin}x^c_e$ and $x_e\tilde{\in}y^c_e, y_e\tilde{\notin}y^c_e$. It follows that (X, τ^s_∞) is a soft T_1-space. On the other hand, we suppose that (X, τ^s_∞) is a soft T_2-space. Then for every x_e, y_e are two soft points $(x \neq y)$ there exists $F_E, G_E \in \tau^s_\infty$ such that $x_e\tilde{\in}F_E$, $y_e\tilde{\in}G_E$ and $F_E\tilde{\cap}G_E = \tilde{\varnothing}$. Hence, $(F_E(e))^c \cup (G_E(e))^c = X$. Since $(F_E(e))^c$ and $(G_E(e))^c$ are two finite subsets of X, X is a finite set which is a contradiction. Hence, (X, τ^s_∞) is not a soft T_2-space.

Definition 3.18. A soft topological space (X, τ, E) is said to be a soft regular space if for all closed soft sets F_E and soft points x_e such that $x_e\tilde{\notin}F_E$ there exists $G_E, H_E \in \tau$ such that $x_e\tilde{\in}G_E, F_E\tilde{\subseteq}H_E$ and $G_E\tilde{\cap}H_E = \tilde{\varnothing}$.

Theorem 3.19. *A soft topological space (X, τ, E) is a soft regular space if and only if for every open soft set G_E, $x_e\tilde{\in}G_E$ there exists $H_E \in \tau$ such that $x_e\tilde{\in}H_E\tilde{\subseteq}\overline{H_E}\tilde{\subseteq}G_E$.*

Proof. "⇒" Let (X, τ, E) be a soft regular space and let G_E be an open soft set, $x_e\tilde{\in}G_E$. Now, G^c_E is a closed soft set, $x_e\tilde{\notin}G^c_E$ (because $G_E\tilde{\cap}G^c_E = \tilde{\varnothing}$). But (X, τ, E) is a soft regular space, then, there exists $H_E, W_E \in \tau$ such that $x_e\tilde{\in}H_E, G^c_E\tilde{\subseteq}W_E$ and $H_E\tilde{\cap}W_E = \tilde{\varnothing}$. Now, we have $x_e\tilde{\in}H_E\tilde{\subseteq}\overline{H_E}\tilde{\subseteq}W^c_E\tilde{\subseteq}G_E$. Hence, there exists $H_E \in \tau$ such that $x_e\tilde{\in}H_E\tilde{\subseteq}\overline{H_E}\tilde{\subseteq}G_E$.

"⇐" Let F_E be a closed soft set, x_e be a soft point such that $x_e\tilde{\notin}F_E$. Now, we have F^c_E is an open soft set ,$x_e \in F^c_E$ (for $F_E\tilde{\cap}F^c_E = \tilde{\varnothing}$). Then, there exists $H_E \in \tau$,$x_e\tilde{\in}H_E\tilde{\subseteq}\overline{H_E}\tilde{\subseteq}F^c_E$. Since $\overline{H_E}\tilde{\subseteq}F^c_E$ and $\overline{H_E} \in \tau^c$, then $F_E\tilde{\subseteq}\overline{H_E}^c$ and $\overline{H_E}^c \in \tau$. Therefore there exists $H_E, \overline{H_E}^c \in \tau$ such that $x_e\tilde{\in}H_E$,$F_E\tilde{\subseteq}\overline{H_E}^c$ and $H_E\tilde{\cap}\overline{H_E}^c = \tilde{\varnothing}$. Hence, (X, τ, E) is a soft regular space. □

Definition 3.20. A soft topological space (X, τ, E) is said to be a soft T_3-space if it is a soft regular space and a soft T_1-space.

Theorem 3.21. *If (X, τ, E) is a soft regular (T_3-) space and x_e is a closed soft set for each $e \in E$, then (X, τ, E) is a soft T_2-space.*

Proof. The proof is the same in two cases. So, let (X, τ, E) be a soft T_3-space and let $x_e \in \tau^c$ for every $e \in E$. We want to show that (X, τ, E) is a soft T_2-space. So, let x_e, y_e be two soft points such that $x \neq y$. By hypothesis, $x_e, y_e \in \tau^c$. Since $x \neq y$, then $y_e\tilde{\nsubseteq}x_e$. Now, $y_e\tilde{\nsubseteq}x_e$ and (X, τ, E) is a soft T_3-space, i.e. (X, τ, E) is a soft regular T_1-space, so, there exists $G_E, H_E \in \tau$ such that $x_e \tilde{\in} G_E, y_e \tilde{\in} H_E$ and $G_E \tilde{\cap} H_E = \tilde{\varnothing}$. Hence, (X, τ, E) is a soft T_2-space. □

Definition 3.22. A soft topological space (X, τ, E) is said to be a soft normal space if for every two non-empty disjoint closed soft sets $(F_1, E), (F_2, E)$ there exists $G_E, H_E \in \tau$ such that $(F_1, E) \tilde{\subseteq} G_E, (F_2, E) \tilde{\subseteq} H_E$ and $G_E \tilde{\cap} H_E = \tilde{\varnothing}$.

Theorem 3.23. *A soft topological space (X, τ, E) is a soft normal space if and only if for every $F_E \in \tau^c, G_E \in \tau$ such that $F_E \tilde{\subseteq} G_E$ there exists $H_E \in \tau$ such that $F_E \tilde{\subseteq} H_E \tilde{\subseteq} \overline{H_E} \tilde{\subseteq} G_E$.*

Proof. It is similar to the proof of Theorem 3.19. □

Definition 3.24. A soft topological space (X, τ, E) is said to be a soft T_4- space if it is a soft normal space and a soft T_1-space.

Theorem 3.25. *If (X, τ, E) is a soft normal (T_4-) space and x_e is a closed soft set $\forall e \in E$, then (X, τ, E) is a soft regular (T_3-) space.*

Proof. The proof is the same in two cases. So, let (X, τ, E) be a soft T_4-space. Then (X, τ, E) is a soft normal T_1-space. It is sufficient to show that (X, τ, E) is a soft regular space, so, let $F_E \in \tau^c, x_e \tilde{\notin} F_E$. Since $x_e \tilde{\notin} F_E$, then, $x_e \tilde{\cap} F_E = \tilde{\varnothing}$. Now, we have $x_e, F_E \in \tau^c$ such that $x_e \tilde{\cap} F_E = \tilde{\varnothing}$. But (X, τ, E) is a soft T_4-space, i.e. (X, τ, E) is a soft normal T_1-space, so, there exists $G_E, H_E \in \tau$ such that $x_e \tilde{\subseteq} G_E, F_E \tilde{\subseteq} H_E$ and $G_E \tilde{\cap} H_E = \tilde{\varnothing}$. It follows that (X, τ, E) is a soft regular space. Since (X, τ, E) is a soft T_1-space, so, (X, τ, E) is a soft T_3-space. □

Example 3.26. In Example 3.8 consider $\tau^c = \{\tilde{X}, \tilde{\varnothing}, F_E^c, G_E^c, H_E^c, W_E^c\}$ where

$$F_E^c(e) = x_{e_1}^c(e) = \begin{cases} \{y\} & \text{if } e = e_1, \\ X & \text{if } e = e_2; \end{cases}$$

$$G_E^c(e) = (x_{e_1} \tilde{\cup} y_{e_2})^c(e) = \begin{cases} \{y\} & \text{if } e = e_1, \\ \{x\} & \text{if } e = e_2; \end{cases}$$

$$H_E^c(e) = (x_{e_2} \tilde{\cup} y_{e_1})^c(e) = \begin{cases} \{x\} & \text{if } e = e_1, \\ \{y\} & \text{if } e = e_2; \end{cases}$$

and

$$W_E^c(e) = (x_{e_1} \tilde{\cup} x_{e_2} \tilde{\cup} y_{e_1})^c(e) = \begin{cases} \varnothing & \text{if } e = e_1, \\ \{y\} & \text{if } e = e_2. \end{cases}$$

For the two non-empty disjoint closed soft sets G_E^c and H_E^c there exists $G_E, H_E \in \tau$ *such that* $H_E^c \tilde{\subseteq} G_E$, $G_E^c \tilde{\subseteq} H_E$ *and* $G_E \tilde{\cap} H_E = \tilde{\varnothing}$. For the two non-empty disjoint closed soft sets G_E^c and W_E^c there exists $G_E, H_E \in \tau$ *such that* $W_E^c \tilde{\subseteq} G_E$, $G_E^c \tilde{\subseteq} H_E$ *and* $G_E \tilde{\cap} H_E = \tilde{\varnothing}$. Then (X, τ, E) is a soft normal space. Also, (X, τ, E) is a soft T_1-space (from Example 3.8) and hence (X, τ, E) is a soft T_4-space.

Since F_E^c is a closed soft set and x_{e_1} is a soft point such that $x_{e_1} \tilde{\notin} F_E^c$ and all open sets which containing x_{e_1} are F_E, G_E and W_E which intersect with the only open soft set $\tilde{X}$ which contains F_E^c, so, (X, τ, E) is not a soft regular and hence (X, τ, E) is not a soft T_3-space.

Definition 3.27. Let (X,τ,E) be a soft topological space and let $(A,E),(B,E)$ be two non-empty soft subsets of X. Then we say that $(A,E),(B,E)$ are two separated sets if: $(A,E)\tilde{\cap}\overline{(B,E)}=\tilde{\varnothing}$ and $\overline{(A,E)}\tilde{\cap}(B,E)=\tilde{\varnothing}$.

Definition 3.28. A soft topological space (X,τ,E) is said to be a soft completely normal space if for all two non-empty separated soft sets $(A,E),(B,E)$ there exists $G_E,H_E\in\tau$ such that $(A,E)\tilde{\subseteq}G_E,(B,E)\tilde{\subseteq}H_E$ and $G_E\tilde{\cap}H_E=\tilde{\varnothing}$.

Definition 3.29. A soft topological space (X,τ,E) is said to be a soft T_5-space if it is a soft completely normal space and a soft T_1-space .

Theorem 3.30. *Every soft completely normal space is a soft normal space and hence every soft T_5-space is a soft T_4-space.*

Proof. Let (X,τ,E) be a soft completely normal space and let $(A,E),(B,E)$ be two non-empty disjoint closed soft sets. Since (A,E) and (B,E) are two closed soft sets. Then, $\overline{(A,E)}=(A,E)$ and $\overline{(B,E)}=(B,E)$. It follows that $(A,E),(B,E)$ are separated sets. But (X,τ,E) is a soft completely normal space, so, there exists $G_E,H_E\in\tau$ such that $(A,E)\tilde{\subseteq}G_E,(B,E)\tilde{\subseteq}H_E$ and $G_E\tilde{\cap}H_E=\tilde{\varnothing}$. Then (X,τ,E) is soft normal space. Hence, every soft completely normal space is a soft normal space. It follows from the definitions that every soft T_5-space is a soft T_4-space. □

Theorem 3.31. *If (X,τ,E) is a soft topological space and for all $G_E\in\tau$, we have (G_E,τ_{G_E}) is soft normal subspace of (X,τ,E). Then (X,τ,E) is a soft completely normal space.*

Proof. Let $(A,E),(B,E)$ be two non-empty separated sets in X,

$$\text{then, } (A,E)\tilde{\cap}\overline{(B,E)}=\tilde{\varnothing} \text{ and } \overline{(A,E)}\tilde{\cap}(B,E)=\tilde{\varnothing}. \tag{3.3}$$

Since $\overline{A_E},\overline{B_E}\in\tau^c$, then $(\overline{A_E}\tilde{\cap}\overline{B_E})^c\in\tau$. Assume that:

$$G_E=(\overline{A_E}\tilde{\cap}\overline{B_E})^c \tag{3.4}$$

Let τ^* denotes the τ-relative topology for G_E. By the given condition, (G_E,τ^*) is a soft normal subspace of (X,τ). Moreover, $G_E\tilde{\cap}\overline{A_E}$ and $G_E\tilde{\cap}\overline{B_E}$ are τ^*-closed subsets of G_E such that $(G_E\tilde{\cap}\overline{A_E})\tilde{\cap}(G_E\tilde{\cap}\overline{B_E})=G_E\tilde{\cap}(\overline{A_E}\tilde{\cap}\overline{B_E})=(\overline{A_E}\tilde{\cap}\overline{B_E})^c\tilde{\cap}(\overline{A_E}\tilde{\cap}\overline{B_E})=\tilde{\varnothing}$, From (3.4).
Now, $G_E\tilde{\cap}\overline{A_E}$ and $G_E\tilde{\cap}\overline{B_E}\in{\tau^*}^c$ such that $(G_E\tilde{\cap}\overline{A_E})\tilde{\cap}(G_E\tilde{\cap}\overline{B_E})=\tilde{\varnothing}$. But (G_E,τ^*) is a soft normal subspace of (X,τ), so, there exists $H_E,W_E\in\tau^*$ such that

$$G_E\tilde{\cap}\overline{A_E}\tilde{\subseteq}H_E,\ G_E\tilde{\cap}\overline{B_E}\tilde{\subseteq}W_E \text{ and } H_E\tilde{\cap}W_E=\tilde{\varnothing}. \tag{3.5}$$

Since $H_E,W_E\in\tau^*,G_E\in\tau$, we have $H_E,W_E\in\tau$. From ((3.3)), $A_E\tilde{\cap}\overline{B_E}=\tilde{\varnothing}$, therefore $A_E\tilde{\subseteq}(\overline{B_E})^c\tilde{\subseteq}(\overline{B_E})^c\tilde{\cup}(\overline{A_E})^c=(\overline{B_E}\tilde{\cap}\overline{A_E})^c=G_E$. Also, $\overline{A_E}\tilde{\cap}B_E=\tilde{\varnothing}$, implies $B_E\tilde{\subseteq}(\overline{A_E})^c\tilde{\subseteq}(\overline{A_E})^c\tilde{\cup}(\overline{B_E})^c=(\overline{A_E}\tilde{\cap}\overline{B_E})^c=G_E$. Now, $A_E\tilde{\subseteq}G_E,A_E\tilde{\subseteq}\overline{A_E}$, implies $A_E=A_E\tilde{\cap}\overline{A_E}\tilde{\subseteq}G_E\tilde{\cap}\overline{A_E}\tilde{\subseteq}H_E$. Also $B_E\tilde{\subseteq}G_E,B_E\tilde{\subseteq}\overline{B_E}$, implies $B_E=B_E\tilde{\cap}\overline{B_E}\tilde{\subseteq}G_E\tilde{\cap}\overline{B_E}\tilde{\subseteq}W_E$.
Consequently, A_E and B_E are two separated subsets of X and there exists $H_E,W_E\in\tau$ such that $A_E\tilde{\subseteq}H_E$, $B_E\tilde{\subseteq}W_E$ and $H_E\tilde{\cap}W_E=\tilde{\varnothing}$. Hence (X,τ,E) is a soft completely normal space. □

Lemma 3.32. *Let (X, τ, E) be a soft topological space, $a \in A$, and $x \in X$. Then $x_a \tilde{\in} (F, A)$ if and only if $x \in_a (F, A)$.*

Proof. "⇒" Let $x_a \tilde{\in} (F, A)$ and let $x \notin_a (F, A)$. By Remark 2.1, $x \notin F(a)$. Then $\{x\} = x_a(a) \nsubseteq F(a)$. It follows that $x_a \tilde{\nsubseteq} (F, A)$ which is a contradiction with $x_a \tilde{\in} (F, A)$. Hence, $x \in_a (F, A)$.

"⇐" Let $x \in_a (F, A)$ and let $x_a \tilde{\notin} (F, A)$. Then $x_a(a) = \{x\} \tilde{\nsubseteq} F(a)$. It follows that $x \notin F(a)$. Consequently, $x \notin_a (F, A)$ which is a contradiction with $x \in_a (F, A)$. Hence, $x_a \tilde{\in} (F, A)$. □

According to Lemma 3.32, we can see that the separation axioms T_i in the sense of [5] and the current separation axioms are equivalent for $i = 0, 1, 2, 3$.

Theorem 3.33. *A soft topological space (X, τ, E) is a soft T_i-space [5] if and only if it is soft T_i-space in our sense ($i = 0, 1, 2, 3$).*

4. Soft Hereditary Property

Theorem 4.1. *Let (X, τ, E) be a soft T_i-space, where $i = 0, 1, 2, 3$. Then every soft subspace (Y, τ_Y, E) of the soft space (X, τ, E) is a soft T_i-space.*

Proof. We prove the theorem for ($i = 2$, for example), the other cases are similar. We want to show that every soft subspace of a soft T_2-space is soft T_2-space. So, let (X, τ, E) be a soft T_2-space, $Y \subseteq X$ such that (Y, τ_Y, E) be a soft subspace of (X, τ, E) and let x_e, y_e be two soft points over Y such that $x \neq y$. Since $Y \subseteq X$, we have x_e, y_e are two soft points over X such that $x \neq y$. But (X, τ, E) is soft T_2-space, so, there exists $G_E, H_E \in \tau$ such that $x_e \tilde{\in} G_E$, $y_e \tilde{\in} H_E$ and $G_E \tilde{\cap} H_E = \tilde{\varnothing}$. It follows that, there exists $Y_E \tilde{\cap} G_E, Y_E \tilde{\cap} H_E \in \tau_Y$ such that $x_e \tilde{\in} Y_E \tilde{\cap} G_E$, $y_e \tilde{\in} Y_E \tilde{\cap} H_E$ and $(Y_E \tilde{\cap} G_E) \tilde{\cap} (Y_E \tilde{\cap} H_E) = Y_E \tilde{\cap} (G_E \tilde{\cap} H_E) = Y_E \tilde{\cap} \tilde{\varnothing} = \tilde{\varnothing}$. Hence, (Y, τ_Y, E) is a soft T_2-space.

□

Theorem 4.2. *Every closed soft subspace of a soft normal space is a soft normal space.*

Proof. Let (X, τ, E) be a soft normal space and (Y, τ_Y, E) be a closed soft subspace of (X, τ, E). We want to show that (Y, τ_Y, E) is a soft normal space. So, let $(F_1, E)^*, (F_2, E)^*$ be two non-empty disjoint closed soft subsets of Y. Since $(F_1, E)^*, (F_2, E)^* \in \tau_Y^c$, from the Definition 2.16 of the soft subspace, there exists $(F_1, E), (F_2, E) \in \tau^c$ such that $(F_1, E)^* = Y_E \tilde{\cap} (F_1, E)$ and $(F_2, E)^* = Y_E \tilde{\cap} (F_2, E)$. Since $Y_E \in \tau^c$ and $(F_1, E), (F_2, E) \in \tau^c$, we have $Y_E \tilde{\cap} (F_1, E), Y_E \tilde{\cap} (F_2, E) \in \tau^c$ and therefore $(F_1, E)^*, (F_2, E)^* \in \tau^c$.

Now, $(F_1, E)^*, (F_2, E)^*$ are two non-empty disjoint closed soft subsets of X, but (X, τ, E) is a soft normal space, so, $\exists G_E, H_E \in \tau$ such that $(F_1, E)^* \tilde{\subseteq} G_E$, $(F_2, E)^* \tilde{\subseteq} H_E$ and $G_E \tilde{\cap} H_E = \tilde{\varnothing}$. It follows that, $\exists Y_E \tilde{\cap} G_E, Y_E \tilde{\cap} H_E \in \tau_Y$ such that $(F_1, E)^* \tilde{\subseteq} Y_E \tilde{\cap} G_E$, $(F_2, E)^* \tilde{\subseteq} Y_E \tilde{\cap} H_E$ and $(Y_E \tilde{\cap} G_E) \tilde{\cap} (Y_E \tilde{\cap} H_E) = Y_E \tilde{\cap} (G_E \tilde{\cap} H_E) = Y_E \tilde{\cap} \tilde{\varnothing} = \tilde{\varnothing}$. Therefore (Y, τ_Y, E) is a soft normal space. □

Theorem 4.3. *Let (X, τ, E) be a soft T_5-space. Then every soft subspace (Y, τ_Y, E) of the soft space (X, τ, E) is a soft T_5-space.*

Proof. Let (X,τ,E) be a soft T_5-space, $Y \subseteq X$ such that (Y,τ_Y,E) be a soft subspace of (X,τ,E). Since (X,τ,E) is a soft T_5-space, it is a soft completely normal T_1-space. Since every soft subspace of a soft T_1-space is a soft T_1-space, (Y,τ_Y,E) is a soft T_1-space. It is sufficient to show that (Y,τ_Y,E) is a soft completely normal space. So, let (A,E),(B,E) be two separated subsets of Y, then

$$\overline{(A,E)}^{\tau_Y}\tilde{\cap}(B,E)=\tilde{\varnothing} \ \ and \ \ (A,E)\tilde{\cap}\overline{(B,E)}^{\tau_Y}=\tilde{\varnothing}. \tag{4.1}$$

$$Since \ \ \overline{(A,E)}^{\tau_Y}=Y_E\tilde{\cap}\overline{(A,E)}^{\tau} \ \ and \ \ \overline{(B,E)}^{\tau_Y}=Y_E\tilde{\cap}\overline{(B,E)}^{\tau}, \tag{4.2}$$

Substituting from (4.2) into (4.1), we have
$(Y_E\tilde{\cap}\overline{(A,E)}^{\tau})\tilde{\cap}(B,E)=\tilde{\varnothing}$ and $(A,E)\tilde{\cap}(Y_E\tilde{\cap}\overline{(B,E)}^{\tau})=\tilde{\varnothing}$.
Since $(A,E),(B,E)\tilde{\subseteq}Y_E$, we have $\overline{(A,E)}^{\tau}\tilde{\cap}(B,E)=\tilde{\varnothing}$ and $(A,E)\tilde{\cap}\overline{(B,E)}^{\tau}=\tilde{\varnothing}$. It follows that (A,E) and (B,E) are two separated subsets of X. But (X,τ,E) is a soft completely normal space, so, there exists $G_E,H_E\in\tau$ such that $A_E\tilde{\subseteq}G_E$, $B_E\tilde{\subseteq}H_E$ and $G_E\tilde{\cap}H_E=\tilde{\varnothing}$. Since $A_E\tilde{\subseteq}Y_E$ and $A_E\tilde{\subseteq}G_E$, we have $A_E\tilde{\subseteq}Y_E\tilde{\cap}G_E$. Also, $B_E\tilde{\subseteq}Y_E$ and $B_E\tilde{\subseteq}H_E$, then, $B_E\tilde{\subseteq}Y_E\tilde{\cap}H_E$. Since $G_E,H_E\in\tau$, implies $Y_E\tilde{\cap}G_E,Y_E\tilde{\cap}H_E\in\tau_Y$. It follows that, there exists $Y_E\tilde{\cap}G_E,Y_E\tilde{\cap}H_E\in\tau_Y$ such that $A_E\tilde{\subseteq}Y_E\tilde{\cap}G_E$, $B_E\tilde{\subseteq}Y_E\tilde{\cap}H_E$ and $(Y_E\tilde{\cap}G_E)\tilde{\cap}(Y_E\tilde{\cap}H_E)=Y_E\tilde{\cap}(G_E\tilde{\cap}H_E)=Y_E\tilde{\cap}\tilde{\varnothing}=\tilde{\varnothing}$. Thus, (Y,τ_Y,E) is a soft completely normal space and hence every soft subspace of a soft T_5-space is a soft T_5-space. □

5. Soft Topological Property

Theorem 5.1. *The property of being soft T_i-space $(i=0,1,2)$ is a topological property.*

Proof. We prove the theorem for ($i=2$, for example), the other cases are similar. Let $f=(\phi,\psi):(X,\tau,E)\to(Y,\tau^*,K)$ be a soft mapping such that:

1. f is 1-1, onto and soft open mapping.
2. (X,τ,E) is a soft T_2-space.

We want to show that (Y,τ^*,K) is a soft T_2-space. So, let x_k,y_k be two soft points in Y such that $x\neq y$. Since f is 1-1 and onto mapping, then there exists two soft points x_e^*,y_e^* in X such that $f(x_e^*)=x_k, f(y_e^*)=y_k$, and $x^*\neq y^*$. But (X,τ,E) is a soft T_2-space, so, there exists $G_E,H_E\in\tau$ such that $x_e^*\tilde{\in}G_E$, $y_e^*\tilde{\in}H_E$ and $G_E\tilde{\cap}H_E=\tilde{\varnothing}_X$. It follows that, $f(x_e^*)=x_k\tilde{\in}f(G_E)$, $f(y_e^*)=y_k\tilde{\in}f(H_E)$ and $f(G_E\tilde{\cap}H_E)=f(G_E)\tilde{\cap}f(H_E)=f(\tilde{\varnothing}_X)=\tilde{\varnothing}_Y$, from Proposition 2.24. Since $G_E,H_E\in\tau$ and f is an soft open mapping, $f(G_E),f(H_E)\in\tau^*$, from Definition 2.27 of soft open mapping. Now, there exists $f(G_E),f(H_E)\in\tau^*$ such that $x_k\tilde{\in}f(G_E)$, $y_k\tilde{\in}f(H_E)$ and $f(G_E)\tilde{\cap}f(H_E)=\tilde{\varnothing}$. Hence, (Y,τ^*,K) is a soft T_2-space.

□

Theorem 5.2. *The property of being soft T_i-space $(i=3,4,5)$ is a soft topological property or it is preserved under a soft homeomorphism mapping.*

Proof. We prove the theorem for ($i=3$, for example), the other cases are similar. Since, the property of being soft T_1-space is a topological property, we only show

that the property of soft regularity is a topological property.
So, let $f = (\phi, \psi) : (X, \tau, E) \to (Y, \tau^*, K)$ be a soft mapping such that:

(1) f is soft homeomorphism from X onto Y (i.e f is a bijection mapping and f, f^{-1} are soft continuous).
(2) (X, τ, E) is a soft regular space.

Let F_K be a τ^*-closed soft subset of Y and let y_k be a soft point in Y such that $y_k \tilde{\notin} F_K$. Since, f is an onto mapping, there exists a soft point x_e in X such that $f(x_e) = y_k$.
Since f is soft continuous mapping and F_K is a τ^*-closed soft subset of Y, we have $f^{-1}(F_K)$ is a τ-closed soft subset of X, from Theorem 2.25. Since $y_k = f(x_e) \tilde{\notin} F_K$, we have $f^{-1}(f(x_e)) = x_e \tilde{\notin} f^{-1}(F_K)$ (as, f is injective). Now, $f^{-1}(F_K)$ is a τ-closed soft subset of X, x_e is a soft point in X such that $x_e \tilde{\notin} f^{-1}(F_K)$. But (X, τ, E) is a soft regular space, so, there exists $G_E, H_E \in \tau$ such that $x_e \tilde{\in} G_E, f^{-1}(F_K) \tilde{\subseteq} H_E$ and $G_E \tilde{\cap} H_E = \tilde{\varnothing}_X$ and thus $f(x_e) = y_k \tilde{\in} f(G_E), f(f^{-1}(F_K)) = F_K \tilde{\subseteq} f(H_E)$ (as, f is surjective) and $f(G_E \tilde{\cap} H_E) = f(G_E) \tilde{\cap} f(H_E) = f(\tilde{\varnothing}_X) = \tilde{\varnothing}_Y$.
Since f^{-1} is soft continuous mapping, i.e. f is an soft open mapping (from Proposition 2.28). Now, $G_E, H_E \in \tau$ and f is an soft open mapping, then $f(G_E), f(H_E) \in \tau^*$, from Definition 2.27 of soft open mapping. Now, there exists $f(G_E), f(H_E) \in \tau^*$ such that $y_k \tilde{\in} f(G_E)$, $F_K \tilde{\subseteq} f(H_E)$ and $f(G_E) \tilde{\cap} f(H_E) = \tilde{\varnothing}_Y$. Thus, (Y, τ^*, K) is a soft regular space. □

Acknowledgements. The authors express their grateful thanks to the referee for reading the manuscript and making helpful comments.

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