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On some fuzzy covering axioms via ultrafilters

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Abstract. The product of two weakly fuzzy countably compact spaces introduced by Afsan [1] need not be weakly fuzzy countably compact spaces in general. The purpose of the present paper is to initiate and investigate some new fuzzy covering axioms related to weakly fuzzy countably compact spaces, namely fuzzy $\mathcal{F}$-compact space, fuzzy $\mathcal{F}$-pseudocompact space and fuzzy densely $\mathcal{F}$-compact space in fuzzy topological spaces via $\mathcal{F}$-convergence of fuzzy sequences in term of ultrafilter on the set of all natural numbers $\mathbf{N}$ which are arbitrarily productive. It has also been shown that the class of fuzzy $\mathcal{F}$-compact spaces is properly contained in the class of weakly fuzzy countably compact spaces [21]. These besides, we have shown that all these covering axioms are "good extension" of similar notions of general topology and we have given several examples to derive proper inclusive relationships among there fuzzy covering axioms.

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1. Introduction

One of the great achievements in general topology is the "Tychonoff's product theorem" for compact spaces which states that arbitrary product of compact spaces is compact. But unfortunately, Tychonoff-like theorem is not true for countably compact spaces. Then it became the big question to the topologists when a product of countably compact spaces is countably compact. In the long process of investigation of this question, several new classes of covering axioms related to countably compact spaces such as pseudocompact, totally countably compact, sequentially compact space etc. were invented. The latest important contribution was done by Bernstein [2]. He initiated the concept of D-compactness in term of a new convergence theory of sequences via filter D of natural numbers that was popularized by Saks [16]. Some recent works related to D-compactness are found in [3, 4, 9, 10, 17, 19].

Following the introduction of fuzzy sets by Zadeh [22], many research has been carried out in the areas of general theories as well as applications. The fuzzy topology was introduced by Chang [5]. Chang [5] defined fuzzy compact topological space in the usual manner. After Chang, several notions of fuzzy compactness [8, 12, 13, 14, 21] were invented. The fuzzy covering axiom weakly fuzzy compact due to Lowen [12] is a good extension of compactness, but fuzzy compactness due to Chang [5] is not a good extension of compactness of general topology. Lowen [14] also established that the Tychonoff-like theorem on (finite) products are fulfilled by weakly fuzzy compact, but not by fuzzy compactness due to Chang.

Observing the serious drawback of the notion of fuzzy countably compactness introduced by Wong [21] that it is not good extension of countably compactness of general topology, Afsan introduced the notion of weakly fuzzy countably compactness [1] which is a good extension of the notion of countably compactness of general topology. The product of (even two) weakly fuzzy countably compact spaces is not weakly fuzzy countably compact. Afsan [1] has achieved several sufficient conditions under which the product of two weakly fuzzy countably compact spaces is weakly fuzzy countably compact.

In the present paper, in section 3, we have introduced some new fuzzy covering axioms, namely fuzzy $\mathcal{F}$-compact space, fuzzy $\mathcal{F}$-pseudocompact space and fuzzy densely $\mathcal{F}$-compact space related to the class of weakly fuzzy countably compact spaces in fuzzy topological spaces via the convergence of fuzzy sequences in terms of ultrafilter $\mathcal{F}$ on the set of all natural numbers $\mathbf{N}$ which are arbitrarily productive. We have driven their mutual relations and relations with weakly fuzzy compact space [12] and weakly countably compact space [1]. We have shown that continuous image of fuzzy $\mathcal{F}$-compact space is fuzzy $\mathcal{F}$-compact. In section 4, we have concentrated on the establishing the good extendedness of all the fuzzy covering axioms introduced in the earlier section along with some counter-examples which establish the inclusive relationships among the classes of these fuzzy covering axioms.

2. Preliminaries

Throughout this paper, spaces (X, σ) and (Y, δ) (or simply X and Y) represent non-empty fuzzy topological spaces due to Chang [5] and the symbols I, $\mathbf{N}$, and I^X have been used for the unit closed interval $[0, 1]$, the set of all natural numbers and the set of all functions with domain X and codomain I respectively. The support of a fuzzy set A is the set $\{x \in X : A(x) > 0\}$ and is denoted by $supp(A)$. A fuzzy set with only non-zero value $\lambda \in (0, 1]$ at only one element $x \in X$ is called a fuzzy point and is denoted by x_λ and the set of all fuzzy points of a fuzzy topological space is denoted by $Pt(X)$. For any two fuzzy sets A and B of X, $A \leq B$ if and only if $A(x) \leq B(x)$ for all $x \in X$. A fuzzy point x_λ is said to be in a fuzzy set A (denoted by $x_\lambda \in A$) if $x_\lambda \leq A$, that is, if $\lambda \leq A(x)$. The set of all fuzzy points of X contained in a fuzzy subset A of X is denoted by $Pt(A)$. The constant fuzzy sets of X with values $\alpha \in [0, 1]$, 0 and 1 are denoted by $\underline{\alpha}$, $\underline{0}$ and $\underline{1}$ respectively. A fuzzy set A is said to be quasi-coincident with B (written as $A\hat{q}B$) [18] if $A(x) + B(x) > 1$ for some $x \in X$. A fuzzy set A is said to be not quasi-coincident with B (written as $A\bar{q}B$) [18] if $A(x) + B(x) \leq 1$, for all $x \in X$. A fuzzy open set A of X is called fuzzy quasi-neighborhood of a fuzzy point x_λ if $x_\lambda \hat{q} A$. The collection of all fuzzy quasi-neighborhood of a fuzzy point $x_\lambda \in Pt(X)$ is denoted by $\mathcal{Q}(X, x_\lambda)$.

A fuzzy topological space X is called fuzzy compact [5] (resp. fuzzy countably compact [21]) if for every family (respectively countable family) Σ of fuzzy open sets of X with $\bigvee\{U : U \in \Sigma\} = \underline{1}$, there exists a finite subfamily $\{U_i : i = 1, 2, ..., n\}$ of Σ such that $\bigvee_{i=1}^{n} U_i = \underline{1}$. Observing several drawbacks, the above two definitions were modified in [12] and [1]. A fuzzy topological space X is called weakly fuzzy compact [12] (resp. weakly fuzzy countably compact [1]) if for every family (resp. countable family) Σ of fuzzy open sets of X with $\bigvee\{U : U \in \Sigma\} = \underline{1}$ and for $\epsilon > 0$, there exists a finite subfamily $\{U_i : i = 1, 2, ..., n\}$ of Σ such that $\bigvee_{i=1}^{n} U_i \geq \underline{1 - \epsilon}$. In [1], It has been shown that the class of weakly fuzzy countably compact spaces properly contains the classes of countably compact and weakly compact spaces.

It is well-known that a filter $\mathcal{F}$ on a non-empty set S is a non-empty family of non-empty subsets of S with the properties (a) $A, B \in \mathcal{F}$ implies $A \cap B \in \mathcal{F}$ and (b) $A \subset B$ and $A \in \mathcal{F}$ implies $B \in \mathcal{F}$. The maximal filters on S are called ultrafilters on S. A filter $\mathcal{F}$ on S is an ultrafilter if and only if for each $A \subset S$, either $A \in \mathcal{F}$ or $S - A \in \mathcal{F}$. An ultrafilter on S of the form $\{A \subset S : s \in A\}$ is called a principal ultrafilter generated by $s \in S$. An ultrafilter $\mathcal{F}$ on S which is not principal is called non-principal. Equivalently, an ultrafilter $\mathcal{F}$ on S is non-principal if and only if (a) $\bigcap\{A \subset S : A \in \mathcal{F}\} = \varnothing$, or (b) $A \in \mathcal{F}$ implies A is not finite. We want to mention that all the basic facts about ultrafilter are found in [6]. Throughout the subsequence sections, $\mathcal{F}$ denotes a non-principal ultrafilter on the set of all natural numbers $\mathbf{N}$. A fuzzy sequence $\mathcal{S} = \{x^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\}$ is said be "in A", where $A \in I^X$ if for each $k \in \mathbf{N}$, $x^k_{\lambda_k} \in A$. If $A = \underline{1}$, then a fuzzy sequence $\mathcal{S} = \{x^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\}$ in A is said to be " in X". For a fuzzy sequence $\mathcal{S} = \{x^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\}$ in X, the collection of all its cluster points is denoted by $\mathcal{A}(\mathcal{S})$. In [1], it has been shown that X is weakly fuzzy countably compact if and only if for every fuzzy sequence $\mathcal{S} = \{x^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\}$ in X, with the property that there exists $\epsilon > 0$ such that for any $k \in \mathbf{N}$, $x^k_{\lambda_k} \not\leq \underline{\epsilon}$, $\mathcal{A}(\mathcal{S}) \neq \underline{0}$.

Let (X, τ) be a given topological space and $\tau_{\mathbf{R}} = \{]r, \infty[: r \in \mathbf{R}\} \cup \{\varnothing\}$. Consider the space $I = [0, 1]$ with subtopology $\tau_{\mathbf{R}} \mid I$ and $\omega(\tau) = \{\mu \in X^I : \mu$ is continuous $\}$. Lowen [12] has shown that $\omega(\tau)$ is a fuzzy topology on X. A fuzzy topological property P is called "good extension" of the similar topological property Q if (X, τ) enjoys the property Q if and only if $(X, \omega(\tau))$ enjoys the property P.

3. Fuzzy covering axioms

Definition 3.1. A fuzzy sequence $\mathcal{S} = \{x^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\}$ in $A \in I^X$ is said to fuzzy $\mathcal{F}$-converge to a fuzzy point $x_\lambda \in Pt(X)$ if for every $U \in \mathcal{Q}(X, x_\lambda)$, $\{k \in \mathbf{N} : x^k_{\lambda_k} \hat{q} U\} \in \mathcal{F}$. Here x_λ is called a fuzzy $\mathcal{F}$-limit of the fuzzy sequence $\mathcal{S}$. We set $\lim(\mathcal{F}, \mathcal{S}) = \bigvee\{x_\lambda \in Pt(X) : x_\lambda$ is a fuzzy $\mathcal{F}$-limit of $\mathcal{S}\}$. A sequence $\mathcal{S} = \{x^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\}$ in X is said to be strictly bounded below if there exists $\epsilon > 0$ such that $x^k_{\lambda_k} \not\leq \underline{\epsilon}$ for all $k \in \mathbf{N}$. Here ϵ is known as a strict lower bound of $\mathcal{S}$.

Remark 3.2. It is clear that $\lim(\mathcal{F}, \mathcal{S}) \leq \mathcal{A}(\mathcal{S})$ for any non-principal ultrafilter $\mathcal{F}$ on $\mathbf{N}$.

Theorem 3.3. *For any fuzzy sequence* $\mathcal{S} = \{x^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\}$ *in* X*, there exists a non-principle filter* $\mathcal{F}$ *on* $\mathbf{N}$ *such that* $\mathcal{A}(\mathcal{S}) \leq \lim(\mathcal{F}, \mathcal{S})$*.*

Proof. Let $x_\lambda \in \mathcal{A}(\mathcal{S})$. Define the set $T_U = \{k \in \mathbf{N} : x^k_{\lambda_k} \hat{q} U\}$ for each $U \in Q(X, x_\lambda)$. Consider the family $\Omega = \{T_U - \{k\} : U \in Q(X, x_\lambda), k \in \mathbf{N}\}$. If possible, let $U, U' \in Q(X, x_\lambda)$ and $k, k' \in \mathbf{N}$ such that $T_U - \{k\}, T_{U'} - \{k'\} \in \Omega$ and $(T_U - \{k\}) \cap (T_{U'} - \{k'\}) = T_{U \wedge U'} - \{k, k'\} = \varnothing$. But then for $n = \max\{k+1, k'+1\}$, there exists no $m (\in \mathbf{N}) \geq n$, such that $x^m_{\lambda^m} \hat{q}(U \wedge U')$ and hence $x_\lambda \notin \mathcal{A}(\mathcal{S})$. So, Ω has finite intersection property. Therefore there exists an ultrafilter $\mathcal{F}$ on $\mathbf{N}$ containing Ω. Since $\bigcap\{T_U - \{k\} : U \in Q(X, x_\lambda), k \in \mathbf{N}\} = \varnothing$, The ultrafilter $\mathcal{F}$ is non-principal. Now it is remain to show that $x_\lambda \in \lim(\mathcal{F}, \mathcal{S})$. Suppose $U \in Q(X, x_\lambda)$. Then $T_U - \{k\} \in \Omega$ and so the inclusion $T_U - \{k\} \subset T_U$ ensures that $T_U = \{k \in \mathbf{N} : x^k_{\lambda_k} \hat{q} U\} \in \mathcal{F}$. Therefore $x_\lambda \in \lim(\mathcal{F}, \mathcal{S})$. □

Theorem 3.4. *If in fuzzy Hausdorff space* X*, a fuzzy sequence fuzzy* $\mathcal{F}$*-converges to two fuzzy* $\mathcal{F}$*-limits, then the supports of the fuzzy points are equal.*

Proof. Let X be a fuzzy Hausdorff space and $\{x^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\}$ be a fuzzy sequence in X fuzzy $\mathcal{F}$-converges to two fuzzy points x_λ and $y_{\lambda'}$ of X. If possible, let $supp(x_\lambda) \neq supp(y_{\lambda'})$. Then there exist $U \in \mathcal{Q}(X, x_\lambda)$ and $V \in \mathcal{Q}(X, y_{\lambda'})$ such that $U \wedge V = \underline{0}$. Since $\{x^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\}$ fuzzy $\mathcal{F}$-converges to two fuzzy points x_λ and $y_{\lambda'}$, $\{k \in \mathbf{N} : x^k_{\lambda_k} \hat{q} U\} \in \mathcal{F}$ and $\{k \in \mathbf{N} : x^k_{\lambda_k} \hat{q} V\} \in \mathcal{F}$. Then $\{k \in \mathbf{N} : x^k_{\lambda_k} \hat{q}(U \wedge V)\} = \{k \in \mathbf{N} : x^k_{\lambda_k} \hat{q} U\} \cap \{k \in \mathbf{N} : x^k_{\lambda_k} \hat{q} V\} \in \mathcal{F}$. So $\underline{0} \in \mathcal{F}$—a contradiction. □

Theorem 3.5. *Let* $\psi : X \to Y$ *be a fuzzy continuous function. Then for any fuzzy sequence* $\mathcal{S} = \{x^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\}$ *in* X*,* $\psi(\lim(\mathcal{F}, \mathcal{S})) \leq \lim(\mathcal{F}, \psi(\mathcal{S}))$*.*

Proof. Let $x_\lambda \in \lim(\mathcal{F}, \mathcal{S})$. Consider $V \in Q(X, \psi(x_\lambda))$. Then there exists $U \in Q(X, x_\lambda)$ such that $\psi(U) \leq V$. Since $x_\lambda \in \lim(\mathcal{F}, \mathcal{S})$, we have $\{k \in \mathbf{N} : x_k \hat{q} U\} \in \mathcal{F}$. So the inclusion $\{k \in \mathbf{N} : \psi(x_k) \hat{q} U\} \subset \{k \in \mathbf{N} : x_k \hat{q} V\}$ ensures that $\{k \in \mathbf{N} : \psi(x_k) \hat{q} V\} \in \mathcal{F}$. Hence $\psi(x_\lambda) \in \lim(\mathcal{F}, \psi(\mathcal{S}))$. □

Definition 3.6. A fuzzy subset A of a fuzzy topological space X is called fuzzy $\mathcal{F}$-compact if every strictly bounded below fuzzy sequence $\mathcal{S} = \{x^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\}$ in A has an $\mathcal{F}$-limit in A. If $A = \underline{1}$ is $\mathcal{F}$-compact, X is called fuzzy $\mathcal{F}$-compact.

Remark 3.7. Every fuzzy $\mathcal{F}$-compact space is weakly fuzzy countably compact [1]. But the converse implication is not valid in general. This will be established latter.

Theorem 3.8. *A fuzzy topological space* X *is fuzzy* $\mathcal{F}$*-compact if for every sequence* $\mathcal{S} = \{x^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\}$*,* $cl(T) = cl(\bigvee\{x^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\})$ *is weakly fuzzy compact.*

Proof. Let X be not fuzzy $\mathcal{F}$-compact. Then there exists a strictly bounded below fuzzy sequence $\mathcal{T} = \{x^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\}$ such that $\lim(\mathcal{F}, \mathcal{S}) = \underline{0}$. Then there exists $\epsilon > 0$ such that for any $k \in \mathbf{N}$, $x^k_{\lambda_k} \not\leq \underline{\epsilon}$. Then for each $x_\lambda \in cl(\mathcal{T})$, there exists an $U(x_\lambda) \in Q(X, x_\lambda)$ such that $\{k \in \mathbf{N} : x^k_{\lambda_k} \hat{q} U(x_\lambda)\} \notin \mathcal{F}$. Since $\mathcal{F}$ is an ultrafilter on $\mathbf{N}$, $\{k \in \mathbf{N} : x^k_{\lambda_k} \bar{q} U(x_\lambda)\} \in \mathcal{F}$. Let $\epsilon > 0$ be a strict lower bound of $\mathcal{T}$. Since $\{U(x_\lambda) : x_\lambda \in cl(\mathcal{T})\}$ is a fuzzy open cover of $cl(\mathcal{T})$, weakly fuzzy compactness of $cl(\mathcal{T})$ ensures the existence of $x^1_{\lambda_1}, x^2_{\lambda_2}, ..., x^n_{\lambda_n} \in cl(\mathcal{T})$ such that $\bigvee_{i=1}^n U(x^i_{\lambda_i}) \geq cl(\mathcal{T}) - \epsilon'$, where $\epsilon' = \frac{\epsilon+1}{2}$. Then $\{k \in \mathbf{N} : x^k_{\lambda_k} \bar{q}(\bigvee_{i=1}^n U(x^i_{\lambda_i}))\} = \varnothing$. It is easy to verify that $\bigcap_{i=1}^n \{k \in \mathbf{N} : x^k_{\lambda_k} \bar{q}(U(x^i_{\lambda_i})\} = \{k \in \mathbf{N} : x^k_{\lambda_k} \bar{q}(\bigvee_{i=1}^n U(x^i_{\lambda_i}))\} = \varnothing$. Thus $\varnothing \in \mathcal{F}$—a contradiction. □

Theorem 3.9. *(i) Fuzzy closed subset of fuzzy $\mathcal{F}$-compact space is fuzzy $\mathcal{F}$-compact.*
(ii) Arbitrary cartesian product of fuzzy $\mathcal{F}$-compact spaces is fuzzy $\mathcal{F}$-compact.
(iii) Fuzzy continuous image of fuzzy $\mathcal{F}$-compact subsets is fuzzy $\mathcal{F}$-compact.

Proof. (i) Let X be fuzzy $\mathcal{F}$-compact and A be a fuzzy closed set of X. Let $\mathcal{S} = \{y^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\}$ be a strictly lower bounded fuzzy sequence in A. Then there exists $\epsilon > 0$ such that for any $k \in \mathbf{N}$, $y^k_{\lambda_k} \not\leq \underline{\epsilon}$. Since X is fuzzy $\mathcal{F}$-compact, the sequence $\{y^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\}$ has $\mathcal{F}$-limit $x_\lambda \in Pt(X)$. We shall show that $x_\lambda \in A$. Let $U \in Q(X, x_\lambda)$. Then $\{k \in \mathbf{N} : y^k_{\lambda_k} \hat{q} U\} \in \mathcal{F}$. Then there exists a $k \in \mathbf{N}$ such that $y^k_{\lambda_k} \hat{q} U$. So $U(y^k) + \lambda_k > 1$ and $A(y^k) \geq \lambda_k$ and so $U(y^k) + A(y^k) > 1$. Thus $A \hat{q} U$ and so $x_\lambda \in cl(A) = A$. Hence A is fuzzy $\mathcal{F}$-compact.

(ii) Let $\{(X_\alpha, \delta_\alpha) : \alpha \in \Delta\}$ be a family of fuzzy $\mathcal{F}$-compact spaces. Let $X = \prod_{\alpha \in \Delta} X_\alpha$. Let $\mathcal{S} = \{x^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\}$ be a strictly lower bounded fuzzy sequence in X. Then there exists $\epsilon > 0$ such that for any $k \in \mathbf{N}$, $x^k_{\lambda_k} \not\leq \underline{\epsilon}$. Since for each $\alpha \in \Delta$, $\mathcal{S}_\alpha = \{P_\alpha(x^k_{\lambda_k}) \in Pt(X_\alpha) : k \in \mathbf{N}\}$, where P_α is the α-th projection is a fuzzy sequence in X_α such that for any $k \in \mathbf{N}$, $P_\alpha(x^k_{\lambda_k}) \not\leq \underline{\epsilon}$, S_α is a strictly lower bounded fuzzy sequence in X_α. Since for each $\alpha \in \Delta$, X_α is fuzzy $\mathcal{F}$-compact, there exists $x^\alpha_{\lambda_\alpha} \in Pt(X_\alpha)$ such that $x^\alpha_{\lambda_\alpha} \in \lim(\mathcal{F}, \mathcal{S}_\alpha)$. We define the fuzzy point $x_\lambda = \Pi_{\alpha \in \Delta} x^\alpha_{\lambda_\alpha}$ of X. We claim that $x_\lambda \in \lim(\mathcal{F}, \mathcal{S})$. Let $U \in Q(X, x_\lambda)$. Then there exist $\alpha_1, \alpha_2, ..., \alpha_n \in \Delta$ such that $U = \bigwedge_{i=1}^n P^{-1}_{\alpha_i}(U_{\alpha_i})$. Clearly, for each $i \in \{1, 2, ..., n\}$, $U_{\alpha_i} \in Q(X_{\alpha_i}, x^{\alpha_i}_{\lambda_{\alpha_i}})$. Since for each $i \in \{1, 2, ..., n\}$, $x^{\alpha_i}_{\lambda_{\alpha_i}} \in \lim(\mathcal{F}, \mathcal{S}_{\alpha_i})$, we have $\{k \in \mathbf{N} : P_{\alpha_i}(x^k_{\lambda_k}) \hat{q} U_{\alpha_i}\} \in \mathcal{F}$ for each $i \in \{1, 2, ..., n\}$. Then the inclusion $\{k \in \mathbf{N} : x^k_{\lambda_k} \hat{q} U\} \subset \bigwedge_{i=1}^n \{k \in \mathbf{N} : P_{\alpha_i}(x^k_{\lambda_k}) \hat{q} U_{\alpha_i}\}$ implies $\{k \in \mathbf{N} : x^k_{\lambda_k} \hat{q} U\} \in \mathcal{F}$. So $x_\lambda \in \lim(\mathcal{F}, \mathcal{S})$. Therefore X is fuzzy $\mathcal{F}$-compact.

(iii) Let A be a fuzzy $\mathcal{F}$-compact subset of X. Let $\mathcal{S} = \{y^k_{\lambda_k} \in Pt(Y) : k \in \mathbf{N}\}$ be a sequence in $\psi(A)$ with the property that there exists $\epsilon > 0$ such that for any $k \in \mathbf{N}$, $y^k_{\lambda_k} \not\leq \underline{\epsilon}$. Then $\psi^{-1}(\mathcal{S}) = \{\psi^{-1}(y^k_{\lambda_k}) \in Pt(X) : k \in \mathbf{N}\}$ is a sequence in A such that for any $k \in \mathbf{N}$, $\psi^{-1}(y^k_{\lambda_k}) \not\leq \underline{\epsilon}$. Since ψ is fuzzy continuous, $\psi(\lim(\mathcal{F}, \psi^{-1}(\mathcal{S}))) \leq \lim(\mathcal{F}, \psi(\psi^{-1}(\mathcal{S}))) \leq \lim(\mathcal{F}, \mathcal{S})$. Again, since A is fuzzy $\mathcal{F}$-compact, there exists a fuzzy point $x_\lambda \in \lim(\mathcal{F}, \psi^{-1}(\mathcal{S})) \bigwedge A$. Therefore $\psi(x_\lambda) \in \lim(\mathcal{F}, \mathcal{S}) \bigwedge \psi(A)$. Thus $\psi(A)$ is fuzzy $\mathcal{F}$-compact. □

Definition 3.10. A fuzzy sequence $\mathcal{R} = \{A_k \in I^X : k \in \mathbf{N}\}$ is said to have fuzzy $\mathcal{F}$-limit $x_\lambda \in Pt(X)$ if for every $U \in \mathcal{Q}(X, x_\lambda)$, $\{k \in \mathbf{N} : A_k \hat{q} U\} \in \mathcal{F}$. We set $\lim(\mathcal{F}, \mathcal{R}) = \bigvee\{x_\lambda \in Pt(X) : x_\lambda$ is a fuzzy $\mathcal{F}$-limit of $\mathcal{R}\}$. A fuzzy sequence $\mathcal{R} = \{A_k \in I^X : k \in \mathbf{N}\}$ is said to be pointwise bounded below if there exists $\epsilon > 0$ such that $A_k \not\leq \underline{\epsilon}$ for all $k \in \mathbf{N}$. A fuzzy topological space X is called fuzzy $\mathcal{F}$-pseudocompact if for every pointwise bounded below fuzzy sequence $\mathcal{R} = \{A_k \in I^X : k \in \mathbf{N}\}$ of fuzzy open subsets of X, $\lim(\mathcal{F}, \mathcal{R}) \neq \underline{0}$. A fuzzy topological space X is called fuzzy pseudocompact if for every pointwise bounded below fuzzy decreasing sequence $\mathcal{R} = \{A_k \in \delta : k \in \mathbf{N}\}$, $\bigwedge\{cl(A_k) \in I^X : k \in \mathbf{N}\} \neq \underline{0}$.

Theorem 3.11. *Let $\psi : X \to Y$ be a fuzzy continuous function. Then for any fuzzy sequence $\mathcal{R} = \{A_k \in I^X : k \in \mathbf{N}\}$ in X, $\psi(\lim(\mathcal{F},\mathcal{R})) \leq \lim(\mathcal{F},\psi(\mathcal{R}))$.*

Proof. The proof is analogous to the proof of Theorem 3.5. □

Theorem 3.12. *Every fuzzy $\mathcal{F}$-pseudocompact space is fuzzy pseudocompact.*

Proof. Let X be a fuzzy $\mathcal{F}$-pseudocompact space. If X is not fuzzy pseudocompact, there exists a pointwise bounded below fuzzy decreasing sequence $\mathcal{R} = \{A_k \in \delta : k \in \mathbf{N}\}$ such that $\bigwedge\{cl(A_k) \in I^X : k \in \mathbf{N}\} = \underline{0}$. Then there exists $\epsilon > 0$ such that for any $k \in \mathbf{N}$, $A_k \not\leq \underline{\epsilon}$. Since X is fuzzy $\mathcal{F}$-pseudocompact, there exists a fuzzy point $x_\lambda \in \lim(\mathcal{F},\mathcal{R})$. But for all $k \in \mathbf{N}$, $x_\lambda \notin cl(A_k)$. Then there exists an $U \in \mathcal{Q}(X, x_\lambda)$ such that $A_1 \bar{q} U$. Since $\mathcal{R}$ is a fuzzy decreasing sequence, $A_k \bar{q} U$ for all $k \in \mathbf{N}$. Therefore $\{k \in \mathbf{N} : A_k \hat{q} U\} = \varnothing$— a contradiction. □

Theorem 3.13. *(i) Every product of $\mathcal{F}$-pseudocompact spaces is $\mathcal{F}$-pseudocompact.*
(ii) Fuzzy continuous image of fuzzy $\mathcal{F}$-compact subsets is fuzzy $\mathcal{F}$-compact.

Proof. The proof is analogous to Theorem 3.9. □

Corollary 3.14. *Every product of $\mathcal{F}$-pseudocompact spaces is pseudocompact.*

Definition 3.15. A fuzzy topological space X is called fuzzy densely $\mathcal{F}$-pseudocompact if there exists a fuzzy dense set $A \in I^X$ such that for every pointwise bounded below fuzzy sequence $\mathcal{S} = \{x^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\}$ in A, $\lim(\mathcal{F},\mathcal{S}) \neq \underline{0}$.

Theorem 3.16. *(i) Every product of fuzzy densely $\mathcal{F}$-compact spaces is densely $\mathcal{F}$-compact.*
(ii) Every fuzzy densely $\mathcal{F}$-compact space is $\mathcal{F}$-pseudocompact.

Proof. (i) The proof is analogous to Theorem 3.9.
(ii) Let X be a fuzzy densely $\mathcal{F}$-compact space and $A \in I^X$ be fuzzy dense in X. Let $\mathcal{R} = \{A_k \in I^X : k \in \mathbf{N}\}$ be a pointwise bounded below fuzzy sequence of fuzzy open subsets of X. Then there exists $\epsilon > 0$ such that for any $k \in \mathbf{N}$, $A_k \not\leq \underline{\epsilon}$. Now we select a fuzzy sequence $\mathcal{S} = \{x^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\}$, $x^k_{\lambda_k} \in A \wedge A_k$. Then for any $k \in \mathbf{N}$, $x^k_{\lambda_k} \not\leq \underline{\epsilon}$, and so the fuzzy densely $\mathcal{F}$-compactness of X ensures the existence of $x_\lambda \in \lim(\mathcal{F},\mathcal{S})$. Now suppose $U \in \mathcal{Q}(X, x_\lambda)$. Then $\{k \in \mathbf{N} : x^k_{\lambda_k} \hat{q} U\} \in \mathcal{F}$. The inclusion $\{k \in \mathbf{N} : x^k_{\lambda_k} \hat{q} U\} \subset \{k \in \mathbf{N} : A_k \hat{q} U\}$ ensures that $\{k \in \mathbf{N} : A_k \hat{q} U\} \in \mathcal{F}$. So $x_\lambda \in \lim(\mathcal{F},\mathcal{R})$ and hence X is $\mathcal{F}$-pseudocompact. □

4. Good extensions and examples

Every new notion of fuzzy topological property demands that it should be a "good extension" of similar concept of general topology. Now we shall establish that fuzzy $\mathcal{F}$-compactness, fuzzy $\mathcal{F}$-pseudocompactness and fuzzy densely $\mathcal{F}$-compactness are "good extensions" of $\mathcal{F}$-compactness [2], $\mathcal{F}$-pseudocompactness [11] and densely $\mathcal{F}$-compactness [11] of general topology.

Theorem 4.1. *The fuzzy topological space $(X, \omega(\tau))$ is fuzzy $\mathcal{F}$-compact if and only if (X, τ) is $\mathcal{F}$-compact.*

Proof. Let $(X, \omega(\tau))$ be fuzzy $\mathcal{F}$-compact. Let $\{x^k : k \in \mathbf{N}\}$ be any sequence in X. Then $\mathcal{S} = \{x^k_{\frac{1}{2}} : k \in \mathbf{N}\}$ is a fuzzy sequence in X. Since X is $(X, \omega(\tau))$ fuzzy $\mathcal{F}$-compact, there exists $x_\lambda \in \lim(\mathcal{F}, \mathcal{S})$. We shall show that x is $\mathcal{F}$-limit of $\{x^k : k \in \mathbf{N}\}$. Let U be an open set containing x. Then χ_U, the characteristic function of U is fuzzy open quasi-neighborhood of x_λ. Then $\{k \in \mathbf{N} : x^k_{\frac{1}{2}} \hat{q} \chi_U\} \in \mathcal{F}$. Therefore the inclusion $\{k \in \mathbf{N} : x^k_{\frac{1}{2}} \hat{q} \chi_U\} \subset \{k \in \mathbf{N} : x^k \in U\}$ ensures that $\{k \in \mathbf{N} : x^k \in U\} \in \mathcal{F}$. So x is $\mathcal{F}$-limit of $\{x^k : k \in \mathbf{N}\}$ and hence X is $\mathcal{F}$-compact.

Conversely, let X be $\mathcal{F}$-compact. Let $\mathcal{S} = \{x^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\}$ be any fuzzy sequence in X with the property that there exists $\epsilon > 0$ such that for any $k \in \mathbf{N}$, $x^k_{\lambda_k} \not\leq \underline{\epsilon}$. Then $\{x^k : k \in \mathbf{N}\}$ is a sequence in X. Since X is $\mathcal{F}$-compact, there exists an $x \in X$ such that $\{x^k : k \in \mathbf{N}\}$ $\mathcal{F}$-converges to x. We claim that $x_1 \in \lim(\mathcal{F}, \mathcal{S})$. If possible, let there exists $\rho \in \mathcal{Q}(X, x_1)$ such that $M = \{k \in \mathbf{N} : x^k_{\lambda_k} \bar{q} \rho\} \in \mathcal{F}$. Let $\mu = \inf\{\lambda_k : k \in M\}$. Since $\rho \in I^X$ is continuous, $U = \{y \in X : \rho(y) > 1 - \mu\}$ is open in X containing x. It is obvious to verify that $M \subset \{k \in \mathbf{N} : x^k \notin U\}$. So $\{k \in \mathbf{N} : x^k \in U\} \notin \mathcal{F}$, which contradicts the fact that X is $\mathcal{F}$-compact. Thus $x_1 \in \lim(\mathcal{F}, \mathcal{S})$ and so X is fuzzy $\mathcal{F}$-compact. □

Now we shall show that weakly fuzzy countably compact spaces need not be fuzzy $\mathcal{F}$-compact space and fuzzy $\mathcal{F}$-compact spaces need not be weakly fuzzy compact.

Example 4.2. (i) Let X_1 and X_2 be two subsets of $\beta\mathbf{N}$, the Stone-Čech compactification of the space of natural numbers $\mathbf{N}$ with usual topology such that $\mathbf{N} \subset X_1 \subset \beta\mathbf{N}$, $\mathbf{N} \subset X_2 \subset \beta\mathbf{N}$ and $X_1 \cap X_2 = \mathbf{N}$ equipped with subspace topologies τ_{X_1} and τ_{X_2}. Then J. Novák [15] and H. Terasaka [20] showed that $X = X_1 \times X_2$ with product topology is not countably compact. Let Z be the disjoint union of X_1 and X_2. Then (Z, τ) is a countably compact space and so $(Z, \omega(\tau))$ is fuzzy weakly countably compact, but $Z \times Z$ with product topology is not countably compact and so not $\mathcal{F}$-compact. Since arbitrary product of $\mathcal{F}$-compact spaces is $\mathcal{F}$-compact space, (Z, τ) is not $\mathcal{F}$-compact. Now Theorem 4.1 implies that $(Z, \omega(\tau))$ is not fuzzy $\mathcal{F}$-compact.
(ii) Let (X, τ) be the space of all ordinals less than the first uncountable ordinal equipped with order topology. Then Bernstein [2] showed that (X, τ) is not compact, but $\mathcal{F}$-compact. Then by Theorem 4.1 in [12] and Theorem 4.1, we can say that $(X, \omega(\tau))$ is not weakly fuzzy compact, but fuzzy $\mathcal{F}$-compact.

Theorem 4.3. *The fuzzy topological space $(X, \omega(\tau))$ is fuzzy $\mathcal{F}$-pseudocompact if and only if (X, τ) is $\mathcal{F}$-pseudocompact.*

Proof. Let $(X, \omega(\tau))$ be fuzzy $\mathcal{F}$-pseudocompact. Let $\mathcal{R} = \{A_k : k \in \mathbf{N}\}$ be any sequence of open subsets of (X, τ). Then $\chi_{\mathcal{R}} = \{\chi_{A_k} : k \in \mathbf{N}\}$ is a fuzzy sequence of fuzzy open sets of $(X, \omega(\tau))$. The fuzzy $\mathcal{F}$-pseudocompactness of $(X, \omega(\tau))$ ensures the existence of a fuzzy $\mathcal{F}$-limit $x_\lambda \in Pt(X)$ of $\chi_{\mathcal{R}} = \{\chi_{A_k} : k \in \mathbf{N}\}$. It is easy to verify that x is a $\mathcal{F}$-limit of $\mathcal{R} = \{A_k : k \in \mathbf{N}\}$. Hence (X, τ) is $\mathcal{F}$-pseudocompact.

Conversely, let $(X, \omega(\tau))$ be not fuzzy $\mathcal{F}$-pseudocompact and let $\mathcal{R} = \{A_k : k \in \mathbf{N}\}$ be a fuzzy sequence of fuzzy open sets of X with the property that there exists $\epsilon > 0$ such that for any $k \in \mathbf{N}$, $A_k \not\leq \underline{\epsilon}$ such that $\lim(\mathcal{F}, \mathcal{R}) = \underline{0}$. Then for each $x \in X$ and for each $x_\lambda \in Pt(X)$, there exists an $\mu \in \mathcal{Q}(X, x_\lambda)$ such that $\{k \in \mathbf{N} : A_k \bar{q} \mu\} \in \mathcal{F}$. The continuity of μ ensures that $U = \{y \in X : \mu(y) > 1 - \lambda\}$ is an open set of X containing x. Again $U_k = \{y \in X : A_k(y) > \lambda\}$ is an open set of X for each $k \in \mathbf{N}$. Now the inclusion $\{k \in \mathbf{N} : A_k \bar{q} \mu\} \subset \{k \in \mathbf{N} : U_k \cap U = \varnothing\}$ ensures that $\{k \in \mathbf{N} : U_k \cap U = \varnothing\} \in \mathcal{F}$. Then (X, τ) is not $\mathcal{F}$-pseudocompact. □

In analogous way, the following theorem can be established.

Theorem 4.4. *The fuzzy topological space $(X, \omega(\tau))$ is fuzzy pseudocompact if and only if (X, τ) is pseudocompact.*

Now we shall show that fuzzy pseudocompact spaces need not be weakly fuzzy countably compact and $\mathcal{F}$-pseudocompact spaces need not be fuzzy weakly fuzzy countably compact.

Example 4.5. Let (X, τ) be a pseudocompact topological space that is not countably compact. For example, let $X = [0, \Omega] \times [0, \omega] - (\omega, \Omega)$. Since weakly fuzzy countably compactness is a good extension of countably compactness [1], $(X, \omega(\tau))$ is not weakly fuzzy countably compact space. Again by Theorem 4.4, $(X, \omega(\tau))$ is fuzzy pseudocompact.

Theorem 4.6. *The fuzzy topological space $(X, \omega(\tau))$ is fuzzy densely $\mathcal{F}$-compact if and only if (X, τ) is densely $\mathcal{F}$-pseudocompact.*

Proof. Let $(X, \omega(\tau))$ be fuzzy densely $\mathcal{F}$-compact. Then there exists a fuzzy dense set $A \in I^X$ such that for every fuzzy sequence $\mathcal{S} = \{x^k_{\lambda_k} : k \in \mathbf{N}\}$ quasi-coincident with A and with the property that there exists $\epsilon > 0$ such that for any $k \in \mathbf{N}$, $x^k_{\lambda_k} \not\leq \underline{\epsilon}$, $\lim(\mathcal{F}, \mathcal{S}) \neq \underline{0}$. Consider the set $G = \{x \in X : A(x) > 0\}$. We claim that G is a dense subset of topological space (X, τ). Let $x \in X$ and an open set U containing x such that $U \cap G = \varnothing$. Then χ_U, the characteristic function of U is fuzzy open quasi-neighborhood of x_λ for any $\lambda \in (0, 1]$ with $\chi_U \bar{q} A$, which is not possible. So, our claim is affirmative. Now consider $\{x^k : k \in \mathbf{N}\}$ be any sequence in G. Then $\{x^k_1 : k \in \mathbf{N}\}$ is a fuzzy sequence quasi-coincident with A. Since $(X, \omega(\tau))$ is fuzzy densely $\mathcal{F}$-compact, there exists a fuzzy point $x_\lambda \in \lim(\mathcal{F}, \mathcal{S})$. We shall show that $\{x^k : k \in \mathbf{N}\}$ $\mathcal{F}$-converges to x. Let V be an open set containing x. Then χ_V, the characteristic function of V is fuzzy open quasi-neighborhood of x_λ. Then $\{k \in \mathbf{N} : x^k_1 \hat{q} \chi_V\} \in \mathcal{F}$. Therefore the inclusion $\{k \in \mathbf{N} : x^k_1 \hat{q} \chi_V\} \subset \{k \in \mathbf{N} : x^k \in V\}$ ensures that $\{k \in \mathbf{N} : x^k \in V\} \in \mathcal{F}$. So x is $\mathcal{F}$-limit of $\{x^k : k \in \mathbf{N}\}$ and hence X is densely $\mathcal{F}$-compact.

Conversely, let (X, τ) be densely $\mathcal{F}$-compact. Let A be a dense subset of (X, τ). Then we shall show that its characteristic function χ_A is a fuzzy dense subset of $(X, \omega(\tau))$. Let $x_\lambda \in Pt(X)$ and $\mu \in \mathcal{Q}(X, x_\lambda)$. The continuity of μ ensures that $U = \{y \in X : \mu(y) > 1 - \lambda\}$ is open in X containing x. Since A is dense in X, $A \cap U \neq \varnothing$. Then there exist $y \in X$ such that $\chi_A(y) + \mu(y) > 1 + (1 - \lambda) > 1$ and so $\chi_A \bar{q} \mu$. Let $\mathcal{S} = \{x^k_{\lambda_k} \in Pt(X) : k \in \mathbf{N}\}$ be any fuzzy sequence quasi-coincident with χ_A and with the property that there exists $\epsilon > 0$ such that for any $k \in \mathbf{N}$, $x^k_{\lambda_k} \not\leq \underline{\epsilon}$. Then $\{x^k : k \in \mathbf{N}\}$ is a sequence in A. Then there exists an $x \in X$ such that $\{x^k : k \in \mathbf{N}\}$ $\mathcal{F}$-converges to x. We claim that $x_1 \in \lim(\mathcal{F}, \mathcal{S})$. If possible, let there exists $\rho \in \mathcal{Q}(X, x_1)$ such that $M = \{k \in \mathbf{N} : x^k_{\lambda_k} \bar{q} \rho\} \in \mathcal{F}$. Let $\mu = \inf\{\lambda_k : k \in M\}$. Since $\rho \in I^X$ is continuous, $U = \{y \in X : \rho(y) > 1 - \mu\}$ is open in X containing x. It is obvious to verify that $M \subset \{k \in \mathbf{N} : x^k \notin U\}$. So $\{k \in \mathbf{N} : x^k \in U\} \notin \mathcal{F}$, which contradicts the fact that X is densely $\mathcal{F}$-compact. Thus $x_1 \in \lim(\mathcal{F}, \mathcal{S})$ and so X is fuzzy densely $\mathcal{F}$-compact. □

In Theorem 3.16, we have seen that every fuzzy densely $\mathcal{F}$-compact is fuzzy $\mathcal{F}$-pseudocompact. Now applying Theorem 4.1 and Theorem 4.6, we can establish the following example that shows that fuzzy$\mathcal{F}$-pseudocompact spaces need not be fuzzy densely $\mathcal{F}$-compact.

Example 4.7. We recall the concept of types of $\beta\mathbf{N} - \mathbf{N}$. It is clear that the relation $\rho = \{(x, y) \in (\beta\mathbf{N} - \mathbf{N}) \times (\beta\mathbf{N} - \mathbf{N}) : \psi(x) = y$ for some homeomorphism $\psi : \beta\mathbf{N} \to \beta\mathbf{N}\}$ is an equivalent relation on $\beta\mathbf{N} - \mathbf{N}$. The equivalent classes are called types of $\beta\mathbf{N} - \mathbf{N}$. The type of $x \in \beta\mathbf{N} - \mathbf{N}$ is denoted by $T(x)$. In [11], Ginsburg and Saks showed that for any non-P-point q of $\beta\mathbf{N} - \mathbf{N}$, $(T(q), \tau)$ is $\mathcal{F}$-pseudocompact, but not densely $\mathcal{F}$-compact. Then by Theorem 4.1 and Theorem 4.6, we conclude that $(T(q), \omega(\tau))$ is fuzzy $\mathcal{F}$-pseudocompact (and so fuzzy pseudocompact), but not fuzzy densely $\mathcal{F}$-compact.

5. Conclusions

In [1], Afsan introduced the notion of weakly fuzzy countably compact space and derived several sufficient conditions under which product of two weakly fuzzy countably compact spaces is weakly fuzzy countably compact space. In this paper we initiated and investigated some new fuzzy covering axioms related to weakly fuzzy countably compact spaces which are productive. We have introduced the the class of fuzzy $\mathcal{F}$-compact space in fuzzy topological spaces which are properly contained in the class of weakly fuzzy countably compact spaces. In fact, we have fuzzified the classical notion $\mathcal{F}$-compactness due to Bernstein [2] of general topological space. Theorem 3.3 and Definition 3.6 ensure that every fuzzy $\mathcal{F}$-compact space is weakly fuzzy countably compact space, but Example 4.2(i) has established that the converse need not be true. Theorem 3.9 shows that fuzzy $\mathcal{F}$-compact spaces are arbitrarily productive and are preserved under fuzzy continuous mappings. Theorem 4.1 gives a bridge result between fuzzy $\mathcal{F}$-compactness and $\mathcal{F}$-compactness [2] of general general topology and hence fuzzy $\mathcal{F}$-compact space is a good extension of $\mathcal{F}$-compactness of Bernstein fuzzy [2]. We have also introduced and investigated some other fuzzy covering axioms related to fuzzy $\mathcal{F}$-compact spaces which are arbitrarily productive.

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