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Feeble compactness of intuitionistic fell topological space

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Abstract. The purpose of this paper is to introduce the concepts of an intuitionistic fell topology and to consider the relations between intuitionistic Fell topology and intuitionistic topological convergence. The main aim of this paper is to introduce the concepts of an intuitionistic regular space and intuitionistic feebly compactness and some interesting properties are discussed.

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1. Introduction

The concept of an intuitionistic set was introduced by D. Coker in [2]. In this paper, we characterize some properties of the Fell topology on 2^X. For a space X, let 2^X be the collection of all intuitionistic closed sets of X. The purpose of this paper is to introduce the concepts of an intuitionistic fell topology and to consider the relations between intuitionistic Fell topology and intuitionistic topological convergence. The main aim of this paper is to introduce the concepts of an intuitionistic regular space and intuitionistic feebly compactness and some interesting properties are discussed.

2. Preliminaries

Definition 2.1 ([1])**.** Let X be a nonempty fixed set.An intuitionistic set(IS for short) A is an object having the form $A = \langle x, A^1, A^2\rangle$ for all $x \in X$, where A^1 and A^2 are subsets of X satisfying $A^1 \cap A^2 = \phi$. The set A^1 is called the set of members of A, while A^2 is called the set of nonmembers of A.

Every crisp set A on a non-empty set X is obviously an intuitionistic set having the form $\langle x, A, A^c \rangle$.

Definition 2.2 ([1])**.** Let X be a nonempty set, $A = \langle x, A^1, A^2 \rangle$ for all $x \in X$, $B = \langle x, B^1, B^2 \rangle$ for all $x \in X$ be intuitionistic sets on X, and let $\{A_i : i \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_i = \langle x, A_i^1, A_i^2 \rangle$ for all $x \in X$.

(i) $A \subseteq B$ if and only if $A_1 \subseteq B_1$ and $B^2 \subseteq A^2$.
(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
(iii) $\overline{A} = \langle X, A^2, A^1 \rangle$.
(iv) $\cup A_i = \langle x, \cup A_i^1, \cap A_i^2 \rangle$.
(v) $\cap A_i = \langle x, \cap A_i^1, \cup A_i^2 \rangle$.
(vi) $A - B = A \cap \overline{B}$.
(vii) $\phi_\sim = \langle x, \phi, X \rangle$ and $X_\sim = \langle x, X, \phi \rangle$.

Definition 2.3 ([1])**.** Let X and Y be two nonempty sets and $f : X \to Y$ be a function

(i) If $B = \langle y, B^1, B^2 \rangle$ for all $y \in X$ is an intuitionistic set in Y,then the *preimage* of B under f,denoted by $f^{-1}(B)$,is an intuitionistic set in X defined by $f^{-1}(B) = \langle x, f^{-1}(B_1), f^{-1}(B_2) \rangle$.
(ii) If $A = \langle x, A^1, A^2 \rangle$ for all $x \in X$ is an intuitionistic set in X,then the *image* of A under f,denoted by $f(A)$,is the intuitionistic set in Y defined by $f(A) = \langle y, f(A^1), f_-(A^2) \rangle$ where $f_-(A^2) = Y - (f(X - A^2))$.

Definition 2.4 ([1])**.** An *intuitionistic topology* (IT for short) on a nonempty set X is a family τ of intuitionistic sets in X satisfying the following axioms:

(i) $\phi_\sim$ and $X_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the ordered pair (X, τ) is called an intuitionistic topological space (ITS for short) and any intuitionistic set in τ is known as an intuitionistic open set (IOS for short) in X. The complement $\overline{A}$ of an intuitionistic open set A is called an intuitionistic closed set (ICS for short) in X.

Definition 2.5 ([1])**.** Let (X, τ) be an intuitionistic topological space and $A = \langle x, A^1, A^2 \rangle$ be an intuitionistic set in X.Then the intuitionistic interior and intuitionistic closure of A are defined by

$Icl(A) = \cap\{K : K$ is an intuitionistic closed set in X and $A \subseteq K\}$,
$Iint(A) = \cup\{G : G$ is an intuitionistic open set in X and $G \subseteq A\}$.

It can be also show that $Icl(A)$ is an intuitionistic closed set and $Iint(A)$ is an intuitionistic open set in X and A is an intuitionistic closed set in X if $Icl(A) = A$ and A is an intuitionistic open set in X if $Iint(A) = A$.

Definition 2.6 ([1])**.** Let (X, τ) be an intuitionistic topological space.

(i) If a family $\{\langle x, G_i^1, G_i^2 \rangle : i \in J\}$ of IOS's in X satisfies the condition $\cup\{\langle x, G_i^1, G_i^2 \rangle : i \in J\} = X_\sim$, then it is called an intuitionistic open cover of X. A finite subfamily of an intuitionistic open cover $\{\langle x, G_i^1, G_i^2 \rangle : i \in J\}$ of X, which is also an intuitionistic open cover of X, is called a finite intuitionistic subcover of $\{\langle x, G_i^1, G_i^2 \rangle : i \in J\}$.

(ii) A family $\{\langle x, K_i^1, K_i^2\rangle : i \in J\}$ of ICS's in X satisfies the finite intersection property (FIP for short) if and only if every finite subfamily $\{\langle x, K_i^1, K_i^2\rangle : i = 1, 2, ...n\}$ of the family satisfies the condition $\cap\langle x, {K_i}^1, {K_i}^2\rangle \neq \phi_\sim$.

Definition 2.7 ([1])**.** An intuitionistic topological space (X, τ) is called intuitionistic compact if and only if each intuitionistic open cover of X has a finite intuitionistic subcover.

Definition 2.8 ([1])**.** Let (X, τ) be an intuitionistic topological space.

(i) If a family $\{\langle x, G_i^1, G_i^2\rangle : i \in J\}$ of IOS's in X satisfies the condition $A \subseteq \cup\{\langle x, G_i^1, G_i^2\rangle : i \in J\}$, then it is called an intuitionistic open cover of A. A finite subfamily of an intuitionistic open cover $\{\langle x, G_i^1, G_i^2\rangle : i \in J\}$ of A, which is also an intuitionistic open cover of A, is called a finite intuitionistic subcover of $\{\langle x, G_i^1, G_i^2\rangle : i \in J\}$.
(ii) An intuitionistic set $A = \langle x, A^1, A^2\rangle$ in an intuitionistic topological space (X, τ) is called intuitionistic compact if and only if each intuitionistic open cover of A has a finite intuitionistic subcover.

Definition 2.9 ([7])**.** A uniform space X with uniformity ξ is a set with a nonempty collection ξ of subsets containing the diagonal Δ_x in XxX satisfying the following properties:

(i) If $E, F \in \xi$, then $E \cap F \in \xi$.
(ii) If $F \subset E$ and $E \in \xi$, then $F \in \xi$.
(iii) If $E \in \xi$, then $E^t = \{(x, y) : (y, x) \in E\} \in \xi$.
(iv) For any $E \in \xi$ there is some $F \in \xi$ such that $F.F \subset E$.

Definition 2.10 ([8])**.** Let (X, τ) be an intuitionistic topological space. Let $x \in X$ and an intuitionistic set $A = \langle x, A^1, A^2\rangle$ of X is called an intuitionistic neighborhood of x if there exists an intuitionistic open set $B = \langle x, B^1, B^2\rangle$ in X such that $x \in B \subseteq A$.

Definition 2.11 ([3])**.** Let X be a topological space and let 2^X be the family of the closed subsets of X. For a subset A of X, we associate the following subsets of 2^X : $A^- = \{F \in 2^X : F \cap A \neq \phi\}$ and $A^+ = \{F \in 2^X : F \subseteq A\}$.

Definition 2.12 ([3])**.** Let X be a topological space. The Fell topology (TF for short) on 2^X has a subbase all sets of the form V^- and the form $(K^c)^+$ where V is an open subset of X, K is a compact subset of X and K^c means the complement of K.

Definition 2.13 ([5])**.** Let X be a topological space. The Vietoris topology (TV for short) on 2^X has a subbase all sets of the form V^- and the form $(K^c)^+$ where V is an open subset of X, K is a closed subset of X and K^c means the complement of K.

Definition 2.14 ([3])**.** A space X is called hemi-compact if, there exists a countable cover of compact sets of X such that any compact subset of X is contained in some element of the cover.

Definition 2.15 ([3])**.** A space X is said to be locally compact if for any $x \in X$ and any open neighborhood V of x there is a compact neighborhood G of x such that $G \subseteq V$.

Definition 2.16 ([7])**.** A space X is said to have a countable basis at the point x if there is a countable collection $\{U_n\}_{n\in\mathbb{Z}_+}$ of neighborhoods of x such that any neighborhood U of x contains atleast one of the sets U_n. A space X that has a countable basis at each of its points is said to satisfy the first countability axiom.

Definition 2.17 ([2])**.** Let (X,τ) be an intuitionistic topological space on X.

(a) A family $\beta \subseteq \tau$ is called an intuitionistic base for (X,τ) if and only if each member of τ can be written as a union of elements of β.
(b) A family $\gamma \subseteq \tau$ is called an intuitionistic subbase for (X,τ) if and only if the family of finite intersection of elements in γ forms a base for (X,τ). In this case the intuitionistic topology τ is said to be generated by γ.

Definition 2.18 ([6])**.** A topological space X is called a regular space if for each pair consisting of a point x and a closed set B disjoint from x, there exist open sets U and V containing x and B, respectively that are disjoint.

Definition 2.19 ([6])**.** If A is a subset of the topological space X and if x is a point of X, then x is a cluster point of A if every neighborhood of x intersects A in some point other than x itself.

3. Properties of nntuitionistic Fell topology

Definition 3.1. Let (X,τ) be an intuituionistic topological space and let 2^X be the family of the intuitionistic closed sets of (X,τ). For an intuitionistic set $A = \langle x, A^1, A^2\rangle$ of (X,τ), we associate the following sets of 2^X : $A^- = \{F \in 2^X : F\cap A \neq \phi_\sim\}$ and $A^+ = \{F \in 2^X : F \subseteq A\}$, where $F = \langle x, F^1, F^2\rangle$.

Example 3.1. Let $X = \{a,b,c\}$. Let $P = \langle\{a\},\{b,c\}\rangle, Q = \langle\{b\},\{a,c\}\rangle, R = \langle\{a,b\},\{c\}\rangle$ and define $\tau = \{X_\sim, \phi_\sim, P, Q, R\}$. Let $A = \langle\{c\},\{a,b\}\rangle$. Then

$$A^- = \{F \in 2^X : F\cap A \neq \phi_\sim\} = \{X_\sim, \phi_\sim, \overline{P}, \overline{Q}, \overline{R}\} \text{ and}$$
$$A^+ = \{F \in 2^X : F \subseteq A\} = \{\phi_\sim, \overline{R}\}$$

Definition 3.2. Let X be a nonempty set. A family $\mathcal{V}$ of intuitionistic G_δ sets in X is said to be intuitionistic Volterra structure on X if it satisfies the following axioms:

(i) $\phi_\sim,\ X_\sim \in \mathcal{V}$,
(ii) $G_1\cap G_2 \in \mathcal{V}$ for any $G_1, G_2 \in \mathcal{V}$,
(iii) $\cup G_i \in \mathcal{V}$ for arbitrary family $\{G_i \mid i \in Z\} \subseteq \mathcal{V}$,
(iv) $Icl(G_i) = X_\sim, i \in Z$ implies $Icl(G_j \cap G_k) = X_\sim$ for any $j,k \in \rho$.

Then the ordered pair $(X,\mathcal{V})$ is called an intuitionistic volterra structure space. Every member of $\mathcal{V}$ is called an intuitionistic volterra open set in $(X,\mathcal{V})$. The complement of an intuitionistic volterra open set is called an intuitionistic volterra closed set in $(X,\mathcal{V})$.

Definition 3.3. Let (X,τ) be an intuitionistic topological space. An intuitionistic fell topology ITF on 2^X has an intuitionistic subbase all intuitionistic sets of the form V^- and the form $(\overline{K})^+$ where $V = \langle x, V^1, V^2\rangle$ is an intuitionistic open set of (X,τ), $K = \langle x, K^1, K^2\rangle$ is an intuitionistic compact set of (X,τ) and $\overline{K} = \langle x, K^2, K^1\rangle$ means the complement of K.

Definition 3.4. Let (X, τ) be an intuitionistic topological space. An intuitionistic Vietoris topology ITV on 2^X has an intuitionistic subbase all intuitionistic sets of the form V^- and the form $(\overline{K})^+$ where $V = \langle x, V^1, V^2 \rangle$ is an intuitionistic open set of (X, τ), $K = \langle x, K^1, K^2 \rangle$ is an intuitionistic closed set of (X, τ) and $\overline{K} = \langle x, K^2, K^1 \rangle$ means the complement of K.

Definition 3.5. An intuitionistic topological space (X, τ) is called intuitionistic hemi-compact if, there exists a countable cover of intuitionistic compact sets of (X, τ) such that any intuitionistic compact set of (X, τ) is contained in some element of an intuitionistic cover.

Definition 3.6. An intuitionistic topological space (X, τ) is said to be intuitionistic locally compact if for any $x \in X$ and any intuitionistic open neighborhood $V = \langle x, V^1, V^2 \rangle$ of (X, τ) there is an intuitionistic compact neighborhood $G = \langle x, G^1, G^2 \rangle$ of (X, τ) such that $G \subseteq V$.

Notation 3.1. Let (X, τ) be an intuitionistic topological space. Let $U = \langle x, U^1, U^2 \rangle$ and $V = \langle x, V^1, V^2 \rangle$ be any two intuitionistic sets in (X, τ). U and V are said to be disjoint if $U \cap V = \phi_\sim$

Definition 3.7. An intuitionistic topological space (X, τ) is called intuitionistic Hausdorff space if for each pair X_1, X_2 of distinct points of (X, τ), there exist intuitionistic neighborhoods $U = \langle x, U^1, U^2 \rangle$ and $V = \langle x, V^1, V^2 \rangle$ of X_1 and X_2, respectively, that are disjoint.

Definition 3.8. If A is an intuitionistic set of an intuitionistic topological space (X, τ) and if x is a point of (X, τ), then x is an *intuitionistic cluster point* of A if every intuitionistic neighborhood of x intersects A in some point other than x itself.

Notation 3.2. $\overline{V}^X = \langle x, (V^2)^X, (V^1)^X \rangle$ denotes the complement of V in (X, τ).

Proposition 3.9. *Let (X, τ) be an intuitionistic topological space. Then*

(a) $\{E \in 2^X : E \subset A\}$ *is intuitionistic closed in* $(2^X, ITF)$ *if* $A \subset X$ *is intuitionistic closed.*

(b) $\{E \in 2^X : E \cap A \notin \phi\}$ *is intuitionistic closed in* $(2^X, ITF)$ *if* $A \subset X$ *is intuitionistic compact and* (X, τ) *is an intuitionistic Hausdorff space, where* $E = \langle x, E^1, E^2 \rangle$ *and* $A = \langle x, A^1, A^2 \rangle$

Proof. The proof is simple. □

Proposition 3.10. *For a function $f : (X_1, \mathcal{V}_1) \to (X_2, \mathcal{V}_2)$ the following statements are equivalent:*

(i) *f is slightly intuitionistic volterra t-continuous function.*

(ii) *inverse image of every intuitionistic volterra clopen set of X_2 is intuitionistic volterra t-open set of X_1.*

(iii) *inverse image of every intuitionistic volterra clopen set of X_2 is intuitionistic volterra t-clopen set of X_1.*

Proof. (a) Let $\mathcal{A} = \{E \in 2^X : E \subset A\}$ and let $B = \langle x, B^1, B^2 \rangle$ be an intuitionistic cluster point of $\mathcal{A}$. Suppose $B \notin \mathcal{A}$. Let $b \in \overline{A}^B$. Since A is intuitionistic closed and $b \notin A$, there is an intuitionistic neighborhood $V = \langle x, V^1, V^2 \rangle$ of b such that $V \cap A = \phi$. Then $B \in V^-$ and $V^- \cap A = \phi$ which contradicts the fact that B ia an intuitionistic cluster point of $\mathcal{A}$.

(b) Let $\mathcal{A} = \{E \in 2^X : E \cap A \notin \phi\}$. Suppose there is an element $B \in 2^X$ which is an intuitionistic cluster point of $\mathcal{A}$ but $B \notin \mathcal{A}$. Then there is an intuitionistic open set V containing B such that $V \cap A = \phi$. Then $(A^c)^+$ is an intuitionistic neighborhood of B which does not meet $\mathcal{A}$, a contradiction. □

Proposition 3.11. *Let (X, τ) be an intuitionistic topological space.If $(CL(X), ITF)$ is an intuitionistic Hausdorff space then (X, τ) is an intuitionistic locally compact space.*

Proof. If (X, τ) is singleton, there is nothing to prove.We assume that (X, τ) contains atleast two points. Take $x \in X$ and an intuitionistic open set $V = \langle x, V^1, V^2 \rangle$ of (X, τ) such that $x \in V$. We claim that V is an intuitionistic compact set, that is every intuitionistic neighborhood of (X, τ) contains all points of (X, τ). Suppose that there exists an intuitionistic open neighborhood $U = \langle x, U^1, U^2 \rangle$ of (X, τ) and $y \in X$ such that $y \notin U$ that is $x \notin cl\{y\}$. Then without loss of generality we may assume that $V \neq X$. Let $E = \overline{V}^X$ and $F = (\overline{V}^X) \cup cl\{x\}$. Then E and F are two different elements of $ICL(X)$. Since $ICL(X)$ is intuitionistic Hausdorff, there exists intuitionistic open sets $U_1 = \langle x, U^1{}_1, U^1{}_2 \rangle, U_2 = \langle x, U^2{}_1, U^2{}_2 \rangle, ...U_n = \langle x, U^n{}_1, U^n{}_2 \rangle, W_1 = \langle x, W^1{}_1, W^1{}_2 \rangle, W_2 = \langle x, W^2{}_1, W^2{}_2 \rangle, ...W_m = \langle x, W^m{}_1, W^m{}_2 \rangle$ of (X, τ) and intuitionistic compact sets $K_1 = \langle x, K^1{}_1, K^1{}_2 \rangle$ and $K_2 = \langle x, K^2{}_1, K^2{}_2 \rangle$ such that

(a) $E \in \bigcap_{i=1}^{n} U_i^- \cap (\overline{K_1})^+$,

(b) $F \in \bigcap_{j=1}^{m} W_j^- \cap (\overline{K_2})^+$,

(c) $(\bigcap_{i=1}^{n} U_i^- \cap (\overline{K_1})^+) \cap (\bigcap_{j=1}^{m} W_j^- \cap (\overline{K_2})^+) = \phi$.

Obviously, $K_1 \cup K_2 \subseteq V$. From (a),(b) and (c), it follows that there is some W_j which meets $cl\{x\}$. Let $G = \langle x, G^1, G^2 \rangle$ be the intersection of all W_j's which meet $cl\{x\}$. Since W_j is intuitionistic open, we have $x \in W_j$. Therefore G is an intuitionistic open neighborhood of (X, τ). Let $W = V \cap G$. If there exists $y \in W$ such that $cl\{y\} \cap (K_1 \cup K_2) = \phi$, then $E \cup cl\{y\} \in (\bigcap_{i=1}^{n} U_i^- \cap (\overline{K_1})^+) \cap (\bigcap_{j=1}^{m} W_j^- \cap (\overline{K_2})^+)$ which contradicts (a). Therefore we have $cl\{y\} \cap (K_1 \cup K_2) \neq \phi$ for every $y \in W$. Now let $K = W \cup K_1 \cup K_2$. By proposition 3.1, K is an intuitionistic compact set of (X, τ). □

Proposition 3.12. *Let (X, τ) be an intuitionistic T_1-space. If $(CL(X), ITF)$ is first-countable, then every intuitionistic open set of (X, τ) is intuitionistic hemicompact.*

Proof. Let V be a proper intuitionistic open set of (X, τ). Let $A = \overline{V}^X$, where $V = \langle x, V^1, V^2 \rangle$ and let $W = \{\alpha_i^- \cap (\overline{K_i})^+ \mid i \in \mathbf{N}\}$ be an intuitionistic base of $(CL(X), ITF)$ at this point A, where each α_i is a finite family of nonempty intuitionistic open sets of (X, τ) and each $K_i = \langle x, K^1{}_i, K^2{}_i \rangle$ is an intuitionistic compact set of (X,τ). Now we claim that $\mathbf{K}_v = \{K_i \mid i \in \mathbf{N}\}$ is cofinal in $\mathbf{K}(V)$. This will be sufficient to conclude that V is intuitionistic hemicompact. Indeed, for each $\mathbf{K} \in \mathbf{K}(V)$, we have $K \cap A = \phi$ (that is $A \in (\overline{K})^+$). Therefore, there exists $i \in \mathbf{N}$ such that $A \in \alpha_i^- \cap (\overline{K_i})^+ \subseteq (\overline{K})^+$. If $\overline{K_i}^K = \phi$, then for $x \in \overline{K_i}^K$, we have $\{x\} \cup A \in \alpha_i^- \cap (\overline{K_i})^+$, but $\{x\} \cup A \notin (\overline{K})^+$, which is a contradiction. Therefore, $K \subseteq K_i$. □

Definition 3.13. An intuitionistic topological space (X,τ) is said to be an intuitionistic R_0 space, if every non-empty intuitionistic open set of (X,τ) contains the intuitionistic closure of each of its points.

Proposition 3.14. *Let (X,τ) be a first-countable intuitionistic $\boldsymbol{R}_0$-space. If each proper intuitionistic open set of (X,τ) is intuitionistic hemicompact, then (X,τ) is intuitionistic locally compact.*

Proof. Take an arbitrary $x \in X$. If every intuitionistic open neighborhood of (X,τ) contains all points of (X,τ), then there is nothing to prove. Now suppose there is an intuitionistic neighborhood $V_x = \langle x, V^1{}_x, V^2{}_x\rangle$ of (X,τ) and for some point y of (X,τ), $y \notin V_x$. Thus $x \notin cl\{y\}$. Let $U = \overline{cl\{y\}}^X$. Now we take an arbitrary intuitionistic open neighborhood $V = \langle x, V^1, V^2\rangle$ of (X,τ). Let $W = U \cap V$. Thus, $W = \langle x, W^1, W^2\rangle$ is a proper intuitionistic open set of (X,τ). Therefore, from the assumption, there is an increasing sequence $\{K_n \mid n \in \mathbf{N}\}$ of intuitionistic compact sets of W such that every intuitionistic compact set of W is contained in some $K_n = \langle x, V^1{}_n, V^2{}_n\rangle$. Again, from the assumptions, let $\mathbf{V} = \{V_n \mid n \in \mathbf{N}\}$ be an intuitionistic base of intuitionistic neighborhoods of (X,τ) such that $\cup\mathbf{V} = W$. We show that there exist $n, m \in \mathbf{N}$ such that $\mathbf{V}_n \subseteq \mathbf{K}_m$ which completes the proof. Let us assume the contradiction. Then for every $n \in \mathbf{N}$, we can choose $x_n \in \overline{K_n}^{V_n}$. It is obvious that this sequence $\{x_n | n \in \mathbf{N}\}$ intuitionistic converges to (X,τ). It follows that there is some $m \in \mathbf{N}$ such that $\{x_n | n \in \mathbf{N}\} \cup \{x\} \subseteq K_m$. Particularly, $x_m \in K_m$ which is a contradiction. □

Definition 3.15. An intuitionistic uniform space (X,τ) with intuitionistic uniformity ξ is an intuitionistic set (X,τ) with a nonempty collection ξ of intuitionistic sets containing an intuitionistic diagonal Δ_x in (X,τ) satisfying the following properties:

(i) If $E, F \in \xi$, then $E \cap F \in \xi$.
(ii) If $F \subset E$ and $E \in \xi$, then $F \in \xi$.
(iii) If $E \in \xi$, then $E^t = \{(x,y) : (y,x) \in E\} \in \xi$.
(iv) For any $E \in \xi$ there is some $F \in \xi$ such that $F.F \subset E$.

Definition 3.16. Let $(X,\mathbf{U})$ be an intuitionistic uniform space. The intuitionistic proximal Fell topology on 2^X has intuitionistic base all intuitionistic sets of the form V^-, where V is an intuitionistic open set of (X,τ) and of the form $(\overline{K})^{++} = \{A \in 2^X :$ there exists $U \in \mathbf{U}$ such that $UA \cap K = \phi_\sim\}$, where $K = \langle x, K^1, K^2\rangle$ is an intuitionistic compact set of (X,τ).

Proposition 3.17. *Let $(X, \boldsymbol{U})$ be an intuitionistic uniform space. Then $(\overline{K})^+ = (\overline{K})^{++}$ for an arbitrary compact subset $K = \langle x, K^1, K^2\rangle$ of (X,τ).*

Proof. It is obvious that $(\overline{K})^{++} \subseteq (\overline{K})^{+}$. Now take $A \in (\overline{K})^{+}$. Then $K \in \overline{A}^{X}$. Since $A = \langle x, A^1, A^2\rangle$ is intuitionistic closed, we have that if $x \in K$, then there is $V_x \in \mathbf{U}$ such that $x \in int(V_x(x)) \subseteq V_x \subseteq \overline{A}^{X}$. On the other hand, there is $U_x \in \mathbf{U}$ such that $U_x.U_x \subseteq V_x$ for all $x \in K$. Now we consider an intuitionistic set $\mathbf{Y} = \{U_x \mid x \in K\}$ and consider that $\mathbf{H} = \{int(U_x(x)) \mid x \in K\}$ is an intuitionistic open cover of K. Since K is intuitionistic compact, there is a finite intuitionistic set $\{x_i \mid i \leq n\}$ such that $K \subseteq \bigcup_{i=1}^{n} U_{x_i}(x_i)$. Let $U = \bigcap_{i=1}^{n} U_{x_i}$. We show that $U(A) \cap K = \phi$. Suppose assume the contradiction. Then we choose $y \in U(A) \cap K$. Thus, there is $a \in A$ such that $(y, a) \in U$, and there is some $i \leq n$ such that $y \in U_{x_i}(x_i)$. Hence, it follows that $(a, x_i) \in V_{x_i}$. Consequently, $a \in V_{x_i}(x_i) \subseteq X \setminus A$, a contradiction. □

4. Feeble compactness of an intuitionistic Fell topology

Definition 4.1. The family $\{F_\alpha : \alpha \in \Lambda\}$ of intuitionistic open set of an intuitionistic topological space (X, τ) is said to be intuitionistic locally finite if for each $x \in X$, there exists an intuitionistic open set V_x containing (X, τ) such that $\{\alpha \in \Lambda : V_x \cap F_\alpha \neq \phi\}$ is finite.

Definition 4.2. An intuitionistic topological space (X, τ) is called an intuitionistic feebly compact if every intuitionistic locally finite family of intuitionistic open sets of (X, τ) is finite.

Definition 4.3. An intuitionistic topological space (X, τ) is called an intuitionistic regular space if for each pair consisting of a point x and an intuitionistic closed set B disjoint from x, there exist disjoint intuitionistic open sets containing x and B, respectively.

Notation 4.1. $\{A_n : n \in \mathbb{N}\}$ denotes a sequence of intuitionistic sets of X and L_sA_n denotes the set of all points x such that every intuitionistic neighborhood of x intersects infinitely many A_n's.

Proposition 4.4. *(X, τ) is an intuitionistic feebly compact space if and only if $L_sG_n \neq \phi$ for every sequence $\{G_n : n \in \mathbb{N}\}$ of non-empty intuitionistic open sets of (X, τ).*

Proof. The proof is obvious. □

Proposition 4.5. *Let $\{A_n : n \in \mathbb{N}\}$ be a sequence of non-empty intuitionistic closed sets of an intuitionistic topological space (X, τ). If $L_sF_n \notin \phi$, then the complement of $\{F_n : n \in \mathbb{N}\}$ in $cl(\{F_n : n \in \mathbb{N}\})$ is non-empty.*

Proof. There are two possibilities:

(a) There is an intuitionistic compact set K such that for infinitely many n, $F_n \in K^-$;
(b) For every intuitionistic compact set K there is $n_0 \in \mathbb{N}$ such that $F_n \notin K^-$ for every $n \geq n_0$.

In (a) $\{F_n : n \in \mathbb{N}\}$ has an intuitionistic cluster point in $(CL(X), T_F)$.

In (b) $cl(\{a\})$ is an intuitionistic cluster point of $\{F_n : n \in \mathbb{N}\}$ for any $a \in L_sF_n$. □

Proposition 4.6. *Let (X, τ) be an intuitionistic regular space. If (X, τ) is not an intuitionistic locally compact, then $(CL(X), T_F)$ is intuitionistic feebly compact.*

Proof. Let $\{\mathcal{V}_n : n \in \mathbb{N}\}$ be a sequence of non-empty intuitionistic open sets of $(CL(X), T_F)$. We have to show that $L_s\mathcal{V}_n \neq \phi$ in $(CL(X), T_F)$. Without loss of generality, we may assume that for each $n \in \mathbb{N}$, $V_n = \cap_{i \leq m_n} V_i(n)^- \cap (K_n)^+$, where $V_i(n)$'s are intuitionistic open sets of (X, τ), K_n is an intuitionistic compact set of (X, τ) and $m_n \in \mathbb{N}$.

Now for any $n \in \mathbb{N}$, choose $F_n \in V_n$.

Case 1 $L_s F_n = \phi$.
Since (X, τ) is not an intuitionistic locally compact space, there exists $x \in X$ and an intuitionistic open neighborhood W of (X, τ) such that W does not contain any intuitionistic compact neighborhood of (X, τ). We show that $cl(x) \in L_s V_n$ in $(CL(X), T_F)$.

Let $\mathcal{V} = \cap_{i \leq n} V_i^- \cap (K^c)^+$ is an intuitionistic neighborhood of $cl(X)$ in $(cl(X), T_F)$, then $cl(x) \cap V_i \neq \phi$ for all $i \leq n$. Let $V = W \cap (\cap_{i \leq n} V_i)$. Then $x \in V$. Since $L_s F_n = \phi$, there exists $n_0 \in \mathbb{N}$ such that $n \geq n_0$ implies $F_n \cap K = \phi$. Fix $n \in \mathbb{N}$ with $n \geq n_0$. We show that there exists $x_n \in V$ such that $cl\{x_n\} \cap (K \cup K_n) = \phi$.

Suppose assume the contradiction. Then we have $cl\{y\} \cap (K \cup K_n) \neq \phi$ for every $y \in V$, hence it follows that $V \cup K \cup K_n$ is intuitionistic compact. Since (X, τ) is an intuitionistic regular space, there is an intuitionistic closed neighborhood G of (X, τ) such that $G \subseteq V \subseteq W \cap V \cup K \cup K_n$. Which is a contradiction. If $A_n = F_n \cup cl\{x_n\}$, then $A_n \in \mathcal{V}_n \cap \mathcal{V}$. This shows that $n \geq n_0$ implies $\mathcal{V} \cap \mathcal{V}_n \neq \phi$.

Case 2 $L_s F_n \neq \phi$.
By the proposition 4.2, we have $L_s \mathcal{V}_n \neq \phi$. □

Proposition 4.7. *If (X, τ) is not an intuitionistic σ-compact, then $(CL(X), T_F)$ is intuitionistic feebly compact.*

Proof. Let $\{\mathcal{V}_n : n \in \mathbb{N}\}$ be a sequence of non-empty intuitionistic open sets of $(CL(X), T_F)$. We have to show that $L_s\mathcal{V}_n \neq \phi$ in $(CL(X), T_F)$. Without loss of generality, we may assume that for each $n \in \mathbb{N}$, $V_n = \cap_{i \leq m_n} V_i(n)^- \cap (K_n)^+$, where $V_i(n)$'s are intuitionistic open sets of (X, τ), K_n is an intuitionistic compact set of (X, τ) and $m_n \in \mathbb{N}$.

Now for any $n \in \mathbb{N}$, choose $F_n \in V_n$.

Case 1 $L_s F_n = \phi$.
Since (X, τ) is not intuitionistic σ-compact, there exists $x \in X$ such that $cl\{x\} \cap (\cup\{K_n : n \in \mathbb{N}\}) = \phi$. Indeed, if such (X, τ) does not exist, then $X = \cup\{X_n : n \in \mathbb{N}\}$ where $X_n = \{x \in X : cl\{x\} \cap K_n \neq \phi\}$. Since K_n's are intuitionistic compact sets of (X, τ). Which is a contradiction to the fact that (X, τ) is not intuitionistic σ-compact. Let $E_n = F_n \cup cl\{x\}$ for each $n \in \mathbb{N}$. Obviously $E_n \in \mathcal{V}_n$ for every $n \in \mathbb{N}$.

We show that the sequence $\{E_n : n \in \mathbb{N}\}$ converges to $cl\{x\}$ which shows that $L_s\mathcal{V}_n \neq \phi$ in $(CL(X), T_F)$. If $\mathcal{V} = \cap_{i \leq n} V_i^- \cap (K^c)^+$ is an intuitionistic neighborhood of $cl(X)$ in $(cl(X), T_F)$, then $cl(x) \cap V_i \neq \phi$ for all $i \leq n$, hence $x \in \cap_{i \leq m} V_i$. Let $V = \cap_{i \leq m} V_i$. Obviously, $E_n \cap V \neq \phi$ for all $n \in \mathbb{N}$. If K is an intuitionistic compact set of (X, τ), then since $L_s F_n = \phi$, there exists $n_0 \in \mathbb{N}$ such that $n \geq n_0$ implies

$F_n \cap K = \phi$. Since $cl\{x_n\} \cap K = \phi$, we have, $E_n \cap K = \phi$, for all $n \geq n_0$. Thus, whenever $n \geq n_0$, we have, $E_n \in \mathcal{V}$.

Case 2 $L_s F_n \neq \phi$.

By the proposition 4.2, we have $L_s \mathcal{V}_n \neq \phi$. □

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