Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 11, No. 3, (March 2016), pp. 475–483
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

A study on IF soft* lower rough approximation and IF soft* upper rough approximation

Sharmistha Bhattacharya Halder, Piyali Debnath

Received 12 August 2011; Revised 29 September 2011; Accepted 7 October 2011

Abstract. Fuzzy set, rough set, and later on IF set became useful mathematical tool for solving various decision making and data mining problems. Molodtsov [4] introduced another concept soft set theory as a general frame work for reasoning about vague concepts. Since most of the data collected are either linguistic variable or consist of vague concepts so IF set and soft set help a lot in data mining problem. The aim of this paper is to introduce the concept of IF soft* lower rough approximation and IF soft* upper rough approximation. Also, some properties of these sets are studied, and also some problems of decision making are cited where these concept may help. Further research will be needed to apply this concept fully in the decision making and data mining problems.

2010 AMS Classification: 03E72, 08A72

Keywords: Rough set, IF set, Soft set, Lower approximations, Upper approximations, IF soft* lower rough approximation, IF soft* upper rough approximation, Oscillation.

Corresponding Author: Piyali Debnath (piyali87maths@rediffmail.com)

1. Introduction

Data mining is one of the areas in which rough set is widely used. Data mining is the process of automatically searching large volumes of data for patterns using tools such as classifications, association, rule mining, and clustering. The rough set theory is well understood format framework for building data mining models in the form of logic rules on the bases of which it is possible to issue predictions that allow classifying new cases. In general whenever data are collected they are linguistic variables. Not only this, the answers are not always in Yes/No form. So, in this case to deal with such type of data IF set is a very important tool. Data are in most of the cases a relation between object and attribute. Soft set is an important tool to deal with such types of data. So, throughout this paper a combined approach of soft set, IF set, rough set is studied. Zadeh in 1965 [7] introduced the concept of fuzzy set. This set contains only a membership function lying between 0 and 1. But while collecting data many cases may be there where data are missing so IF sets are required which consists of both membership value and nonmembership value. Atanassov [1] introduced the concept of IF set and he named it intuitionistic fuzzy set. But nowadays a problem arose due to the already introduced concept of intuitionistic logic. Hence, instead of intuitionistic fuzzy set, throughout this paper we are using the nomenclature IF set. Rough sets introduced by Pawlak [5, 6] are also a very useful tool for data mining problems where vagueness is the key factor. Molodtsov [4] introduced the concept of soft set, and Feng et al. [3] introduced a combined notion of fuzzy set, rough set, and soft set to deal with complex data which arises in the most social science problems. In this paper our aim to introduce the concept of IF soft* lower rough approximation and IF soft* upper rough approximation and some of its properties and applications. B. Davvaz and S. Bhattacharya (Halder) [2] in 2012 introduce the concept of IF soft rough approximation space which is very useful in data mining and decision making processes.

2. Preliminaries

In this section, we first recall some concepts and definitions which would be needed in the sequel.

Definition 2.1 ([1])**.** Let X be a nonempty set and let I be the unit interval [0, 1]. Then an intuitionistic fuzzy set U is an object having the form
$U = \{\langle x, \mu_U(x), \nu_U(x)\rangle : x \in X\}$
where the functions $\mu_U : X \to [0,1]$ and $\nu_U : X \to [0,1]$ denote, respectively, the degree of membership and the degree of non- membership of each element $x \in X$ to the set U, and $0 \leq \mu_U(x) + \nu_U(x) \leq 1$ for each $x \in X$.

Lemma 2.2. *Let X be a nonempt set and let IF sets U and V are in the following forms:* $U = \{\langle x, \mu_U(x), \nu_U(x)\rangle : x \in X\}$ *and* $V = \{\langle x, \mu_V(x), \nu_V(x)\rangle : x \in X\}$ *then*

(i) $U^C = \{\langle x, \nu_U(x), \mu_U(x)\rangle : x \in X\}$
(ii) $U \cap V = \{\langle x, \mu_U(x) \wedge \mu_V(x), \nu_U(x) \vee \nu_V(x)\rangle : x \in X\}$,
(iii) $U \cup V = \{\langle x, \mu_U(x) \vee \mu_V(x), \nu_U(x) \wedge \nu_V(x)\rangle : x \in X\}$,
(iv) $0_\sim = \{\langle x, 0, 1\rangle : x \in X\}, 1_\sim = \{\langle x, 1, 0\rangle : x \in X\}$,
(v) $(U^C)^C = U$, $0_\sim^C = 1_\sim$, $1_\sim^C = 0_\sim$.

Let U be a finite nonempty set, called universe and R an equivalence relation on U, called indiscernibility relation. The pair (U, R) is called an approximation space. By R(x) we mean that the set of all y such that xRy, that is, $R(x) = [x]_R$ is containing the element x. Let X be a subset of U. We want to characterize the set X with respect to R. According to Pawlak [6], the lower approximation of a set X with respect to R is the set of all objects, which surely belong to X, that is, $R_*(X) = \{x : R(x) \subseteq X\}$, and the upper approximation of X with respect to R is the set of all objects, which are partially belonging to X, that is, $R^*(X) = \{x : R(x) \cap X \neq \phi\}$. For an approximation space (U, R), by a rough approximation in (U, R) we mean a mapping

$Apr : \wp(U) \to \wp(U) \times \wp(U)$ defined by for every $X \in \wp(U)$,

$$Apr(X) = (R_*(X), R^*(X)).$$

Given an approximation space (U, R), a pair $(A, B) \in \wp(U) \times \wp(U)$ is called a rough set in (U, R) if (A, B)= Apr(X) for some $X \in \wp(U)$.

Fuzzy set is defined by employing the fuzzy membership function, whereas rough set is defined by approximations. The difference of the upper and the lower approximation is a boundary region. Any rough set has a nonempty boundary region whereas any crisp set has an empty boundary region.

Let (U, R) be a Pawlak [6] approximation space. For a fuzzy set $\mu \in F(U)$, the lower and upper rough approximations of μ in (U, R) are denoted by $\underline{R}(\mu)$ and $\overline{R}(\mu)$, respectively, which are fuzzy sets defined by for all $x \in U$,

$$\underline{R}(\mu)(x) = \wedge\{\mu(y) : y \in [x]_R\},$$
$$\overline{R}(\mu)(x) = \vee\{\mu(y) : y \in [x]_R\}.$$

The operators $\underline{R}$ and $\overline{R}$ are called the lower and upper rough approximation operators on fuzzy sets. If $\underline{R} = \overline{R}$ the fuzzy set μ is said to be definable, otherwise μ is called a rough fuzzy set.

Definition 2.3 ([2])**.** A soft set (F, A) over U is called a full soft set if $\cup_{a\in A}F(a) = U$. Let (F, A) be a full soft set over U, and let S = (U, E) be a soft approximation space. For a fuzzy set $\mu \in F(U)$ the lower and upper soft rough approximations of μ with respect to S are denoted by $\underline{Sap}_S(\mu)$ and $\overline{Sap}_S(\mu)$ respectively, which are fuzzy sets in U given by for all $x \in U$,

$$\underline{Sap}_S(\mu)(x) = \wedge\{\mu(y) : \exists a \in A, \{x, y\} \subseteq F(a)\},$$
$$\overline{Sap}(\mu)(x) = \vee\{\mu(y) : \exists a \in A, \{x, y\} \subseteq F(a)\}.$$

The operators $\underline{Sap}_S(\mu)$ and $\overline{Sap}_S(\mu)$ are called the lower and upper soft rough approximation operators on fuzzy sets. If both the operators are the same then μ is said to be soft definable, otherwise μ is said to be soft rough fuzzy set.

Definition 2.4 ([2])**.** Let $\varepsilon = (F, A)$ be a full soft set over U and S =(U, E) a soft approximation space. For an IF set (μ, ν), the IF soft lower rough approximation and IF soft upper rough approximation with respect to the soft approximation space S are denoted by $\underline{Sap}^*_S(\langle\mu, \nu\rangle)$ and $\overline{Sap}^*_S(\langle\mu, \nu\rangle)$ are defined as follows:

$$\underline{Sap}^*_S(\langle\mu, \nu\rangle)(x) = sup_\nu inf_\mu\{\langle\mu(y), \nu(y)\rangle : \exists a \in A, \{x, y\} \subseteq F(a)\},$$
$$\overline{Sap}^*_S(\langle\mu, \nu\rangle) = inf_\mu sup_\nu\langle\mu(y), \nu(y)\rangle : \exists a \in A, \{x, y\} \subseteq F(a)\ ,$$

for all $x \in U$.

Remark 2.5 ([2])**.** (1) If $\underline{Sap}^*_S(\langle\mu, \nu\rangle)(x) = \overline{Sap}^*_S(\langle\mu, \nu\rangle)(x)$, then the IF soft rough approximation is said to be simply IF soft approximation.

(2) If for some of the object $\underline{Sap}^*_S(\langle\mu, \nu\rangle)(x) = \overline{Sap}^*_S(\langle\mu, \nu\rangle)(x)$, then the IF soft rough approximation is said to be simply IF soft oscillating approximation.

(3) If for none of the object $\underline{Sap}^*_S(\langle\mu, \nu\rangle)(x) = \overline{Sap}^*_S(\langle\mu, \nu\rangle)(x)$, then the IF soft rough approximation is said to be completely IF soft rough approximation. For this case we may consider two more definitions which are known as IF soft stable lower rough approximation and IF soft stable upper rough approximations and are denoted by $\underline{Sap}(\langle\mu, \nu\rangle)$ and $\underline{sap}(\langle\mu, \nu\rangle)$.

Definition 2.6 ([2]). The positive difference between $\underline{Sap}^*_S(\langle\mu,\nu\rangle)$ and $\overline{Sap}^*_S(\langle\mu,\nu\rangle)$ is denoted by O_S is said to oscillate in the approximation space, that is,

$$O_S = |\overline{Sap}^*_S(\langle\mu,\nu\rangle) - \underline{Sap}^*_S(\langle\mu,\nu\rangle)|,$$

where $'|\cdot|'$ is required since otherwise the membership value of the difference may be negative.

Theorem 2.7 ([2]). (1) *O_S can never be $\langle 1, 0\rangle$ for any object.*

(2) *O_S can never be $\langle 0, 1\rangle$ for any object.*

Definition 2.8 ([2]). An IF soft stable lower[resp. upper] rough approximation of $\langle\mu,\nu\rangle$ with respect to S, denoted by $\underline{Sap}_S(\langle\mu,\nu\rangle)$ and $[\overline{Sap}_S(\langle\mu,\nu\rangle)]$ is defined by

$$\underline{Sap}_S(\langle\mu,\nu\rangle) = \underline{Sap}^*_S(\langle\mu,\nu\rangle) \cap \overline{Sap}^*_S(\langle\mu,\nu\rangle)$$
$$[\text{resp. } \overline{Sap}_S(\langle\mu,\nu\rangle) = \underline{Sap}^*_S(\langle\mu,\nu\rangle) \cup \overline{Sap}^*_S(\langle\mu,\nu\rangle)].$$

Remark 2.9 ([2]). (1) Here $B_S = |\underline{Sap}_S(\langle\mu,\nu\rangle) - \overline{Sap}_S(\langle\mu,\nu\rangle)|$ is the IF soft rough boundary region.

(a) If $B_S = \langle 0,0\rangle$ then the data is IF soft set. If $B_S \neq \langle 0,1\rangle$ then the data are IF soft rough set.

(b) $B_S = \langle 0,0\rangle$ if and only if $O_S = \langle 0,0\rangle$. (2) Here $N_S = 1_\sim - \overline{Sap}^*_S\langle\mu,\nu\rangle$, where $1_\sim$ denotes that the value $\langle 1,0\rangle$ for every object is the IF soft rough negative region.

(a) If $N_S = 0_\sim$, then $\overline{Sap}^*_S\langle\mu,\nu\rangle = 1_\sim$ if and only if $\langle\mu,\nu\rangle = 1_\sim$.

(b) $N_S = 1_\sim$ if $\overline{Sap}^*_S\langle\mu,\nu\rangle = 0_\sim$ if and only if $\langle\mu,\nu\rangle = 0_\sim$, where $0_\sim$ is the value $\langle 0,0\rangle$ for every object.

3. On IF soft* lower rough approximation and IF soft* upper rough approximation

In this section, we introduce the concept of IF soft* rough approximation and some of its properties are studied and examples are presented.

Definition 3.1. Let $\epsilon = (F, A)$ be a full soft set over U and S = (U,E) be a soft approximation space. For an IF set $\langle\mu,\nu\rangle$, the IF soft* lower rough approximation and IF soft* upper rough approximation with respect to the soft approximation space S are denoted by $\underline{SR}^*_S(\langle\mu,\nu\rangle)$ and $\overline{SR}^*_S(\langle\mu,\nu\rangle)$ and are defined as follows:

$$\underline{SR}^*_S(\langle\mu,\nu\rangle) = sup_\mu inf_\nu\{\langle\mu(y),\nu(y)\rangle : \exists a \in A, \{x,y\} \subseteq F(a)\},$$
$$\overline{SR}^*_S(\langle\mu,\nu\rangle) = inf_\nu sup_\mu\{\langle\mu(y),\nu(y)\rangle : \exists a \in A, \{x,y\} \subseteq F(a)\},$$

for all $x \in U$.

Example 3.2. Suppose that $U = \{d_1, d_2, d_3, d_4, d_5, d_6, d_7\}$ is the universe of the days of a week and the set of parameters are given by $E = \{t_1, t_2, t_3, t_4, t_5, t_6\}$, where t_i(i = 1,2, . . . , 6) stands for hot, medium, cold, heavy rain, medium rainy, and not raining. Let us consider a soft set (F, E) describing the weather.

$F(t_1) = \{d_1, d_3\}$, $F(t_2) = \{d_2, d_6, d_7\}$, $F(t_3) = \{d_5, d_7\}$, $F(t_4) = \{d_1, d_6\}$, $F(t_5) = \{d_4, d_6\}$, $F(t_6) = \{d_2, d_3, d_5\}$ and we take
$\langle\mu,\nu\rangle = \{\frac{\langle 0.6,0.3\rangle}{d_1}, \frac{\langle 0.6,0.4\rangle}{d_2}, \frac{\langle 0.4,0.3\rangle}{d_3}, \frac{\langle 0.7,0.2\rangle}{d_4}, \frac{\langle 0.5,0.2\rangle}{d_5}, \frac{\langle 0.8,0.1\rangle}{d_6}, \frac{\langle 0.9,0.1\rangle}{d_7}\}$,

$\underline{SR}^*_S(\langle\mu,\nu\rangle) = \{\frac{\langle 0.8,0.1\rangle}{d_1}, \frac{\langle 0.9,0.1\rangle}{d_2}, \frac{\langle 0.6,0.4\rangle}{d_3}, \frac{\langle 0.8,0.1\rangle}{d_4}, \frac{\langle 0.9,0.1\rangle}{d_5}, \frac{\langle 0.9,0.1\rangle}{d_6}, \frac{\langle 0.9,0.1\rangle}{d_7}\}$,

$\overline{SR}^*_S(\langle\mu,\nu\rangle) = \{\frac{\langle 0.8,0.1\rangle}{d_1}, \frac{\langle 0.9,0.1\rangle}{d_2}, \frac{\langle 0.5,0.2\rangle}{d_3}, \frac{\langle 0.8,0.1\rangle}{d_4}, \frac{\langle 0.9,0.1\rangle}{d_5}, \frac{\langle 0.9,0.1\rangle}{d_6}, \frac{\langle 0.9,0.1\rangle}{d_7}\}$.

Remark 3.3. (1) (a)$\underline{SR}^*_S(\langle\mu,\nu\rangle) \nleq \langle\mu,\nu\rangle$ which follows from the above example. But the membership and non membership value of $\underline{SR}^*_S(\langle\mu,\nu\rangle)$ are taken from the same set $\langle\mu,\nu\rangle$ which depending on the mapping F.

(b) If any object is of the form $\langle 1,0\rangle$, then $\underline{SR}^*_S(\langle 1,0\rangle) = \overline{SR}^*_S(\langle 1,0\rangle = \langle 1,0\rangle$ since 0 is the infimum of all members and 1 is the supremum of all non members.

(c) If any object is of the form $\langle 0,1\rangle$, $\underline{SR}^*_S(\langle 0,1\rangle)$ need not be $\langle 0,1\rangle$, since there may exist many other elements whose membership value is less than 1. Similarly, $\overline{SR}^*_S\langle 0,1\rangle \neq \langle 0,1\rangle$.

(d) Let any object d be of the form $\langle 0,0\rangle$. Now, if $\underline{SR}^*_S\langle 0,0\rangle = \langle 0,0\rangle$, then also there does not exist any object in the same mapping with membership 0 but if other object exists with membership nonzero its nonmembership must be zero.

(2) (a) If $\underline{SR}^*_S(\langle\mu,\nu\rangle)(x) = \overline{SR}^*_S(\langle\mu,\nu\rangle)(x)$, then the IF soft* rough approximation is said to be simply IF soft* approximation.

(b) If for some of the object $\underline{SR}^*_S(\langle\mu,\nu\rangle)(x) = \overline{SR}^*_S(\langle\mu,\nu\rangle)(x)$, then the IF soft rough approximation is said to be simply IF soft* oscillating approximation.

(c) If for none of the object $\underline{SR}^*_S(\langle\mu,\nu\rangle)(x) = \overline{SR}^*_S(\langle\mu,\nu\rangle)(x)$, then the IF soft* rough approximation is said to be completely IF soft* rough approximation. For this case we may consider two more definitions which are known as IF soft* stable lower rough approximation and IF soft* stable upper rough approximations and are denoted by $\underline{SR}(\langle\mu,\nu\rangle)$ and $\overline{SR}(\langle\mu,\nu\rangle)$.

Definition 3.4. The positive difference between $\underline{SR}^*_S(\langle\mu,\nu\rangle)$ and $\overline{SR}^*_S(\langle\mu,\nu\rangle)$ is denoted by O_R and is said to oscillate in the soft approximation space, that is

$$O_R = |\overline{SR}^*_S(\langle\mu,\nu\rangle) - \underline{SR}^*_S(\langle\mu,\nu\rangle)|,$$

where $'|\circ|'$ is required since otherwise the membership value of the difference may be negative.

Example 3.5. Consider the example 3.2, we have

$O_R = \{\frac{\langle 0,0\rangle}{d_1}, \frac{\langle 0,0\rangle}{d_2}, \frac{\langle 0.1,0.2\rangle}{d_3}, \frac{\langle 0,0\rangle}{d_4}, \frac{\langle 0,0\rangle}{d_5}, \frac{\langle 0,0\rangle}{d_6}, \frac{\langle 0,0\rangle}{d_7}\}$.

Theorem 3.6. *(1) O_R can never be $\langle 1,0\rangle$ for any object.*

(2) O_R can be never be $\langle 0,1\rangle$ for any object.

Proof. (1) Suppose that $O_R = \langle 1,0\rangle$, then from the definition we have $O_R = |\overline{SR}^*_S(\langle\mu,\nu\rangle) - \underline{SR}^*_S(\langle\mu,\nu\rangle)| = \langle 1,0\rangle$ that is,
$sup_\mu inf_\nu\{\langle\mu(y),\nu(y)\rangle : \exists a \in A, \{x,y\} \subseteq F(a)\}$ - $inf_\nu sup_\mu\{\langle\mu(y),\nu(y)\rangle : \exists a \in A, \{x,y\} \subseteq F(a)\} = \langle 1,0\rangle$.
Let $\langle a,b\rangle - \langle c,d\rangle = \langle 1,0\rangle$, that is, $a-c=1$, $d-b=0$, that is, $d=b$. Since $a \ngtr 1$, so $a-c=1$ gives a =1 and c = 0. Since a = 1, b = 0 gives d = 0, that is, $\underline{SR}_S\langle\mu,\nu\rangle = \langle 1,0\rangle$ and $\overline{SR}_S\langle\mu,\nu\rangle = \langle 0,0\rangle$. But if $\langle 1,0\rangle$ and $\langle 0,0\rangle$ are members of the same mapping F, then $\overline{SR}_S\langle\mu,\nu\rangle = \langle 0,0\rangle$, which is a contradiction. Hence $O_R \neq \langle 1,0\rangle$.

(2) can be proved similarly. ∎

Definition 3.7. An IF soft* stable lower rough approximation of $\langle\mu,\nu\rangle$ with respect to S, denoted by $\underline{SR}(\langle\mu,\nu\rangle)$, is defined

$$\underline{SR}(\langle\mu,\nu\rangle) = \underline{SR}^*_S\langle\mu,\nu\rangle \cap \overline{SR}^*_S\langle\mu,\nu\rangle.$$

An IF soft* stable upper rough approximation of $\langle\mu,\nu\rangle$ with respect to S, denoted by $\overline{SR}(\langle\mu,\nu\rangle)$, is defined by

$$\overline{SR}(\langle\mu,\nu\rangle) = \underline{SR}^*_S\langle\mu,\nu\rangle \cup \overline{SR}^*_S\langle\mu,\nu\rangle.$$

Example 3.8. Consider the Example 3.9. Then

$$\underline{SR}(\langle\mu,\nu\rangle) = \{\frac{\langle 0.8,0.1\rangle}{d_1}, \frac{\langle 0.9,0.1\rangle}{d_2}, \frac{\langle 0.5,0.4\rangle}{d_3}, \frac{\langle 0.8,0.1\rangle}{d_4}, \frac{\langle 0.9,0.1\rangle}{d_5}, \frac{\langle 0.9,0.1\rangle}{d_6}, \frac{\langle 0.9,0.1\rangle}{d_7}\},$$

$$\overline{SR}(\langle\mu,\nu\rangle = \{\frac{\langle 0.8,0.1\rangle}{d_1}, \frac{\langle 0.9,0.1\rangle}{d_2}, \frac{\langle 0.6,0.2\rangle}{d_3}, \frac{\langle 0.8,0.1\rangle}{d_4}, \frac{\langle 0.9,0.1\rangle}{d_5}, \frac{\langle 0.9,0.1\rangle}{d_6}, \frac{\langle 0.9,0.1\rangle}{d_7}\}.$$

Definition 3.9. The positive difference between $\underline{SR}(\langle\mu,\nu\rangle$ and $\overline{SR}(\langle\mu,\nu\rangle)$ is denoted by B_R and said to be the IF soft* rough boundary region and is denoted by

$$B_R = |\overline{SR}\langle\mu,\nu\rangle - \underline{SR}\langle\mu,\nu\rangle|.$$

If we consider the example 3.9, then

$$B_R = \{\frac{\langle 0,0\rangle}{d_1}, \frac{\langle 0,0\rangle}{d_2}, \frac{\langle 0.1,0.2\rangle}{d_3}, \frac{\langle 0,0\rangle}{d_4}, \frac{\langle 0,0\rangle}{d_5}, \frac{\langle 0,0\rangle}{d_6}, \frac{\langle 0,0\rangle}{d_7}\}.$$

Theorem 3.10. *If (F, A) be a full soft set over U, and let S = (U, E) be a soft approximation space. Then, we have* $O_R = B_R$.

Proof. We know that
$B_R = |\overline{Sap}\langle\mu,\nu\rangle - \underline{Sap}\langle\mu,\nu\rangle|$
$=|(\underline{SR}^*_S\langle\mu,\nu\rangle \cup \overline{SR}^*_S\langle\mu,\nu\rangle) - (\underline{SR}^*_S\langle\mu,\nu\rangle \cap \overline{SR}^*_S\langle\mu,\nu\rangle)|$
$=|sup\{\underline{SR}^*_S\langle\mu,\nu\rangle, \overline{SR}^*_S\langle\mu,\nu\rangle\} - inf\{\underline{SR}^*_S\langle\mu,\nu\rangle \cap \overline{SR}^*_S\langle\mu,\nu\rangle\}|$
$=|\langle max\{\mu_{\underline{SR}^*_S}, \mu_{\overline{SR}^*_S}\}, min\{\nu_{\underline{SR}^*_S}, \nu_{\overline{SR}^*_S}\}\rangle - \langle min\{\mu_{\underline{SR}^*_S}, \mu_{\overline{SR}^*_S}\}, max\{\nu_{\underline{SR}^*_S}, \nu_{\overline{SR}^*_S}\}\rangle|.$
Then four cases may be arise :

(i) $\mu_{\underline{SR}^*_S} \leq \mu_{\overline{SR}^*_S}$ and $\nu_{\underline{SR}^*_S} \leq \nu_{\overline{SR}^*_S}$,

(ii) $\mu_{\underline{SR}^*_S} \geq \mu_{\overline{SR}^*_S}$ and $\nu_{\underline{SR}^*_S} \geq \nu_{\overline{SR}^*_S}$,

(ii) $\mu_{\underline{SR}^*_S} \geq \mu_{\overline{SR}^*_S}$ and $\nu_{\underline{SR}^*_S} \leq \nu_{\overline{SR}^*_S}$,

(iv) $\mu_{\underline{SR}^*_S} \leq \mu_{\overline{SR}^*_S}$ and $\nu_{\underline{SR}^*_S} \geq \nu_{\overline{SR}^*_S}$.

Case (i) : If $\mu_{\underline{SR}^*_S} \leq \mu_{\overline{SR}^*_S}$ and $\nu_{\underline{SR}^*_S} \leq \nu_{\overline{SR}^*_S}$, then
$B_R = |\langle\mu_{\overline{SR}^*_S}, \nu_{\underline{SR}^*_S}\rangle - \langle\mu_{\underline{SR}^*_S}, \nu_{\overline{SR}^*_S}\rangle|$
$=|\langle\mu_{\overline{SR}^*_S} - \mu_{\underline{SR}^*_S}, \nu_{\underline{SR}^*_S} - \nu_{\overline{SR}^*_S}\rangle|$
$=|\overline{SR}^*_S - \underline{SR}^*_S| = O_R.$

Case (ii) : If $\mu_{\underline{SR}^*_S} \geq \mu_{\overline{SR}^*_S}$ and $\nu_{\underline{SR}^*_S} \geq \nu_{\overline{SR}^*_S}$, then
$B_R = |\langle\mu_{\underline{SR}^*_S}, \nu_{\overline{SR}^*_S}\rangle - \langle\mu_{\overline{SR}^*_S}, \nu_{\underline{SR}^*_S}\rangle|$

$=|\langle \mu_{\underline{SR}^*_S} - \mu_{\overline{SR}^*_S}, \nu_{\overline{SR}^*_S} - \nu_{\underline{SR}^*_S}\rangle|$
$=|\overline{SR}^*_S - \underline{SR}^*_S| = O_R.$

case (iii) : If $\mu_{\underline{SR}^*_S} \geq \mu_{\overline{SR}^*_S}$ and $\nu_{\underline{SR}^*_S} \leq \nu_{\overline{SR}^*_S}$, then
$B_R = |\langle \mu_{\underline{SR}^*_S}, \nu_{\underline{SR}^*_S}\rangle - \langle \mu_{\overline{SR}^*_S}, \nu_{\overline{SR}^*_S}\rangle|$
$=|\langle \mu_{\underline{SR}^*_S} - \mu_{\overline{SR}^*_S}, \nu_{\underline{SR}^*_S} - \nu_{\overline{SR}^*_S}\rangle|$
$=|\overline{SR}^*_S - \underline{SR}^*_S| = O_R.$

Case (iv) : If $\mu_{\underline{SR}^*_S} \leq \mu_{\overline{SR}^*_S}$ and $\nu_{\underline{SR}^*_S} \geq \nu_{\overline{SR}^*_S}$, then
$B_R = |\langle \mu_{\overline{SR}^*_S}, \nu_{\overline{SR}^*_S}\rangle - \langle \mu_{\overline{SR}^*_S}, \nu_{\overline{SR}^*_S}\rangle|$
$=|\langle \mu_{\overline{SR}^*_S} - \mu_{\underline{SR}^*_S}, \nu_{\overline{SR}^*_S} - \nu_{\underline{SR}^*_S}\rangle|$
$=|\overline{SR}^*_S - \underline{SR}^*_S| = O_R$ □

Remark 3.11. $N_R = 1_\sim - \overline{SR}\langle \mu, \nu\rangle$, where $1_\sim$ denotes the value $\langle 1, 0\rangle$, for every objects is the IF soft* rough negative region.

(a) If $N_R = 0_\sim$, then $\overline{SR}\langle \mu, \nu\rangle = 1_\sim$ if and only if $\langle \mu, \nu\rangle = 1_\sim$, by Remark 3.3(1)(a).

(b) If $N_R = 1_\sim$ and $\overline{SR}\langle \mu, \nu\rangle = 0_\sim$, if and if only $\langle \mu, \nu\rangle = 0_\sim$, where $1_\sim$ is the value $\langle 0, 0\rangle$ for every object.

4. Oscillation on IF soft lower rough approximation and IF soft* lower rough approximation

In this section we introduce the concept of oscillation on IF soft lower rough approximation and IF soft* lower rough approximation and some of its properties.

Definition 4.1. The positive difference between $\underline{Sap}^*_S(\langle \mu, \nu\rangle)$ and $\underline{SR}^*_S(\langle \mu, \nu\rangle)$ is denoted by L_R and is said to be oscillate in the lower rough approximation spaces, that is

$$L_R = |\underline{Sap}^*_S(\langle \mu, \nu\rangle) - \underline{SR}^*_S(\langle \mu, \nu\rangle)|.$$

Example 4.2. Suppose that $U = \{d_1, d_2, d_3, d_4, d_5, d_6, d_7\}$ is the universe of the dresses and $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ be the set of parameters, where e_1 stands for cotton, e_2 stands for woolen, e_3 stands for synthetic, e_4 stand for expensive beautiful, e_5 stands for cheap, e_6 stands for expensive. Let us consider a soft set (F,E) describing the attractiveness of the dresses.

Now $F(e_1) = \{d_1, d_4, d_7\}$, $F(e_2) = \{d_3, d_5\}$, $F(e_3) = \{d_6, d_7\}$, $F(e_4) = \{d_1, d_6\}$, $F(e_5 = \{d_3, d_7\}$, $F(e_6) = \{d_2, d_3, d_6\}$.

$\langle \mu, \nu\rangle = \{\frac{\langle 0.5,0.2\rangle}{d_1}, \frac{\langle 0.4,0.3\rangle}{d_2}, \frac{\langle 0.4,0.2\rangle}{d_3}, \frac{\langle 0.7,0.2\rangle}{d_4}, \frac{\langle 0.1,0.6\rangle}{d_5}, \frac{\langle 0.2,0.6\rangle}{d_6}, \frac{\langle 0.1,0.4\rangle}{d_7}\}$,

$\underline{Sap}^*_S(\langle \mu, \nu\rangle) = \{\frac{\langle 0.1,0.4\rangle}{d_1}, \frac{\langle 0.2,0.6\rangle}{d_2}, \frac{\langle 0.1,0.6\rangle}{d_3}, \frac{\langle 0.1,0.4\rangle}{d_4}, \frac{\langle 0.1,0.6\rangle}{d_5}, \frac{\langle 0.1,0.4\rangle}{d_6}, \frac{\langle 0.1,0.4\rangle}{d_7}\}$,

$\underline{SR}^*_S(\langle\mu,\nu\rangle) = \{\frac{\langle 0.7,0.2\rangle}{d_1}, \frac{\langle 0.4,0.2\rangle}{d_2}, \frac{\langle 0.4,0.2\rangle}{d_3}, \frac{\langle 0.7,0.2\rangle}{d_4}, \frac{\langle 0.4,0.2rangle}{d_5}, \frac{\langle 0.5,0.2\rangle}{d_6}, \frac{\langle 0.4,0.2\rangle}{d_7}\}$,

$L_R = \{\frac{\langle 0.6,0.2\rangle}{d_1}, \frac{\langle 0.2,0.4\rangle}{d_2}, \frac{\langle 0.3,0.4\rangle}{d_3}, \frac{\langle 0.6,0.2\rangle}{d_4}, \frac{\langle 0.3,0.4\rangle}{d_5}, \frac{\langle 0.4,0.2\rangle}{d_6}, \frac{\langle 0.3,0.2\rangle}{d_7}\}$.

Theorem 4.3. *(1) L_R can never be $\langle 1,0\rangle$ for any object.*
(2) L_R can never be $\langle 0,1\rangle$ for any object.

Proof. (1) Suppose that $L_S = \langle 1,0\rangle$, then from the definition we have $L_R = |\underline{Sap}^*_S(\langle\mu,\nu\rangle) - \underline{SR}^*_S(\langle\mu,\nu\rangle)| = \langle 1,0\rangle$, that is,

$sup_\nu inf_\mu\{\mu(y),\nu(y)\rangle : \exists a \in A, \{x,y\} \subseteq F(a)\}$ - $sup_\mu inf_\nu\{\langle\mu(y),\nu(y)\rangle : \exists a \in A, \{x,y\} \subseteq F(a)\} = \langle 1,0\rangle$.

Let $\langle a,b\rangle - \langle c,d\rangle = \langle 1,0\rangle$, that is a - c = 1, d - b = 0, that is d=b. Since , so a - c = 1 gives a = 1 and c = 0. Since $a \ngeq 1$, b = 0 gives d = 0, that is,

$\underline{Sap}^*_S(\langle\mu,\nu\rangle) = \langle 1,0\rangle$, $\underline{SR}^*_S(\langle\mu,\nu\rangle) = \langle 0,0\rangle$.

But if $\langle 1,0\rangle$ and $\langle 0,0\rangle$ are members of the same mapping F, then $\underline{Sap}^*_S(\langle\mu,\nu\rangle) = \langle 0,0\rangle$, which is a contradiction. Hence $L_R \neq \langle 1,0\rangle$.
(2) It can be proved similarly. □

Example 4.4. Suppose that $U = \{1,2,3,4,5,6,7\}$ is the universe consisting of six persons and the set of parameters are given by $E = \{S_1,S_2,S_3,S_4,S_5,S_6\}$ where S_1 implies Fever, S_2 implies headache,S_3 implies muscle pain, S_4 implies weakness, S_5 implies Extreme Exhaustion, S_6 implies cough. Let us consider a soft set (F, E) describing the ”flu infected person”.

Let $F(S_1) = \{1,5,7\}$, $F(S_2) = \{2,3,6\}$, $F(S_3) = \{4,5,6\}$, $F(S_4) = \{1,6\}$, $F(S_5) = \{4,7\}$, $F(S_6) = \{2,7\}$.

Let us now consider the IF set a flu infected person as per our choice as

$\langle\mu,\nu\rangle = \{\frac{\langle 0.3,0.5\rangle}{1}, \frac{\langle 0.2,0.7\rangle}{2}, \frac{\langle 0.1,0.8\rangle}{3}, \frac{\langle 0.2,0.8\rangle}{4}, \frac{\langle 0.3,0.6\rangle}{5}, \frac{\langle 0.2,0.7\rangle}{6}, \frac{\langle 0.3,0.4\rangle}{7}\}$.
Then

$\underline{Sap}^*_S(\langle\mu,\nu\rangle) = \{\frac{\langle 0.2,0.8\rangle}{1}, \frac{\langle 0.1,0.8\rangle}{2}, \frac{\langle 0.1,0.8\rangle}{3}, \frac{\langle 0.2,0.9\rangle}{4}, \frac{\langle 0.2,0.9\rangle}{5}, \frac{\langle 0.1,0.8\rangle}{6}, \frac{\langle 0.2,0.9\rangle}{7}\}$,

$\overline{sap}^*_S(\langle\mu,\nu\rangle) = \{\frac{\langle 0.3,0.9\rangle}{1}, \frac{\langle 0.3,0.9\rangle}{2}, \frac{\langle 0.1,0.8\rangle}{3}, \frac{\langle 0.2,0.9\rangle}{4}, \frac{\langle 0.2,0.9\rangle}{5}, \frac{\langle 0.2,0.9\rangle}{6}, \frac{\langle 0.2,0.9\rangle}{7}\}$,

$\underline{SR}^*_S(\langle\mu,\nu\rangle) = \{\frac{\langle 0.3,0.5\rangle}{1}, \frac{\langle 0.4,0.7\rangle}{2}, \frac{\langle 0.4,0.7\rangle}{3}, \frac{\langle 0.4,0.6\rangle}{4}, \frac{\langle 0.3,0.5\rangle}{5}, \frac{\langle 0.3,0.5\rangle}{6}, \frac{\langle 0.3,0.5\rangle}{7}\}$,

$O_R = \{\frac{\langle 0.1,0.1\rangle}{1}, \frac{\langle 0.2,0.1\rangle}{2}, \frac{\langle 0,0\rangle}{3}, \frac{\langle 0,0\rangle}{4}, \frac{\langle 0,0\rangle}{5}, \frac{\langle 0.1,0.1\rangle}{6}, \frac{\langle 0.1,0\rangle}{7}\}$,

$L_R = \{\frac{\langle 0.1,0.3\rangle}{1}, \frac{\langle 0.3,0.1\rangle}{2}, \frac{\langle 0.3,0.1\rangle}{3}, \frac{\langle 0.2,0.3\rangle}{4}, \frac{\langle 0.1,0.4\rangle}{5}, \frac{\langle 0.2,0.3\rangle}{6}, \frac{\langle 0.1,0.4\rangle}{7}\}$.
Therefore $O_R \neq L_R$.

Remark 4.5. Let $L_R = |\underline{Sap}^*_S(\langle\mu,\nu\rangle) - \underline{SR}^*_S(\langle\mu,\nu\rangle)| < \epsilon$ be a relation on $\langle\mu,\nu\rangle$.

(i) If $|\underline{Sap}^*_S(\langle\mu,\mu\rangle) - \underline{SR}^*_S(\langle\mu,\mu\rangle)| = 0$, then it is reflexive.

(ii) If $|\underline{Sap}^*_S(\langle\mu,\nu\rangle) - \underline{SR}^*_S(\langle\mu,\nu\rangle)| < \epsilon$, then $|\underline{SR}^*_S(\langle\mu,\nu\rangle) - \underline{Sap}^*_S(\langle\mu,\nu\rangle)| < \epsilon$. Thus it is symmetric.

(iii) If $|\underline{Sap}^*_S(\langle\mu,\gamma\rangle) - \underline{SR}^*_S(\langle\mu,\gamma\rangle)| < \epsilon$ and $|\underline{Sap}^*_S(\langle\gamma,\nu\rangle) - \underline{SR}^*_S(\langle\gamma,\nu\rangle)| < \epsilon$, then $|\underline{Sap}^*_S(\langle\mu,\nu\rangle) - \underline{SR}^*_S(\langle\mu,\nu\rangle)| < \epsilon$. Thus it is transitive.

4.1. **Note:** Since L_R is reflexive, symmetric and transitive, therefore L_R is an equivalence relation on $\langle\mu,\nu\rangle$. For the different value of ϵ, L_R can be subdivided in some equivalence classes. Let $[\epsilon]_R$ denote the equivalence class on L_R . The set of distinct equivalence classes for the relation L_R form a partition on $\langle\mu,\nu\rangle$.

References

[1] K. T. Atanassov , Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87–96.
[2] S. Bhattacharya (Halder) and B. Davvaz, Some More Results on IF Soft Rough Approximation Space 2011 (2011) 1–10.
[3] F. Feng, C. Li, B. Davvaz and M. I. Ali, Soft sets combined with fuzzy sets and rough sets a tentative approach, Soft Computing 11 (1982) 341–356.
[4] D. Molodtsov, Soft set theory first result, An International Journal of Computers and mathematics with Applications 37 (1999) 19–31.
[5] Z. Pawlak, Rough sets and fuzzy sets, Fuzzy Sets and Systems 17 (1985) 99–102.
[6] Z. Pawlak, Rough sets, International Journal of Computer and Information Sciences 11 (1983) 341–356.
[7] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.

Sharmistha bhattacharya Halder (halder_731@rediffmail.com)
Department of Mathematics, Tripura University-799022, Tripura, India

Piyali Debnath (piyali87maths@rediffmail.com)
Department of Mathematics, Tripura University-799022, Tripura, India