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Some remarks on interval valued anti fuzzy ideals of near-rings

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Abstract. In this paper, some characterizations of interval valued anti fuzzy ideals of near-ring and fuzzy relations on near-rings are discussed. Also, homomorphism and anti-homomorphism of interval valued anti fuzzy ideals in near-rings are studied.

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1. Introduction

The study concept of fuzzy set was first initiated by Zadeh[14] in 1965. After a decade he also introduced the notion of interval valued fuzzy subset[15](in short i-v fuzzy subset) in 1975. In [2] Biswas first discussed the concept of anti fuzzy subgroups. Abou-Zaid[1] proposed the concept of fuzzy subnear-rings and ideals of near-ring. The concept of fuzzy subgroups first introduced by Rosenfeld[9]. Kim et al.[8] initiated the notion of anti fuzzy ideals of near-rings. The idea of anti fuzzy ideals of Γ-near-rings was studied by Srinivas et al.[10]. In [7], the notion of fuzzy ideals in near-rings was introduced by Kim and Kim. Davvaz[4] has discussed the concept of fuzzy ideals of near-rings with interval valued membership functions. Jianming Zhan et al.[6] initiated the concept of interval valued fuzzy ideals of hypernear-rings. The concept of interval valued fuzzy quasi-ideals of semigroups was first introduced by Thillaigovindan and Chinnadurai in [11]. Thillaigovindan et al.[12] have introduced the notion of interval valued anti fuzzy ideals in near-rings. Jun et al.[13] have discussed some basic properties of fuzzy h-ideals in hemiring. In this paper, some characterizations of interval valued anti fuzzy ideals of near-ring and fuzzy relations on near-rings are discussed. Homomorphism and anti-homomorphism of interval valued anti fuzzy ideals in near-rings are also studied.

2. Preliminaries

Through out this paper R denotes left near-ring unless otherwise specified. In this section we recall some basic definitions and results.

A near-ring is an algebraic system $(R, +, \cdot)$ consisting of a non empty set R together with two binary operations called $+$ and $\cdot$ such that $(R, +)$ is a group not necessarily abelian and $(R, \cdot)$ is a semigroup connected by the distributive law: $x \cdot (y + z) = x \cdot y + x \cdot z$ valid for all $x, y, z \in R$. We use the word 'near-ring 'to mean a 'left near-ring '. We denote xy instead of $x \cdot y$. An ideal I of a near-ring R is the subset of R such that (i) $(I, +)$ is a normal subgroup of $(R, +)$, (ii) $RI \subseteq I$, (iii) $(x + a)y - xy \in I$, for any $a \in I$ and $x, y \in R$. Note that I is a left ideal if it satisfies (i) and (ii), and a right ideal if it satisfies (i) and (iii).

Definition 2.1. Let I be an ideal of R. For each $a + I, b + I$ in the factor group R/I, we define $(a + I) + (b + I) = (a + b) + I$ and $(a + I)(b + I) = ab + I$. Then R/I is a near-ring which we call the residue class near-ring of R with respect to I.

Notation 2.2 ([11, 5]). By an interval number $\overline{a}$, we mean an interval $[a^-, a^+]$ such that $0 \leq a^- \leq a^+ \leq 1$ where a^- and a^+ are the lower and upper limits of $\overline{a}$ respectively. The set of all closed subintervals of $[0, 1]$ is denoted by $D[0, 1]$. We also identify the interval $[a, a]$ by the number $a \in [0, 1]$. For any interval numbers $\overline{a}_j = [a_j^-, a_j^+], \overline{b}_j = [b_j^-, b_j^+] \in D[0, 1], j \in J$ an index set we define

$$\max{}^i\{\overline{a}_j, \overline{b}_j\} = [\max{}^i\{a_j^-, b_j^-\}, \max{}^i\{a_j^+, b_j^+\}],$$
$$\min{}^i\{\overline{a}_j, \overline{b}_j\} = [\min{}^i\{a_j^-, b_j^-\}, \min{}^i\{a_j^+, b_j^+\}],$$
$$\inf{}^i \overline{a}_j = \left[\inf_{j\in I} a_j^-, \inf_{j\in I} a_j^+\right], \sup{}^i \overline{a}_j = \left[\sup_{j\in I} a_j^-, \sup_{j\in I} a_j^+\right]$$

and put

(1) $\overline{a} \leq \overline{b} \iff a^- \leq b^-$ and $a^+ \leq b^+$,
(2) $\overline{a} = \overline{b} \iff a^- = b^-$ and $a^+ = b^+$,
(3) $\overline{a} < \overline{b} \iff \overline{a} \leq \overline{b}$ and $\overline{a} \neq \overline{b}$
(4) $k\overline{a} = [ka^-, ka^+]$, for $0 \leq k \leq 1$.

Definition 2.3 ([11]). Let X be a non-empty set. A mapping $\overline{\mu} : X \to D[0, 1]$ is called an i-v fuzzy subset of X. For all $x \in X$, $\overline{\mu}(x) = [\mu^-(x), \mu^+(x)]$, where μ^- and μ^+ are fuzzy subsets of X such that $\mu^-(x) \leq \mu^+(x)$. Thus $\overline{\mu}(x)$ is an interval(a closed subset of $[0, 1]$) and not a number from the interval $[0, 1]$ as in the case of a fuzzy set.

Let $\overline{\mu}, \overline{\nu}$ be i-v fuzzy subsets of X. The following are defined by

(1) $\overline{\mu} \leq \overline{\nu} \Leftrightarrow \overline{\mu}(x) \leq \overline{\nu}(x)$. (2) $\overline{\mu} = \overline{\nu} \Leftrightarrow \overline{\mu}(x) = \overline{\nu}(x)$.
(3) $(\overline{\mu} \cup \overline{\nu})(x) = \max{}^i\{\overline{\mu}(x), \overline{\nu}(x)\}$. (4) $(\overline{\mu} \cap \overline{\nu})(x) = \min{}^i\{\overline{\mu}(x), \overline{\nu}(x)\}$.
(5) $\overline{\mu}^c(x) = \overline{1} - \overline{\mu}(x) = [1 - \mu^+(x), 1 - \mu^-(x)]$.

Definition 2.4 ([11]). Let $\overline{\mu}$ be an i-v fuzzy subset of X and $[t_1, t_2] \in D[0, 1]$. Then the set
$\overline{U}(\overline{\mu} : [t_1, t_2]) = \{x \in X | \ \overline{\mu}(x) \geq [t_1, t_2]\}$, is called the upper level set of $\overline{\mu}$ and $\overline{L}(\overline{\mu} : [t_1, t_2]) = \{x \in X | \ \overline{\mu}(x) \leq [t_1, t_2]\}$, is called the lower level set of $\overline{\mu}$.

Definition 2.5 ([12]). An i-v fuzzy subset $\overline{\mu}$ of a near-ring R is called an i-v anti fuzzy subnear-ring of R if

(1) $\overline{\mu}(x-y) \leq \max^i\{\overline{\mu}(x), \overline{\mu}(y)\}$,
(2) $\overline{\mu}(xy) \leq \max^i\{\overline{\mu}(x), \overline{\mu}(y)\}$, for all $x, y \in R$.

Definition 2.6 ([12]). An i-v fuzzy subset $\overline{\mu}$ of a near-ring R is called an i-v anti fuzzy ideal of R if $\overline{\mu}$ is an i-v anti fuzzy subnear-ring of R and
(3) $\overline{\mu}(y+x-y) \leq \overline{\mu}(x)$,
(4) $\overline{\mu}(xy) \leq \overline{\mu}(y)$,
(5) $\overline{\mu}((x+z)y-xy) \leq \overline{\mu}(z)$ for any $x, y, z \in R$.
Note that $\overline{\mu}$ is an i-v anti fuzzy left ideal of R if it satisfies (1), (2), (3) and (4), and $\overline{\mu}$ is an i-v anti fuzzy right ideal of R if it satisfies (1), (2), (3) and (5).

3. Characterizations of interval valued anti fuzzy ideals

In this section we first define the direct product of i-v anti fuzzy ideals of near-rings and provide an example. It is established that the anti direct product of i-v anti fuzzy ideals of near-rings is also an i-v anti fuzzy ideals. We obtain the necessary and sufficient condition for the strongest i-v anti fuzzy relation on a near-ring R with respect to an i-v fuzzy subset $\overline{\mu}$ to be an i-v anti fuzzy ideal of R.

Definition 3.1. Let $\overline{\mu}_i$ be an i-v anti fuzzy ideal of near-ring R_i, for $i = 1, 2, ..., n$. Then the anti direct product of $\overline{\mu}_i, (i = 1, 2, ..., n)$ is a function $\overline{\mu}_1 \times \overline{\mu}_2 \times ... \times \overline{\mu}_n : R_1 \times R_2 \times ... \times R_n \longrightarrow D[0,1]$ defined by $\overline{\mu}_1 \times \overline{\mu}_2 \times ... \times \overline{\mu}_n(x_1, x_2, ..., x_n) = \max^i\{\overline{\mu}_1(x_1), \overline{\mu}_2(x_2), ..., \overline{\mu}_n(x_n)\}$.

Example 3.2. Let $R_1, R_2, R_3 = \{0, 1, 2\}$. Addition is modulo 3 and $\cdot_{R_1}, \cdot_{R_2}$ and $\cdot_{R_3}$ be defined as follows:

$+$	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

$\cdot_{R_1}$	0	1	2
0	0	1	2
1	0	1	2
2	0	1	2

$\cdot_{R_2}$	0	1	2
0	0	0	0
1	0	1	2
2	0	1	2

$\cdot_{R_3}$	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

Then clearly, $(R_1, +, \cdot_{R_1}), (R_2, +, \cdot_{R_2})$ and $(R_3, +, \cdot_{R_3})$ are left near-rings. Let $\overline{\mu}_1 : R_1 \to D[0,1], \overline{\mu}_2 : R_2 \to D[0,1]$ and $\overline{\mu}_3 : R_3 \to D[0,1]$ be i-v fuzzy subsets of R_1, R_2 and R_3, respectively defined by,
$\overline{\mu}_1(0) = [0.1, 0.2]$, $\overline{\mu}_1(1) = [0.5, 0.6] = \overline{\mu}_1(2)$,
$\overline{\mu}_2(0) = [0.3, 0.4]$, $\overline{\mu}_2(1) = [0.7, 0.8] = \overline{\mu}_2(2)$,
$\overline{\mu}_3(0) = [0.5, 0.6]$, $\overline{\mu}_3(1) = [0.8, 0.9] = \overline{\mu}_3(2)$.
Clearly, μ_1, μ_2 and μ_3 are i-v anti fuzzy ideals of R_1, R_2 and R_3.
Let $R = R_1 \times R_2 \times R_3$ and $\overline{\mu} = \overline{\mu}_1 \times \overline{\mu}_2 \times \overline{\mu}_3$. Let $\overline{\mu} : R \to D[0,1]$ defined by $\overline{\mu}(x_1, x_2, x_3) = \max^i\{\overline{\mu}_1(x_1), \overline{\mu}_2(x_2), \overline{\mu}_3(x_3)\}$, for all $x_1 \in R_1, x_2 \in R_2$ and $x_3 \in R_3$

is an anti direct product of μ as shown below.

$$\begin{array}{lll}
\overline{\mu}(0,0,0)=[0.5,0.6] & \overline{\mu}(0,0,1)=[0.8,0.9] & \overline{\mu}(0,0,2)=[0.8,0.9]\\
\overline{\mu}(0,1,0)=[0.7,0.8] & \overline{\mu}(0,1,1)=[0.8,0.9] & \overline{\mu}(0,1,2)=[0.8,0.9]\\
\overline{\mu}(0,2,0)=[0.7,0.8] & \overline{\mu}(0,2,1)=[0.7,0.8] & \overline{\mu}(0,2,2)=[0.8,0.9]\\
\overline{\mu}(1,0,0)=[0.5,0.6] & \overline{\mu}(1,0,1)=[0.8,0.9] & \overline{\mu}(1,0,2)=[0.8,0.9]\\
\overline{\mu}(1,1,0)=[0.7,0.8] & \overline{\mu}(1,1,1)=[0.8,0.9] & \overline{\mu}(1,1,2)=[0.8,0.9]\\
\overline{\mu}(1,2,0)=[0.8,0.9] & \overline{\mu}(1,2,1)=[0.8,0.9] & \overline{\mu}(1,2,2)=[0.8,0.9]\\
\overline{\mu}(2,0,0)=[0.5,0.6] & \overline{\mu}(2,0,1)=[0.8,0.9] & \overline{\mu}(2,0,2)=[0.8,0.9]\\
\overline{\mu}(2,1,0)=[0.7,0.8] & \overline{\mu}(2,1,1)=[0.8,0.9] & \overline{\mu}(2,1,2)=[0.8,0.9]\\
\overline{\mu}(2,2,0)=[0.7,0.8] & \overline{\mu}(2,2,1)=[0.8,0.9] & \overline{\mu}(2,2,2)=[0.8,0.9].
\end{array}$$

Definition 3.3. Let $\overline{\mu}$ be an i-v fuzzy subset in a set R, the strongest i-v anti fuzzy relation on R, that is an i-v anti fuzzy relation $\overline{\gamma}$ with respect to $\overline{\mu}$ given by $\overline{\gamma}(x,y)=\max^i\{\overline{\mu}(x),\overline{\mu}(y)\}$ for all $x,y\in R$.

Definition 3.4 ([4])**.** Let f be a mapping from a set R to a set S. Let $\overline{\mu}$ be an i-v fuzzy subset of R and $\overline{\lambda}$ be an i-v fuzzy subset of S. Then, the pre-image $f^{-1}(\overline{\lambda})$ is an i-v fuzzy subset of R defined by $f^{-1}(\overline{\lambda})(x)=\overline{\lambda}(f(x))$, for all $x\in R$. The image $f(\overline{\mu})$ is an i-v fuzzy subset of S defined by

$$f(\overline{\mu})(x)=\begin{cases}\sup\limits_{y\in f^{-1}(x)}{}^{i}\overline{\mu}(y) & \text{if} f^{-1}(x)\neq\varnothing\\ 0 & \text{otherwise.}\end{cases}$$

The anti-image $f_{-}(\overline{\mu})$ is an i-v fuzzy subset of S defined by

$$f_{-}(\overline{\mu})(x)=\begin{cases}\inf\limits_{y\in f^{-1}(x)}{}^{i}\overline{\mu}(y) & \text{if} f^{-1}(x)\neq\varnothing\\ 1 & \text{otherwise.}\end{cases}$$

Definition 3.5 ([7])**.** Let R and S be near-rings. A map $f:R\to S$ is called a (near-ring) homomorphism if $f(x+y)=f(x)+f(y)$ and $f(xy)=f(x)f(y)$, for all $x,y\in R$.

Definition 3.6 ([3])**.** Let R and S be near-rings. A map $f:R\to S$ is called a (near-ring) anti-homomorphism if $f(x+y)=f(y)+f(x)$ and $f(xy)=f(y)f(x)$, for all $x,y\in R$.

Theorem 3.7. *The anti direct product of i-v anti fuzzy ideals of near-rings is i-v anti fuzzy ideals of a near-ring.*

Proof. Let $\overline{\mu}_i$ be i-v anti fuzzy ideals of near-rings R_i, i=1,2,...,n. Let $x=(x_1,x_2,...,x_n)$ $y=(y_1,y_2,...,y_n)$ and $z=(z_1,z_2,...,z_n)\in R_1\times R_2\times...\times R_n$ and let $\overline{\mu}_1\times\overline{\mu}_2\times...\times\overline{\mu}_n=$

$\overline{\mu}$. Then

$$\begin{aligned}
\overline{\mu}(x-y) &= \overline{\mu}((x_1,x_2,...,x_n)-(y_1,y_2,...,y_n)) \\
&= \overline{\mu}(x_1-y_1,x_2-y_2,...,x_n-y_n) \\
&= \max^i\{\overline{\mu}_1(x_1-y_1),\overline{\mu}_2(x_2-y_2),...,\overline{\mu}_n(x_n-y_n)\} \\
&\leq \max^i\{\max^i\{\overline{\mu}_1(x_1),\overline{\mu}_1(y_1)\},\max^i\{\overline{\mu}_2(x_2),\overline{\mu}_2(y_2)\},..., \\
&\qquad \max^i\{\overline{\mu}_n(x_n),\overline{\mu}_n(y_n)\}\} \\
&= \max^i\{\max^i\{\overline{\mu}_1(x_1),\overline{\mu}_2(x_2),...,\overline{\mu}_n(x_n),\},\max^i\{\overline{\mu}_1(y_1),\overline{\mu}_2(y_2),..., \\
&\qquad \overline{\mu}_n(y_n),\}\} \\
&= \max^i\{\overline{\mu}_1\times\overline{\mu}_2\times,...,\overline{\mu}_n(x_1,x_2,...,x_n),\overline{\mu}_1\times\overline{\mu}_2\times,...,\overline{\mu}_n(y_1,y_2,...,y_n)\} \\
&= \max^i\{\overline{\mu}(x),\overline{\mu}(y)\}.
\end{aligned}$$

Similarly, $\overline{\mu}(xy)\leq\max^i\{\overline{\mu}(x),\overline{\mu}(y)\}$.

$$\begin{aligned}
\overline{\mu}(y+x-y) &= \overline{\mu}((y_1,y_2,...,y_n)+(x_1,x_2,...,x_n)-(y_1,y_2,...,y_n)) \\
&= \overline{\mu}(y_1+x_1-y_1,y_2+x_2-y_2,...,y_n+x_n-y_n) \\
&= \max^i\{\overline{\mu}_1(y_1+x_1-y_1),\overline{\mu}_2(y_2+x_2-y_2),...,\overline{\mu}_n(y_n+x_n-y_n)\} \\
&\leq \max^i\{\overline{\mu}_1(x_1),\overline{\mu}_2(x_2),...,\overline{\mu}_n(x_n)\} \\
&= \overline{\mu}_1\times\overline{\mu}_2\times,...,\overline{\mu}_n(x_1,x_2,...,x_n) \\
&= \overline{\mu}(x).
\end{aligned}$$

$$\begin{aligned}
\overline{\mu}(xy) &= \overline{\mu}((x_1,x_2,...,x_n)(y_1,y_2,...,y_n)) \\
&= \overline{\mu}(x_1y_1,x_2y_2,...,x_ny_n) \\
&= \max^i\{\overline{\mu}_1(x_1y_1),\overline{\mu}_2(x_2y_2),...,\overline{\mu}_n(x_ny_n)\} \\
&\leq \max^i\{\overline{\mu}_1(y_1),\overline{\mu}_2(y_2),...,\overline{\mu}_n(y_n)\} \\
&= \overline{\mu}_1\times\overline{\mu}_2\times,...,\overline{\mu}_n(y_1,y_2,...,y_n) \\
&= \overline{\mu}(y).
\end{aligned}$$

$$\begin{aligned}
\overline{\mu}((x+z)y-xy) &= \overline{\mu}(((x_1,x_2,...,x_n)+(z_1,z_2,...,z_n))(y_1,y_2,...,y_n)- \\
&\qquad (x_1,x_2,...,x_n)(y_1,y_2,...,y_n)) \\
&= \overline{\mu}(((x_1+z_1)y_1-x_1y_1),((x_2+z_2)y_2-x_2y_2),..., \\
&\qquad ((x_n+z_n)y_n-x_ny_n)) \\
&= \max^i\{\overline{\mu}_1((x_1+z_1)y_1-x_1y_1),\overline{\mu}_2((x_2+z_2)y_2-x_2y_2),..., \\
&\qquad \overline{\mu}_n((x_n+z_n)y_n-x_ny_n)\} \\
&\leq \max^i\{\overline{\mu}_1(z_1),\overline{\mu}_2(z_2),...,\overline{\mu}_n(z_n)\}=\overline{\mu}(z).
\end{aligned}$$

Therefore $\overline{\mu}$ is an i-v anti fuzzy ideal of R. □

Theorem 3.8. *Let $\overline{\mu}$ be an i-v fuzzy subset of a near-ring R and $\overline{\gamma}$ be the strongest i-v anti fuzzy relation of R with respect to $\overline{\mu}$. Then $\overline{\mu}$ is an i-v anti fuzzy ideal of R if and only if $\overline{\gamma}$ is an i-v anti fuzzy ideal of $R\times R$.*

Proof. Let us assume that $\overline{\mu}$ is an i-v anti fuzzy ideal of R. Let $x_1, y_1, x_2, y_2, z_1, z_2 \in R$. Then for $x = (x_1, x_2), y = (y_1, y_2), z = (z_1, z_2) \in R \times R$, we have

$$\begin{aligned}
\overline{\gamma}(x-y) &= \overline{\gamma}((x_1, x_2) - (y_1, y_2)) \\
&= \overline{\gamma}(x_1 - y_1,\ x_2 - y_2) \\
&= \max{}^i\{\overline{\mu}(x_1 - y_1), \overline{\mu}(x_2 - y_2)\} \\
&\leq \max{}^i\{\max{}^i\{\overline{\mu}(x_1), \overline{\mu}(y_1)\}, \max{}^i\{\overline{\mu}(x_2), \overline{\mu}(y_2)\}\} \\
&= \max{}^i\{\max{}^i\{\overline{\mu}(x_1), \overline{\mu}(x_2)\}, \max{}^i\{\overline{\mu}(y_1), \overline{\mu}(y_2)\}\} \\
&= \max{}^i\{\overline{\gamma}(x_1, x_2), \overline{\gamma}(y_1, y_2)\} \\
&= \max{}^i\{\overline{\gamma}(x), \overline{\gamma}(y)\}.
\end{aligned}$$

Similarly, $\overline{\gamma}(xy) \leq \max^i\{\overline{\gamma}(x), \overline{\gamma}(y)\}$.

$$\begin{aligned}
\overline{\gamma}(y + x - y) &= \overline{\gamma}((y_1, y_2) + (x_1, x_2) - (y_1, y_2)) \\
&= \overline{\gamma}(y_1 + x_1 - y_1, y_2 + x_2 - y_2) \\
&= \max{}^i\{\overline{\mu}(y_1 + x_1 - y_1), \overline{\mu}(y_2 + x_2 - y_2)\} \\
&\leq \max{}^i\{\overline{\mu}(x_1), \overline{\mu}(x_2)\} = \overline{\gamma}(x_1, x_2) = \overline{\gamma}(x). \\
\overline{\gamma}(xy) &= \overline{\gamma}((x_1, x_2)(y_1, y_2)) \\
&= \overline{\gamma}(x_1 y_1, x_2 y_2) \\
&= \max{}^i\{\overline{\mu}(x_1 y_1), \overline{\mu}(x_2 y_2)\} \\
&\leq \max{}^i\{\overline{\mu}(y_1), \overline{\mu}(y_2)\} = \overline{\gamma}(y_1, y_2) = \overline{\gamma}(y).
\end{aligned}$$

$$\begin{aligned}
\overline{\gamma}((x+z)y - xy) &= \overline{\gamma}(((x_1, x_2) + (z_1, z_2))(y_1 y_2) - (x_1, x_2)(y_1, y_2)) \\
&= \overline{\gamma}((x_1 + z_1)y_1 - x_1 y_1, (x_2 + z_2)y_2 - x_2 y_2) \\
&= \max{}^i\{\overline{\mu}((x_1 + z_1)y_1 - x_1 y_1), \overline{\mu}((x_2 + z_2)y_2 - x_2 y_2)\} \\
&\leq \max{}^i\{\overline{\mu}(z_1), \overline{\mu}(z_2)\} = \overline{\gamma}(z_1, z_2) = \overline{\gamma}(z).
\end{aligned}$$

Therefore $\overline{\gamma}$ is an i-v anti fuzzy ideal of $R \times R$.

Conversely, assume that $\overline{\gamma}$ is an i-v anti fuzzy ideal of $R \times R$. Then, for $x = (x_1, x_2),\ y = (y_1, y_2),\ z = (z_1, z_2) \in R \times R$,

$$\begin{aligned}
\max{}^i\{\overline{\mu}(x_1 - y_1), \overline{\mu}(x_2 - y_2)\} &= \overline{\gamma}(x_1 - y_1, x_2 - y_2) \\
&= \overline{\gamma}((x_1, x_2) - (y_1, y_2)) \\
&= \overline{\gamma}(x - y) \\
&\leq \max{}^i\{\overline{\gamma}(x), \overline{\gamma}(y)\} \\
&= \max{}^i\{\overline{\gamma}(x_1, x_2), \overline{\gamma}(y_1, y_2)\} \\
&= \max{}^i\{\max{}^i\{\overline{\mu}(x_1), \overline{\mu}(x_2)\}, \max{}^i\{\overline{\mu}(y_1), \overline{\mu}(y_2)\}\}.
\end{aligned}$$

If $\overline{\mu}(x_1 - y_1) \geq \overline{\mu}(x_2 - y_2)$, then $\overline{\mu}(x_1) \geq \overline{\mu}(x_2)$, $\overline{\mu}(y_1) \geq \overline{\mu}(y_2)$, we get $\overline{\mu}(x_1 - y_1) \leq \max^i\{\overline{\mu}(x_1), \overline{\mu}(y_1)\}$. Similarly, $\overline{\mu}(x_1 y_1) \leq \max^i\{\overline{\mu}(x_1), \overline{\mu}(y_1)\}$.

$$\begin{aligned} \max^i\{\overline{\mu}(y_1 + x_1 - y_1), \overline{\mu}(y_2 + x_2 - y_2)\} &= \overline{\gamma}(y_1 + x_1 - y_1, y_2 + x_2 - y_2) \\ &= \overline{\gamma}((y_1, y_2) + (x_1, x_2) - (y_1, y_2)) \\ &= \overline{\gamma}(y + x - y) \\ &\leq \overline{\gamma}(x) = \overline{\gamma}(x_1, x_2) \\ &= \max^i\{\overline{\mu}(x_1), \overline{\mu}(x_2)\}. \end{aligned}$$

If $\overline{\mu}(y_1 + x_1 - y_1) \geq \overline{\mu}(y_2 + x_2 - y_2)$, then $\overline{\mu}(x_1) \geq \overline{\mu}(x_2)$, we get $\overline{\mu}(y_1 + x_1 - y_1) \leq \overline{\mu}(x_1)$.

$$\begin{aligned} \max^i\{\overline{\mu}(x_1 y_1), \overline{\mu}(x_2 y_2)\} &= \overline{\gamma}(x_1 y_1, x_2 y_2) \\ &= \overline{\gamma}((x_1, x_2)(y_1, y_2)) \\ &= \overline{\gamma}(xy) \\ &\leq \overline{\gamma}(y) = \overline{\gamma}(y_1, y_2) \\ &= \max^i\{\overline{\mu}(y_1), \overline{\mu}(y_2)\}. \end{aligned}$$

If $\overline{\mu}(x_1 y_1) \geq \overline{\mu}(x_2 y_2)$, then $\overline{\mu}(x_1) \geq \overline{\mu}(x_2)$ and $\overline{\mu}(y_1) \geq \overline{\mu}(y_2)$, we get $\overline{\mu}(x_1 y_1) \leq \overline{\mu}(y_1)$.

$$\begin{aligned} &\max^i\{\overline{\mu}((x_1 + z_1)y_1 - x_1 y_1), \overline{\mu}((x_2 + z_2)y_2 - x_2 y_2)\} \\ &\qquad = \overline{\gamma}((x_1 + z_1)y_1 - x_1 y_1, (x_2 + z_2)y_2 - x_2 y_2) \\ &\qquad = \overline{\gamma}(((x_1, x_2) + (z_1, z_2))(y_1 y_2) - (x_1, x_2)(y_1, y_2)) \\ &\qquad = \overline{\gamma}((x + z)y - xy) \\ &\qquad \leq \overline{\gamma}(z) = \overline{\gamma}(z_1, z_2) \\ &\qquad = \max^i\{\overline{\mu}(z_1), \overline{\mu}(z_2)\}. \end{aligned}$$

If $\overline{\mu}((x_1 + z_1)y_1 - x_1 y_1) \geq \overline{\mu}((x_2 + z_2)y_2 - x_2 y_2)$, then $\overline{\mu}(x_1) \geq \overline{\mu}(x_2)$, $\overline{\mu}(y_1) \geq \overline{\mu}(y_2)$ and $\overline{\mu}(z_1) \geq \overline{\mu}(z_2)$, we get $\overline{\mu}((x_1 + z_1)y_1 - x_1 y_1) \leq \overline{\mu}(z_1)$.
Therefore, $\overline{\mu}$ is an i-v anti fuzzy ideal of R. □

In the next theorem we prove the necessary and sufficient condition for an i-v fuzzy subset of a near-ring R to be an i-v anti fuzzy ideal of R.

Theorem 3.9. *An i-v fuzzy subset $\overline{\mu}$ of a near-ring R is an i-v anti fuzzy ideal of R if and only if its complement $\overline{\mu}^c$ is an i-v fuzzy ideal of R.*

Proof. Assume that $\overline{\mu}$ is an i-v anti fuzzy ideal of R. For $x, y, z \in R$

$$\begin{aligned} \overline{\mu}^c(x - y) &= \overline{1} - \overline{\mu}(x - y) \\ &\geq \overline{1} - \max^i\{\overline{\mu}(x), \overline{\mu}(y)\} \\ &= \min^i\{\overline{1} - \overline{\mu}(x), \overline{1} - \overline{\mu}(y)\} \\ &= \min^i\{\overline{\mu}^c(x), \overline{\mu}^c(y)\}. \end{aligned}$$

Similarly, $\overline{\mu}^c(xy) \geq \min^i\{\overline{\mu}^c(x), \overline{\mu}^c(y)\}$ and,

$$\begin{aligned} \overline{\mu}^c(y+x-y) &= \overline{1} - \overline{\mu}(y+x-y) \\ &\geq \overline{1} - \overline{\mu}(x) = \overline{\mu}^c(x). \\ \overline{\mu}^c(xy) &= \overline{1} - \overline{\mu}(xy) \\ &\geq \overline{1} - \overline{\mu}(y) = \overline{\mu}^c(y). \end{aligned}$$

Further, $\overline{\mu}^c((x+z)y - xy) = \overline{1} - \overline{\mu}((x+z)y - xy) \geq \overline{1} - \overline{\mu}(z) = \overline{\mu}^c(z)$.
Therefore, $\overline{\mu}^c$ is an i-v fuzzy ideal of R.
Conversely, Assume that $\overline{\mu}^c$ is an i-v fuzzy ideal of R. Let $x, y, z \in R$. Then

$$\begin{aligned} \overline{1} - \overline{\mu}(x-y) &= \overline{\mu}^c(x-y) \\ &\geq \min^i\{\overline{\mu}^c(x), \overline{\mu}^c(y)\} \\ &= \overline{1} - \max^i\{\overline{1} - \overline{\mu}^c(x), \overline{1} - \overline{\mu}^c(y)\}, \end{aligned}$$

which implies that $\overline{1} - \overline{\mu}(x-y) \geq \overline{1} - \max^i\{\overline{\mu}(x), \overline{\mu}(y)\}$. Thus $\overline{\mu}(x-y) \leq \max^i\{\overline{\mu}(x), \overline{\mu}(y)\}$.
Similarly, $\overline{\mu}(xy) \leq \max^i\{\overline{\mu}(x), \overline{\mu}(y)\}$ and

$$\begin{aligned} \overline{1} - \overline{\mu}(y+x-y) &= \overline{\mu}^c(y+x-y) \\ &\geq \overline{\mu}^c(x) = \overline{1} - \overline{\mu}(x). \end{aligned}$$

Therefore $\overline{\mu}(y+x-y) \leq \overline{\mu}(x)$. Next, $\overline{1} - \overline{\mu}(xy) = \overline{\mu}^c(xy) \geq \overline{\mu}^c(y) = \overline{1} - \overline{\mu}(y)$. Thus $\overline{\mu}(xy) \leq \overline{\mu}(y)$. Further, $\overline{1} - \overline{\mu}((x+z)y - xy) = \overline{\mu}^c((x+z)y - xy) \geq \overline{\mu}^c(z) = \overline{1} - \overline{\mu}(z)$. Therefore $\overline{\mu}((x+z)y - xy) \leq \overline{\mu}(z)$. Thus $\overline{\mu}$ is an i-v anti fuzzy ideal of R. □

Example 3.10. Let $Z_3 = \{0, 1, 2\}$. Addition is modulo 3 and . is defined as follows:

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

·	0	1	2
0	0	0	0
1	0	1	2
2	0	1	2

Then clearly, $(R, +, \cdot)$ is a left near-ring. Let $\overline{\mu} : R \to D[0,1]$ be an i-v fuzzy subset defined by $\overline{\mu}(0) = [0.2, 0.3]$ and $\overline{\mu}(1) = [0.6, 0.7] = \overline{\mu}(2)$. Clearly, $\overline{\mu}$ is an i-v anti fuzzy ideal of R. Further $\overline{\mu}^c(0) = [0.7, 0.8]$ and $\overline{\mu}^c(1) = [0.3, 0.4] = \overline{\mu}^c(2)$. It is easy to check that $\overline{\mu}^c$ is an i-v fuzzy ideal of R.

Theorem 3.11. *Let I be an ideal of R. Then for any $\bar{t} \in D[0,1]$ with $\bar{t} \neq [1,1]$ there exists an i-v anti fuzzy ideal $\overline{\mu}$ of R such that $\overline{L}(\overline{\mu} : \bar{t}) = I$.*

Proof. Let $\overline{\mu} : R \to D[0,1]$ be an i-v fuzzy set of R defined by

$$\overline{\mu}(x) = \begin{cases} \bar{t} & \text{if } x \in I \\ \overline{1} & \text{if } x \notin I. \end{cases}$$

Where $\bar{t}$ is a fixed i-v number in $D[0,1]$ with $\bar{t} \neq [1,1]$ and clearly $\overline{L}(\overline{\mu} : \bar{t}) = I$. Let $x, y \in R$. Suppose that $\overline{\mu}(x-y) > \max^i\{\overline{\mu}(x), \overline{\mu}(y)\}$. Then $\overline{\mu}(x-y) = \overline{1}$ and $\max^i\{\overline{\mu}(x), \overline{\mu}(y)\} = \bar{t}$. This implies that $\overline{\mu}(x) = \bar{t} = \overline{\mu}(y)$ and so $x, y \in I$. Clearly, $x - y \in I$, because I is an ideal of R, which implies that, $\overline{\mu}(x-y) = \bar{t}$. This leads to a contradiction. Thus $\overline{\mu}(x-y) \leq \max^i\{\overline{\mu}(x), \overline{\mu}(y)\}$. Similarly, $\overline{\mu}(xy) \leq \max^i\{\overline{\mu}(x), \overline{\mu}(y)\}$. If there exists $x, y \in R$ such that $\overline{\mu}(y+x-y) > \overline{\mu}(x)$. Then

$\overline{\mu}(y+x-y) = \overline{1}$ and $\overline{\mu}(x) = \overline{t}$. Thus $x \in I$ and so $y+x-y \in I$, which implies that $\overline{\mu}(y+x-y) = \overline{t}$, a contradiction. Hence $\overline{\mu}(y+x-y) \leq \overline{\mu}(x)$, for all $x, y \in R$. Similarly, we have to prove that $\overline{\mu}(xy) \leq \overline{\mu}(y)$ and $\overline{\mu}((x+z)y - xy) \leq \overline{\mu}(z)$, for all $x, y, z \in R$. Therefore, $\overline{\mu}$ is an i-v anti fuzzy ideal of R. □

The final result of this section brings out the connection between the residue class near-ring R/I with respect to an ideal I of R and the i-v fuzzy subset of R/I.

Theorem 3.12. *Let I be an ideal of R. If $\overline{\mu}$ is an i-v anti fuzzy ideal of R, then the i-v fuzzy subset $\overline{\mu}^*$ of R/I defined by $\overline{\mu}^*(a+I) = \inf_{x\in I}{}^i\overline{\mu}(a+x)$ is an i-v anti fuzzy ideal of the residue class near-ring R/I of R with respect to I.*

Proof. Let $a, b \in R$ be such that $a+I = b+I$. Then $b = a+y$ for some $y \in I$ and so $\overline{\mu}^*(b+I) = \inf_{x\in I}{}^i\overline{\mu}(b+x) = \inf_{x\in I}{}^i\overline{\mu}(a+y+x) = \inf_{x+y=z\in I}{}^i\overline{\mu}(a+z) = \overline{\mu}^*(a+I)$. Hence, $\overline{\mu}^*$ is well defined. For any $x+I, y+I, z+I \in R/I$, we have

$$\begin{aligned}
\overline{\mu}^*((x+I)-(y+I)) &= \overline{\mu}^*((x-y)+I) \\
&= \inf_{z\in I}{}^i\overline{\mu}((x-y)+z) \\
&= \inf_{z=u-v\in I}{}^i\{\overline{\mu}((x-y)+(u-v))\} \\
&= \inf_{u,v\in I}{}^i\{\overline{\mu}((x+u)-(y+v))\} \\
&\leq \inf_{u,v\in I}{}^i \max{}^i\{\overline{\mu}(x+u), \overline{\mu}(y+v)\} \\
&= \max{}^i\{\inf_{x\in I}{}^i\overline{\mu}(x+u), \inf_{v\in I}{}^i\overline{\mu}(y+v)\} \\
&= \max{}^i\{\overline{\mu}^*(x+I), \overline{\mu}^*(y+I)\}.
\end{aligned}$$

Similarly, $\overline{\mu}^*((x+I)(y+I)) \leq \max^i\{\overline{\mu}^*(x+I), \overline{\mu}^*(y+I)\}$ and,

$$\begin{aligned}
\overline{\mu}^*((y+I)+(x+I)-(y+I)) &= \overline{\mu}^*((y+x-y)+I) \\
&= \inf_{z\in I}{}^i\overline{\mu}((y+x-y)+z) \\
&= \inf_{z=(v+u-v)\in I}{}^i\overline{\mu}((y+x-y)+(v+u-v)) \\
&= \inf_{u\in I, v\in I}{}^i\overline{\mu}((y+v)+(x+u)-(y+v)) \\
&\leq \inf_{z\in I}{}^i\overline{\mu}(x+u) \\
&= \overline{\mu}^*(x+I).
\end{aligned}$$

Again,

$$\begin{aligned}\overline{\mu}^*((x+I)(y+I)) &= \overline{\mu}^*((xy)+I)\\ &= \inf_{z\in I}{}^i\overline{\mu}((xy)+z)\\ &= \inf_{z=(uv)\in I}{}^i\overline{\mu}((xy)+(uv))\\ &= \inf_{u\in I, v\in I}{}^i\overline{\mu}((x+u)(y+v))\\ &\leq \inf_{v\in I}{}^i\overline{\mu}(y+v)\\ &= \overline{\mu}^*(y+I).\end{aligned}$$

Similarly, $\overline{\mu}^*(((x+I)+(z+I))(y+I)-(x+I)(y+I)) \leq \overline{\mu}^*(z+I)$.
Therefore, $\overline{\mu}^*$ is an i-v anti fuzzy ideal of R/I with respect to I. □

4. Homomorphism of interval valued anti fuzzy ideals

In this section, we give some characterizations of homomorphism of interval valued anti fuzzy ideals of near-rings.

Theorem 4.1. *Let $f : R \to S$ be an onto homomorphism of near-rings.*
(1) If $\overline{\lambda}$ is an i-v fuzzy subset in S, then $f^{-1}(\overline{\lambda}^c) = (f^{-1}(\overline{\lambda}))^c$.
(2) If $\overline{\mu}$ is an i-v fuzzy subset in R, then $f(\overline{\mu}^c) = (f_-(\overline{\mu}))^c$ and $f_-(\overline{\mu}^c) = (f(\overline{\mu}))^c$.

Proof. (1) Suppose $\overline{\lambda}$ is an i-v fuzzy subset in S. For $x \in R$,

$$\begin{aligned}f^{-1}(\overline{\lambda}^c)(x) &= \overline{\lambda}^c(f(x))\\ &= \overline{1}-\overline{\lambda}(f(x))\\ &= \overline{1}-f^{-1}(\overline{\lambda})(x)\\ &= (f^{-1}(\overline{\lambda}))^c(x).\end{aligned}$$

Thus $f^{-1}(\overline{\lambda}^c) = (f^{-1}(\overline{\lambda}))^c$.

(2) Suppose $\overline{\mu}$ is an fuzzy subset in R. Let $x \in R$. Then

$$\begin{aligned} f(\overline{\mu}^c)(y) &= \sup_{x\in f^{-1}(y)}{}^i\overline{\mu}^c(x) \\ &= \sup_{x\in f^{-1}(y)}{}^i(\overline{1}-\overline{\mu}(x)) \\ &= \overline{1}-\inf_{x\in f^{-1}(y)}{}^i\overline{\mu}(x) \\ &= \overline{1}-f_-(\overline{\mu})(y) \\ &= (f_-(\overline{\mu}))^c(y). \\ \text{And } f_-(\overline{\mu}^c)(y) &= \inf_{x\in f^{-1}(y)}{}^i\overline{\mu}^c(x) \\ &= \inf_{x\in f^{-1}(y)}{}^i(\overline{1}-\overline{\mu}(x)) \\ &= \overline{1}-\sup_{x\in f^{-1}(y)}{}^i\overline{\mu}(x) \\ &= \overline{1}-f(\overline{\mu})(y) \\ &= (f(\overline{\mu}))^c(y). \end{aligned}$$

□

Theorem 4.2. *Let $f : R \to S$ be a homomorphism of near-rings R and S. If $\overline{\lambda}$ is an i-v anti fuzzy ideal of S, then $f^{-1}(\overline{\lambda})$ is an i-v anti fuzzy ideal of R.*

Proof. Let $\overline{\lambda}$ is an i-v anti fuzzy ideal of S. Let $x, y, z \in R$. Then

$$\begin{aligned} f^{-1}(\overline{\lambda})(x-y) &= \overline{\lambda}(f(x-y)) \\ &= \overline{\lambda}(f(x)-f(y)) \\ &\leq \max{}^i\{\overline{\lambda}(f(x)),\overline{\lambda}(f(y))\} \\ &= \max{}^i\{f^{-1}(\overline{\lambda})(x), f^{-1}(\overline{\lambda})(y)\}. \end{aligned}$$

Similarly, $f^{-1}(\lambda)(xy) \leq \max^i\{f^{-1}(\overline{\lambda})(x), f^{-1}(\overline{\lambda})(y)\}$. And

$$\begin{aligned} f^{-1}(\overline{\lambda})(y+x-y) &= \overline{\lambda}(f(y+x-y)) \\ &= \overline{\lambda}(f(y)+f(x)-f(y)) \\ &\leq \overline{\lambda}(f(x)) \\ &= f^{-1}(\overline{\lambda})(x). \end{aligned}$$

Again,

$$\begin{aligned} f^{-1}(\overline{\lambda})(xy) &= \overline{\lambda}(f(xy)) \\ &= \overline{\lambda}(f(x)f(y)) \\ &\leq \overline{\lambda}(f(y)) \\ &= f^{-1}(\overline{\lambda})(y). \end{aligned}$$

$$\begin{aligned} f^{-1}(\overline{\lambda})((x+z)y-xy) &= \overline{\lambda}(f((x+z)y-xy)) \\ &= \overline{\lambda}((f(x)+f(z))f(y)-f(x)f(y)) \\ &\leq \overline{\lambda}(f(z)) \\ &= f^{-1}(\overline{\lambda})(z). \end{aligned}$$

Therefore $f^{-1}(\overline{\lambda})$ is an i-v anti fuzzy ideal of R. □

The converse of the Theorem 4.2 with stronger condition on f can be stated as follows.

Theorem 4.3. *Let $f : R \to S$ be an epimorphism of near-rings R and S. If $\overline{\lambda}$ is an i-v fuzzy subset of S, such that $f^{-1}(\overline{\lambda})$ is an i-v anti fuzzy ideal of R then $\overline{\lambda}$ is an i-v anti fuzzy ideal of S.*

Proof. Let $x, y, z \in S$. Then $f(a) = x$, $f(b) = y$ and $f(c) = z$ for some $a, b, c \in R$. It follows that

$$\begin{aligned} \overline{\lambda}(x-y) &= \overline{\lambda}(f(a)-f(b)) = \overline{\lambda}(f(a-b)) \\ &= f^{-1}(\overline{\lambda})(a-b) \\ &\leq \max{}^i\{f^{-1}(\overline{\lambda})(a), f^{-1}(\overline{\lambda})(b)\} \\ &= \max{}^i\{\overline{\lambda}(f(a)), \overline{\lambda}(f(b))\} \\ &= \max{}^i\{\overline{\lambda}(x), \overline{\lambda}(y)\}. \end{aligned}$$

Similarly, $\overline{\lambda}(xy) \leq \max^i\{\overline{\lambda}(x), \overline{\lambda}(y)\}$.

$$\begin{aligned} \overline{\lambda}(y+x-y) &= \overline{\lambda}(f(b)+f(a)-f(b)) \\ &= \overline{\lambda}(f(b+a-b)) \\ &= f^{-1}(\overline{\lambda})(b+a-b) \leq f^{-1}(\overline{\lambda})(a) \\ &= \overline{\lambda}(f(a)) = \overline{\lambda}(x). \\ \overline{\lambda}(xy) &= \overline{\lambda}(f(a)f(b)) = \overline{\lambda}(f(ab)) \\ &= f^{-1}(\overline{\lambda})(ab) \leq f^{-1}(\overline{\lambda})(b) \\ &= \overline{\lambda}(f(b)) = \overline{\lambda}(y). \\ \overline{\lambda}((x+z)y-xy) &= \overline{\lambda}((f(a)+f(c))f(b)-f(a)f(b)) \\ &= \overline{\lambda}(f((a+c)b-ab)) \\ &= f^{-1}(\overline{\lambda})((a+c)b-ab) \\ &\leq f^{-1}(\overline{\lambda})(c) = \overline{\lambda}(f(c)) = \overline{\lambda}(z). \end{aligned}$$

Hence $\overline{\lambda}$ is an i-v anti fuzzy ideal of R. □

Theorem 4.4. *Let $f : R \to S$ be an epimorphism of near-rings R and S. If $\overline{\mu}$ is an i-v anti fuzzy ideal of R, then $f_-(\overline{\mu})$ is an i-v anti fuzzy ideal of S.*

Proof. Let $\overline{\mu}$ be an i-v anti fuzzy ideal of R. Since $f_-(\overline{\mu})(x') = \inf\limits_{f(x)=x'}{}^i(\overline{\mu}(x))$, for $x' \in S$. So $\overline{\mu}$ is nonempty. Let $x', y', z' \in S$. Then we have

$\{x|x \in f^{-1}(x'-y')\} \supseteq \{x-y|x \in f^{-1}(x') \text{ and } y \in f^{-1}(y')\}$ and
$\{x|x \in f^{-1}(x'y')\} \supseteq \{xy|x \in f^{-1}(x') \text{ and } y \in f^{-1}(y')\}$.

$$\begin{aligned} f_-(\overline{\mu})(x'-y') &= \inf_{f(z)=x'-y'}{}^i\{\overline{\mu}(z)\} \leq \inf_{f(x)=x',f(y)=y'}{}^i\{\overline{\mu}(x-y)\} \\ &\leq \inf_{f(x)=x',f(y)=y'}{}^i\{\max{}^i\{\overline{\mu}(x),\overline{\mu}(y)\}\} \\ &= \max{}^i\{\inf_{f(x)=x'}{}^i\{\overline{\mu}(x)\}, \inf_{f(y)=y'}{}^i\{\overline{\mu}(y) \\ &= \max{}^i\{f_-(\overline{\mu})(x'), f_-(\overline{\mu})(y')\}. \end{aligned}$$

Similarly, $f_-(\overline{\mu})(x'y') \leq \max^i\{f_-(\overline{\mu})(x'), f_-(\overline{\mu})(y')\}$.

$$\begin{aligned} f_-(\overline{\mu})(y'+x'-y') &= \inf_{f(z)=y'+x'-y'}{}^i\{\overline{\mu}(z)\} \\ &\leq \inf_{f(x)=x',f(y)=y'}{}^i\{\overline{\mu}(y+x-y)\} \\ &\leq \inf_{f(x)=x'}{}^i\{\overline{\mu}(x)\} = f_-(\overline{\mu})(x'). \\ f_-(\overline{\mu})(x'y') &= \inf_{f(z)=y'+x'-y'}{}^i\{\overline{\mu}(z)\} \\ &\leq \inf_{f(x)=x',f(y)=y'}{}^i\{\overline{\mu}(xy)\} \\ &\leq \inf_{f(y)=y'}{}^i\{\overline{\mu}(y)\} = f_-(\overline{\mu})(y'). \\ f_-(\overline{\mu})((x'+z')y'-x'y') &= \inf_{f(c)=(x'+z')y'-x'y'}{}^i\{\overline{\mu}(c)\} \\ &\leq \inf_{f(x)=x',f(z)=z',f(y)=y'}{}^i\{\overline{\mu}((x+z)y-xy)\} \\ &\leq \inf_{f(z)=z'}{}^i\{\overline{\mu}(z)\} = f_-(\overline{\mu})(z'). \end{aligned}$$

Therefore $f_-(\overline{\mu})$ is an i-v anti fuzzy ideal of S. □

5. Anti-homomorphism of interval valued anti fuzzy ideals

In this section, we characterize of anti-homomorphism of i-v anti fuzzy ideals.

Theorem 5.1. *Let $f: R \to S$ be an anti-homomorphism of near-rings R and S. If $\overline{\lambda}$ is an i-v anti fuzzy ideal of S, then $f^{-1}(\overline{\lambda})$ is an i-v anti fuzzy ideal of R.*

Proof. The proof is straightforward from Theorem 4.2. □

The converse of the Theorem 5.1 with stronger condition on f is stated as follows.

Theorem 5.2. *Let $f: R \to S$ be an anti-epimorphism of near-rings R and S. If $\overline{\lambda}$ is an i-v fuzzy subset of S, such that $f^{-1}(\overline{\lambda})$ is an i-v anti fuzzy ideal of R then $\overline{\lambda}$ is an i-v anti fuzzy ideal of S.*

Proof. The proof is straightforward from Theorem 4.3. □

Theorem 5.3. *Let $f: R \to S$ be an anti-epimorphism of near-rings R and S. If $\overline{\mu}$ is an i-v anti fuzzy ideal of R, then $f(\overline{\mu})$ is an i-v anti fuzzy ideal of S.*

Proof. The proof is straight forward from Theorem 4.4. □

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