Annals of Fuzzy Mathematics and Informatics
Volume 11, No. 3, (March 2016), pp. 425–438
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr



# Application of soft sets in decision making based on game theory

Irfan Deli, Naim Çağman

Received 9 June 2015; Revised 21 August 2015; Accepted 23 September 2015

Abstract. In this work, after given the definition of soft sets and their basic operations we define two person soft games which can apply to problems contain vagueness and uncertainty. We then give four solution methods of the games which are soft saddle points, soft lower and soft upper values, soft dominated strategies and soft Nash equilibrium. We also give an example from the real world. Finally, we extended the two person soft games to n-person soft games.

2010 AMS Classification: 03E72, 08A72

Keywords: Soft sets, Two person soft games, Soft payoff functions, Soft dominated strategies, Soft lower and soft upper values, Soft Nash equilibrium.

Corresponding Author: Irfan Deli (irfandeli@kilis.edu.tr)

## 1. Introduction

In 1999, Molodtsov [29] introduced soft set theory for modeling vagueness and uncertainty. In [29], Molodtsov pointed out several directions for the applications of soft sets, such as stability and regularization, game theory, and operations research and soft analysis. After Molodtsov, works on soft set theory have been progressing rapidly. For instance; on the theory of soft sets [3, 13, 14, 17, 26, 30, 31, 32], on the soft decision making [15, 16, 25, 39, 42], on the algebraic structures of soft sets [2, 33, 35, 36, 37] are some of the selected works.

Game theory is originally the mathematical study of competition and cooperation, in other words, game theory is a study of strategic decision making [27]. Game theory was introduced in 1944 with the publication of von Neumann and Morgenstern [34]. They started modern game theory with the two-person zero-sum games and its proof. Game theory is mainly used in many fields such as; economics, political science, psychology in [5, 44]. Ferguson [20] presented various mathematical models of game theory. Binmore [8] focused the cooperative and noncooperative game theory. Aliprantis and Chakrabarti [4] gave games with decision making.

In 1965, Zadeh [47] developed the theory of fuzzy sets that is the most appropriate theory for dealing with uncertainties. In recent years, many interesting applications of game theory have been expanded by embedding the ideas of fuzzy sets. The two person zero-sum games with fuzzy payoffs and fuzzy goals game theory have been studied by many authors (e.g. [6, 10, 11, 12, 21, 23, 24, 43]). The max-min solution with respect to a degree of attainment of a fuzzy goal has also been studied (e.g. [1, 19, 40, 41, 45, 46]). Many study of game theory have been expanded by using the ideas of interval value (e.g. [19, 22, 28]). Theory for linear programming problems with fuzzy parameters is introduced (e.g. [6, 7, 9]).

In the classical and fuzzy games, the payoff functions are real valued and therefore the solutions of such games are obtained by using arithmetic operations. Especially, fuzzy games depend on the fuzzy set that is described by its membership function. It is mentioned in [29], there exists a difficulty to set the membership function in each particular case and also the fuzzy set operations based on the arithmetic operations with membership functions do not look natural since the nature of the membership function is extremely individual.

In this work, we propose a game model for dealing with uncertainties which is free of the difficulties mentioned above. The proposed new game is called a soft game since it is based on soft sets theory. To construct a soft set we can use any parametrization with the help of words and sentences, real numbers, functions, mappings and so on. Therefore, payoff functions of the soft game are set valued function and solution of the soft games obtained by using the operations of sets that make this game very convenient and easily applicable in practice. The present expository paper is a condensation of part of the dissertation [18].

This work is organized as follows. In the next section, most of the fundamental definitions of the operations of soft sets are presented. In Section 3, we construct two person soft games and then give four solution methods for the games which are soft saddle points, soft lower and soft upper value, soft dominated strategies and soft Nash equilibrium. In section 4, we give an application for two person soft games. In section 5, we give n-person soft games that is extension of the two person soft games. In final Section, we concluded the work.

## 2. Soft sets

In this section, we present the basic definitions and results of soft set theory [13]. More detailed explanations related to this subsection may be found in [13, 26, 29].

**Definition 2.1** ([29])**.** Let $U$ be a universe, $P(U)$ be the power set of $U$ and $E$ be a set of parameters that are describe the elements of $U$. A soft set $S$ over $U$ is a set defined by a set valued function $S$ representing a mapping

$$f_S : E \to P(U).$$

It is noting that the soft set is a parametrized family of subsets of the set $U$ and therefore it can be written a set of ordered pairs

$$S = \{(x, f_S(x)) : x \in E\}.$$

Here, $f_S$ is called approximate function of the soft set $S$ and $f_S(x)$ is called $x$-approximate value of $x \in E$. The subscript $S$ in the $f_S$ indicates that $f_S$ is the approximate function of $S$.

Generally, $f_S$, $f_T$, $f_V$, ... will be used as an approximate functions of $S$, $T$, $V$, ..., respectively.

Note that if $f_S(x) = \emptyset$, then the element $(x, f_S(x))$ is not appeared in $S$.

**Example 2.2.** Suppose that $U = \{u_1, u_2, u_3, u_4\}$ is the universe contains four cars under consideration in an auto agent and $E = \{x_1, x_2, x_3, x_4\}$ is the set of parameters, where $x_i$ $(i = 1, 2, 3, 4)$ stand for 'safety", "cheap", "modern" and "large", respectively.

A customer to select a car from the auto agent, can construct a soft set $S$ that describes the characteristic of cars according to own requests. Assume that $f_S(x_1) = \{u_1, u_2\}$, $f_S(x_2) = \{u_1, u_2, u_4\}$, $f_S(x_3) = \emptyset$, $f_S(x_4) = U$ then the soft-set $S$ is written by

$$S = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2, u_4\}), (x_4, U)\}.$$

By using same parameter set $E$, another customer to select a car from the same auto agent, can construct a soft set $T$ according to own requests. Here $T$ may be different then $S$. Assume that $f_T(x_1) = \{u_1, u_2\}$, $f_T(x_2) = \{u_1, u_2, u_3\}$, $f_T(x_3) = \{u_1, u_2\}$, $f_T(x_4) = \{u_1\}$ then the soft-set $T$ is written by

$$T = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2, u_3\}), (x_3, \{u_1, u_2\}), (x_4, \{u_1\})\}.$$

**Definition 2.3** ([13])**.** Let $S$ and $T$ be two soft sets. Then,

(1) If $f_S(x) = \emptyset$ for all $x \in E$, then $S$ is called a empty soft set, denoted by $S_\Phi$.
(2) If $f_S(x) \subseteq f_T(x)$ for all $x \in E$, then $S$ is a soft subset of $T$, denoted by $S \tilde{\subseteq} T$.

**Definition 2.4** ([13])**.** Let $S$ and $T$ be two soft sets. Then,

(1) Complement of $S$ is denoted by $S^{\tilde{c}}$. Its approximate function $f_{S^{\tilde{c}}}$ is defined by
$$f_{S^{\tilde{c}}}(x) = U \setminus f_S(x) \quad \text{for all } x \in E.$$
(2) Union of $S$ and $T$ is denoted by $S \tilde{\cup} T$. Its approximate function $f_{S \tilde{\cup} T}$ is defined by
$$f_{S \tilde{\cup} T}(x) = f_S(x) \cup f_T(x) \quad \text{for all } x \in E.$$
(3) Intersection of $S$ and $T$ is denoted by $S \tilde{\cap} T$. Its approximate function $f_{S \tilde{\cap} T}$ is defined by
$$f_{S \tilde{\cap} T}(x) = f_S(x) \cap f_T(x) \quad \text{for all } x \in E.$$

## 3. Two person soft games

In this section, we construct two person soft games with soft payoffs. We then give four solution methods for the games. The basic definitions and preliminaries of the game theory we refer to [4, 20, 29, 34, 38, 44].

**Definition 3.1.** Let $X, Y$ are a sets of strategies. A choice of behavior is called an action. The elements of $X \times Y$ are called action pairs. That is, $X \times Y$ is the set of available actions.

**Definition 3.2.** Let $U$ be a set of alternatives, $P(U)$ be the power set of $U$, $X, Y$ are sets of strategies. Then, a set valued function

$$f_S : X \times Y \to P(U)$$

is called a soft payoff function. For each $(x, y) \in X \times Y$, the value $f_S(x, y)$ is called a soft payoff.

**Definition 3.3.** Let $X$ and $Y$ be a set of strategies of Player 1 and 2, respectively, $U$ be a set of alternatives and $f_{S_k} : X \times Y \to P(U)$ be a soft payoff function for player $k$, $(k = 1, 2)$. Then, for each Player $k$, a two person soft game (*tps*-game) is defined by a soft set over $U$ as

$$S_k = \{((x, y), f_{S_k}(x, y)) : (x, y) \in X \times Y\}.$$

The *tps*-game is played as follows : at a certain time Player 1 chooses a strategy $x_i \in X$, simultaneously Player 2 chooses a strategy $y_j \in Y$ and once this is done each player k (k=1,2) receives the soft payoff $f_{S_k}(x_i, y_j)$.

If $X = \{x_1, x_2, ..., x_m\}$ and $Y = \{y_1, y_2, ..., y_n\}$, then the soft payoffs of $S_k$ can be arranged in the form of the $m \times n$ matrix shown in Table 1.

| $S_k$ | $y_1$ | $y_2$ | ... | $y_n$ |
|---|---|---|---|---|
| $x_1$ | $f_{S_k}(x_1, y_1)$ | $f_{S_k}(x_1, y_2)$ | ... | $f_{S_k}(x_1, y_n)$ |
| $x_2$ | $f_{S_k}(x_2, y_1)$ | $f_{S_k}(x_2, y_2)$ | ... | $f_{S_k}(x_2, y_n)$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $x_m$ | $f_{S_k}(x_m, y_1)$ | $f_{S_k}(x_m, y_2)$ | ... | $f_{S_k}(x_m, y_n)$ |

Table 1*: The two person soft game*

Now, we can give an example for *tps*-game.

**Example 3.4.** Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}\}$be a set of alternatives, $P(U)$ be the power set of $U$, $X = \{x_1, x_3, x_5\}$ and $Y = \{x_1, x_2, x_4\}$ be a set of the strategies Player 1 and 2, respectively.

If Player 1 constructs a *tps*-games as follows,

$$\begin{aligned} S_1 = \ & \Big\{((x_1, x_1), \{u_1, u_2, u_5, u_8\}), (x_1, x_2), \{u_1, u_2, u_3, u_4, u_5, u_8\}), (x_1, x_4), \\ & \{u_3, u_8\}), ((x_3, x_1), \{u_1, u_3, u_7\}), (x_3, x_2), \{u_1, u_2, u_3, u_5, u_6, u_7\}), \\ & (x_3, x_4), \{u_1, u_2, u_3\}), ((x_5, x_1), \{u_3, u_4, u_5, u_8\}), (x_5, x_2), \{u_1, u_2, u_3, \\ & u_4, u_5, u_6, u_8\}), (x_5, x_4), \{u_1, u_2, u_3, u_8\})\Big\}, \end{aligned}$$

then the soft payoffs of the game can be arranged as in Table 2.

| $S_1$ | $x_1$ | $x_2$ | $x_4$ |
|---|---|---|---|
| $x_1$ | $\{u_1, u_2, u_5, u_8\}$ | $\{u_1, u_2, u_3, u_4, u_5, u_8\}$ | $\{u_3, u_8\}$ |
| $x_3$ | $\{u_1, u_3, u_7\}$ | $\{u_1, u_2, u_3, u_5, u_6, u_7\}$ | $\{u_1, u_2, u_3\}$ |
| $x_5$ | $\{u_3, u_4, u_5, u_8\}$ | $\{u_1, u_2, u_3, u_4, u_5, u_6, u_8\}$ | $\{u_1, u_2, u_3, u_8\}$ |

*Table 2*

Let us explain some elements of this game; if Player 1 select $x_3$ and Player 2 select $x_2$, then the value of game will be a set $\{u_1, u_2, u_3, u_5, u_6, u_7\}$, that is,

$$f_{S_1}(x_3, x_2) = \{u_1, u_2, u_3, u_5, u_6, u_7\}.$$

In this case, Player 1 wins the set of alternatives $\{u_1, u_2, u_3, u_5, u_6, u_7\}$ and Player 2 lost the same set of alternatives.

Similarly, if Player 2 constructs a *tps*-game as follows,

$$\begin{aligned} S_2 = \ & \Big\{((x_1, x_1), \{u_3, u_4, u_6, u_7\}), (x_1, x_2), \{u_6, u_7\}), (x_1, x_4), \{u_1, u_2, u_4, \\ & u_5, u_6, u_7\}), ((x_3, x_1), \{u_2, u_4, u_5, u_6, u_8\}), (x_3, x_2), \{u_4, u_8\}), (x_3, x_4), \\ & \{u_4, u_5, u_6, u_7, u_8\}), ((x_5, x_1), \{u_1, u_2, u_6 u_7\}), (x_5, x_2), \{u_7\}), \\ & (x_5, x_4), \{u_4, u_5, u_6, u_7\})\Big\}, \end{aligned}$$

then the soft payoffs of the game can be arranged as in Table 3.

| $S_2$ | $x_1$ | $x_2$ | $x_4$ |
|---|---|---|---|
| $x_1$ | $\{u_3, u_4, u_6, u_7\}$ | $\{u_6, u_7\}$ | $\{u_1, u_2, u_4, u_5, u_6, u_7\}$ |
| $x_3$ | $\{u_2, u_4, u_5, u_6, u_8\}$ | $\{u_4, u_8\}$ | $\{u_4, u_5, u_6, u_7, u_8\}$ |
| $x_5$ | $\{u_1, u_2, u_6, u_7\}$ | $\{u_7\}$ | $\{u_4, u_5, u_6, u_7\}$ |

*Table 3*

Let us explain some element of this *tps*-game; if Player 1 select $x_3$ and Player 2 select $x_2$, then the value of game will be a set $\{u_4, u_8\}$, that is,

$$f_{S_2}(x_3, x_2) = \{u_4, u_8\}$$

In this case, Player 1 wins the set of alternatives $\{u_4, u_8\}$ and Player 2 lost $\{u_4, u_8\}$.

**Definition 3.5.** Let $S_k = \{((x, y), f_{S_k}(x, y)) : (x, y) \in X \times Y\}$ be a two person soft game and $(x_i, y_j)$, $(x_r, y_s) \in X \times Y$. Then, Player $k$ is called rational, if the player's soft payoff satisfies the following conditions :

(1) Either $f_{S_k}(x_i, y_j) \supseteq f^k_{X\times Y}(x_r, y_s)$ or $f_{S_k}(x_r, y_s) \supseteq f^k_{X\times Y}(x_i, y_j)$.
(2) When $f_{S_k}(x_i, y_j) \supseteq f^k_{X\times Y}(x_r, y_s)$ and $f_{S_k}(x_r, y_s) \supseteq f^k_{X\times Y}(x_i, y_j)$, then $f_{S_k}(x_i, y_j) = f^k_{X\times Y}(x_r, y_s)$.

**Definition 3.6.** Let $S_k = \{((x, y), f_{S_k}(x, y)) : (x, y) \in X \times Y\}$ be a two person soft game. Then, an action $(x^*, y^*) \in X \times Y$ is called an optimal action if

$$f_{S_k}(x^*, y^*) \supseteq f_{S_k}(x, y) \text{ for all } (x, y) \in X \times Y.$$

**Definition 3.7.** Let $S_k = \{((x, y), f_{S_k}(x, y)) : (x, y) \in X \times Y\}$ be a two person soft game. Then,

(1) if $f_{S_k}(x_i, y_j) \supset f_{S_k}(x_r, y_s)$, we says that a player strictly prefers action pair $(x_i, y_j)$ over action $(x_r, y_s)$,
(2) if $f_{S_k}(x_i, y_j) = f_{S_k}(x_r, y_s)$, we says that a player is indifferent between the two actions,
(3) if $f_{S_k}(x_i, y_j) \supseteq f_{S_k}(x_r, y_s)$, we says that a player either prefers $(x_i, y_j)$ to $(x_r, y_s)$ or is indifferent between the two actions.

**Definition 3.8.** Let $S_k = \{((x, y), f_{S_k}(x, y)) : (x, y) \in X \times Y\}$ be a two person soft game for $k = 1, 2$. Then,

(1) If $f_{S_k}(x, y) = \varnothing$ for all $(x, y) \in X \times Y$, then $S_k$ is called a empty soft game, denoted by $\check{S}_\Phi$.
(2) If $f_{S_k}(x, y) = U$ for all $(x, y) \in X \times Y$, then $S_k$ is called a full soft game, denoted by $\check{S}_E$.

Now the two person zero sum game on the classical game theory will be a two person disjoint game on the soft game theory. It is given in following definition.

**Definition 3.9.** A *tps*-game is called a two person disjoint soft game if intersection of the soft payoff of players is empty set for each action pair.

For instance, Example 3.4 is a two person disjoint soft game.

**Proposition 3.10.** *Let $S_k = \{((x, y), f_{S_k}(x, y)) : (x, y) \in X \times Y\}$ be a two person disjoint soft game for $k = 1, 2$. Then,*

(1) $({S_1}^c)^c = S_1$,
(2) $({S_2}^c)^c = S_2$,
(3) $S_1 \setminus S_2 = S_1$,
(4) $S_2 \setminus S_1 = S_2$,
(5) $S_1 \cap S_2 = \check{S}_\phi$.

*Proof.* Proof is straightforward. □

**Definition 3.11.** A *tps*-game is called a two person universal soft game if union of the soft payoff of players is universal set for each action pair.

For instance, Example 3.4 is a two person universal soft game.

**Proposition 3.12.** *Let $S_k = \{((x, y), f_{S_k}(x, y)) : (x, y) \in X \times Y\}$ be a two person universal soft game for $k = 1, 2$. Then,*

(1) $({S_k}^c)^c = S_k$, $k = 1, 2$
(2) $S_1 \cup S_2 = \check{S}_E$
(3) $({S_1}^c)^c = S_1$,
(4) $({S_2}^c)^c = S_2$,
(5) ${S_1}^c = S_2$,
(6) ${S_2}^c = S_1$.

*Proof.* Proof is straightforward. □

**Proposition 3.13.** *Let $S_k = \{((x, y), f_{S_k}(x, y)) : (x, y) \in X \times Y\}$ be a two person both universal and disjoint soft game for $k = 1, 2$. Then,*

(1) $S_1 \setminus S_2 = S_1$,
(2) $S_2 \setminus S_1 = S_2$,
(3) $S_1 \cap S_2 = \check{S}_\phi$
(4) $S_1 \cup S_2 = \check{S}_E$.

*Proof.* Proof is straightforward. □

**Definition 3.14.** Let $f_{S_k}$ be a soft payoff function of a *tps*-game $S_k$. If the following properties hold :

(1) $\bigcup_{i=1}^{m} f_{S_k}(x_i, y_j) = f_{S_k}(x, y)$,
(2) $\bigcap_{j=1}^{n} f_{S_k}(x_i, y_j) = f_{S_k}(x, y)$.

then $f_{S_k}(x, y)$ is called a a soft saddle point value and $(x, y)$ is called a a soft saddle point of Player k's in the *tps*-game.

Note that if $f_{S_1}(x, y)$ is a soft saddle point of a *tps*-game $S_1$, then Player 1 can then win at least by choosing the strategy $x \in X$ and Player 2 can keep her/his loss to at most $f_{S_1}(x, y)$ by choosing the strategy $y \in Y$. Hence the soft saddle poind is a value of the *tps*-game.

**Example 3.15.** Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}\}$ be a set of alternatives, $X = \{x_1, x_2, x_3, x_4\}$ and $Y = \{y_1, y_2, y_3\}$ be the strategies for Player 1 and 2, respectively. Then, *tps*-game of Player 21 is given as in Table 4.

| $S_1$ | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|
| $x_1$ | $\{u_2, u_4, u_7\}$ | $\{u_4\}$ | $\{u_4\}$ |
| $x_2$ | $\{u_5\}$ | $\{u_7\}$ | $\{u_4, u_7\}$ |
| $x_3$ | $\{u_2, u_4, u_5, u_7, u_8, u_{10}\}$ | $\{u_4, u_8\}$ | $\{u_7, u_8\}$ |
| $x_4$ | $\{u_2, u_4, u_5, u_7, u_8\}$ | $\{u_1, u_4, u_7, u_8\}$ | $\{u_4, u_7, u_8\}$ |

*Table 4*

Clearly,

$$\bigcup_{i=1}^{4} f_{S_1}(x_i, y_1) = \{u_2, u_4, u_5, u_7, u_8, u_{10}\},$$
$$\bigcup_{i=1}^{4} f_{S_1}(x_i, y_2) = \{u_1, u_4, u_7, u_8\},$$
$$\bigcup_{i=1}^{4} f_{S_1}(x_i, y_3) = \{u_4, u_7, u_8\},$$

and

$$\bigcap_{j=1}^{3} f_{S_1}(x_1, y_j) = \{u_4\},$$
$$\bigcap_{j=1}^{3} f_{S_1}(x_2, y_j) = \phi,$$
$$\bigcap_{j=1}^{3} f_{S_1}(x_3, y_j) = \{u_8\},$$
$$\bigcap_{j=1}^{3} f_{S_1}(x_4, y_j) = \{u_4, u_7, u_8\}.$$

Therefore, $\{u_4, u_7, u_8\}$ is a soft saddle point of the *tps*-game, since the intersection of the forth row is equal to the union of the third column. So, the value of the *tps*-game is $\{u_4, u_7, u_8\}$.

Note that every *tps*-game has not a soft saddle point. (For instance, in the above example, if $\{u_4, u_7, u_8\}$ is replaced with $\{u_4, u_7, u_8, u_9\}$ in soft payoff $f_{S_1}(x_4, y_3)$, then a soft saddle point of the game can not be found.) Saddle point can not be used for a *tps*-game, soft upper and soft lower values of the *tps*-game may be used is given in the following definition.

**Definition 3.16.** Let $f_{S_k}$ be a soft payoff function of a *tps*-game $S_k$. Then,

(1) Soft upper value of the *tps*-game, denoted $\overline{v}$, is defined by
$$\overline{v} = \cap_{y \in Y}(\cup_{x \in X}(f_{S_k}(x, y))).$$
(2) Soft lower value of the *tps*-game, denoted $\underline{v}$, is defined by
$$\underline{v} = \cup_{x \in X}(\cap_{y \in Y}(f_{S_k}(x, y))).$$
(3) If soft upper and soft lower value of a *tps*-game are equal, they are called value of the *tps*-game, noted by $v$. That is $v = \underline{v} = \overline{v}$.

**Example 3.17.** Let us consider Table 4 in Example 3.15. It is clear that soft upper value $\overline{v} = \{u_4, u_7, u_8\}$ and soft lower value $\underline{v} = \{u_4, u_7, u_8\}$, hence $\underline{v} = \overline{v}$. It means that value of the *tps*-game is $\{u_4, u_7, u_8\}$.

**Theorem 3.18.** *$\underline{v}$ and $\overline{v}$ be a soft lower and soft upper value of a tps-game, respectively. Then, the soft lower value is subset or equal to the soft upper value, that is,*

$$\underline{v} \subseteq \overline{v}.$$

*Proof.* Assume that $\underline{v}$ be a soft lower value, $\overline{v}$ be a soft upper value of a *tps*-game and $X = \{x_1, x_2, ..., x_m\}$ and $Y = \{y_1, y_2, ..., y_n\}$ are sets of the strategies for Player 1 and 2, respectively.

We choose $x_i^* \in X$ and $y_j^* \in Y$. Then,

$$\begin{aligned} \underline{v} &= \cup_{x\in X}(\cap_{y\in Y}(f_{X\times Y}(x,y))) \\ &\subseteq \cap_{y\in Y}(f_{X\times Y}(x^*,y)) \\ &\subseteq f_{X\times Y}(x^*,y^*) \\ &\subseteq \cup_{x\in X}(f_{X\times Y}(x,y^*)) \\ &\subseteq \cap_{y\in Y}(\cup_{x\in X}(f_{X\times Y}(x,y))), \end{aligned}$$

i.e.:

$$\underline{v} = \cup_{x\in X}(\cap_{y\in Y}(f_{X\times Y}(x,y))) \subseteq \overline{v} = \cap_{y\in Y}(\cup_{x\in X}(f_{X\times Y}(x,y))).$$

The proof is valid. □

**Example 3.19.** Let us consider soft upper value $\overline{v}$ and soft lower value $\underline{v}$ in Example 3.17. It is clear that $\overline{v} = \{u_4, u_7, u_8\} \subseteq \underline{v} = \{u_4, u_7, u_8\}$, hence $\underline{v} \subseteq \overline{v}$.

**Theorem 3.20.** *Let $f_{S_k}(x, y)$ be a soft saddle point, $\underline{v}$ be a soft lower value and $\overline{v}$ be a soft upper value of a tps-game. Then,*

$$\underline{v} \subseteq f_{S_k}(x^*, y^*) \subseteq \overline{v}$$

*Proof.* Assume that $f_{S_k}(x^*, y^*)$ be a soft saddle point, $\underline{v}$ be a soft lower value, $\overline{v}$ be a soft upper value of a *tps*-game and $X = \{x_1, x_2, ..., x_m\}$ and $Y = \{y_1, y_2, ..., y_n\}$ are sets of the strategies for Player 1 and 2, respectively.

We choose $x_i^* \in X$ and $y_j^* \in Y$.

Since $f_{S_k}(x^*, y^*)$ is a soft saddle point, we have

$$\bigcup_{i=1}^{m} f_{S_k}(x_i, y_j) = \bigcap_{j=1}^{n} f_{S_k}(x_i, y_j) = f_{S_k}(x^*, y^*).$$

Clearly,

$$eV_L = \cup_{x\in X}(\cap_{y\in Y}(f_{X\times Y}(x,y))) \subseteq \bigcup_{i=1}^{m} f_{S_k}(x_i, y_j) = f_{S_k}(x^*, y^*) \tag{3.1}$$

and

$$f_{S_k}(x^*, y^*) = \bigcap_{j=1}^{n} f_{S_k}(x_i, y_j) \subseteq eV_U = \cap_{y\in Y}(\cup_{x\in X}(f_{X\times Y}(x,y))). \tag{3.2}$$

Then, from (3.1) and (3.2)

$$eV_L \subseteq f_{X\times Y}(x, y) \subseteq eV_U.$$

The proof is valid. □

**Corollary 3.21.** *Let $f_{S_k}(x,y)$ be a soft saddle point, $\underline{v}$ be a soft lower value and $\overline{v}$ be a soft upper value of a tps-game. If $v = \underline{v} = \overline{v}$, then $f_{S_k}(x,y)$ is exactly $v$.*

**Example 3.22.** Let us consider Table 4 in Example 3.15 and soft upper value $\overline{v}$ and soft lower value $\underline{v}$ in Example 3.17. It is clear that soft saddle point $f_{S_k}(x,y)$ is exactly $v = \underline{v} = \overline{v} = \{u_4, u_7, u_8\}$.

Note that in every *tps*-game, the soft lower value $\underline{v}$ can not be equals to the soft upper value $\overline{v}$. (For instance, in the above example, if $\{u_4, u_7, u_8\}$ is replaced with $\{u_4, u_7, u_8, u_9\}$ in soft payoff $f_{S_1}(x_4, y_3)$, then the soft lower value $\underline{v}$ can not be equals to the soft upper value $\overline{v}$.) If in a *tps*-game $\underline{v} \neq \overline{v}$, then to get the solution of the game soft dominated strategy may be used. We define soft dominated strategy for *tps*-game as follows.

**Definition 3.23.** Let $S_1$ be a *tps*-game with its soft payoff function $f_{S_1}$. Then,

(1) a strategy $x_i \in X$ is called a soft dominated to another strategy $x_r \in X$, if $f_{S_1}(x_i, y) \supseteq f_{S_1}(x_r, y)$ for all $y \in Y$,

(2) a strategy $y_j \in Y$ is called a soft dominated to another strategy $y_s \in Y$, if $f_{S_1}(x, y_j) \subseteq f_{S_1}(x, y_s)$ for all $x \in X$

By using soft dominated strategy, *tps*-games may be reduced by deleting rows and columns that are obviously bad for the player who uses them. This process of eliminating soft dominated strategies sometimes leads us to a solution of a *tps*-game. Such a method of solving *tps*-game is called a soft elimination method.

The following *tps*-game can be solved by using the method.

**Example 3.24.** Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}\}$ be a set of alternatives, $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2, y_3\}$ be the strategies for Player 1 and 2, respectively. Then, *tps*-game of Player 1 is given as in Table 5.

| $S_1$ | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|
| $x_1$ | $\{u_2, u_4, u_7\}$ | $\{u_4\}$ | $\{u_4\}$ |
| $x_2$ | $\{u_5\}$ | $\{u_7\}$ | $\{u_4, u_7\}$ |
| $x_3$ | $\{u_2, u_4, u_5, u_7, u_8, u_{10}\}$ | $\{u_4, u_7, u_8\}$ | $\{u_4, u_7, u_8\}$ |

*Table 5*

The last column is dominated by the middle column. Deleting the last column we can obtain Table 6 as :

| $S_1$ | $y_1$ | $y_2$ |
|---|---|---|
| $x_1$ | $\{u_2, u_4, u_7\}$ | $\{u_4\}$ |
| $x_2$ | $\{u_5\}$ | $\{u_7\}$ |
| $x_3$ | $\{u_2, u_4, u_5, u_7, u_8, u_{10}\}$ | $\{u_4, u_7, u_8\}$ |

*Table 6*

Now, in Table 6, the top row is dominated by the bottom row. (Note that this is not the case in Table 5). Deleting the top row we obtain Table 7 as :

| $S_1$ | $y_1$ | $y_2$ |
|---|---|---|
| $x_2$ | $\{u_5\}$ | $\{u_7\}$ |
| $x_3$ | $\{u_2, u_4, u_5, u_7, u_8, u_{10}\}$ | $\{u_4, u_7, u_8\}$ |

*Table 7*

In Table 7, Player 1 has a soft dominant strategy $x_3$ so that $x_2$ is now eliminated. Player 2 can now choose between $y_1$ and $y_2$ and she/he will clearly choose $y_2$. The solution using the method is $(x_3, y_2)$, that is, value of the *tps*-game is $\{u_4, u_7, u_8\}$.

Note that the soft elimination method cannot be used for some *tps*-games which do not have a soft dominated strategies. In this case, we can use soft Nash equilibrium that is defined as follows.

**Definition 3.25.** Let $S_k$ be a *tps*-game with its soft payoff function $f_{S_k}$ for $k = 1, 2$. If the following properties hold

(1) $f_{S_1}(x^*, y^*) \supseteq f_{S_1}(x, y^*)$ for each $x \in X$
(2) $f_{S_2}(x^*, y^*) \supseteq f_{S_2}(x^*, y)$ for each $y \in Y$

then, $(x^*, y^*) \in X \times Y$ is called a soft Nash equilibrium of a *tps*-game.

Note that if $(x^*, y^*) \in X \times Y$ is a soft Nash equilibrium of a *tps*-game, then Player 1 can then win at least $f_{S_1}(x^*, y^*)$ by choosing strategy $x^* \in X$ and Player 2 can win at least $f_{S_2}(x^*, y^*)$ by choosing strategy $y^* \in Y$. Hence the soft Nash equilibrium is an optimal action for *tps*-game, therefore, $f_{S_k}(x^*, y^*)$ is the solution of the *tps*-game for Player $k$, $k = 1, 2$.

Following game, given in Example 3.26, can be solved by soft Nash equilibrium, but it is very difficult to solve by using the others methods.

**Example 3.26.** Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}\}$ be a set of alternatives, $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2, y_3\}$ be the strategies Player 1 and 2, respectively. Then, *tps*-game of Player 1 is given as in Table 8.

| $S_1$ | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|
| $x_1$ | $\{u_1, u_2, u_4, u_7, u_8, u_9\}$ | $\{u_1, u_2, u_4, u_7, u_8\}$ | $\{u_1, u_2, u_3, u_4, u_7, u_8\}$ |
| $x_2$ | $\{u_1, u_2, u_3, u_5\}$ | $\{u_1, u_4, u_7, u_8\}$ | $\{u_1, u_2, u_3, u_4, u_5, u_7\}$ |
| $x_3$ | $\{u_2, u_5, u_7, u_8, u_{10}\}$ | $\{u_2, u_4, u_7, u_8\}$ | $\{u_4, u_5, u_7, u_8, u_{10}\}$ |

*Table 8*

and *tps*-game of Player 2 is given as in Table 9.

| $S_2$ | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|
| $x_1$ | $\{u_3, u_5, u_6, u_{10}\}$ | $\{u_3, u_5, u_6, u_9, u_{10}\}$ | $\{u_5, u_6, u_9, u_{10}\}$ |
| $x_2$ | $\{u_4, u_6, u_7, u_8, u_9, u_{10}\}$ | $\{u_2, u_3, u_5, u_6, u_9, u_{10}\}$ | $\{u_6, u_8, u_9, u_{10}\}$ |
| $x_3$ | $\{u_1, u_3, u_4, u_6, u_9\}$ | $\{u_1, u_3, u_5, u_6, u_9, u_{10}\}$ | $\{u_1, u_2, u_3, u_6, u_9\}$ |

*Table 9*

From the tables, we have

(1) $f_{S_1}(x_1, y_2) \supseteq f_{S_1}(x, y_2)$ for each $x \in X$, and
(2) $f_{S_2}(x_1, y_2) \supseteq f_{S_2}(x_1, y)$ for each $y \in Y$.

Thus, $(x_1, y_2) \in X \times Y$ is a soft Nash equilibrium. So, $f_{S_1}(x_1, y_2) = \{u_1, u_2, u_4, u_7, u_8\}$ and $f_{S_2}(x_1, y_2) = \{u_3, u_5, u_6, u_9, u_{10}\}$ is the solution of the *tps*-game for Player 1 and Player 2, respectively.

## 4. An application

In this section, we give a financial problem that is solved by using both soft dominated strategy and soft saddle point methods.

There are two companies, say Player 1 and Player 2, who competitively want to increase sale of produces in the country. Therefore, they give advertisements. Assume that two companies have a set of different products $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ where for $i = 1, 2, ..., 8$, the product $u_i$ stand for "oil", "salt",, "honey", " jam", " cheese", "sugar", "cooker" and "jar", respectively. The products can be characterized by a set of strategy $X = Y = \{x_i : i = 1, 2, 3\}$ which contains styles of advertisement where for $j = 1, 2, 3$, the strategies $x_j$ stand for "TV", "radio" and " newspaper", respectively.

Suppose that $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1 = x_1, y_2 = x_2, y_3 = x_3\}$ are strategies of Player 1 and 2, respectively. Then, a *tps*-game of Player 1 is given as in Table 10.

| $S_1$ | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|
| $x_1$ | $\{u_1, u_2, u_3, u_5, u_8\}$ | $\{u_1, u_2, u_3, u_4, u_5, u_8\}$ | $\{u_3\}$ |
| $x_2$ | $\{u_1, u_3, u_7\}$ | $\{u_1, u_2, u_3, u_5, u_6, u_7\}$ | $\{u_2, u_3\}$ |
| $x_3$ | $\{u_1, u_2, u_3, u_4, u_5\}$ | $\{u_1, u_2, u_3, u_4, u_5, u_6, u_8\}$ | $\{u_1, u_2, u_3\}$ |

*Table 10*

In Table 10, let us explain action pair $(x_1, y_1)$; if Player 1 select $x_1 = "TV"$ and Player 2 select $y_1 = "TV"$, then the soft payoff of Player 1 is a set $\{u_1, u_2, u_3, u_5, u_8\}$, that is,

$$f_{S_1}(x_1, y_1) = \{u_1, u_2, u_3, u_5, u_8\}.$$

In this case, Player 1 increase sale of $\{u_1, u_2, u_3, u_5, u_8\}$ and Player 2 decrease sale of $\{u_1, u_2, u_3, u_5, u_8\}$.

We can now solve the game. It is seen in Table 10,

$$\begin{aligned}
\{u_1, u_2, u_3, u_5, u_8\} &\subseteq \{u_1, u_2, u_3, u_4, u_5, u_8\}\\
\{u_1, u_3, u_7\} &\subseteq \{u_1, u_2, u_3, u_5, u_6, u_7\}\\
\{u_1, u_2, u_3, u_4, u_5\} &\subseteq \{u_1, u_2, u_3, u_4, u_5, u_6, u_8\}.
\end{aligned}$$

the middle column is dominated by the right column. We then deleting the middle column we obtain Table 11.

| $S_1$ | $y_1$ | $y_3$ |
|---|---|---|
| $x_1$ | $\{u_1, u_2, u_3, u_5, u_8\}$ | $\{u_3\}$ |
| $x_2$ | $\{u_1, u_3, u_7\}$ | $\{u_2, u_3\}$ |
| $x_3$ | $\{u_1, u_2, u_3, u_4, u_5\}$ | $\{u_1, u_2, u_3\}$ |

*Table 11*

In Table 11, there is no another soft dominated strategy, we can use soft saddle point method.

$$\begin{aligned}
\bigcup_{i=1}^{3} f_{S_1}(x_i, y_1) &= \{u_1, u_2, u_3, u_4, u_5, u_7, u_8\}, \\
\bigcup_{i=1}^{3} f_{S_1}(x_i, y_3) &= \{u_1, u_2, u_3\}, \\
\bigcap_{j=1,3} f_{S_1}(x_1, y_j) &= \{u_3\}, \\
\bigcap_{j=1,3} f_{S_1}(x_2, y_j) &= \{u_3\}, \\
\bigcap_{j=1,3} f_{S_1}(x_3, y_j) &= \{u_1, u_2, u_3\}.
\end{aligned}$$

Here, optimal strategy of the game is $(x_3, y_3)$, since

$$\bigcup_{i=1}^{3} f_{S_1}(x_i, y_3) = \bigcap_{j=1,3} f_{S_1}(x_3, y_j).$$

Therefore, value of the *tps*-game is $\{u_1, u_2, u_3\}$.

## 5. $n$-PERSON SOFT GAMES

In many applications the soft games can be often played between more than two players. Therefore, *tps*-games can be extended to $n$-person soft games.

**Definition 5.1.** Let $U$ be a set of alternatives, $P(U)$ be the power set of $U$ and $X_k$ is the set of strategies of Player $k$, $(k = 1, 2, ..., n)$. Then, for each Player $k$, an $n$-person soft game (*nps*-game) is defined by a soft set over $U$ as

$$S_k^n = \{((x_1, x_2, ..., x_n), f_{S_k^n}(x_1, x_2, ..., x_n)) : (x_1, x_2, ..., x_n) \in X_1 \times X_2 \times ... \times X_n\}$$

where $f_{S_k^n}$ is a soft payoff function of Player $k$.

The *nps*-game is played as follows: at a certain time Player 1 chooses a strategy $x_1 \in X_1$ and simultaneously each Player $k$ $(k = 2, ..., n)$ chooses a strategy $x_k \in X_k$ and once this is done each player $k$ receives the soft payoff $f_{S_k^n}(x_1, x_2, ..., x_n)$.

**Definition 5.2.** Let $S_k^n = \{((x_1, x_2, ..., x_n), f_{S_k^n}(x_1, x_2, ..., x_n)) : (x_1, x_2, ..., x_n) \in X_1 \times X_2 \times ... \times X_n\}$ be an *nps*-game. Then, a strategy $x_k \in X_k$ is called a soft dominated to another strategy $x \in X_k$, if

$$f_{S_k^n}(x_1, ..., x_{k-1}, x_k, x_{k+1}, ..., x_n) \supseteq f_{S_k^n}(x_1, ..., x_{k-1}, x, x_{k+1}, ..., x_n)$$

for each strategy $x_i \in X_i$ of player $i$ $(i = 1, 2, ...k-1, k+1, ..., n)$, respectively.

**Definition 5.3.** Let $S_k^n = \{((x_1, x_2, ..., x_n), f_{S_k^n}(x_1, x_2, ..., x_n)) : (x_1, x_2, ..., x_n) \in X_1 \times X_2 \times ... \times X_n\}$ be an *nps*-game. If for each player $k$ (k=1,2,...,n) the following properties hold

$$f_{S_k^n}(x_1^*, ..., x_{k-1}^*, x_k^*, x_{k+1}^*, ..., x_n^*) \supseteq f_{S_k^n}(x_1^*, ..., x_{k-1}^*, x, x_{k+1}^*, ..., x_n^*)$$

for each $x \in X_k$, then $(x_1^*, x_2^*, ..., x_n^*) \in S_k^n$ is called a soft Nash equilibrium of an *nps*-game.

## 6. Conclusion

In this study, we construct tps-games with soft payofs. We also give four solution methods for the tps-games and give an example which shows the methods can be successfully applied to a financial problem. Finally, we extended the tps-games to n-person soft games.

## References


[1] S. Abbasbandy, E. Babolian and M. Allame, Numerical solution of fuzzy max-min systems, Applied Mathematics and Computation 174 (2006) 1321–1328.

[2] H. Aktaş and N. Çağman, Soft sets and soft groups, Inform. Sci. 177 (2007) 2726–2735.

[3] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, Comput. Math. Appl. 57 (2009) 1547–1553.

[4] C. D. Aliprantis and S. K. Chakrabarti, Games and Decision making, Oxford University Press 2000.

[5] R. J. Aumann, "game theory," Introduction, The New Palgrave Dictionary of Economics, 2nd Edition 2008.

[6] C. R. Bector, S. Chandra and V. Vijay, Duality in linear programming with fuzzy parameters and matrix games with fuzzy pay-offs, Fuzzy Sets and Systems 146 (2004) 253–269.

[7] C. R. Bector, Fuzzy Mathematical Programming and Fuzzy Matrix Games, Springer-Verlag Berlin Heidelberg 2005.

[8] K. Binmore, Fun and Games, A Text on Game Theory, Chancellor Press, London 1982.

[9] L. Campos, Fuzzy linear programming models to solve fuzzy matrix games, Fuzzy Sets and Systems 32 (1989) 275–289.

[10] A. C. Cevikel and M. Ahlatçıoğlu, A New Solution Concept in Fuzzy Matrix Games, World Applied Sciences Journal 7 (7) (2009) 866–871.

[11] A. C. Cevikel and M. Ahlatçıoğlu, Solutions for fuzzy matrix games , Computers and Mathematics with Applications 60 (3) (2010) 399–410.

[12] Y. W. Chena and M. Larbanib , Two-person zero-sum game approach for fuzzy multiple attribute decision making problems, Fuzzy Sets and Systems 157 (2006) 34–51.

[13] N. Çağman and S. Enginoglu, Soft set theory and uni-int decision making, Eur. J. Oper. Res. 207 (2010) 848–855.

[14] N. Çağman and S. Enginoglu, Soft matrix theory and its decision making, Comput. Math. Appl. 59 (2010) 3308–3314.

[15] N. Çağman and I. Deli, Means of FP-Soft Sets and its Applications, Hacettepe Journal of Mathematics and Statistics 41 (5) (2012) 615–625.

[16] N. Çağman and I. Deli, Product of FP-Soft Sets and its Applications, Hacettepe Journal of Mathematics and Statistics 41 (3) (2012) 365–374.

[17] I. Deli and S. Broumi, Neutrosophic soft relations and some properties, Ann. Fuzzy Math. Inform. 9 (1) (2015) 169–182.

[18] Deli, I. Applications of Soft And Fuzzy Soft Sets to Game Theory (In Turkish), PhD Thesis, Gaziosmanpasa University, Graduate School of Natural and Applied Science 2013.

[19] C. Dexin, C. Meirong, W. Haijun and J. Jianxiong, Interval method for global solutions of a class of min-max-min problems, Applied Mathematics and Computation 196 (2008) 594–602.

[20] T. S. Ferguson, Game Theory, UCLA 2008.

[21] M. Larbani, Solving bimatrix games with fuzzy payoffs by introducing Nature as a third player, Fuzzy Sets and Systems 160 (2009) 657–666.

[22] S. T Liu and C. Kao, Matrix games with interval data, Computers and Industrial Engineering 56 (2009) 1697–1700.

[23] T. Maeda, On characterization of equilibrium strategy of two person zero-sum games with fuzzy payoffs, Fuzzy Sets and Systems 139 (2003) 283–296.

[24] T. Maeda, Characterization of the Equilibrium Strategy of the Bimatrix Game with Fuzzy Payoff, J. Math. Anal. Appl. 251 (2000) 885–896.

[25] P. K. Maji, A. R. Roy and R. Biswas, An application of soft sets in a decision making problem, Computers and Mathematics with Applications 44 (2002) 1077–1083.
[26] P. K. Maji, R. Bismas and A. R. Roy, Soft set theory, Computers and Mathematics with Applications 45 (2003) 555–562.
[27] R. B. Myerson, Game Theory: Analysis of Conflict, Harvard University Press 1991.
[28] L. Mallozzi, V. Scalzo and S. Tijs, Fuzzy interval cooperative games, Fuzzy Sets and Systems 165 (2011) 98–105.
[29] D. A. Molodtsov, Soft set theory-first results, Computers and Mathematics with Applications 37 (1999) 19–31.
[30] D. A. Molodtsov, The description of a dependence with the help of soft sets, J. Comput. Sys. Sc. Int. 40 (6) (2001) 977–984.
[31] D. A. Molodtsov, The Theory of Soft Sets (in Russian), URSS Publishers, Moscow 2004.
[32] D. A. Molodtsov, V. Yu. Leonov and D. V. Kovkov, Soft sets sechnique and its application, Nechetkie Sistemi i Myakie Vychisleniya, 1 (1) (2006) 8–39.
[33] Sk. Nazmul and S. K. Samanta, On soft multigroups, Ann. Fuzzy Math. Inform. 10 (2) (2015) 271–285.
[34] J. V. Neumann and O. Morgenstern, The Theory of Games and Economic Behavior, Princeton University Press 1944.
[35] A. Sezgin and A. O. Atagün, On operations of soft sets, Comput. Math. Appl. 61/5 (2011) 1457-1467.
[36] A. Sezgin, A. O. Atagün and N. Çağman, Soft intersection near-rings with its applications, Neural Computing and Applications 21 (2012) 221–229.
[37] A. Sezgin, A. O. Atagün and N. Çağman, Union soft substructures of near-rings and N-groups, Neural Computing and Applications 21 (2012) 133–143.
[38] G. Owen, Game theory, San Diego, Academic Press 1995.
[39] Rajarajeswari and P. Dhanalakshmi, An application of interval valued intuitionistic fuzzy soft matrix theory in medical diagnosis, Ann. Fuzzy Math. Inform. 9 (3) (2015) 463–472.
[40] B. Rustem and R. G. Becker, W. Marty, Robust min-max portfolio strategies for rival forecast and risk scenarios, Journal of Economic Dynamics and Control 24 (2000) 1591–1621.
[41] M. Sakawa and I. Nishizaki, Max-min solutions for fuzzy multiobjective matrix games, Fuzzy Sets and Systems 67 (1994) 53–69.
[42] R. Thakur and S. K. Samanta, Soft Banach Algebra, Ann. Fuzzy Math. Inform. 10 (3) (2015) 397–412.
[43] V. Vijaya, S. Chandraa and C. R. Bectorb, Matrix games with fuzzy goals and fuzzy payoffs, Omega 33 (2005) 425–429.
[44] J. N. Webb, Game Theory Decisions, Interaction and Evolution, Springer Undergraduate Mathematics Ser. 2006.
[45] C. T. Yeh, On the minimal solutions of max-min fuzzy relational equations, Fuzzy Sets and Systems 159 (2008) 23–39.
[46] Y. Yoshida, A Minimax Theorem for Zero-Sum Stopping Games in Dynamic Fuzzy Systems, Mathematical and Computer Modelling 29 (1999) 19–33.
[47] L. A. Zadeh, Fuzzy Sets, Information and Control 8 (1965) 338–353.

Irfan Deli (irfandeli@kilis.edu.tr)
Muallim Rıfat Faculty of Education, 7 Aralık University, 79000 Kilis, Turkey

Naim Çağman (naim.cagman@gop.edu.tr)
Department of Mathematics, Faculty of Arts and Sciences, Gaziosmanpaşa University, 60250 Tokat, Turkey