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# Soft intersectional ideals of Γ-semirings

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Abstract. The aim of this paper is to investigate the ideal theory of Γ-semirings based on intersectional soft sets. Some basic operations are investigated and some related properties are also studied. Finally, some characterizations of regular and intra-regular Γ-semirings are obtained by means of soft intersectional ideals (bi-ideals, quasi-ideals).

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## 1. Introduction

The traditional classical models often fail to overcome the complexities arising in the modeling of uncertain data in many fields like economics, engineering, environmental science, sociology, medical science etc. In order to overcome these difficulties, Molodtsov [18] introduced the concept of soft sets as a new mathematical tool for dealing with uncertainties. Maji et al. [16] discussed further soft set theory. Ali et al. [2] proposed some new operations on soft sets. As a continuation, several authors (for example, see [5, 6, 17]) described the application of soft set theory to decision making problems. Recently, the algebraic structures of soft sets [7, 24] have been studied increasingly, such as soft groups [19, 25], soft-int groups [4], soft-int semigroups [12, 27], soft rings [1, 3], soft semirings [8, 14, 15, 29, 30], soft near-rings [26, 28] and so on.

Semirings which are regarded as a generalization of rings have been found useful for dealing with problems in different areas of applied mathematics and information sciences, as the semiring structure provides an algebraic framework for modeling and investigating the key factors in these problems. We know that ideals of the semiring [9, 11] plays a central role in the structure theory and useful for many purposes and they do not in general coincide with the usual ring ideals and so many results in ring theory have no analogues in semirings using only ideals. In [20], Nobusawa

introduced the notion of Γ-ring as more general than ring. After this many mathematicians, for example [10, 13, 21], made works on this subject. As a continuation of this Rao [22] introduced the concept of Γ-semiring which is a generalization of semiring as well as Γ-ring. To make Γ-semiring more effective several authors investigated some of its related properties.
In this paper, we have investigated the ideal theory of Γ-semirings [23] based on intersectional soft sets and obtain some related properties. Some basic operations are also investigated. Finally, we describe some characterizations of regular and intra-regular Γ-semirings by means of soft intersectional ideals.

## 2. Preliminaries

We recall the following definitions for subsequent use.

**Definition 2.1** ([22])**.** Let $S$ and $\Gamma$ be two additive commutative semigroups with zero. Then $S$ is called a Γ-semiring if there exists a mapping $S \times \Gamma \times S \to S$ ( (a,α,b) $\mapsto$ aαb) satisfying the following conditions:

(i) $(a+b)\alpha c = a\alpha c + b\alpha c$
(ii) $a\alpha(b+c) = a\alpha b + a\alpha c$
(iii) $a(\alpha+\beta)b = a\alpha b + a\beta b$
(iv) $a\alpha(b\beta c) = (a\alpha b)\beta c$
(v) $0_S\alpha a = 0_S = a\alpha 0_S$
(vi) $a 0_\Gamma b = 0_S = b 0_\Gamma a$

where $a, b, c \in S$, $\alpha, \beta \in \Gamma$, $0_S$ is the zero element of $S$, $0_\Gamma$ is the zero element of $\Gamma$. For simplification we write 0 instead of $0_S$ and $0_\Gamma$.

**Definition 2.2** ([22])**.** A left ideal $I$ of Γ-semiring $S$ is a nonempty subset of $S$ satisfying the following conditions:

(i) If $a, b \in I$ then $a + b \in I$
(ii) If $a \in I$, $s \in S$ and $\gamma \in \Gamma$ then $s\gamma a \in I$
(iii) $I \neq S$.

A right ideal of $S$ is defined in an analogous manner and an ideal of $S$ is a nonempty subset which is both a left ideal and a right ideal of $S$.

**Definition 2.3** ([18])**.** Let $U$ be an initial universe set and $E$ be a set of parameters. Let $P(U)$ denotes the power set of $U$. Consider a nonempty set $A$, $A \subseteq E$. A pair $(f, A)$ is called a soft set over $U$, where $f$ is a mapping given by $f : A \to P(U)$.
It is clear to see that a soft set is a parameterized family of subsets of the set $U$. Note that the set of all soft sets over $U$ will be denoted by $S(U)$.

**Definition 2.4** ([1])**.** Let $(f, A)$ be a soft set. The set $Supp(f, A) = \{x \in A | f(x) \neq \phi\}$ is called the support of the soft set $(f, A)$. The soft set is said to be non-empty if its support is not equal to the empty set.

**Definition 2.5** ([6])**.** Let $f, g \in S(U)$, then

(i) The intersection of $f$ and $g$, denoted by $f \cap g$, is defined as $(f \cap g)(x) = f(x) \cap g(x)$ for all $x \in E$.
(ii) The union of $f$ and $g$, denoted by $f \cup g$, is defined as $(f \cup g)(x) = f(x) \cup g(x)$ for all $x \in E$.

(iii) The $\wedge$-product of $f$ and $g$, denoted by $f \wedge g$, is defined as $(f \wedge g)(x, y) = f(x) \cap g(y)$ for all $x,\ y \in E$.
(iv) The $\vee$-product of $f$ and $g$, denoted by $f \vee g$, is defined as $(f \vee g)(x, y) = f(x) \cup g(y)$ for all $x,\ y \in E$.

**Definition 2.6.** Let $A \subseteq E$, the set of parameters and $U$ be the universe. We denote $\chi_A$, the soft characteristic function of $A$ and define as

$$\begin{aligned} \chi_A(x) \quad &= U \text{ if } x \in A \\ &= \phi \text{ if } x \notin A. \end{aligned}$$

## 3. Soft intersectional ideals with some operations

Throughout this paper unless otherwise mentioned $S$ denotes the $\Gamma$-semiring, $\chi_S$ denotes its characteristic function.

**Definition 3.1.** Let $f$ and $g$ be two soft sets of an $\Gamma$-semiring $S$ over $U$. We define composition of $f$ and $g$ as follows:

$$\begin{aligned} fog(x) \quad &= \cup[\bigcap_i\{\cap\{f(a_i), f(c_i), g(b_i), g(d_i)\}\}] \\ &\quad x+\sum_{i=1}^{n} a_i\alpha_i b_i = \sum_{i=1}^{n} c_i\beta_i d_i \\ &= \phi,\ \text{if } x \text{ cannot be expressed as above} \\ &\quad \text{where } x, z, a_i, b_i, c_i, d_i \in S \text{ and } \alpha_i, \beta_i \in \Gamma. \end{aligned}$$

**Definition 3.2.** Let $f$ be a non-empty soft set of an $\Gamma$-semiring $S$ (i.e. $f(x) \neq \phi$ for some $x \in S$) over $U$. Then $f$ is called a soft intersectional left ideal [resp. soft intersectional right ideal] of $S$ over $U$ if

(i) $f(x + y) \supseteq f(x) \cap f(y)$
(ii) $f(x\gamma y) \supseteq f(y)$ [resp. $f(x\gamma y) \supseteq f(x)$]

for all $x, y \in S$ and $\gamma \in \Gamma$.
Note that for soft intersectional $k$-ideal the following additional relation must holds: For $x, a, b \in S$ with $x + a = b \Rightarrow f(x) \supseteq f(a) \cap f(b)$.

**Example 3.3.** Let $U = S = Z_4 = \{0, 1, 2, 3\}$ and $\Gamma = Z_2 = \{0, 1\}$. Then $S$ forms a $\Gamma$-semiring with respect to usual addition and multiplication of integers. Define $f(0) = f(2) = \{0, 1, 2, 3\}$ and $f(1) = f(3) = \{0, 2\}$. Then $f$ is a soft intersectional ideal of $S$ over $U$.

**Example 3.4.** Let $U = Z^+$, $S = Z_4 = \{0, 1, 2, 3\}$ and $\Gamma = Z_2 = \{0, 1\}$. Define soft set $f$ of $S$ over $U$ by $f(0) = \{n|n \in Z^+\}$, $f(1) = f(3) = \{4n|n \in Z^+\}$ and $f(2) = \{2n|n \in Z^+\}$. Then we can easily check that $f$ is a soft intersectional $k$-ideal of $S$ over $U$.

**Proposition 3.5.** *Intersection of a non-empty collection of soft intersectional left (resp. right) ideals is also a soft intersectional left (resp. right) ideal of $S$ over $U$.*

*Proof.* Let $\{f_i : i \in I\}$ be a non-empty family of soft intersectional left ideals of $S$ over $U$ and $x, y \in S$, $\gamma \in \Gamma$.

Then

$$\begin{aligned}(\bigcap_{i\in I} f_i)(x+y) &= \bigcap_{i\in I}\{f_i(x+y)\} \supseteq \bigcap_{i\in I}\{f_i(x)\cap f_i(y)\}\\ &= \cap\{\bigcap_{i\in I} f_i(x), \bigcap_{i\in I} f_i(y)\} = (\bigcap_{i\in I} f_i)(x) \cap (\bigcap_{i\in I} f_i)(y).\end{aligned}$$

Again

$$(\bigcap_{i\in I} f_i)(x\gamma y) = \bigcap_{i\in I}\{f_i(x\gamma y)\} \supseteq \bigcap_{i\in I} f_i(y) = (\bigcap_{i\in I} f_i)(y).$$

Hence $\bigcap_{i\in I} f_i$ is a soft intersectional left ideal of $S$.
Similarly we can prove the result for soft intersectional right ideal also. □

**Theorem 3.6.** *Let $f$ and $g$ be soft intersectional left ideals of an $\Gamma$-semiring $S$ over $U$. Then $f \wedge g$ is a soft intersectional left ideal of $S \times S$ over $U$.*

*Proof.* Let $(x_1, x_2), (y_1, y_2) \in S \times S$ and $\gamma \in \Gamma$. Then

$$\begin{aligned}(f\wedge g)((x_1,x_2)+(y_1,y_2)) &= (f\wedge g)(x_1+y_1, x_2+y_2)\\ &= (f(x_1+y_1)\cap g(x_2+y_2))\\ &= (f(x_1+y_1))\cap(g(x_2+y_2))\\ &\supseteq (f(x_1)\cap f(y_1))\cap(g(x_2)\cap g(y_2))\\ &= (f(x_1)\cap g(x_2))\cap(f(y_1)\cap g(y_2))\\ &= (f\wedge g)(x_1,x_2)\cap(f\wedge g)(y_1,y_2)\end{aligned}$$

and

$$\begin{aligned}(f\wedge g)((x_1,x_2)\gamma(y_1,y_2)) &= (f\wedge g)(x_1\gamma y_1, x_2\gamma y_2)\\ &= (f(x_1\gamma y_1)\cap g(x_2\gamma y_2))\\ &= (f(x_1\gamma y_1))\cap(g(x_2\gamma y_2))\\ &\supseteq f(y_1)\cap g(y_2) = (f\wedge g)(y_1,y_2).\end{aligned}$$

Therefore $f \wedge g$ is a soft intersectional left ideal of $S \times S$ over $U$. □

**Theorem 3.7.** *Let $f$ be a soft set of an $\Gamma$-semiring $S$ over $U$. Then $f$ is a soft intersectional left ideal of $S$ if and only if $f \wedge f$ is a soft intersectional left ideal of $S \times S$ over $U$.*

*Proof.* Assume that $f$ is a soft intersectional left ideal of $S$ over $U$. Then by Theorem 3.6, $f \wedge f$ is a soft intersectional left ideal of $S \times S$ over $U$.
Conversely, suppose that $f \wedge f$ is a soft intersectional left ideal of $S \times S$ over $U$. Let $x_1, x_2, y_1, y_2 \in S$ and $\gamma \in \Gamma$. Then

$$\begin{aligned}(f(x_1+y_1)\cap f(x_2+y_2)) &= (f\wedge f)(x_1+y_1, x_2+y_2)\\ &= (f\wedge f)((x_1,x_2)+(y_1,y_2))\\ &\supseteq (f\wedge f)(x_1,x_2)\cap(f\wedge f)(y_1,y_2)\\ &= (f(x_1)\cap f(x_2))\cap(f(y_1)\cap f(y_2)).\end{aligned}$$

Now, putting $x_1 = x,\ x_2 = 0,\ y_1 = y$ and $y_2 = 0$, in this inequality and noting that $f(0) \supseteq f(x)$ for all $x \in S$, we obtain $f(x+y) \supseteq f(x) \cap f(y)$.
Next, we have

$$\begin{aligned}(f(x_1\gamma y_1)\cap f(x_2\gamma y_2)) &= (f\wedge f)(x_1\gamma y_1, x_2\gamma y_2)\\ &= (f\wedge f)((x_1,x_2)\gamma(y_1,y_2))\\ &\supseteq (f\wedge f)(y_1,y_2)\\ &= f(y_1)\cap f(y_2).\end{aligned}$$

Taking $x_1 = x,\ y_1 = y$ and $y_2 = 0$, we obtain $f(x\gamma y) \supseteq f(y)$.
Hence $f$ is a soft intersectional left ideal of $S$ over $U$. □

**Theorem 3.8.** *Let $f_1,\ f_2$ be any two soft intersectional $k$-ideals of $S$ over $U$. Then $f_1 o f_2$ is a soft intersectional ideal of $S$ over $U$.*

*Proof.* Let $f_1,\ f_2$ be any two soft intersectional $k$-ideals of $S$ over $U$ and $x,\ y \in S$ and $\gamma \in \Gamma$. Then

$$\begin{aligned}
&(f_1 o f_2)(x+y)\\
&= \bigcup_{x+y+\sum a_i\alpha_i b_i=\sum c_i\beta_i d_i} \{\bigcap_i \{f_1(a_i), f_2(b_i),\ f_1(c_i), f_2(d_i)\}\}\\
&\supseteq \bigcup_{x+y+\sum x_{1i}\alpha_{1i}x_{2i}+\sum y_{1i}\alpha_{2i}y_{2i}=\sum x_{3i}\beta_{1i}x_{4i}+\sum y_{3i}\beta_{2i}y_{4i}} \{\bigcap_i \{f_1(x_{1i}), f_1(x_{3i}), f_1(y_{1i}), f_1(y_{3i}), \quad f_2(x_{2i}), f_2(x_{4i}), f_2(y_{2i}), f_2(y_{4i})\}\}\\
&\supseteq \cap\{ \bigcup_{x+\sum x_{1i}\alpha_{1i}x_{2i}=\sum x_{3i}\beta_{1i}x_{4i}} \{\bigcap_i \{f_1(x_{1i}), f_1(x_{3i}),\ \ f_2(x_{2i}), f_2(x_{4i})\}\},\ \bigcup_{y+\sum y_{1i}\alpha_{2i}y_{2i}=\sum y_{3i}\beta_{2i}y_{4i}} \{\bigcap_i \{f_1(y_{1i}), f_1(y_{3i}),\ \ f_2(y_{2i}), f_2(y_{4i})\}\}\}\\
&= (f_1 o f_2)(x) \cap (f_1 o f_2)(y).
\end{aligned}$$

Now assuming $f_1,\ f_2$ are as soft intersectional right ideals we have

$$\begin{aligned}
(f_1 o f_2)(x\gamma y) &= \bigcup_{x\gamma y+\sum a_i\gamma_i b_i=\sum c_i\delta_i d_i} \{\bigcap_i \{f_1(a_i), f_2(b_i),\ f_1(c_i), f_2(d_i)\}\}\\
&\supseteq \bigcup_{x\gamma y+\sum x_{1i}\alpha_i x_{2i}\gamma y=\sum x_{3i}\beta_i x_{4i}\gamma y} \{\bigcap_i \{f_1(x_{1i}), f_1(x_{3i}), \quad f_2(x_{2i}\gamma y), f_2(x_{4i}\gamma y)\}\}\\
&\supseteq \bigcup_{x+\sum x_{1i}\alpha_i x_{2i}=\sum x_{3i}\beta_i x_{4i}} \{\bigcap_i \{f_1(x_{1i}), f_1(x_{3i}),\ f_2(x_{2i}), f_2(x_{4i})\}\}\\
&= (f_1 o f_2)(x).
\end{aligned}$$

Similarly, assuming $f_1,\ f_2$ are as soft intersectional left $k$-ideals we can show that $(f_1 o f_2)(x\gamma y) \supseteq (f_1 o f_2)(y)$.
Hence $f_1 o f_2$ is a soft intersectional ideal of $S$ over $U$. □

## 4. Soft intersectional ideals of regular Γ-semiring

**Definition 4.1.** A soft set $f$ of a Γ-semiring $S$ over $U$ is called soft intersectional bi-ideal if for all $x, y \in S$ and $\alpha,\ \beta \in \Gamma$ we have

(i) $f(x+y) \supseteq f(x) \cap f(y)$
(ii) $f(x\alpha y) \supseteq f(x) \cap f(y)$
(iii) $f(x\alpha y\beta z) \supseteq f(x) \cap f(z)$

**Definition 4.2.** A soft set $f$ of a Γ-semiring $S$ over $U$ is called soft intersectional quasi-ideal if for all $x, y \in S$ we have

(i) $f(x+y) \supseteq f(x) \cap f(y)$
(ii) $((f o \chi_S) \cap (\chi_S o f))(x) \subseteq f(x)$

**Proposition 4.3.** *Intersection of a non-empty collection of soft intersectional bi-ideals of $S$ over $U$ is also a soft intersectional bi-ideal of $S$ over $U$.*

*Proof.* The proof follows by routine verifications. □

**Proposition 4.4.** *Let $\{f_i : i \in I\}$ be a family of soft intersectional bi-ideals of $S$ over $U$ such that $f_i \subseteq f_j$ or $f_j \subseteq f_i$ for $i, j \in I$. Then $\underset{i\in I}{\cup} f_i$ is a soft intersectional bi-ideal of $S$ over $U$.*

*Proof.* Straightforward. □

**Lemma 4.5.** *In a $\Gamma$-semiring every soft intersectional quasi ideals are soft intersectional bi-ideals.*

*Proof.* Let $f$ be a soft intersectional quasi ideal of $S$ over $U$. It is sufficient to prove that $f(x\alpha y\beta z) \supseteq f(x) \cap f(z)$ for all $x,\ y,\ z \in S$ and $\alpha,\ \beta \in \Gamma$. Since $f$ is a soft intersectional quasi ideal of $S$ over $U$, we have

$$\begin{aligned}
f(x\alpha y\beta z) &\supseteq ((f o\chi_S) \cap (\chi_S o f))(x\alpha y\beta z) \\
&= \{(f o\chi_S)(x\alpha y\beta z) \cap (\chi_S o f)(x\alpha y\beta z)\} \\
&= \cap\{ \bigcup_{x\alpha y\beta z+\sum_{i=1}^{n} a_i\gamma_i b_i = \sum_{i=1}^{n} c_i\delta_i d_i} (\bigcap_i (f(a_i) \cap f(c_i))) , \bigcup_{x\alpha y\beta z+\sum_{i=1}^{n} a_i\gamma_i b_i = \sum_{i=1}^{n} c_i\delta_i d_i} (\bigcap_i (f(b_i) \cap f(d_i))) \} \\
&\supseteq f(0) \cap f(x) \cap f(0) \cap f(z))(\text{since } x\alpha y\beta z + 0\gamma 0 + 0 = x\alpha y\beta z + 0) \\
&= f(x) \cap f(z).
\end{aligned}$$

Similarly, we can show that $f(x\alpha y) \supseteq f(x) \cap f(y)$ for all $x, y \in S$ and $\alpha \in \Gamma$. □

**Example 4.6.** Let $U = Z,\ S = \Gamma = \{0, a, b\}$. Then $S$ forms a $\Gamma$-semiring with respect to addition and multiplication defined as:

| + | 0 | $a$ | $b$ |
|---|---|---|---|
| 0 | 0 | $a$ | $b$ |
| $a$ | $a$ | $a$ | $a$ |
| $b$ | $b$ | $a$ | $b$ |

and

| . | 0 | $a$ | $b$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| $a$ | 0 | $a$ | $a$ |
| $b$ | 0 | $a$ | $a$ |

Now define soft set $f$ on $S$ over $U$ as: $f(0) = Z,\ f(a) = 2Z,\ f(b) = 4Z$. Then $f$ is a soft intersectional bi-ideal of $S$ over $U$ which is not a soft intersectional quasi-ideal because $b$ has three representations: $b + 0 = b$, $b + b = b$ and $b + a = a$. For the first two cases we have nothing to show as $b$ cannot be expressed as $b = xyz$ for $x, z \in S$ and $y \in \Gamma$. For the last case, the expression can be written as $b + a \cdot a \cdot a = a \cdot a \cdot a$ and for this $f(b) = 4Z \not\supseteq 2Z = ((f o\chi_S) \cap (\chi_S o f))(b)$.

**Definition 4.7.** A $\Gamma$-semiring $S$ is said to be $k$-regular if for each $x \in S$, there exist $a, b \in S$ and $\alpha, \beta, \gamma, \delta \in \Gamma$ such that $x + x\alpha a\beta x = x\gamma b\delta x$.

**Definition 4.8.** A $\Gamma$-semiring $S$ is said to be $k$-intra-regular if for each $x \in S$, there exist $z, a_i, a_i^{'}, b_i, b_i^{'} \in S$, $\alpha_{1i}, \alpha_{2i}, \alpha_{3i}, \beta_{1i}, \beta_{2i}, \beta_{3i} \in \Gamma$, $i \in \mathbf{N}$, the set of natural numbers, such that $x + \sum_{i=1}^{n} a_i\alpha_{1i}x\alpha_{2i}x\alpha_{3i}a_i^{'} = \sum_{i=1}^{n} b_i\beta_{1i}x\beta_{2i}x\beta_{3i}b_i^{'}$.

**Theorem 4.9.** *Let $S$ be a $k$-regular $\Gamma$-semiring and $x \in S$. Then*

(i) *$f(x) \subseteq (f o\chi_S o f)(x)$ for every soft intersectional $k$-bi-ideal $f$ of $S$ over $U$.*
(ii) *$f(x) \subseteq (f o\chi_S o f)(x)$ for every soft intersectional $k$-quasi-ideal $f$ of $S$ over $U$.*

*Proof.* Let $S$ be a $k$-regular $\Gamma$-semiring and $x$ be any element of $S$. Suppose $f$ be any soft intersectional $k$-bi-ideal of $S$. Since $S$ is $k$-regular there exist $a, b \in S$ and $\alpha, \beta, \gamma, \delta \in \Gamma$ such that $x + x\alpha a\beta x = x\gamma b\delta x$. Now

$$
\begin{aligned}
&(fo\chi_S of)(x)\\
&= \bigcup_{x+\sum_{i=1}^{n} a_i\alpha_i b_i = \sum_{i=1}^{n} c_i\beta_i d_i} \Big(\bigcap_i \{(fo\chi_S)(a_i), (fo\chi_S)(c_i), f(b_i), f(d_i)\}\Big)\\
&\supseteq \cap\{(fo\chi_S)(x\alpha a), (fo\chi_S)(x\gamma b), f(x)\}\\
&\qquad [\text{since } x + x\alpha a\beta x = x\gamma b\delta x]\\
&= \cap\Big\{ \bigcup_{x\alpha a+\sum_{i=1}^{n} a_i\alpha_{1i} b_i = \sum_{i=1}^{n} c_i\beta_{1i} d_i}\Big(\bigcap_i\{(f(a_i), f(c_i))\}\Big), \bigcup_{x\gamma b+\sum_{i=1}^{n} a_i\alpha_{2i} b_i = \sum_{i=1}^{n} c_i\beta_{2i} d_i}\Big(\bigcap_i\{(f(a_i), f(c_i))\}\Big), f(x)\Big\}\\
&\supseteq \cap\{f(x), f(x), f(x)\}\\
&\qquad [\text{since } x\alpha a + x\alpha a\beta x\alpha a = x\gamma b\delta x\alpha a \text{ and } x\gamma b + x\alpha a\beta x\gamma b = x\gamma b\delta x\gamma b]\\
&= f(x).
\end{aligned}
$$

This implies that $f(x) \subseteq (fo\chi_S of)(x)$.
$(i) \Rightarrow (ii)$ is straightforward from Lemma 4.5. □

**Theorem 4.10.** *Let $S$ be a $k$-regular $\Gamma$-semiring and $x \in S$. Then*

(i) $(f \cap g)(x) \subseteq (fogof)(x)$ *for every soft intersectional $k$-bi-ideal $f$ and every soft intersectional $k$-ideal $g$ of $S$ over $U$ .*

(ii) $(f \cap g)(x) \subseteq (fogof)(x)$ *for every soft intersectional $k$-quasi-ideal $f$ and every soft intersectional $k$-ideal $g$ of $S$ over $U$ .*

*Proof.* Assume that $S$ is a $k$-regular $\Gamma$-semiring. Let $f$ and $g$ be any soft intersectional $k$-bi-ideal and soft intersectional $k$-ideal of $S$ over $U$, respectively and $x$ be any element of $S$. Since $S$ is $k$-regular, there exist $a, b \in S$ and $\alpha, \beta, \gamma, \delta \in \Gamma$ such that $x + x\alpha a\beta x = x\gamma b\delta x$. Then

$$
\begin{aligned}
&(fogof)(x)\\
&= \bigcup_{x+\sum_{i=1}^{n} a_i\alpha_i b_i = \sum_{i=1}^{n} c_i\beta_i d_i} \Big(\bigcap_i \{(fog)(a_i), (fog)(c_i), f(b_i), f(d_i)\}\Big)\\
&\supseteq \cap\{(fog)(x\alpha a), (fog)(x\gamma b), f(x)\}\\
&\qquad [\text{since } x + x\alpha a\beta x = x\gamma b\delta x]\\
&= \cap\Big\{\bigcup_{x\alpha a+\sum_{i=1}^{n} a_i\alpha_i b_i = \sum_{i=1}^{n} c_i\beta_i d_i}\Big(\bigcap_i\{(f(a_i), f(c_i), g(b_i), g(d_i))\}\Big), \bigcup_{x\gamma b+\sum_{i=1}^{n} a_i\gamma_i b_i = \sum_{i=1}^{n} c_i\delta_i d_i}\Big(\bigcap_i\{(f(a_i), f(c_i), g(b_i), g(d_i))\}\Big), f(x)\Big\}\\
&\supseteq \cap\{\cap\{f(x), f(a\beta x\alpha a), g(b\delta x\alpha a)\}, \cap\{f(x), g(a\beta x\gamma b), g(b\delta x\gamma b)\}, f(x)\}\\
&\qquad [\text{since } x\alpha a + x\alpha a\beta x\alpha a = x\gamma b\delta x\alpha a \text{ and } x\gamma b + x\alpha a\beta x\gamma b = x\gamma b\delta x\gamma b]\\
&\supseteq f(x) \cap g(x) = (f \cap g)(x).
\end{aligned}
$$

(i)⇒(ii) is straightforward from Lemma 4.5. □

**Theorem 4.11.** *Let $S$ is both $k$-regular and $k$-intra-regular $\Gamma$-semiring and $x \in S$. Then*

(i) $f(x) = (fof)(x)$ *for every soft intersectional $k$-bi-ideal $f$ of $S$ over $U$.*
(ii) $f(x) = (fof)(x)$ *for every soft intersectional $k$-quasi-ideal $f$ of $S$ over $U$.*

*Proof.* Suppose $S$ is both $k$-regular and $k$-intra-regular $\Gamma$-semiring. Let $x \in S$ and $f$ be any soft intersectional $k$-bi-ideal of $S$ over $U$. Since $S$ is both $k$-regular and $k$-intra-regular there exist $a_i, b_i, c_i, d_i \in S$, $\alpha_{1i}, \alpha_{2i}, \alpha_{3i}, \alpha_{4i}, \alpha_{5i}, \beta_{1i}, \beta_{2i}, \beta_{3i}, \beta_{4i}, \beta_{5i} \in \Gamma$, $i \in \mathbf{N}$ such that $x + \sum_{i=1}^{n} x\alpha_{1i}a_i\alpha_{2i}x\alpha_{3i}x\alpha_{4i}b_i\alpha_{5i}x = \sum_{i=1}^{n} x\beta_{1i}c_i\beta_{2i}x\beta_{3i}x\beta_{4i}d_i\beta_{5i}x$. Therefore

$$\begin{aligned}
(fof)(x) &= \bigcup_{x+\sum_{i=1}^{n} a_i\alpha_i b_i \subseteq \sum_{i=1}^{n} c_i\beta_i d_i}\Big[\bigcap_i\{\cap\{f(a_i), f(c_i), f(b_i), f(d_i)\}\}\Big] \\
&\supseteq \bigcap_i\Big[\cap\{f(x\alpha_{1i}a_i\alpha_{2i}x), f(x\alpha_{4i}b_i\alpha_{5i}x), f(x\beta_{1i}c_i\beta_{2i}x), f(x\beta_{4i}d_i\beta_{5i}x)\}\Big] \\
&\qquad x+\sum_{i=1}^{n} x\alpha_{1i}a_i\alpha_{2i}x\alpha_{3i}x\alpha_{4i}b_i\alpha_{5i}x = \sum_{i=1}^{n} x\beta_{1i}c_i\beta_{2i}x\beta_{3i}x\beta_{4i}d_i\beta_{5i}x \\
&\supseteq f(x).
\end{aligned}$$

Now $(fof)(x) \subseteq (fo\chi_S)(x) \subseteq f(x)$. Hence $f(x) = (fof)(x)$ for every soft intersectional $k$-bi-ideal $f$ of $S$.
$(i) \Rightarrow (ii)$ is straightforward from the Lemma 4.5. □

**Theorem 4.12.** *Let $S$ is both $k$-regular and $k$-intra-regular $\Gamma$-semiring and $x \in S$. Then*

(i) $(f \cap g)(x) \subseteq (fog)(x)$ *for all soft intersectional $k$-bi-ideals $f$ and $g$ of $S$ over $U$.*
(ii) $(f \cap g)(x) \subseteq (fog)(x)$ *for every soft intersectional $k$-bi-ideals $f$ and every soft intersectional $k$-quasi-ideal $g$ of $S$ over $U$.*
(iii) $(f \cap g)(x) \subseteq (fog)(x)$ *for every soft intersectional $k$-quasi-ideals $f$ and every soft intersectional $k$-bi-ideal $g$ of $S$ over $U$.*
(iv) $(f \cap g)(x) \subseteq (fog)(x)$ *for all soft intersectional $k$-quasi-ideals $f$ and $g$ of $S$ over $U$.*

*Proof.* Assume that $S$ is both $k$-regular and $k$-intra-regular $\Gamma$-semiring. Let $x \in S$ and $f$, $g$ be any soft intersectional $k$-bi-ideals of $S$ over $U$. Since $S$ is both $k$-regular and $k$-intra-regular there exist $a_i, b_i, c_i, d_i \in S$, $\alpha_{1i}, \alpha_{2i}, \alpha_{3i}, \alpha_{4i}, \alpha_{5i}, \beta_{1i}, \beta_{2i}, \beta_{3i}, \beta_{4i}, \beta_{5i} \in \Gamma$, $i \in \mathbf{N}$ such that $x + \sum_{i=1}^{n} x\alpha_{1i}a_i\alpha_{2i}x\alpha_{3i}x\alpha_{4i}b_i\alpha_{5i}x \subseteq \sum_{i=1}^{n} x\beta_{1i}c_i\beta_{2i}x\beta_{3i}x\beta_{4i}d_i\beta_{5i}x$.

Therefore

$$\begin{aligned}
&(fog)(x)\\
&= \cup[\bigcap_i\{\cap\{f(a_i), f(c_i), g(b_i), g(d_i)\}\}]\\
&\quad {\scriptstyle x+}\sum_{i=1}^{n} a_i\alpha_i b_i = \sum_{i=1}^{n} c_i\beta_i d_i\\
&\supseteq \bigcap_i[\cap\{f(x\alpha_{1i}a_i\alpha_{2i}x), g(x\alpha_{4i}b_i\alpha_{5i}x), f(x\beta_{1i}c_i\beta_{2i}x), g(x\beta_{4i}d_i\beta_{5i}x\}]\\
&\supseteq f(x)\cap g(x) = (f\cap g)(x).
\end{aligned}$$

$(i) \Rightarrow (ii) \Rightarrow (iv)$ and $(i) \Rightarrow (iii) \Rightarrow (iv)$ are obvious from Lemma 4.5. □

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