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An intuitionistic normal fuzzy soft $k-$ideal over a $\Gamma-$semiring

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Abstract. In this paper, we introduce the notion of intuitionistic fuzzy soft ideals, intuitionistic fuzzy soft $k-$ideals, intuitionistic normal fuzzy soft $k-$ideals, intuitionistic $k-$fuzzy soft ideals over a $\Gamma-$semiring and study their properties and relations between them. We prove that if (f, A) is an intuitionistic fuzzy soft $k-$ideal over a $\Gamma-$semiring M then (f^+, A) is an intuitionistic normal fuzzy soft $k-$ideal over a $\Gamma-$semiring M and if (f, A) is a normal then $(f, A) = (f^+, A)$.

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Keywords: $\Gamma-$semiring, fuzzy soft set, Intuitionistic fuzzy soft set, Intuitionistic fuzzy soft ideal, Intuitionistic fuzzy soft $k-$ideal, Intuitionistic normal fuzzy soft $k-$ideal, Intuitionistic $k-$ fuzzy soft ideal.

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1. Introduction

The notion of a semiring is an algebraic structure with two associative binary operations where one distributes over the other, was first introduced by Vandiver [20] in 1934 but semirings had appeared in studies on the theory of ideals of rings.An universal algebra $(S, +, \cdot)$ is called a semiring if and only if $(S, +), (S, \cdot)$ are semigroups which are connected by distributive laws, *i.e.*, $a(b+c) = ab+ac$, $(a+b)c = ac+bc$, for all $a, b, c \in S$. Though semiring is a generalization of a ring, ideals of semiring do not coincide with ring ideals. For example an ideal of semiring needs not be the kernel of some semiring homomorphism. To solve this problem, Herniksen [7] defined $k-$ideals in semirings to obtain analogous of ring results for semiring. In structure, semirings lie between semigroups and rings. The results which hold in rings but not in semigroups hold in semirings, since semiring is a generalization of ring. The study of rings shows that multiplicative structure of ring is an independent of additive structure whereas in semiring multiplicative structure of semiring is not an independent of additive structure of semiring. The additive and the multiplicative structure of a semiring play an important role in determining the structure of a semiring. The theory of rings and theory of semigroups have considerable impact on the development of theory of semirings. Semirings play an important role in studying matrices and determinants. Semirings are useful in the areas of theoretical computer science as well as in the solutions of graph theory, optimization theory, in particular for studying automata,coding theory and formal languages. Semiring theory has many applications in other branches.

The notion of $\Gamma-$ring was introduced by Nobusawa [16] as a generalization of ring in 1964. Sen [18] introduced the notion of $\Gamma-$semigroup in 1981. The notion of ternary algebraic system was introduced by Lehmer [8] in 1932, Lister [9] introduced ternary ring. Dutta & Kar [4] introduced the notion of ternary semiring which is a generalization of ternary ring and semiring.The notion of $\Gamma-$semiring was introduced by Murali Krishna Rao [14] not only generalizes the notion of semiring and $\Gamma-$ring but also the notion of ternary semiring. The natural growth of gamma semiring is influenced by two things. One is the generalization of results of gamma rings and another is the generalization of results of semirings and ternary semirings. This notion provides an algebraic back ground to the non positive cones of the totally ordered rings.

The theory of fuzzy sets is the most appropriate theory for dealing with uncertainty was first introduced by Zadeh [21]. The concept of fuzzy subgroup was introduced by Rosenfeld [17]. Many papers on fuzzy sets appeared showing the importance of the concept and its applications to logic, set theory, group theory, ring theory, real analysis, topology, measure theory etc . Uncertain data in many important applications in the areas such as economics, engineering, environment ,medical sciences and business management could be caused by data randomness, information incompleteness, limitations of measuring instrument, delayed data updates etc.

Molodtsov [13] introduced the concept of soft set theory as a new mathematical tool for dealing with uncertainties ,only partially resolves the problem is that objects in universal set often does not precisely satisfy the parameters associated to each of the elements in the set. Then Maji et al.[10] extended soft set theory to fuzzy soft set theory. Aktas and Cagman [1] defined the soft set and soft groups. Majumdar and Samantha [12] extended soft sets to fuzzy soft set. Acar et al. [2], gave the basic concept of soft ring. Furthermore, Shah and Medhit [19] gave the concept of primary decomposition in a soft ring and a soft module. Jayanth Ghosh et al. [6] initiated the study of fuzzy soft rings and fuzzy soft ideals. Feng et al. [5] studied soft semirings by using the soft set theory. Atanasso [3] studied intuitionistic fuzzy sets. Zhou et al.[22] extended the concept of intuitionistic fuzzy soft set to semigroup theory in 2011. Maji et al.[11] introduced the concept of intuitionistic fuzzy soft set which is an extension to soft set and intuitionistic fuzzy set. Murali Krishna Rao [15] introduced and studied fuzzy soft ideals and fuzzy soft $k-$ideals over a $\Gamma-$semiring. In this paper, we introduce the notion of an intuitionistic fuzzy soft ideals, intuitionistic fuzzy soft $k-$ideals and intuitionistic normal fuzzy soft $k-$ideals over a $\Gamma-$semiring and their properties are studied.

2. Preliminaries

In this section, we recall some of the concepts and definitions from [6, 10, 11, 14, 15].

Definition 2.1 ([14])**.** A set S together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$ respectively) will be called a semiring provided

(i) addition is a commutative operation,
(ii) there exists $0 \in S$ such that $x + 0 = x$ and $x \cdot 0 = 0 \cdot x = 0$ for each $x \in S$,
(iii) multiplication distributes over addition both from the left and from the right.

Definition 2.2 ([14])**.** Let $(M, +)$ and $(\Gamma, +)$ be commutative semigroups. Then we call M as a $\Gamma-$semiring,if there exists a mapping $M \times \Gamma \times M \to M$ is written (x, α, y) as $x\alpha y$ such that it satisfies the following axioms : for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$,

(i) $x\alpha(y + z) = x\alpha y + x\alpha z$,
(ii) $(x + y)\alpha z = x\alpha z + y\alpha z$,
(iii) $x(\alpha + \beta)y = x\alpha y + x\beta y$,
(iv) $x\alpha(y\beta z) = (x\alpha y)\beta z$.

Definition 2.3 ([14])**.** Let S be a $\Gamma-$semiring and A be a non-empty subset of S. A is called a $\Gamma-$subsemiring of S if A is a sub-semigroup of $(S, +)$ and $A\Gamma A \subseteq A$.

Definition 2.4 ([14])**.** Let S be a $\Gamma-$semiring. A subset A of S is called a left(right) ideal of S if A is closed under addition and $S\Gamma A \subseteq A(A\Gamma S \subseteq A)$. A is called an ideal of S if it is both a left ideal and a right ideal.

Definition 2.5 ([21])**.** Let S be a non-empty set. A mapping $f : S \to [0, 1]$ is called a fuzzy subset of S.

Definition 2.6 ([21])**.** Let f be a fuzzy subset of a non-empty set S. For $t \in [0, 1]$ the set $f_t = \{x \in S \mid f(x) \geq t\}$ is called a level subset of S with respect to a fuzzy subset f.

Definition 2.7 ([15])**.** Let S be a $\Gamma-$semiring. A fuzzy subset μ of S is said to be fuzzy $\Gamma-$subsemiring of S if it satisfies the following conditions :

(i) $\mu(x + y) \geq \min\ \{\mu(x), \mu(y)\}$,
(ii) $\mu(x\alpha y) \geq \min\ \{\mu(x), \mu(y)\}$ for all $x, y \in S, \alpha \in \Gamma$.

Definition 2.8 ([15])**.** A fuzzy subset μ of a $\Gamma-$semiring S is called a fuzzy left(right) ideal of S if for all $x, y \in S, \alpha \in \Gamma$

(i) $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$,
(ii) $\mu(x\alpha y) \geq \mu(y)\ (\mu(x))$.

Definition 2.9. [15] A fuzzy subset μ of a $\Gamma-$semiring S is called a fuzzy ideal of S if for all $x, y \in S, \alpha \in \Gamma$

(i) $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$,
(ii) $\mu(x\alpha y) \geq \max\ \{\mu(x), \mu(y)\}$.

Definition 2.10. [15] An ideal I of a $\Gamma-$semiring S is called $k-$ideal if for $x, y \in S,\ x + y \in I, y \in I \Rightarrow x \in I$.

Definition 2.11 ([15])**.** Let f and g be fuzzy subsets of S. Then $f \cup g, f \cap g$ are fuzzy subsets of S defined by
$f \cup g(x) = \max\{f(x), g(x)\}, f \cap g(x) = \min\{f(x), g(x)\}$ for all $x \in S$.

Definition 2.12 ([15])**.** A fuzzy subset $\mu : S \to [0,1]$ is called a non-empty if μ is not the constant function.

Definition 2.13 ([15])**.** For any two fuzzy subsets λ and μ of S, $\lambda \subseteq \mu$ means $\lambda(x) \leq \mu(x)$ for all $x \in S$.

Definition 2.14 ([15])**.** Let U be an initial universe set, E be the set of parameters and $P(U)$ denotes the power set of U. A pair (f, E) is called a soft set over U where f is a mapping given by $f : E \to P(U)$.

Definition 2.15 ([15])**.** For a soft set (f, A), the set $\{x \in A \mid f(x) \neq \varnothing\}$ is called a support of (f, A) denoted by $Supp(f, A)$. If $\mathrm{Supp}(f, A) \neq \varnothing$ then (f, A) is called a non null soft set.

Definition 2.16 ([10])**.** Let U be an initial universe set, E be the set of parameters and $A \subseteq E$. A pair (f, A) is called a fuzzy soft set over U where f is a mapping given by $f : A \to I^U$ where I^U denotes the collection of all fuzzy subsets of U.

Definition 2.17 ([6])**.** Let X be a group and (f, A) be a soft set over X. Then (f, A) is said to be a soft group over X if and only if $f(a)$ is a subgroup of X for each $a \in A$.

Definition 2.18 ([6])**.** Let X be a group and (f, A) be fuzzy soft set over X. Then (f, A) is said to be fuzzy soft group over X if and only if for each $a \in A, x, y \in X$
(i) $f_a(x * y) \geq f_a(x) * f_a(y)$,
(ii) $f_a(x^{-1}) \geq f_a(x)$,
where f_a is the fuzzy subset of X corresponding to the parameter $a \in A$.

Definition 2.19 ([11])**.** An intuitionistic fuzzy set f of a non-empty set X is an object having the form $f = (\mu_f, \lambda_f) = \{x, \mu_f(x), \lambda_f(x) \mid x \in X\}$ where $\mu_f : X \to [0,1], \lambda_f : X \to [0,1]$ are membership functions, $\mu_f(x)$ is a degree of membership, $\lambda_f(x)$ is a degree of non membership and $0 \leq \mu_f(x) + \lambda_f(x) \leq 1$ for all $x \in X$.

Definition 2.20 ([11])**.** Let f and g be intuitionistic fuzzy sets of X. Then $f \cap g$ and $f \cup g$ are defined as

$$f \cap g = (\mu_{f \cap g}, \lambda_{f \cap g}) = \{x, \mu_f \cap \mu_g(x), \lambda_f \cap \lambda_g(x) \mid x \in X\}$$
$$f \cup g = (\mu_{f \cup g}, \lambda_{f \cup g},) = \{x, \mu_f \cup \mu_g(x), \lambda_f \cup \lambda_g(x) \mid x \in X\}$$

respectively.

Definition 2.21 ([11])**.** Let f and g be intuitionistic fuzzy sets of a non-empty set X. $f \subseteq g$ means $\mu_f(x) \leq \mu_g(x)$ and $\lambda_f(x) \geq \lambda_g(x)$, for all $x \in X$.

Definition 2.22 ([11])**.** Let U be an initial universe set, E be the set of parameters and $A \subseteq E$. A pair (f, A) is called an intuitional fuzzy soft set over U where f is a mapping given by $f : A \to I^U$ where I^U denotes the collection of all fuzzy subsets of U.

Definition 2.23 ([11])**.** Let $(f, A), (g, B)$ be intuitionistic fuzzy soft sets over U. Then (f, A) is said to be intuitionistic fuzzy soft set of (g, B), denoted by $(f, A) \subseteq (g, B)$ if $A \subseteq B$ and $f_a \subseteq g_a$ for all $a \in A$.

Definition 2.24 ([11])**.** Let $(f, A), (g, B)$ be intuitionistic fuzzy soft sets. The intersection of intuitionistic fuzzy soft sets (f, A) and (g, B), denoted by $(f, A) \cap (g, B) = (h, C)$ where $C = A \cup B$, is defined as

$$h_c = \begin{cases} f_c, & \text{if } c \in A \setminus B; \\ g_c, & \text{if } c \in B \setminus A; \\ f_c \cap g_c, & \text{if } c \in A \cap B. \end{cases}$$

Definition 2.25 ([11])**.** Let $(f, A), (g, B)$ be intuitionistic fuzzy soft sets. The union of intuitionistic fuzzy soft sets (f, A) and (g, B), denoted by $(f, A) \cup (g, B) = (h, C)$ where $C = A \cup B$, is defined as

$$h_c = \begin{cases} f_c, & \text{if } c \in A \setminus B; \\ g_c, & \text{if } c \in B \setminus A; \\ f_c \cup g_c, & \text{if } c \in A \cap B. \end{cases}$$

Definition 2.26 ([11])**.** Let $(f, A), (g, B)$ be intuitionistic fuzzy soft sets over U. (f, A) AND (g, B), denoted by $(f, A) \wedge (g, B)$, is defined by $(f, A) \wedge (g, B) = (h, C)$ where $C = A \times B$ and $h_c(x) = \min\{f_a(x), g_b(x)\}$ for all $c = (a, b) \in A \times B$, $x \in U$.

Definition 2.27. [11] Let $(f, A), (g, B)$ be intuitionistic fuzzy soft sets over U. (f, A) OR (g, B), denoted by $(f, A) \vee (g, B)$, is defined by $(f, A) \vee (g, B) = (h, C)$ where $C = A \times B$ and $h_c(x) = \max\{f_a(x), g_b(x)\}$ for all $c = (a, b) \in A \times B, x \in U$.

Definition 2.28 ([15])**.** Let S be a $\Gamma-$semiring, E be a parameter set, $A \subseteq E$ and f be a mapping given by $f : A \to P(S)$ where $P(S)$ is the power set of S. Then (f, A) is called a soft $\Gamma-$semiring over S if and only if for each $a \in A$, $f(a)$ is $\Gamma-$subsemiring of S. i.e. (i). $x, y \in S \Rightarrow x + y \in f(a)$ (ii). $x, y \in S, \alpha \in \Gamma \Rightarrow x\alpha y \in f(a)$.

Definition 2.29 ([15])**.** Let S be a $\Gamma-$semiring, E be a parameter set, $A \subseteq E$ and f be a mapping given by $f : A \to [0, 1]^S$ where $[0, 1]^S$ denotes the collection of all fuzzy subsets of S. Then (f, A) is called a fuzzy soft $\Gamma-$semiring over S if and only if for each $a \in A, f(a) = f_a$ is the fuzzy $\Gamma-$subsemiring of S. i.e.,(i) $f_a(x + y) \geq \min\{f_a(x), f_a(y)\}$ (ii) $f_a(x\alpha y) \geq \min\{f_a(x), f_a(y)\}$ for all $x, y \in S, \alpha \in \Gamma$.

Definition 2.30 ([15])**.** Let S be a $\Gamma-$semiring, E be a parameter set, $A \subseteq E$ and f be a mapping given by $f : A \to P(S)$. Then (f, A) is called a soft left(right) ideal over S if and only if for each $a \in A, f(a)$ is a left(right) ideal of S. i.e., (i) $x, y \in f(a) \Rightarrow x + y \in f(a)$ (ii) $x, y \in f(a), \alpha \in \Gamma, r \in S \Rightarrow r\alpha x(x\alpha r) \in f(a)$.

Definition 2.31 ([15])**.** Let S be a $\Gamma-$semiring, E be a parameter set, $A \subseteq E$ and $f : A \to P(S)$. Then (f, A) is called a soft ideal over S if and only if for each $a \in A, f(a)$ is an ideal of S. i.e., (i) $x, y \in f(a) \Rightarrow x + y \in f(a)$ (ii) $x \in f(a), \alpha \in \Gamma, r \in S \Rightarrow r\alpha x \in f(a)$ and $x\alpha r \in f(a)$.

Definition 2.32 ([15])**.** Let S be a $\Gamma-$semiring, E be a parameter set, $A \subseteq E$ and f be a mapping given by $f : A \to [0, 1]^S$ where $[0, 1]^S$ denotes the collection of all fuzzy subsets of S. Then (f, A) is called a fuzzy soft left(right) ideal over S if and only if for each $a \in A$, the corresponding fuzzy subset $f_a : S \to [0,1]$ is a fuzzy left(right) ideal of S. i.e., (i) $f_a(x+y) \geq \min\{f_a(x), f_a(y)\}$ (ii) $f_a(x\alpha y) \geq f_a(y)(f_a(x))$ for all $x, y \in S, \alpha \in \Gamma$.

Definition 2.33 ([15]). Let S be a $\Gamma-$semiring, E be a parameter set and $A \subseteq E$. Let f be a mapping given by $f : A \to [0,1]^S$ where $[0,1]^S$ denotes the collection of all fuzzy subsets of S. Then (f, A) is called a fuzzy soft ideal over S if and only if for each $a \in A$, the corresponding fuzzy subset $f_a : S \to [0,1]$ is a fuzzy ideal of S. i.e.,

(i) $f_a(x+y) \geq \min\{f_a(x), f_a(y)\}$,
(ii) $f_a(x\alpha y) \geq \max\{f_a(x), f_a(y)\}$ for all $x, y \in S, \alpha \in \Gamma$.

3. Intuitionistic fuzzy soft ideals

In this section, we introduce the notion of an intuitionistic fuzzy soft ideal over a $\Gamma-$semiring and study their properties.

Definition 3.1. An intuitionistic fuzzy set f of a $\Gamma-$semiring M is an object having the form $f = (\mu_f, \lambda_f) = \{x, \mu_f(x), \lambda_f(x) \mid x \in M\}$, where $\mu_f : M \to [0,1], \lambda_f : M \to [0,1]$ are membership functions, $\mu_f(x)$ is a degree of membership, $\lambda_f(x)$ is a degree of non membership and $0 \leq \mu_f(x) + \lambda_f(x) \leq 1$ for all $x \in M$.

Definition 3.2. An intuitionistic fuzzy set $f = (\mu_f, \lambda_f)$ of a $\Gamma-$semiring M is called an intuitionistic fuzzy ideal if f satisfies the following conditions :
(i) $\mu_f(x+y) \geq \min\{\mu_f(x), \mu_f(y)\}$,
(ii) $\mu_f(x\alpha y) \geq \max\{\mu_f(x), \mu_f(y)\}$,
(iii) $\lambda_f(x+y) \leq \max\{\lambda_f(x), \lambda_f(y)\}$.
(iv) $\lambda_f(x\alpha y) \leq \min\{\lambda_f(x), \lambda_f(y)\}$, for all $x, y \in M$ and $\alpha \in \Gamma$.

Obviously, $\mu_f(0) \geq \mu_f(x)$ and $\lambda_f(0) \leq \lambda_f(x)$, for all $x \in M$.

Example 3.3. Let M be the additive commutative semigroup of all non negative integers and Γ be the additive semigroup of all natural numbers. Then M is a $\Gamma-$semiring if $a\gamma b$ is defined as usual multiplication of integers a, γ, b, where $a, b \in M, \gamma \in \Gamma$. Define

$$\mu_f(x) = \begin{cases} \frac{3}{4}, & \text{if } x = 0; \\ \frac{1}{2}, & \text{if } x \in \{2,4,6,\cdots\}; \\ 0, & \text{otherwise.} \end{cases} \quad \lambda_f(x) = \begin{cases} \frac{1}{4}, & \text{if } x = 0; \\ \frac{1}{3}, & \text{if } x \in \{2,4,6,\cdots\}; \\ 1, & \text{otherwise.} \end{cases}$$

for all $x \in M$. Then intuitionistic fuzzy set $f = (\mu_f, \lambda_f)$ of $\Gamma-$semiring M is an intuitionistic fuzzy ideal.

Definition 3.4. Let M be a $\Gamma-$semiring , E be a parameter set and $A \subseteq E$. Then (f, A) is called an intuitionistic fuzzy soft ideal over a $\Gamma-$semiring M if and only if for each $a \in A$, the corresponding intuitionistic fuzzy set $f_a = (\mu_{f_a}, \lambda_{f_a})$ is an intuitionistic fuzzy ideal of $\Gamma-$semiring M.

Example 3.5. Let M be the additive commutative semigroup of all non negative integers and Γ be the additive semigroup of all natural numbers. Then M is a $\Gamma-$semiring if $a\gamma b$ is defined as usual multiplication of integers a, γ, b, where $a, b \in M, \gamma \in \Gamma$. Let $A = M$. Define

$$\mu_{f_a}(x) = \begin{cases} 0.9, & \text{if } x = 0; \\ \frac{1}{a}, & \text{if } x \in \{a, 2a, 3a, \cdots\}; \\ 0, & \text{otherwise.} \end{cases} \quad \lambda_{f_a}(x) = \begin{cases} 0.1, & \text{if } x = 0; \\ 1 - \frac{1}{a}, & \text{if } x \in \{a, 2a, 3a, \cdots\}; \\ 1, & \text{otherwise.} \end{cases}$$

for all $a, x \in M$. Then (f, A) is an intuitionistic fuzzy soft ideal over $\Gamma-$semiring M.

The proof of the following lemma is a straightforward verification.

Lemma 3.1. *Let* $R = [0, 1]$. *If* $x, y, z, w \in R$, *then*
(i) $\max\{\max\{x, y\}, \max\{z, w\}\} = \max\{\max\{x, z\}, \max\{y, w\}\}$.
(ii) $\min\{\min\{x, y\}, \min\{z, w\}\} = \min\{\min\{x, z\}, \min\{y, w\}\}$.
(iii) $\min\{\max\{x, y\}, \max\{z, w\}\} = \max\{\min\{x, z\}, \min\{y, w\}\}$.
(iv) $\min\{\max\{x, y\}, \max\{z, w\}\}$ *need not be equal to* $\max\{\min\{x, y\}, \min\{z, w\}\}$.

Theorem 3.2. *Let* (f, A) *and* (g, B) *be two intuitionistic fuzzy soft ideals over a* $\Gamma-$*semiring* M. *Then* $(f, A) \wedge (g, B)$ *is an intuitionistic fuzzy soft ideal over a* $\Gamma-$*semiring* M.

Proof. Let (f, A) and (g, B) be two intuitionistic fuzzy soft ideals over a $\Gamma-$semiring M. By Definition 2.26, $(f, A) \wedge (g, B) = (h, A \times B)$, where $h_c = f_a \cap g_b$ for all $c = (a, b) \in A \times B$. Let $x, y \in M, \alpha \in \Gamma$. Then

$$\begin{aligned}
\mu_{f_a \cap g_b}(x + y) &= \min\left\{\mu_{f_a}(x + y), \mu_{g_b}(x + y)\right\} \\
&\geq \min\left\{ \min\{\mu_{f_a}(x), \mu_{f_a}(y)\}, \min\{\mu_{g_b}(x), \mu_{g_b}(y)\}\right\} \\
&= \min\left\{ \min\{\mu_{f_a}(x), \mu_{g_b}(x)\}, \min\{\mu_{f_a}(y), \mu_{g_b}(y)\}\right\} \\
&= \min\left\{\mu_{f_a \cap g_b}(x), \mu_{f_a \cap g_b}(y)\right\} \\
\lambda_{f_a \cap g_b}(x + y) &= \min\left\{\lambda_{f_a}(x + y), \lambda_{g_b}(x + y)\right\} \\
&\leq \min\left\{ \max\{\lambda_{f_a}(x), \lambda_{f_a}(y)\}, \max\{\lambda_{g_b}(x), \lambda_{g_b}(y)\}\right\} \\
&= \max\left\{ \min\{\lambda_{f_a}(x), \lambda_{g_b}(x)\}, \min\{\lambda_{f_a}(y), \lambda_{g_b}(y)\}\right\} \\
&= \max\left\{\lambda_{f_a \cap g_b}(x), \lambda_{f_a \cap g_b}(y)\right\} \\
\mu_{f_a \cap g_b}(x\alpha y) &= \min\left\{\mu_{f_a}(x\alpha y), \mu_{g_b}(x\alpha y)\right\} \\
&\geq \min\left\{ \max\{\mu_{f_a}(x), \mu_{f_a}(y)\}, \max\{\mu_{g_b}(x), \mu_{g_b}(y)\}\right\} \\
&= \max\left\{ \min\{\mu_{f_a}(x), \mu_{g_b}(x)\}, \min\{\mu_{f_a}(y), \mu_{g_b}(y)\}\right\} \\
&= \max\left\{\mu_{f_a \cap g_b}(x), \mu_{f_a \cap g_b}(y)\right\} \\
\lambda_{f_a \cap g_b}(x\alpha y) &= \min\left\{\lambda_{f_a}(x\alpha y), \lambda_{g_b}(x\alpha y)\right\} \\
&\leq \min\left\{ \min\{\lambda_{f_a}(x), \lambda_{f_a}(y)\}, \min\{\lambda_{g_b}(x), \lambda_{g_b}(y)\}\right\} \\
&= \min\left\{ \min\{\lambda_{f_a}(x), \lambda_{g_b}(x)\}, \min\{\lambda_{f_a}(y), \lambda_{g_b}(y)\}\right\} \\
&= \min\left\{\lambda_{f_a \cap g_b}(x), \lambda_{f_a \cap g_b}(y)\right\}.
\end{aligned}$$

Thus $f_a \cap g_b$ is an intuitionistic fuzzy ideal. So $(f, A) \wedge (g, B)$ is an intuitionistic fuzzy soft ideal over a $\Gamma-$semiring M. □

Theorem 3.3. *Let (f, A) and (g, B) be two intuitionistic fuzzy soft ideals over a $\Gamma-$semiring M. Then $(f, A) \vee (g, B)$ is an intuitionistic fuzzy soft ideal over a $\Gamma-$semiring M.*

Proof. Let (f, A) and (g, B) be two intuitionistic fuzzy soft ideals over a $\Gamma-$semiring M. By Definition 2.27, $(f, A) \vee (g, B) = (h, A \times B)$, where $h_c = f_a \cup g_b$ for all $c = (a, b) \in A \times B$. Let $x, y \in M, \alpha \in \Gamma$. Then

$$\begin{aligned}
\mu_{f_a \cup g_b}(x + y) &= \max\left\{\mu_{f_a}(x + y), \mu_{g_b}(x + y)\right\}\\
&\geq \max\left\{ \min\{\mu_{f_a}(x), \mu_{f_a}(y)\}, \min\{\mu_{g_b}(x), \mu_{g_b}(y)\}\right\}\\
&= \min\left\{ \max\{\mu_{f_a}(x), \mu_{g_b}(x)\}, \max\{\mu_{f_a}(y), \mu_{g_b}(y)\}\right\}\\
&= \min\left\{\mu_{f_a \cup g_b}(x), \mu_{f_a \cup g_b}(y)\right\}\\
\lambda_{f_a \cup g_b}(x + y) &= \max\left\{\lambda_{f_a}(x + y), \lambda_{g_b}(x + y)\right\}\\
&\leq \max\left\{ \max\{\lambda_{f_a}(x), \lambda_{f_a}(y)\}, \max\{\lambda_{g_b}(x), \lambda_{g_b}(y)\}\right\}\\
&= \max\left\{ \max\{\lambda_{f_a}(x), \lambda_{g_b}(x)\}, \max\{\lambda_{f_a}(y), \lambda_{g_b}(y)\}\right\}\\
&= \max\left\{\lambda_{f_a \cup g_b}(x), \lambda_{f_a \cup g_b}(y)\right\}\\
\mu_{f_a \cup g_b}(x\alpha y) &= \max\left\{\mu_{f_a}(x\alpha y), \mu_{g_b}(x\alpha y)\right\}\\
&\geq \max\left\{ \max\{\mu_{f_a}(x), \mu_{f_a}(y)\}, \max\{\mu_{g_b}(x), \mu_{g_b}(y)\}\right\}\\
&= \max\left\{ \max\{\mu_{f_a}(x), \mu_{g_b}(x)\}, \max\{\mu_{f_a}(y), \mu_{g_b}(y)\}\right\}\\
&= \max\left\{\mu_{f_a \cup g_b}(x), \mu_{f_a \cup g_b}(y)\right\}\\
\lambda_{f_a \cup g_b}(x\alpha y) &= \max\left\{\lambda_{f_a}(x\alpha y), \lambda_{g_b}(x\alpha y)\right\}\\
&\leq \max\left\{ \min\{\lambda_{f_a}(x), \lambda_{f_a}(y)\}, \min\{\lambda_{g_b}(x), \lambda_{g_b}(y)\}\right\}\\
&= \min\left\{ \max\{\lambda_{f_a}(x), \lambda_{g_b}(x)\}, \max\{\lambda_{f_a}(y), \lambda_{g_b}(y)\}\right\}\\
&= \min\left\{\lambda_{f_a \cup g_b}(x), \lambda_{f_a \cup g_b}(y)\right\}.
\end{aligned}$$

Thus $f_a \cup g_b$ is an intuitionistic fuzzy ideal of $\Gamma-$semiring M. So $(f, A) \vee (g, B)$ is an intuitionistic fuzzy soft ideal over a $\Gamma-$semiring M. □

Theorem 3.4. *Let (f, A) and (g, B) be intuitionistic fuzzy soft ideals over a $\Gamma-$semiring M. Then $(f, A) \cap (g, B)$ is an intuitionistic fuzzy soft ideal over a $\Gamma-$semiring M.*

Proof. Let (f, A) and (g, B) be intuitionistic fuzzy soft ideals over a $\Gamma-$semiring M. By Definition 2.24, $(f, A) \cap (g, B) = (h, C)$, where $C = A \cup B$.

Case(i): Suppose $A \cap B = \phi$. Let $c \in C = A \cup B$. Then $c \in A$ or $c \in B$. If $c \in A$, then $h_c = f_c$. If $c \in B$, then $h_c = g_c$. Since (f, A) and (g, B) be intuitionistic fuzzy soft ideals over a $\Gamma-$semiring M, in both cases h_c is an intuitionistic fuzzy ideal over a $\Gamma-$semiring M.

Case(ii): If $A \cap B \neq \phi$ and $c \in C = A \setminus B$, then $h_c = f_c$ is an intuitionistic fuzzy soft ideal. If $c \in C = B \setminus A$, then $h_c = g_c$ is an intuitionistic fuzzy soft ideal.

Case(iii): If $c \in A \cap B$, then $h_c = f_c \cap g_c$. Let $x, y \in M, \alpha \in \Gamma$. Then

$$\begin{aligned}
\mu_{f_c \cap g_c}(x+y) &= \min\left\{\mu_{f_c}(x+y), \mu_{g_c}(x+y)\right\} \\
&\geq \min\left\{\min\{\mu_{f_c}(x), \mu_{f_c}(y)\}, \min\{\mu_{g_c}(x), \mu_{g_c}(y)\}\right\} \\
&= \min\left\{\min\{\mu_{f_c}(x), \mu_{g_c}(x)\}, \min\{\mu_{f_c}(y), \mu_{g_c}(y)\}\right\} \\
&= \min\left\{\mu_{f_c \cap g_c}(x), \mu_{f_c \cap g_c}(y)\right\} \\
\mu_{f_c \cap g_c}(x\alpha y) &= \min\left\{\mu_{f_c}(x\alpha y), \mu_{g_c}(x\alpha y)\right\} \\
&\geq \min\left\{\max\{\mu_{f_c}(x), \mu_{f_c}(y)\}, \max\{\mu_{g_c}(x), \mu_{g_c}(y)\}\right\} \\
&= \max\left\{\min\{\mu_{f_c}(x), \mu_{g_c}(x)\}, \min\{\mu_{f_c}(y), \mu_{g_c}(y)\}\right\} \\
&= \max\left\{\mu_{f_c \cap g_c}(x), \mu_{f_c \cap g_c}(y)\right\} \\
\lambda_{f_c \cap g_c}(x+y) &= \min\left\{\lambda_{f_c}(x+y), \lambda_{g_c}(x+y)\right\} \\
&\leq \min\left\{\max\{\lambda_{f_c}(x), \lambda_{f_c}(y)\}, \max\{\lambda_{g_c}(x), \lambda_{g_c}(y)\}\right\} \\
&= \max\left\{\min\{\lambda_{f_c}(x), \lambda_{g_c}(x)\}, \min\{\lambda_{f_c}(y), \lambda_{g_c}(y)\}\right\} \\
&= \max\left\{\lambda_{f_c \cap g_c}(x), \lambda_{f_c \cap g_c}(y)\right\} \\
\lambda_{f_c \cap g_c}(x\alpha y) &= \min\left\{\lambda_{f_c}(x\alpha y), \lambda_{g_c}(x\alpha y)\right\} \\
&\leq \min\left\{\min\{\lambda_{f_c}(x), \lambda_{f_c}(y)\}, \min\{\lambda_{g_c}(x), \lambda_{g_c}(y)\}\right\} \\
&= \min\left\{\min\{\lambda_{f_c}(x), \lambda_{g_c}(x)\}, \min\{\lambda_{f_c}(y), \lambda_{g_c}(y)\}\right\} \\
&= \min\left\{\lambda_{f_c \cap g_c}(x), \lambda_{f_c \cap g_c}(y)\right\}.
\end{aligned}$$

Thus $f_c \cap g_c$ is an intuitionistic fuzzy ideal over a $\Gamma-$semiring M. So $(f, A) \cap (g, B)$ is an intuitionistic fuzzy soft ideal over a $\Gamma-$semiring M. □

The proof of the following theorem is similar to proof of Theorem 3.4

Theorem 3.5. *Let (f, A) and (g, B) be two intuitionistic fuzzy soft ideals over a $\Gamma-$semiring M. Then $(f, A) \cup (g, B)$ is an intuitionistic fuzzy soft ideal over a $\Gamma-$semiring M.*

4. Intuitionistic fuzzy soft $k-$ideals and intuitionistic $k-$fuzzy soft ideals

In this section, we introduce the notion of an intuitionistic fuzzy soft $k-$ideal, an intuitionistic $k-$fuzzy soft ideal over a $\Gamma-$semiring M and study their basic properties.

Definition 4.1. An intuitionistic fuzzy ideal $f = (\mu_f, \lambda_f)$ of a $\Gamma-$semiring M is called an intuitionistic fuzzy $k-$ideal if f satisfies the following the conditions :
(i) $\mu_f(x) \geq \min\{\mu_f(x+y), \mu_f(y)\}$,
(ii) $\lambda_f(x) \leq \max\{\lambda_f(x+y), \lambda_f(y)\}$, for all $x, y \in M, \alpha \in \Gamma$.

Definition 4.2. Let M be a $\Gamma-$semiring , E be a parameter set and $A \subseteq E$. Then (f, A) is called an intuitionistic fuzzy soft $k-$ideal over a $\Gamma-$semiring M if and only if for each $a \in A$, the corresponding intuitionistic fuzzy set $f_a = (\mu_{f_a}, \lambda_{f_a})$ is an intuitionistic fuzzy $k-$ideal of $\Gamma-$semiring M.

Example 4.3. Let M be the additive commutative semigroup of all integers and Γ be the additive semigroup of all natural numbers. Then M is a $\Gamma-$semiring if $a\gamma b$ is defined as usual multiplication of integers a, γ, b, where $a, b \in M, \gamma \in \Gamma$. Let $A = M$. Define

$$\mu_{f_a}(x) = \begin{cases} 0.8, & \text{if } x = 0; \\ \frac{1}{a}, & \text{if } x \in \{\cdots, -3a, -2a, -a, a, 2a, 3a, \cdots\}; \\ 0, & \text{otherwise.} \end{cases}$$

$$\lambda_{f_a}(x) = \begin{cases} 0.2, & \text{if } x = 0; \\ 1 - \frac{1}{a}, & \text{if } x \in \{\cdots, -3a, -2a, -a, a, 2a, 3a, \cdots\}; \\ 1, & \text{otherwise.} \end{cases}$$

for all $a, x \in M$. Then (f, A) is an intuitionistic fuzzy soft ideal over $\Gamma-$semiring M.

Example 4.4. Let M be the additive commutative semigroup of all 2×2 matrices over non negative integers and $M = \Gamma$. Then M is a $\Gamma-$semiring if $a\gamma b$ is defined as usual multiplication of matrices, where $a, b, \gamma \in M$. Define

$$\mu_f(x) = \begin{cases} 0.6, & \text{if } x = \begin{pmatrix} 0 & 0 \\ a & b \end{pmatrix}; \\ 0.3, & \text{if } x = \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}; \\ 1, & \text{otherwise,} \end{cases}$$

$$\lambda_f(x) = \begin{cases} 0.3, & \text{if } x = \begin{pmatrix} 0 & 0 \\ a & b \end{pmatrix}; \\ 0.6, & \text{if } x = \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}; \\ 0, & \text{otherwise,} \end{cases}$$

where a, b are even integers, for all $x \in M$. Then $f = (\mu_f, \lambda_f)$ is an intuitionistic fuzzy ideal but not an intuitionistic fuzzy $k-$ideal.

Theorem 4.1. *Let (f, A) and (g, B) be intuitionistic fuzzy soft $k-$ideals over a $\Gamma-$semiring M. Then $(f, A) \wedge (g, B)$ is an intuitionistic fuzzy soft $k-$ideal over a $\Gamma-$semiring M.*

Proof. Let (f, A) and (g, B) be intuitionistic fuzzy soft $k-$ideals over a $\Gamma-$semiring M. By Definition 2.26, $(f, A) \wedge (g, B) = (h, C)$ where $C = A \times B$. Let $c = (a, b) \in$

$C = A \times B$. Then, by Theorem 3.2, $(f, A) \wedge (g, B)$ is an intuitionistic fuzzy soft ideal over a $\Gamma-$semiring M. $h_c(x) = (f_a \cap g_b)(x)$.

$$\begin{aligned}
\mu_{f_a \cap g_b}(x) &= \min\{\mu_{f_a}(x), \mu_{g_b}(x)\} \\
&\geq \min\big\{ \min\{\mu_{f_a}(x+y), \mu_{f_a}(y)\}, \min\{\mu_{g_b}(x+y), \mu_{g_b}(y)\}\big\} \\
&= \min\big\{ \min\{\mu_{f_a}(x+y), \mu_{g_b}(x+y)\}, \min\{\mu_{f_a}(y), \mu_{g_b}(y)\}\big\} \\
&= \min\big\{\mu_{f_a \cap g_b}(x+y), \quad \mu_{f_a \cap g_b}(y)\big\}, \text{ for all } \; x, y \in M. \\
\lambda_{f_a \cap g_b}(x) &= \min\{\lambda_{f_a}(x), \lambda_{g_b}(x)\} \\
&\leq \min\big\{ \max\{\lambda_{f_a}(x+y), \lambda_{f_a}(y)\}, \max\{\lambda_{g_b}(x+y), \lambda_{g_b}(y)\}\big\} \\
&= \max\big\{ \min\{\lambda_{f_a}(x+y), \lambda_{g_b}(x+y)\}, \min\{\lambda_{f_a}(y), \lambda_{g_b}(y)\}\big\} \\
&= \min\big\{\lambda_{f_a \cap g_b}(x+y), \lambda_{f_a \cap g_b}(y)\big\}, \text{ for all } \; x, y \in M.
\end{aligned}$$

Thus $f_a \cap g_b$ is an intuitionistic fuzzy $k-$ideal over a $\Gamma-$semiring M. So $(f, A) \wedge (g, B)$ is an intuitionistic fuzzy soft $k-$ideal over a $\Gamma-$semiring M. □

Theorem 4.2. *Let (f, A) and (g, B) be intuitionistic fuzzy soft $k-$ideals over a $\Gamma-$semiring M. Then $(f, A) \vee (g, B)$ is an intuitionistic fuzzy soft $k-$ideal over a $\Gamma-$semiring M.*

Proof. Let (f, A) and (g, B) be intuitionistic fuzzy soft $k-$ideals over a $\Gamma-$semiring M. By Definition 2.27, $(f, A) \vee (g, B) = (h, C)$, where $C = A \times B$. Let $c = (a, b) \in C = A \times B$. Then $h_c = f_a \cup g_b$. Thus, by Theorem 3.3, $(f, A) \vee (g, B)$ is an intuitionistic fuzzy soft ideal over a $\Gamma-$semiring M. Since $h_c(x) = (f_a \cup g_b)(x)$.

$$\begin{aligned}
\mu_{f_a \cup g_b}(x) &= \max\{\mu_{f_a}(x), \mu_{g_b}(x)\} \\
&\geq \max\big\{ \min\{\mu_{f_a}(x+y), \mu_{f_a}(y)\}, \min\{\mu_{g_b}(x+y), \mu_{g_b}(y)\}\big\} \\
&= \min\big\{ \max\{\mu_{f_a}(x+y), \mu_{g_b}(x+y)\}, \max\{\mu_{f_a}(y), \mu_{g_b}(y)\}\big\} \\
&= \min\big\{\mu_{f_a \cup g_b}(x+y), \mu_{f_a \cup g_b}(y)\big\}, \text{ for all } \; x, y \in M. \\
\lambda_{f_a \cup g_b}(x) &= \max\{\lambda_{f_a}(x), \lambda_{g_b}(x)\} \\
&\leq \max\big\{ \max\{\lambda_{f_a}(x+y), \lambda_{f_a}(x)\}, \max\{\lambda_{g_b}(x+y), \lambda_{g_b}(y)\}\big\} \\
&= \max\big\{ \max\{\lambda_{f_a}(x+y), \lambda_{g_b}(x+y)\}, \max\{\lambda_{f_a}(y), \lambda_{g_b}(y)\}\big\} \\
&= \max\big\{\lambda_{f_a \cup g_b}(x+y), \lambda_{f_a \cup g_b}(y)\}\big\}, \text{ for all } \; x, y \in M.
\end{aligned}$$

So h_c is an intuitionistic fuzzy $k-$ideal over a $\Gamma-$semiring M. Hence $(f, A) \vee (g, B)$ is an intuitionistic fuzzy soft $k-$ideal over a $\Gamma-$semiring M. □

Theorem 4.3. *Let (f, A) and (g, B) be intuitionistic fuzzy soft $k-$ideals over a $\Gamma-$semiring M. Then $(f, A) \cap (g, B)$ is an intuitionistic fuzzy soft $k-$ideal over a $\Gamma-$semiring M.*

Proof. Let (f, A) and (g, B)be intuitionistic fuzzy soft $k-$ideals over a $\Gamma-$semiring M. Then, by Theorem 3.4, $(f, A) \cap (g, B)$ is an intuitionistic fuzzy soft ideal over a $\Gamma-$semiring M. By Definition 2.24, $(f, A) \cap (g, B) = (h, C)$, where $C = A \cup B$.
If $c \in A \setminus B$, then $h_c = f_c$, h_c is an intuitionistic fuzzy $k-$ideal over a $\Gamma-$semiring M.

If $c \in B \setminus A$, then $h_c = g_c,\ h_c$ is an intuitionistic fuzzy $k-$ideal over a $\Gamma-$semiring M.
If $c \in B \cap A$, then $h_c = f_c \cap g_c$. Thus

$$\begin{aligned}
\mu_{f_c \cap g_c}(x) =& \min\{\mu_{f_c}(x), \mu_{g_c}(x)\} \\
\geq& \min\big\{ \min\{\mu_{f_c}(x+y), \mu_{f_c}(y)\}, \min\{\mu_{g_c}(x+y), \mu_{g_c}(y)\}\big\} \\
=& \min\big\{ \min\{\mu_{f_c}(x+y), \mu_{g_c}(x+y)\}, \min\{\mu_{f_c}(y), \mu_{g_c}(y)\}\big\} \\
=& \min\big\{\mu_{f_c \cap g_c}(x+y), \mu_{f_c \cap g_c}(y)\big\}, \text{ for all } \ x, y \in M. \\
\lambda_{f_c \cap g_c}(x) =& \min\{\lambda_{f_c}(x), \lambda_{g_c}(x)\} \\
\leq& \min\big\{ \max\{\lambda_{f_c}(x+y), \lambda_{f_c}(x)\}, \max\{\lambda_{g_c}(x+y), \lambda_{g_c}(y)\}\big\} \\
=& \max\big\{ \min\{\lambda_{f_c}(x+y), \lambda_{g_c}(x+y)\}, \min\{\lambda_{f_c}(y), \lambda_{g_c}(y)\}\big\} \\
=& \max\big\{\lambda_{f_c \cap g_c}(x+y), \lambda_{f_c \cap g_c}(y)\}\big\}, \text{ for all } \ x, y \in M.
\end{aligned}$$

So h_c is an intuitionistic fuzzy $k-$ideal over a $\Gamma-$semiring M. Hence $(f, A) \cap (g, B)$ is an intuitionistic fuzzy soft $k-$ideal over a $\Gamma-$semiring M. □

The proof of the following theorem is similar to Theorem 4.3.

Theorem 4.4. *Let (f, A) and (g, B) be intuitionistic fuzzy soft $k-$ideals over a $\Gamma-$semiring M. Then $(f, A) \cup (g, B)$ is an intuitionistic fuzzy soft $k-$ideal over a $\Gamma-$semiring M.*

Definition 4.5. Let M be a $\Gamma-$semiring , E be a parameter set and $A \subseteq E$. Then (f, A) is called an intuitionistic $k-$fuzzy soft ideal over a $\Gamma-$semiring M if and only if for each $a \in A$, the corresponding intuitionistic fuzzy set $f_a = (\mu_{f_a}, \lambda_{f_a})$ is an intuitionistic $k-$fuzzy ideal of $\Gamma-$semiring M and

(i) $\mu_{f_a}(x+y) = \mu_{f_a}(0), \mu_{f_a}(y) = \mu_{f_a}(0) \Rightarrow \ \mu_{f_a}(x) = \mu_{f_a}(0)$.
(ii) $\lambda_{f_a}(x+y) = \lambda_{f_a}(0), \lambda_{f_a}(y) = \lambda_{f_a}(0) \Rightarrow \ \lambda_{f_a}(x) = \lambda_{f_a}(0), forall\ x, y \in M.$

Theorem 4.5. *Let (f, A) be an intuitionistic fuzzy soft $k-$ideal over a $\Gamma-$semiring M. Then (f, A) is an intuitionistic $k-$fuzzy soft ideal over a $\Gamma-$semiring M.*

Proof. Let (f, A) be an intuitionistic fuzzy soft $k-$ideal over a $\Gamma-$semiring M and $a \in A$. Since (f, A) is an intuitionistic fuzzy soft $k-$ideal, f_a is an intuitionistic fuzzy $k-$ideal. Let $x, y \in M$ such that

$$\begin{aligned}
\mu_{f_a}(x+y) =& \mu_{f_a}(0) \text{ and } \mu_{f_a}(y) = \mu_{f_a}(0) \\
\lambda_{f_a}(x+y) =& \lambda_{f_a}(0) \text{ and } \lambda_{f_a}(y) = \mu_{f_a}(0). \\
\mu_{f_a}(x) \geq& \min\{\mu_{f_a}(x+y), \mu_{f_a}(y)\} \\
=& \min\{\mu_{f_a}(0), \mu_{f_a}(0)\} \\
=& \mu_{f_a}(0).
\end{aligned}$$

Then $\mu_{f_a}(0) \geq \mu_{f_a}(x),$ for all $x \in M$. Thus $\mu_{f_a}(x) = \mu_{f_a}(0)$. So

$$\begin{aligned}
\lambda_{f_a}(x) \leq& \max\{\lambda_{f_a}(x+y), \lambda_{f_a}(y)\} \\
=& \max\{\lambda_{f_a}(0), \lambda_{f_a}(0)\} \\
=& \lambda_{f_a}(0).
\end{aligned}$$

Hence $\lambda_{f_a}(0) \leq \lambda_{f_a}(x)$, for all $x \in M$ and thus $\lambda_{f_a}(x) = \lambda_{f_a}(0)$. Therefore (f, A) is an intuitionistic $k-$fuzzy soft ideal over a $\Gamma-$semiring M. $\square$

5. Intuitionistic normal fuzzy soft $k-$ideal over a $\Gamma-$semiring

In this section, we introduce the notion of intuitionistic $k-$fuzzy ideal, intuitionistic normal fuzzy soft $k-$ideal over a $\Gamma-$semiring M and study their properties.

Definition 5.1. An intuitionistic fuzzy soft set (f, A) over a $\Gamma-$semiring M is said to be intuitionistic normal fuzzy soft set if $\mu_{f_a}(0) = 1$, for all $a \in A$.

Example 5.2. Let M be the additive commutative semigroup of all integers and Γ be the additive semigroup of all natural numbers. Then M is a $\Gamma-$semiring if $a\gamma b$ is defined as usual multiplication of integers a, γ, b, where $a, b \in M, \gamma \in \Gamma$. Let $A =$ be set of natural numbers. Define

$$\mu_{f_a}(x) = \begin{cases} 1, & \text{if } x = 0; \\ \frac{1}{a}, & \text{if } x \in \{\cdots, -3a, -2a, -a, a, 2a, 3a, \cdots\}; \\ 0, & \text{otherwise.} \end{cases}$$

$$\lambda_{f_a}(x) = \begin{cases} 0, & \text{if } x = 0; \\ 1 - \frac{1}{a}, & \text{if } x \in \{\cdots, -3a, -2a, -a, a, 2a, 3a, \cdots\}; \\ 1, & \text{otherwise.} \end{cases}$$

for all $a, x \in M$. Then (f, A) is an intuitionistic normal fuzzy soft ideal over $\Gamma-$semiring M.

Theorem 5.1. *Let (f, A) be an intuitionistic fuzzy soft $k-$ideal over a $\Gamma-$semiring M and each $a \in A$, f_a^+ is defined by*

$$\begin{aligned} f_a^+ &= \{(x, \mu_{f_a^+}(x), \lambda_{f_a^+}(x)\}, \\ \textit{where} \quad \mu_{f_a^+}(x) &= \mu_{f_a}(x) + 1 - \mu_{f_a}(0), \\ \lambda_{f_a^+}(x) &= \lambda_{f_a}(x) - \lambda_{f_a}(0), \quad \textit{for all } x \in M. \end{aligned}$$

Then (f^+, A) is an intuitionistic normal fuzzy $k-$soft ideal over a $\Gamma-$semiring M and (f, A) is an intuitionistic fuzzy soft subset of (f^+, A).

Proof. Let (f, A) be an intuitionistic fuzzy soft $k-$ideal over a $\Gamma-$semiring $M, a \in A$ $x, y \in M$ and $\alpha \in \Gamma$. Then

$$0 \leq \mu_{f_a}(x) + \lambda_{f_a}(x) \leq 1 \text{ and } 0 \leq \mu_{f_a}(0) + \lambda_{f_a}(0) \leq 1.$$

Thus

$$\mu_{f_a}(x) + \lambda_{f_a}(x) - \mu_{f_a}(0) - \lambda_{f_a}(0) \leq 0$$

and

$$\mu_{f_a}(x) + \lambda_{f_a}(x) + 1 - \mu_{f_a}(0) - \lambda_{f_a}(0) \leq 1.$$

So

$$0 \leq \mu_{f_a^+}(x) + \lambda_{f_a^+}(x) \leq 1.$$

On the other hand,

$$\begin{aligned}\mu_{f_a^+}(x+y) &= \mu_{f_a}(x+y) + 1 - \mu_{f_a}(0)\\ &\geq \min\{\mu_{f_a}(x), \mu_{f_a}(y)\} + 1 - \mu_{f_a}(0)\\ &= \min\{\mu_{f_a}(x) + 1 - \mu_{f_a}(0), \mu_{f_a}(y) + 1 - \mu_{f_a}(0)\}\\ &= \min\{\mu_{f_a^+}(x), \mu_{f_a^+}(y)\},\\ \mu_{f_a^+}(x\alpha y) &= \mu_{f_a}(x\alpha y) + 1 - \mu_{f_a}(0)\\ &\geq \max\{\mu_{f_a}(x), \mu_{f_a}(y)\} + 1 - \mu_{f_a}(0)\\ &= \max\{\mu_{f_a}(x) + 1 - \mu_{f_a}(0), \mu_{f_a}(y) + 1 - \mu_{f_a}(0)\}\\ &= \max\{\mu_{f_a^+}(x), \mu_{f_a^+}(y)\},\\ \lambda_{f_a^+}(x+y) &= \lambda_{f_a}(x+y) - \lambda_{f_a}(0)\\ &\leq \max\{\lambda_{f_a}(x), \lambda_{f_a}(y)\} - \lambda_{f_a}(0)\\ &= \max\{\lambda_{f_a}(x) - \lambda_{f_a}(0), \lambda_{f_a}(y) - \lambda_{f_a}(0)\}\\ &= \max\{\lambda_{f_a^+}(x), \lambda_{f_a^+}(y)\},\\ \lambda_{f_a^+}(x\alpha y) &= \lambda_{f_a}(x\alpha y) - \lambda_{f_a}(0)\\ &\leq \min\{\lambda f_a(x), \lambda_{f_a}(y)\} - \lambda_{f_a}(0)\\ &= \min\{\lambda_{f_a}(x) - \lambda_{f_a}(0), \lambda_{f_a}(y) - \lambda_{f_a}(0)\}\\ &= \min\{\lambda_{f_a^+}(x), \lambda_{f_a^+}(y)\}.\end{aligned}$$

Thus $\mu_{f_a^+}(0) = \mu_{f_a}(0) + 1 - \mu_{f_a}(0) = 1$. So

$$\mu_{f_a}(x) \leq \mu_{f_a^+}(x) \text{ and } \lambda_{f_a}(x) \geq \lambda_{f_a^+}(x), \quad \text{for all } x \in M.$$

Hence $\mu_{f_a^+}$ is a normal and fuzzy ideal of $\Gamma-$semiring M.

$$\begin{aligned}\mu_{f_a^+}(x) &= \mu_{f_a}(x) + 1 - \mu_{f_a}(0)\\ &\geq \min\{\mu_{f_a}(x+y),\ \mu_{f_a}(y)\} + 1 - \mu_{f_a}(0)\\ &= \min\{\mu_{f_a}(x+y) + 1 - \mu_{f_a}(0),\ \mu_{f_a}(y) + 1 - \mu_{f_a}(0)\}\\ &= \min\{\mu_{f_a^+}(x+y),\ \mu_{f_a^+}(y)\}, \quad \text{for all } x, y \in M\end{aligned}$$

and

$$\begin{aligned}\lambda_{f_a^+}(x) &= \lambda_{f_a}(x) - \lambda_{f_a}(0)\\ &\leq \max\{\lambda_{f_a}(x+y),\ \lambda_{f_a}(y)\} - \lambda_{f_a}(0)\\ &= \max\{\lambda_{f_a}(x+y) - \lambda_{f_a}(0),\ \lambda_{f_a}(y) - \lambda_{f_a}(0)\}\\ &= \max\{\lambda_{f_a^+}(x+y),\ \lambda_{f_a^+}(y)\}, \quad \text{for all } x, y \in M.\end{aligned}$$

Therefore the proof is complete. □

Theorem 5.2. *Let (f, A) be an intuitionistic fuzzy soft $k-$ideal over a $\Gamma-$semiring M. Then the following hold.*

(a) *(f, A) is a normal if and only if $(f, A) = (f^+, A)$.*
(b) *$(f^{++}, A) = (f^+, A)$.*
(c) *If $\mu_{f_a^+}(x) = 0$ then $\mu_{f_a}(x) = 0$.*

Proof. (a) Suppose (f, A) is a normal and $a \in A$. Then $\mu_{f_a^+}(x) = \mu_{f_a}(x) + 1 - \mu_{f_a}(0)$. Thus

$$\begin{aligned}\mu_{f_a^+}(x) =& \mu_{f_a}(x) + 1 - 1\\ =& \mu_{f_a}(x) \text{ and } \lambda_{f_a^+}(x) = \lambda_{f_a}(x), \text{ for all } x \in M.\end{aligned}$$

So $(f, A) = (f^+, A)$. The converse is obvious.

(b) Clearly $\mu_{f_a^{++}}(x) = \mu_{f_a^+}(x) + 1 - \mu_{f_a^+}(0)$.

$$\begin{aligned}\mu_{f_a^{++}}(x) =& \mu_{f_a^+}(x) + 1 - 1\\ =& \mu_{f_a^+}(x) \text{ and } \lambda_{f_a^{++}}(x) = \lambda_{f_a^+}(x), \text{ for all } x \in M.\end{aligned}$$

Thus $(f^{++}, A) = (f^+, A)$.

(c) Suppose $\mu_{f_a^+}(x) = 0,\ a \in A, x \in M$. Then $\mu_{f_a}(x) \leq \mu_{f_a^+}(x)$. Thus $\mu_{f_a}(x) \leq \mu_{f_a^+}(x) = 0$. So $\mu_{f_a}(x) = 0$. Hence the proof is complete. □

6. Conclusion

In this paper, we introduced the notion of intuitionistic fuzzy soft ideals, intuitionistic fuzzy soft $k-$ideals, intuitionistic normal fuzzy soft $k-$ideals, intuitionistic $k-$fuzzy soft ideals over a $\Gamma-$semiring and we studied their properties and relations between them. In continuous of this paper we study intuitionistic fuzzy soft prime ideals over $\Gamma-$semiring and intuitionistic fuzzy soft ideals over ordered $\Gamma-$semiring.

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