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Supra regular generalized closed soft sets in supra soft topological spaces

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Abstract. Many researchers defined some basic notions on supra soft topology and studied many properties of this concept. In this paper, we define supra regular generalized closed (open) soft sets in supra soft topological spaces and study their properties in detail. Also we introduce the concept of supra regular generalized closed soft sets with respect to a soft ideal in supra soft topological spaces, which is the extension of the concept of supra regular generalized closed soft sets. We discuss their relationships with different types of subsets of supra soft topological spaces with the help of counterexamples. We introduce these concepts in supra soft topological spaces which are defined over an initial universe with a fixed set of parameters.

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1. Introduction

In 1999, Molodtsov [9] proposed the concept of a soft set which can be seen as a new mathematical approach to vagueness. In fact, a soft set is a parameterized family of subsets of a given universe set. The way of parameterization in problem solving makes soft set theory convenient and simple for application. Maji et al. [8] carried out Molodtsov's idea by introducing several operations in soft set theory. Shabir and Naz [12] presented soft topological spaces and they investigated some properties of them. Later, many researchers [3, 4, 6, 7, 10, 11, 14] studied some of basic concepts and properties of soft topological spaces.

El-Sheikh and Abd El-Latif [1] introduced the concept of supra soft topological spaces, which is wider and more general than the class of soft topological spaces.

They introduced a unification of some types of different kinds of subsets of supra soft topological spaces using the notion of γ-operation and studied the decompositions of some forms of supra soft continuity. After then Kandil et al. [5] studied the concepts of supra generalized closed soft sets and supra generalized closed soft sets with respect to a soft ideal in a supra topological space.

Yüksel et al. [13] defined some new concepts in supra soft topological spaces which are defined over an initial universe with a fixed set of parameters such as soft regular closed (open) sets and soft regular generalized closed (open) sets. In this paper we extend these different types of subsets of soft topological spaces to supra soft topological spaces. Also we introduce the concept of supra regular generalized closed soft sets with respect to a soft ideal in supra soft topological spaces, which is the extension of the concept of supra regular generalized closed soft sets. We investigate many basic properties of these concepts. Moreover, we also establish several interesting results and present their fundamental properties with the help of some examples.

2. Preliminaries

In this section, we present the basic definitions and results of soft set theory which may be found in earlier studies.

Let X be an initial universe set and E be the set of all possible parameters with respect to X. Parameters are often attributes, characteristics or properties of the objects in X. Let $P(X)$ denote the power set of X. Then a soft set over X is defined as follows.

Definition 2.1 ([9])**.** Let X be an initial universe set and E be the set of parameters. A pair (F, A) is called a soft set over X where $A \subseteq E$ and $F : A \longrightarrow P(X)$ is a set valued mapping.

In other words, a soft set over X is a parameterized family of subsets of the universe X. For $\forall \varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F, A). Clearly, a soft set is not a set.

Definition 2.2 ([8])**.** A soft set (F, A) over X is said to be a null soft set denoted by Φ if for all $e \in A$, $F(e) = \varnothing$. A soft set (F, A) over X is said to be an absolute soft set denoted by $\widetilde{A}$ if for all $e \in A$, $F(e) = X$.

Definition 2.3 ([12])**.** Let Y be a nonempty subset of X, then $\widetilde{Y}$ denotes the soft set (Y, E) over X for which $Y(e) = Y$, for all $e \in E$. In particular, (X, E) will be denoted by $\widetilde{X}$.

Definition 2.4 ([8])**.** For two soft sets (F, A) and (G, B) over X, we say that (F, A) is a soft subset of (G, B) if $A \subseteq B$ and for all $e \in A$, $F(e)$ and $G(e)$ are identical approximations. We write $(F, A) \sqsubseteq (G, B)$. (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(G, B) \sqsubseteq (F, A)$. Then (F, A) and (G, B) are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.5 ([8])**.** The union of two soft sets (F,A) and (G,B) over X is the soft set (H,C), where $C = A \cup B$ and for all $e \in C$, $H(e) = F(e)$ if $e \in A \backslash B$, $H(e) = G(e)$ if $e \in B \backslash A$, $H(e) = F(e) \cup G(e)$ if $e \in A \cap B$. We write $(F,A) \sqcup (G,B) = (H,C)$. [2] The intersection (H,C) of (F,A) and (G,B) over X, denoted $(F,A) \sqcap (G,B)$, is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

Definition 2.6 ([12])**.** The difference (H,E) of two soft sets (F,E) and (G,E) over X, denoted by $(F,E) \backslash (G,E)$, is defined as $H(e) = F(e) \backslash G(e)$ for all $e \in E$.

Definition 2.7 ([12])**.** The relative complement of a soft set (F,E) over X is denoted by $(F,E)^c$ and is defined by $(F,E)^c = (F^c, E)$ where $F^c : E \longrightarrow P(X)$ is a mapping given by $F^c(e) = X \backslash F(e)$ for all $e \in E$.

Proposition 2.8 ([8, 12, 14])**.** *Let (F,E), (G,E), (H,E) and (K,E) be the soft sets over X. Then*

(1) $(F,E) \sqcap (F,E) = (F,E)$, $(F,E) \sqcap \Phi = \Phi$, $(F,E) \sqcap \widetilde{X} = (F,E)$.

(2) $(F,E) \sqcup (F,E) = (F,E)$, $(F,E) \sqcup \Phi = (F,E)$, $(F,E) \sqcup \widetilde{X} = \widetilde{X}$.

(3) $(F,E) \sqcap (G,E) = (G,E) \sqcap (F,E)$, $(F,E) \sqcup (G,E) = (G,E) \sqcup (F,E)$.

(4) $(F,E) \sqcup ((G,E) \sqcup (H,E)) = ((F,E) \sqcup (G,E)) \sqcup (H,E)$, $(F,E) \sqcap ((G,E) \sqcap (H,E)) = ((F,E) \sqcap (G,E)) \sqcap (H,E)$.

(5) $(F,E) \sqcup ((G,E) \sqcap (H,E)) = ((F,E) \sqcup (G,E)) \sqcap ((F,E) \sqcup (H,E))$, $(F,E) \sqcap ((G,E) \sqcup (H,E)) = ((F,E) \sqcap (G,E)) \sqcup ((F,E) \sqcap (H,E))$.

(6) $(F,E) \sqsubseteq (G,E)$ *if and only if* $(F,E) \sqcap (G,E) = (F,E)$.

(7) $(F,E) \sqsubseteq (G,E)$ *if and only if* $(F,E) \sqcup (G,E) = (G,E)$.

(8) *If* $(F,E) \sqcap (G,E) = \Phi$, *then* $(F,E) \sqsubseteq (G,E)^c$.

(9) *If* $(F,E) \sqsubseteq (G,E)$ *and* $(G,E) \sqsubseteq (H,E)$, *then* $(F,E) \sqsubseteq (H,E)$.

(10) *If* $(F,E) \sqsubseteq (G,E)$ *and* $(H,E) \sqsubseteq (K,E)$, *then* $(F,E) \sqcap (H,E) \sqsubseteq (G,E) \sqcap (K,E)$.

(11) $(F,E) \sqcup (F,E)^c = \widetilde{X}$.

(12) $(F,E) \sqsubseteq (G,E)$ *if and only if* $(G,E)^c \sqsubseteq (F,E)^c$.

(13) $((F,E) \sqcup (G,E))^c = (F,E)^c \sqcap (G,E)^c$.

(14) $((F,E) \sqcap (G,E))^c = (F,E)^c \sqcup (G,E)^c$.

Definition 2.9 ([14])**.** Let I be an arbitrary index set and $\{(F_i,E)\}_{i \in I}$ be a subfamily of soft sets over X. The union of these soft sets is the soft set (G,E), where $G(e) = \cup_{i \in I} F_i(e)$ for all $e \in E$. We write $\sqcup_{i \in I}(F_i,E) = (G,E)$. The intersection of these soft sets is the soft set (H,E), where $H(e) = \cap_{i \in I} F_i(e)$ for all $e \in E$. We write $\sqcap_{i \in I}(F_i,E) = (H,E)$.

Definition 2.10 ([12])**.** Let $\widetilde{\tau}$ be the collection of soft sets over a universe X with a fixed set of parameters E, then $\widetilde{\tau}$ is said to be a soft topology on X if

(1) $\Phi, \widetilde{X} \in \widetilde{\tau}$.

(2) the union of any number of soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$.

(3) the intersection of any two soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$.

The triplet $(X, \widetilde{\tau}, E)$ is called a soft topological space. Every member of $\widetilde{\tau}$ is called a soft open set. A soft set (F,E) is called soft closed in X if $(F,E)^c \in \widetilde{\tau}$.

Definition 2.11 ([1])**.** Let μ be the collection of soft sets over a universe X with a fixed set of parameters E, then μ is said to be a supra soft topology on X if

(1) $\Phi, \widetilde{X} \in \mu$.

(2) the union of any number of soft sets in μ belongs to μ.

The triplet (X, μ, E) is called a supra soft topological space. Every member of μ is called a supra open soft set. A soft set (F, E) is called supra closed soft in X if $(F, E)^c \in \mu$.

Remark 2.12 ([1])**.** Every soft topological space is supra soft topological space, but the converse is not true in general as shown in the following example.

Example 2.13 ([1])**.** Let $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2\}$ and $\mu = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E)\}$, where

$F_1(e_1) = \{x_1, x_2\}, \ F_1(e_2) = \{x_2, x_3\},$

$F_2(e_1) = \{x_1, x_3\}, \ F_2(e_2) = \{x_1, x_2\}.$

Then (X, μ, E) is a supra soft topology, but it is not soft topology.

Definition 2.14 ([1])**.** Let (X, μ, E) be a supra soft topological space and (F, E) be a soft set over X.

(1) The supra soft closure of (F, E) is the soft set $(F, E)^{-s} = \cap\{(G, E) : \ (G, E)$ is supra closed soft and $(F, E) \sqsubseteq (G, E)\}$.

(2) The supra soft interior of (F, E) is the soft set $(F, E)^{\circ s} = \cup\{(H, E) : (H, E)$ is supra open soft and $(H, E) \sqsubseteq (F, E)\}$.

Clearly, $(F, E)^{-s}$ is the smallest supra closed soft set over X which contains (F, E) and $(F, E)^{\circ s}$ is the largest supra open soft set over X which is contained in (F, E).

Theorem 2.15 ([1])**.** *Let (X, μ, E) be a supra soft topological space and (F, E) and (G, E) soft sets over X. Then*

(1) $\Phi^{\circ s} = \Phi$ *and* $(\widetilde{X})^{\circ s} = \widetilde{X}$.

(2) $(F, E)^{\circ s} \sqsubseteq (F, E)$.

(3) $((F, E)^{\circ s})^{\circ s} = (F, E)^{\circ s}$.

(4) $(F, E) \sqsubseteq (G, E)$ *implies* $(F, E)^{\circ s} \sqsubseteq (G, E)^{\circ s}$.

(5) $((F, E) \sqcap (G, E))^{\circ s} \sqsubseteq (F, E)^{\circ s} \sqcap (G, E)^{\circ s}$.

Theorem 2.16 ([1])**.** *Let (X, μ, E) be a supra soft topological space and (F, E) and (G, E) soft sets over X. Then*

(1) $\Phi^{-s} = \Phi$ *and* $(\widetilde{X})^{-s} = \widetilde{X}$.

(2) $(F, E) \sqsubseteq (F, E)^{-s}$.

(3) $((F, E)^{-s})^{-s} = (F, E)^{-s}$.

(4) $(F, E) \sqsubseteq (G, E)$ *implies* $(F, E)^{-s} \sqsubseteq (G, E)^{-s}$.

(5) $(F, E)^{-s} \sqcup (G, E)^{-s} \sqsubseteq ((F, E) \sqcup (G, E))^{-s}$.

Definition 2.17 ([5])**.** Let (X, μ, E) be a supra soft topological space. A soft set (F, E) is called a supra generalized closed soft set (supra g-closed soft) in X if $(F, E)^{-s} \sqsubseteq (G, E)$ whenever $(F, E) \sqsubseteq (G, E)$ and (G, E) is a supra open soft set in X.

Remark 2.18 ([5])**.** Every supra closed soft set is supra g-closed soft.

Definition 2.19 ([10, 4]). Let I be a nonempty collection of soft subsets over a universe X with a fixed set of parameters E, then I is said to be a soft ideal on X if the following holds :

(1) If $(F,E) \in I$ and $(G,E) \sqsubseteq (F,E)$, then $(G,E) \in I$ (heredity).

(2) If (F,E) and $(G,E) \in I$, then $(F,E) \sqcup (G,E) \in I$ (additivity).

Definition 2.20 ([5]). Let (X,μ,E) be a supra soft topological space. A soft set (F,E) is called a supra generalized closed soft set with respect to a soft ideal I (supra I-g-closed soft) in X if $(F,E)^{-s} \backslash (G,E) \in I$ whenever $(F,E) \sqsubseteq (G,E)$ and (G,E) is a supra open soft set in X.

3. Supra regular generalized closed (open) soft sets

In this section we introduce supra regular generalized closed soft sets which are weaker than the supra generalized closed soft sets and study some of their properties.

Definition 3.1. Let (X,μ,E) be a supra soft topological space. A soft set (F,E) is called a supra regular open soft set (supra regular closed soft set) in X if $(F,E) = ((F,E)^{-s})^{\circ s}$ $((F,E) = ((F,E)^{\circ s})^{-s})$.

Remark 3.2. Every supra regular closed soft set in a supra soft topological space (X,μ,E) is a supra closed soft set.

Now we give an example to show that the converse of above remark does not hold.

Example 3.3. Let $X = \{x_1,x_2,x_3,x_4\}$, $E = \{e_1,e_2\}$ and
$\mu = \{\Phi, \widetilde{X}, (F_1,E), (F_2,E), (F_3,E), (F_4,E)\}$, where
$F_1(e_1) = \{x_1,x_2\}$, $F_1(e_2) = \{x_1,x_4\}$,
$F_2(e_1) = \{x_2\}$, $F_2(e_2) = \{x_4\}$,
$F_3(e_1) = \{x_3,x_4\}$, $F_3(e_2) = \{x_2,x_3\}$,
$F_4(e_1) = \{x_2,x_3,x_4\}$, $F_4(e_2) = \{x_2,x_3,x_4\}$.
Then (X,μ,E) is a supra soft topological space over X. Clearly, $(F_4,E)^c$ is a supra closed soft set in (X,μ,E), but it is not a supra regular closed soft set.

Definition 3.4. Let (X,μ,E) be a supra soft topological space. A soft set (F,E) is called a supra regular generalized closed soft set (supra rg-closed soft) in X if $(F,E)^{-s} \sqsubseteq (G,E)$ whenever $(F,E) \sqsubseteq (G,E)$ and (G,E) is a supra regular open soft set in X.

Proposition 3.5. *Every supra g-closed soft set in a supra soft topological space (X,μ,E) is a supra rg-closed soft set.*

Proof. Suppose that $(F,E) \sqsubseteq (G,E)$, where (G,E) is supra regular open soft. If (G,E) is supra regular open soft, then (G,E) is supra open soft. Thus $(F,E) \sqsubseteq (G,E)$ and (G,E) is supra open soft. Since (F,E) is supra g-closed soft, $(F,E)^{-s} \sqsubseteq (G,E)$. Thus (F,E) is a supra rg-closed soft set. □

Example 3.6. The supra soft topological space (X,μ,E) is the same as in Example 3.3. Let (H,E) be a soft set over X such that $H(e_1) = \{x_2,x_3\}$, $H(e_2) = \{x_4\}$. Clearly, (H,E) is supra rg-closed soft in (X,μ,E), but it is not supra g-closed soft.

Remark 3.7. The intersection (resp. union) of two supra rg-closed soft sets is generally not a supra rg-closed soft set.

Example 3.8. Let $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$, $E = \{e_1, e_2\}$ and
$\mu = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E), ..., (F_{11}, E)\}$, where
$F_1(e_1) = \{x_2, x_5, x_6\},\ F_1(e_2) = \{x_4, x_5, x_6\}$,
$F_2(e_1) = \{x_5\},\ F_2(e_2) = \{x_5\}$,
$F_3(e_1) = \{x_1, x_2, x_3\},\ F_3(e_2) = \{x_1\}$,
$F_4(e_1) = \{x_2\},\ F_4(e_2) = \{x_4, x_6\}$,
$F_5(e_1) = \{x_1, x_2, x_3, x_5, x_6\},\ F_5(e_2) = \{x_1, x_4, x_5, x_6\}$,
$F_6(e_1) = \{x_1, x_2, x_3, x_5\},\ F_6(e_2) = \{x_1, x_5\}$,
$F_7(e_1) = \{x_2, x_5\},\ F_7(e_2) = \{x_4, x_5, x_6\}$,
$F_8(e_1) = \{x_1, x_2, x_3\},\ F_8(e_2) = \{x_1, x_4, x_6\}$,
$F_9(e_1) = \{x_1, x_2, x_3, x_5\},\ F_9(e_2) = \{x_1, x_4, x_5, x_6\}$,
$F_{10}(e_1) = X,\ F_{10}(e_2) = \{x_1, x_2, x_3, x_4, x_6\}$,
$F_{11}(e_1) = \{x_1, x_2, x_3, x_4, x_6\},\ F_{11}(e_2) = X$.
Then (X, μ, E) is a supra soft topological space over X. Let (H, E) and (G, E) be two soft sets over X such that $H(e_1) = \varnothing,\ H(e_2) = \{x_5\}$ and $G(e_1) = \{x_5\},\ G(e_2) = \varnothing$. Clearly, (H, E) and (G, E) are supra rg-closed soft sets in (X, μ, E) but $(H, E) \sqcup (G, E)$ is not a supra rg-closed soft set.

Example 3.9. The supra soft topological space (X, μ, E) is the same as in Example 3.8. Let (H, E) and (G, E) be two soft sets over X such that

$$H(e_1) = \{x_1, x_2, x_3, x_4\},\ H(e_2) = \{x_1, x_4, x_6\}$$

and

$$G(e_1) = \{x_1, x_2, x_3\},\ G(e_2) = \{x_1, x_4, x_5, x_6\}.$$

Clearly, (H, E) and (G, E) are supra rg-closed soft sets in (X, μ, E) but $(H, E) \sqcap (G, E)$ is not a supra rg-closed soft set.

Theorem 3.10. *Let (X, μ, E) be a supra soft topological space and (F, E) be a soft set over X. If a soft set (F, E) is a supra rg-closed soft set, then $(F, E)^{-s} \backslash (F, E)$ contains only null supra regular closed soft set.*

Proof. Suppose that (F, E) is a supra rg-closed soft set. Let (H, E) be a supra regular closed soft subset of $(F, E)^{-s} \backslash (F, E)$. Then $(H, E) \sqsubseteq (F, E)^{-s} \sqcap (F, E)^c$. Thus $(F, E) \sqsubseteq (H, E)^c$. But (F, E) is a supra rg-closed soft set. So $(F, E)^{-s} \sqsubseteq (H, E)^c$. Consequently

$$(H, E) \sqsubseteq ((F, E)^{-s})^c\ . \tag{3.1}$$

We have already

$$(H, E) \sqsubseteq (F, E)^{-s}\ . \tag{3.2}$$

From (3.1) and (3.2)

$$(H, E) \sqsubseteq (F, E)^{-s} \sqcap ((F, E)^{-s})^c = \Phi.$$

Thus $(H, E) = \Phi$. Therefore $(F, E)^{-s} \backslash (F, E)$ contains only null supra regular closed soft set. □

The converse of this theorem is not true in general as can be seen from the following example.

Example 3.11. The supra soft topological space (X, μ, E) is the same as in Example 3.8. $(F_8, E)^{-s} \backslash (F_8, E)$ contains only null supra regular closed soft set. But (F_8, E) is not a supra rg-closed soft set in (X, μ, E).

Corollary 3.12. *Let (X, μ, E) be a supra soft topological space and (F, E) be a supra rg-closed soft set in X. (F, E) is supra regular closed soft if and only if $((F, E)^{\circ s})^{-s} \backslash (F, E)$ is supra regular closed soft.*

Proof. Let (F, E) be a supra rg-closed soft set. If (F, E) is supra regular closed soft, then $((F, E)^{\circ s})^{-s} \backslash (F, E) = \Phi$. Since Φ is always supra regular closed soft, $((F, E)^{\circ s})^{-s} \backslash (F, E)$ is supra regular closed soft.

Conversely, suppose that $((F, E)^{\circ s})^{-s} \backslash (F, E)$ is supra regular closed soft. Since (F, E) is supra rg-closed soft and $(F, E)^{-s} \backslash (F, E)$ contains the supra regular closed soft set $((F, E)^{\circ s})^{-s} \backslash (F, E)$, $((F, E)^{\circ s})^{-s} \backslash (F, E) = \Phi$ by Theorem 3.10. Thus $((F, E)^{\circ s})^{-s} = (F, E)$. So (F, E) is supra regular closed soft. □

Theorem 3.13. *Let (X, μ, E) be a supra soft topological space, (F, E) and (G, E) soft sets over X. If (F, E) is supra rg-closed soft and $(F, E) \sqsubseteq (G, E) \sqsubseteq (F, E)^{-s}$, then $(G, E)^{-s} \backslash (G, E)$ contains only null supra regular closed soft set.*

Proof. Since $(F, E) \sqsubseteq (G, E)$, $(G, E)^c \sqsubseteq (F, E)^c$. Since $(G, E) \sqsubseteq (F, E)^{-s}$, $(G, E)^{-s} \sqsubseteq ((F, E)^{-s})^{-s} = (F, E)^{-s}$. Then $(G, E)^{-s} \sqsubseteq (F, E)^{-s}$. Thus

$$((G, E)^{-s} \sqcap (G, E)^c) \sqsubseteq ((F, E)^{-s} \sqcap (F, E)^c).$$

This implies

$$((G, E)^{-s} \backslash (G, E)) \sqsubseteq ((F, E)^{-s} \backslash (F, E)).$$

Now (F, E) is supra rg-closed soft. Hence $(F, E)^{-s} \backslash (F, E)$ contains only null supra regular closed soft, neither does $(G, E)^{-s} \backslash (G, E)$. □

Definition 3.14. Let (X, μ, E) be a supra soft topological space. A soft set (F, E) is called supra regular generalized open soft (supra rg-open soft) in X if $(F, E)^c$ is supra rg-closed soft.

Theorem 3.15. *A soft set (F, E) is supra rg-open soft in a supra soft topological space (X, μ, E) if and only if $(H, E) \sqsubseteq (F, E)^{\circ s}$ whenever (H, E) is supra regular closed soft in X and $(H, E) \sqsubseteq (F, E)$.*

Proof. Let $(H, E) \sqsubseteq (F, E)^{\circ s}$ whenever (H, E) is supra regular closed soft in X, $(H, E) \sqsubseteq (F, E)$ and $(K, E) = (F, E)^c$. Suppose that $(K, E) \sqsubseteq (G, E)$ where (G, E) is supra regular open soft.

Now $(F, E)^c \sqsubseteq (G, E)$ implies $(H, E) = (G, E)^c \sqsubseteq (F, E)$ and (H, E) is supra regular closed soft which implies $(H, E) \sqsubseteq (F, E)^{\circ s}$. Also $(H, E) \sqsubseteq (F, E)^{\circ s}$ implies $((F, E)^{\circ s})^c \sqsubseteq (H, E)^c = (G, E)$. This inturn implies $(((K, E)^c)^{\circ s})^c \sqsubseteq (G, E)$. Or equivalently $(K, E)^{-s} \sqsubseteq (G, E)$. Thus (K, E) is supra rg-closed soft. So we obtain (F, E) is supra rg-open soft.

Conversely, suppose that (F, E) is supra rg-open soft, $(H, E) \sqsubseteq (F, E)$ and (H, E) is supra regular closed soft. Then $(H, E)^c$ is supra regular open soft. Thus $(F, E)^c \sqsubseteq$

$(H,E)^c$. Since $(F,E)^c$ is supra rg-closed soft, $((F,E)^c)^{-s} \sqsubseteq (H,E)^c$. So $(H,E) \sqsubseteq (((F,E)^c)^{-s})^c = (F,E)^{\circ s}$. □

Remark 3.16. The intersection (resp. union) of two supra rg-open soft sets is generally not a supra rg-open soft set. (See Example 3.8 and Example 3.9)

4. Supra regular generalized closed (open) soft sets with respect to a soft ideal

In this section we study the concept of supra regular generalized closed soft sets with respect to a soft ideal in a supra soft topological space (X,μ,E), which is the extension of the concept of supra generalized closed soft sets.

Definition 4.1. A soft set (F,E) is called a supra regular generalized closed soft set with respect to a soft ideal I (supra I-rg-closed soft) in a supra soft topological space (X,μ,E) if $(F,E)^{-s}\backslash(G,E) \in I$ whenever $(F,E) \sqsubseteq (G,E)$ and (G,E) is supra regular open soft in (X,μ,E).

Example 4.2. Let $X = \{x_1,x_2,x_3\}$, $E = \{e_1,e_2\}$ and $\mu = \{\Phi, \widetilde{X}, (F_1,E), (F_2,E), (F_3,E)\}$, where
$F_1(e_1) = \{x_3\},\ F_1(e_2) = \{x_3\}$,
$F_2(e_1) = \{x_2,x_3\},\ F_2(e_2) = \{x_3\}$,
$F_3(e_1) = \{x_2\},\ F_3(e_2) = \varnothing$.
Then (X,μ,E) is a supra soft topological space. Let $I = \{\Phi, (I_1,E), (I_2,E), (I_3,E), ..., (I_7,E)\}$ be a soft ideal on X, where
$I_1(e_1) = \varnothing,\ I_1(e_2) = \{x_2\}$,
$I_2(e_1) = \varnothing,\ I_2(e_2) = \{x_1,x_2\}$,
$I_3(e_1) = \{x_1\},\ I_3(e_2) = \varnothing$,
$I_4(e_1) = \{x_1\},\ I_4(e_2) = \{x_1,x_2\}$,
$I_5(e_1) = \{x_1\},\ I_5(e_2) = \{x_1\}$,
$I_6(e_1) = \varnothing,\ I_6(e_2) = \{x_1\}$,
$I_7(e_1) = \{x_1\},\ I_7(e_2) = \{x_2\}$.
Clearly, (F_3,E) is supra I-rg-closed soft. In fact, $(F_3,E) \sqsubseteq (F_3,E)$ and (F_3,E) is supra regular open soft. Thus we obtain $(F_3,E)^{-s}\backslash(F_3,E) \in I$.

Proposition 4.3. *Every supra rg-closed soft set is supra I-rg-closed soft.*

Proof. Let (F,E) be a supra rg-closed soft set in a supra soft topological space (X,μ,E). Let $(F,E) \sqsubseteq (G,E)$ and (G,E) be supra regular open soft in (X,μ,E). Since (F,E) is supra rg-closed soft, $(F,E)^{-s} \sqsubseteq (G,E)$. Thus $(F,E)^{-s}\backslash(G,E) = \Phi \in I$. So (F,E) is a supra I-rg-closed soft set. □

The converse of this proposition is not true in general as can be seen from the following example.

Example 4.4. Let us take the supra soft topology μ on X in Example 4.2. Let (H,E) be a soft set over X such that $H(e_1) = \{x_3\},\ H(e_2) = \varnothing$. Clearly, (H,E) is supra I-rg-closed soft but it is not supra rg-closed soft.

Theorem 4.5. *Every supra I-g-closed soft set is supra I-rg-closed soft.*

Proof. Let (F, E) be a supra I-g-closed soft set in a supra soft topological space (X, μ, E). We show that (F, E) is supra I-rg-closed soft. Suppose that $(F, E) \sqsubseteq (G, E)$, where (G, E) is supra regular open soft. If (G, E) is supra regular open soft, then (G, E) is supra open soft. Thus $(F, E) \sqsubseteq (G, E)$ and (G, E) is supra open soft. Since (F, E) is supra I-g-closed soft, $(F, E)^{-s} \backslash (G, E) \in I$. So (F, E) is a supra I-rg-closed soft set. □

Example 4.6. Let $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2\}$ and $\mu = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$, where
$F_1(e_1) = \{x_1, x_2, x_4\}$, $F_1(e_2) = X$,
$F_2(e_1) = \{x_1\}$, $F_2(e_2) = \{x_2\}$,
$F_3(e_1) = \{x_1, x_2\}$, $F_3(e_2) = \{x_1, x_2, x_3\}$.
Then (X, μ, E) is a supra soft topological space over X. Let $I = \{\Phi, (I_1, E), (I_2, E), (I_3, E)\}$ be a soft ideal on X, where
$I_1(e_1) = \{x_2\}$, $I_1(e_2) = \varnothing$,
$I_2(e_1) = \varnothing$, $I_2(e_2) = \{x_1\}$,
$I_3(e_1) = \{x_2\}$, $I_3(e_2) = \{x_1\}$.
Clearly, (F_1, E) is supra I-rg-closed soft in (X, μ, E), but it is not supra I-g-closed soft.

Remark 4.7. The intersection (resp. union) of two supra I-rg-closed soft sets is generally not a supra I-rg-closed soft set.

Example 4.8. The supra soft topological space (X, μ, E) is the same as in Example 3.8. Let $I = \{\Phi, (I_1, E), (I_2, E), (I_3, E)\}$ be a soft ideal on X, where
$I_1(e_1) = \{x_3\}$, $I_1(e_2) = \varnothing$,
$I_2(e_1) = \varnothing$, $I_2(e_2) = \{x_1\}$,
$I_3(e_1) = \{x_3\}$, $I_3(e_2) = \{x_1\}$.
Let (H, E) and (G, E) be two soft sets over X such that $H(e_1) = \varnothing$, $H(e_2) = \{x_5\}$ and $G(e_1) = \{x_5\}$, $G(e_2) = \varnothing$. Clearly, (H, E) and (G, E) are supra I-rg-closed soft sets in (X, μ, E) but $(H, E) \sqcup (G, E)$ is not a supra I-rg-closed soft set. Also, let (H, E) and (G, E) be two soft sets over X such that

$$H(e_1) = \{x_1, x_2, x_3, x_4\}, H(e_2) = \{x_1, x_4, x_6\}$$

and

$$G(e_1) = \{x_1, x_2, x_3\}, G(e_2) = \{x_1, x_4, x_5, x_6\}.$$

Clearly, (H, E) and (G, E) are supra I-rg-closed soft sets in (X, μ, E) but $(H, E) \sqcap (G, E)$ is not a supra I-rg-closed soft set.

Theorem 4.9. *Let (X, μ, E) be a supra soft topological space and (F, E) be a soft set over X. If a soft set (F, E) is supra I-rg-closed soft, then $(H, E) \sqsubseteq (F, E)^{-s} \backslash (F, E)$ and (H, E) is supra regular closed soft implies $(H, E) \in I$.*

Proof. Suppose that (F, E) is supra I-rg-closed soft. Let (H, E) be a supra regular closed soft subset of $(F, E)^{-s} \backslash (F, E)$. Then $(H, E) \sqsubseteq (F, E)^{-s} \sqcap (F, E)^c$. Thus $(F, E) \sqsubseteq (H, E)^c$. Since (F, E) is a supra I-rg-closed soft set, $(F, E)^{-s} \backslash (H, E)^c \in I$. But $(H, E) \sqsubseteq (F, E)^{-s} \backslash (H, E)^c$, by the properties of soft ideal $(H, E) \in I$. □

We note that the converse of this theorem is not true in general as shown in the following example.

Example 4.10. The supra soft topological space (X, μ, E) and soft ideal I are the same as in Example 4.8. $(F_8, E)^{-s} \backslash (F_8, E)$ contains soft set (G, E) such that $(G, E) = \Phi$ is only supra regular closed soft set implies $(G, E) = \Phi \in I$. But (F_8, E) is not a supra I-rg-closed soft set in (X, μ, E).

Theorem 4.11. *Let (X, μ, E) be a supra soft topological space, (F, E) and (G, E) soft sets over X. If (F, E) is supra I-rg-closed soft and $(F, E) \sqsubseteq (G, E) \sqsubseteq (F, E)^{-s}$, then (G, E) is supra I-rg-closed soft.*

Proof. Let (F, E) be a supra I-rg-closed soft set and $(F, E) \sqsubseteq (G, E) \sqsubseteq (F, E)^{-s}$. Suppose that $(G, E) \sqsubseteq (H, E)$ and (H, E) is supra regular open soft. Then $(F, E) \sqsubseteq (H, E)$. Since (F, E) is supra I-rg-closed soft, $(F, E)^{-s} \backslash (H, E) \in I$. If $(G, E) \sqsubseteq (F, E)^{-s}$, then $(G, E)^{-s} \sqsubseteq ((F, E)^{-s})^{-s} = (F, E)^{-s}$. So we have $[(G, E)^{-s} \backslash (H, E)] \sqsubseteq [(F, E)^{-s} \backslash (H, E)]$ and we obtain $(G, E)^{-s} \backslash (H, E) \in I$. Hence (G, E) is supra I-rg-closed soft. □

Definition 4.12. A soft set (F, E) is called supra regular generalized open soft set with respect to a soft ideal I (supra I-rg-open soft) in a supra soft topological space (X, μ, E) if and only if $(F, E)^c$ is supra I-rg-closed soft in (X, μ, E).

Theorem 4.13. *A soft set (F, E) is a supra I-rg-open soft set in a supra soft topological space (X, μ, E) if and only if $(G, E) \backslash (H, E) \sqsubseteq (F, E)^{\circ s}$ for some $(H, E) \in I$, whenever $(G, E) \sqsubseteq (F, E)$ and (G, E) is supra regular closed soft in (X, μ, E).*

Proof. Suppose that (F, E) is supra I-rg-open soft, $(G, E) \sqsubseteq (F, E)$ and (G, E) is supra regular closed soft. Then $(G, E)^c$ is supra regular open soft. Thus $(F, E)^c \sqsubseteq (G, E)^c$. Since $(F, E)^c$ is supra I-rg-closed soft, $((F, E)^c)^{-s} \backslash (G, E)^c \in I$. That is

$$((F, E)^c)^{-s} \backslash (G, E)^c = ((F, E)^c)^{-s} \sqcap (G, E) = (H, E) \in I.$$

So $[((F, E)^c)^{-s} \sqcap (G, E)] \sqcup (G, E)^c = (H, E) \sqcup (G, E)^c$. Hence $((F, E)^c)^{-s} \sqsubseteq (G, E)^c \sqcup (H, E)$ for some $(H, E) \in I$. Therefore $[(G, E)^c \sqcup (H, E)]^c \sqsubseteq (((F, E)^c)^{-s})^c = (F, E)^{\circ s}$ and thus$(G, E) \backslash (H, E) = (G, E) \sqcap (H, E)^c \sqsubseteq (F, E)^{\circ s}$.

Conversely, assume that $(G, E) \sqsubseteq (F, E)$ and (G, E) is supra regular closed soft in (X, μ, E). Then $(G, E) \backslash (H, E) \sqsubseteq (F, E)^{\circ s}$ for some $(H, E) \in I$. Now we show that $(F, E)^c$ is supra I-rg-closed soft. Let $(F, E)^c \sqsubseteq (K, E)$ and (K, E) is supra regular open soft. Then $(K, E)^c \sqsubseteq (F, E)$. By assumption, $(K, E)^c \backslash (H, E) \sqsubseteq (F, E)^{\circ s} = (((F, E)^c)^{-s})^c$ for some $(H, E) \in I$. This gives that $[(K, E) \sqcup (H, E)]^c \sqsubseteq (((F, E)^c)^{-s})^c$. Then $((F, E)^c)^{-s} \sqsubseteq (K, E) \sqcup (H, E)$ for some $(H, E) \in I$. Thus

$$\begin{aligned} ((F, E)^c)^{-s} \backslash (K, E) &\sqsubseteq ((K, E) \sqcup (H, E)) \backslash (K, E) \\ &= ((K, E) \sqcup (H, E)) \sqcap (K, E)^c = (H, E) \sqcap (K, E)^c \\ &\sqsubseteq (H, E) \in I. \end{aligned}$$

So $((F, E)^c)^{-s} \backslash (K, E) \in I$ (from the properties of soft ideal). Hence $(F, E)^c$ is supra I-rg-closed soft and (F, E) is supra I-rg-open soft. □

Remark 4.14. The intersection (resp. union) of two supra I-rg-open soft sets is generally not supra I-rg-open soft. (See Example 4.8).

Theorem 4.15. *Let (X, μ, E) be a supra soft topological space and (F, E) be a soft set over X. If a soft set (F, E) is supra I-rg-closed soft, then $(F, E)^{-s} \backslash (F, E)$ is supra I-rg-open soft.*

Proof. Suppose that (F, E) is supra I-rg-closed and $(H, E) \sqsubseteq (F, E)^{-s} \backslash (F, E)$, where (H, E) is supra regular closed soft. Then $(H, E) \in I$. Thus $(H, E) \backslash (G, E) = \Phi$ for some $(G, E) \in I$. Clearly $(H, E) \backslash (G, E) \sqsubseteq ((F, E)^{-s} \backslash (F, E))^{\circ s}$. So ,by Theorem 4.13, $(F, E)^{-s} \backslash (F, E)$ is supra I-rg-open soft. □

The converse of this theorem is not true in general as can be seen from the following example.

Example 4.16. The supra soft topological space (X, μ, E) and soft ideal I are the same as in Example 4.8. Let (H, E) be a soft set over X such that $H(e_1) = \{x_5\}$, $H(e_2) = \{x_5\}$. $(H, E)^{-s} \backslash (H, E)$ is a supra I-rg-open soft set. But (H, E) is not supra I-rg-closed soft in (X, μ, E).

5. Conclusions

In the present work, we have studied some new concepts in supra soft topological spaces which are defined over an initial universe with a fixed set of parameters such as supra regular generalized closed (open) soft sets and supra regular generalized closed (open) soft sets with respect to a soft ideal. Also, we have investigated many basic properties of these concepts. In future more general types of supra regular generalized closed soft sets may be defined and using of them characterizations related with supra soft separation axioms and supra soft continuity may be studied. Hence we expect that some research teams will be actively working on supra regular generalized closed (open) soft sets and different types of subsets of supra soft topological spaces with respect to a soft ideal.

References

[1] S. A. El-Sheikh and A. M. Abd El-Latif, Decompositions of some types of supra soft sets and soft continuity, International Journal of Mathematics Trends and Technology 9 (1) (2014) 37–56.
[2] F. Feng, Y. B. Jun and X. Zhao, Soft semirings, Comput. Math. Appl. 56 (2008) 2621–2628.
[3] S. Hussain and B. Ahmad, Some properties of soft topological spaces, Comput. Math. Appl. 62 (2011) 4058–4067.
[4] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-Latif, Soft ideal theory soft local function and generated soft topological spaces, Appl. Math. Inf. Sci. 8 (4) (2014) 1595–1603.
[5] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-Latif, Supra generalized closed soft sets with respect to an soft ideal in supra soft topological spaces, Appl. Math. Inf. Sci. 8 (4) (2014) 1731–1740.
[6] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-Latif, γ -operation and decompositions of some forms of soft continuity in soft topological spaces, Ann. Fuzzy Math. Inform. 7 (2) (2014) 181–196.
[7] J. Mahanta and P. K. Das, On soft topological space via semiopen and semiclosed soft sets, Kyungpook Math. J. 54 (2) (2014) 221–236.
[8] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555–562.
[9] D. Molodtsov, Soft set theory-first results, Comput. Math. Appl. 37 (1999) 19–31.
[10] H. I. Mustafa and F. M. Sleim, Soft generalized closed sets with respect to an ideal in soft topological spaces, Appl. Math. Inf. Sci. 8 (2) (2014) 665–671.

[11] B. Pazar Varol and H. Aygün, On soft Hausdorff spaces, Ann. Fuzzy Math. Inform. 5 (1) (2013) 15–24.
[12] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl. 61 (2011) 1786–1799.
[13] S. Yüksel, N. Tozlu and Z. Güzel Ergül, Soft regular generalized closed sets in soft topological spaces, International Journal of Mathematical Analysis, 8 (8) 355–367.
[14] İ. Zorlutuna, M. Akdağ, W. K. Min and S. Atmaca, Remarks on soft topological spaces, Ann. Fuzzy Math. Inform. 3 (2) (2012) 171–185.

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