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Magic labeling of interval-valued fuzzy graph

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Abstract. In this paper, we introduced the concept of magic labeling of interval-valued fuzzy graph. It is well-known about the concept of magic square, its beauty and applications, from which the concept of magic labeling is inspired. Here we discussed the significant perception over magic labeling of interval-valued fuzzy graph, we obtained some of its properties and also incur some structures and bring it into the operation of interval-valued fuzzy magic labeling graph. We have established neighborhood intervals and obtained some bounds over the size and shape of the interval-valued fuzzy graph and confine the membership values of the nodes and edges.

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1. Introduction

For more adequate description of uncertainty Zadeh [26] introduced in 1975 the concept of interval-valued fuzzy set which is a generalization of traditional fuzzy set [25, 26]. It is therefore prominent to use interval-valued fuzzy set in applications, such as in fuzzy control, etc. As we realize that defuzzification is an intensive part of fuzzy control but the work on approximate reasoning by Gorzalczany [9, 10] shows the wide range of application of interval-valued fuzzy set. Many researchers adopted the concept of interval-valued fuzzy set and applied it in the various areas like on medical diagnosis. Roy and Biswas [20] admits, rather than crisp, fuzzy set provide more apparent and immaculate result. Further, Turksen on multivalued logic [24] and Mendelś works on intelligent control [14] reveals ponder ability of fuzzy set.

Simultaneously, according to enhancement of usual set the changes reflects in graph theory and fuzzy graph theory was introduced by Rosenfeld [19] in 1975 and developed the structure by obtaining analogs of several graph theoretical concepts. Later, Mordeson and Peng [15] introduced some operations on fuzzy graphs and further

Sunitha and Vijayakumar [22], Bhutani and Rosenfeld [4], Hongmei and Lianhua [12], Pramanik, Samanta and Pal [16, 17] and others studied it and obtained various properties and applied it into several areas of graph theory, network and optimization. In crisp graph, we are familiar with the concept of labeling which trace their identity to one introduced by Rosa [18] in 1967, or by Graham and Sloane [11] in 1980. Among several researcher Beineke and Hegde [3] assumes that labeling of discrete structure is a frontier between graph theory and theory of numbers, it attracts many researchers, some more types of labeling and its applications gets existence into the literature like Stanley [21] and Avadayappan's [2] work on magic labeling. Afterward Gani et al. [7, 8] introduced the concept of labeling and magic labeling of fuzzy graph. Only fuzzy graph remain scanty to solve all the problems existed in real life, henceforth we are interested to materialize the concept of interval-valued fuzzy graph which is defined by Akram and Dudek [1] and many other researchers, Ismayil and Ali [13], Talebi and Rashmanlou [23], Debnath [5, 6] obtained various properties over it. There are promiscuous real life applications of interval-valued fuzzy graph, as for example suppose we want to draw status of a running train for scheduling in winter season which passes through some junctions. Weather, distortion of track, signal communication all becomes parameter and are uncertain which effect the speed of the train between the stations. Therefore, due to its time dependency possible speed of the train admits interval-valued membership. Thus arrival of train at each junction is not certain and hence the complete event can be represented using interval-valued fuzzy number. For this problem if we represent junctions by interval-valued fuzzy nodes and route between two junctions by interval-valued fuzzy edges, then problem will be modeled into interval-valued fuzzy graph and can be optimize to get the proper result.

Similarly, for any natural disaster numerous people and animals are trapped in the effected region, and most of things in the region becomes completely unknown. Many roads and buildings are destroyed, landmarks and identification of places becomes difficult. For this situation shortest and fastest routes are required for evacuation. Thus to find such routes for the rescue, becomes challenging one for the operational team. However, the information about the region is not precise, it is collected from different individuals, e.g., villagers, trekkers, doctors, military personals, etc. Different type of uncertainty are associated with it which are listed below:

(i) Major uncertainty is associated with the meaning of words used in the description. As every people have freedom to use the words according to their amenities, which are linguistic uncertainty.

(ii) As data (information) varies with respect to time due to the new construction by rescue team and repair of damaged regions.

Hence the application of an interval-valued membership for different operations are more significant than any other approach. Here in this paper, we introduced the concept of magic labeling for interval-valued fuzzy graph which has a wider applications in the various area of research, among them network, optimization, medical diagnostic system and remote sensing are the most favorable.

2. Preliminaries

In [7, 8] Gani et al. proposed the concept of labeling of fuzzy graph, in which they defined fuzzy labeling, magic labeling of a graph and also obtained various results over it. But, they were unable to claim that whether every fuzzy graph will get label or not. In the consequence of that work we are trying to solve that problem upto some extent by defining interval-valued fuzzy labeling of a graph.

Throughout this work, $\tilde{G}$ is a fuzzy graph, G^* is an interval-valued fuzzy graph and G is a underlying crisp graph so for $\tilde{G}$ both V and E are fuzzy sets where V be a fuzzy set with membership value $\mu_v : V \to [0,1]$ and E is a fuzzy relation on $V \times V$ and its membership values are $\mu_e : V \times V \to [0,1]$ where $\mu_e(uv) \leq \mu_v(u) \wedge \mu_v(v)$.

Definition 2.1 ([1])**.** By an interval-valued fuzzy graph of a graph G we mean a pair $G^* = (A, B)$, where $A = [{\mu_A}^-, {\mu_A}^+]$ is an interval-valued fuzzy set on V and $B = [{\mu_B}^-, {\mu_B}^+]$ is an interval-valued fuzzy relation on E such that

$$\begin{aligned} {\mu_B}^-(xy) &\leq min({\mu_A}^-(x), {\mu_A}^-(y)),\\ {\mu_B}^+(xy) &\leq min({\mu_A}^+(x), {\mu_A}^+(y)), \end{aligned}$$

for all $xy \in E$.

Definition 2.2 ([7])**.** A graph $\tilde{G} = (\mu_v, \mu_e)$ is said to be a fuzzy labeling graph, if $\mu_v : V \to [0,1]$ and $\mu_e : V \times V \to [0,1]$ are bijective such that the membership value of nodes and edges are distinct and $\mu_e(x,y) \leq \mu_v(x) \wedge \mu_v(y) \;\; \forall x, y \in V$.

Definition 2.3 ([7])**.** A fuzzy labeling graph is said to be a fuzzy magic graph if $\mu_v(u) + \mu_e(u,v) + \mu_v(v)$ has a same magic value for all $u, v \in V$.

Definition 2.4. A graph $G^* = (A, B)$ is said to be an interval-valued fuzzy labeling graph, if $\mu_A^-, \mu_A^+, \mu_B^-, \mu_B^+ \in [0,1]$ all are distinct for each nodes and edges, where μ_A^- is lower limit and μ_A^+ is the upper limit of the interval membership of nodes, similarly, μ_B^-, μ_B^+ are lower and upper limit respectively of the interval membership of edges.

Definition 2.5. An interval $[\mu - \epsilon, \mu + \epsilon]$ is said to be an ϵ-neighborhood of any membership value(i.e.,corresponding to any nodes or edges) μ for any ϵ satisfying the following conditions :

(i) $\epsilon \ngtr \min\limits_{i,j}\{\mu_v(v_i), \mu_e(e_{ij})\}$,

(ii)$\epsilon \ngtr 1 - \max\limits_{i,j}\{\mu_v(v_i), \mu_e(e_{ij})\}$,

(iii)$\epsilon \neq d(\mu(x), \mu(y)) or \frac{1}{2} d(\mu(x), \mu(y))$,

where $d(\mu(x), \mu(y)) = |\mu(x) - \mu(y)|$ and $\mu(x), \mu(y)$ are the membership of nodes or edges.

Theorem 2.6. *Any fuzzy graph can be converted into interval-valued fuzzy labeling graph.*

Proof. As we know that every fuzzy graph is not a fuzzy labeling graph. Thus for label any fuzzy graph we take interval-valued membership of all nodes and edges in such a way so that the obtained graph get labeled. For this a proper approach is to take an ϵ-neighborhood corresponding to each node and edge, here we claim that it gives an interval-valued labeling graph. For any fuzzy graph only three cases are

there, either all nodes and edges have same membership value, or only few nodes and edges have same membership value or all nodes and edges have distinct membership values.
case(i): If all the nodes and edges have same membership values, then if the sum of the number of nodes and edges is n then we take n distinct ϵ as defined above and on assigning ϵ-neighborhood, we get an interval-valued fuzzy labeling graph.
case(ii): If only few nodes and edges have same membership value, then firstly we make a list of all membership values and corresponding nodes or edges, from it we take a set of membership values and assign one $\epsilon = \epsilon_1$ for them and strike off those membership values and corresponding nodes or edges from the list. Again we take another set of membership values from the remaining element of the list and assign another $\epsilon = \epsilon_2$ to them and again strike off the assigned membership values and corresponding nodes or edges from the list. Continue this process till the last element in the list. Thus corresponding to each $\epsilon_i, i = 1, 2, ...$ we get distinct ϵ-neighborhood interval, assigning it to the corresponding nodes and edges, we get an interval-valued fuzzy graph satisfies the condition of interval-valued fuzzy labeling.
case(iii): If all nodes and edges have distinct membership value, then we can take only one ϵ and we get distinct ϵ-neighborhood corresponding to each nodes and edges.
Hence in each case we can convert a fuzzy graph into interval-valued fuzzy labeling graph. □

Example 2.7. Let $\tilde{G}$ be a fuzzy graph with four nodes and four edges with some membership. We have to obtain corresponding interval-valued fuzzy labeling graph G^*. In this example, list of membership values of $\tilde{G}$ is $\{0.5, 0.7, 0.6, 0.6, 0.2, 0.4, 0.3, 0.4\}$

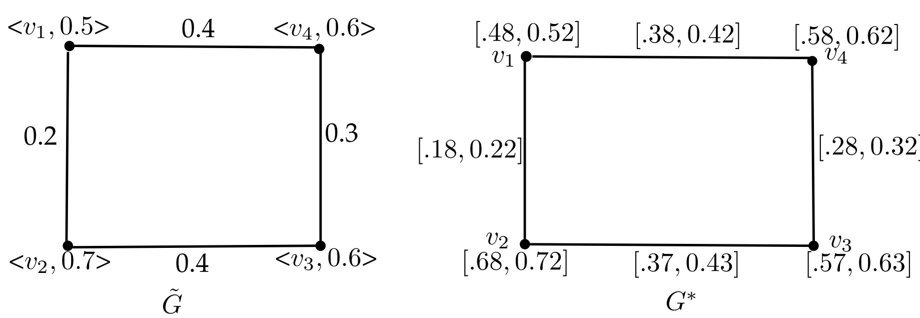

FIGURE 1. A fuzzy graph and corresponding interval-valued fuzzy labeling graph

corresponding to $v_1, v_2, v_3, v_4, v_1v_2, v_2v_3, v_3v_4, v_4v_1$. A set W is formed from all distinct membership in the list $\tilde{G}$ is $W = \{0.5, 0.7, 0.6, 0.2, 0.4, 0.3\}$ which is corresponding to $v_1, v_2, v_3, v_1v_2, v_3v_4, v_4v_1$, set $\epsilon = .02$ for W and strike off those element from the list $\tilde{G}$ which are in W. Again we consider another set W_1 formed from $\tilde{G} - W$ so, $W_1 = \{0.6, 0.4\}$ corresponding to v_4, v_2v_3, set $\epsilon = .03$ for it and on striking off W_1 list $\tilde{G}$ becomes empty obtained interval-valued fuzzy labeling graph is shown in FIGURE 1 as G^*.

3. Interval-valued fuzzy magic labeling graph

Definition 3.1. An interval-valued fuzzy labeling graph is said to be an interval-valued fuzzy magic graph if the lower magic membership value (i.e.,$\mu_A^-(x)+\mu_B^-(x,y)+\mu_A^-(y)$) remains equal for all $x, y \in V$ and the upper magic membership value (i.e.,$\mu_A^+(x) + \mu_B^+(x,y) + \mu_A^+(y)$) remains equal for all $x, y \in V$. The lower magic membership value is denoted as $m_0^-(G^*)$ and upper magic membership value is denoted by $m_0^+(G^*)$ and we denote an interval-valued fuzzy magic graph by $M_0(G^*)$.

Example 3.2. An example of an interval-valued fuzzy magic labeling cycle graph with three nodes and three edges is given in Figure 2. In this graph $\mu_A^-(v_i)$ +

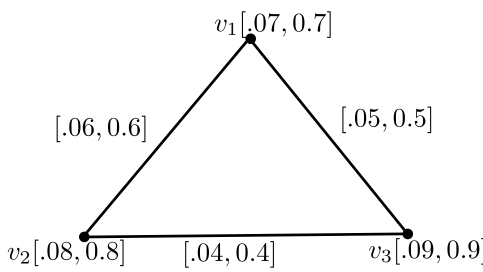

Figure 2. An Interval-valued fuzzy magic graph

$\mu_B^-(v_i, v_j) + \mu_A^-(v_j) = 0.08 + 0.04 + 0.09 = 0.21$, for all $v_1, v_2, v_3 \in V$ and $\mu_A^+(v_i) + \mu_B^+(v_i, v_j) + \mu_A^+(v_j) = 0.8 + 0.4 + 0.9 = 2.1$ for all $v_1, v_2, v_3 \in V$. Here $m_0^-(G^*) = 0.21$ and $m_0^+(G^*) = 2.1$. Thus graph is magic labeled.

Theorem 3.3. *Every fuzzy magic graph can be converted into interval-valued fuzzy magic graph.*

Proof. In fuzzy magic graph all the nodes and edges assigns distinct membership values. Let the membership value of nodes are $\mu_v(v_i)$ and edges are $\mu_e(e_{ij})$ for nodes v_i, v_j. Let the sum of membership values for each pair of nodes and corresponding edges is S, i.e., $\mu_v(v_i)+\mu_e(e_{ij})+\mu_v(v_j) = S$. Now, we find $M = \max_{i,j}\{\mu_v(v_i), \mu_e(e_{ij})\}$ and $m = \min_{i,j}\{\mu_v(v_i), \mu_e(e_{ij})\}$, then we choose any ϵ which satisfy additional conditions $M + \epsilon \leq 1$ and $m - \epsilon \geq 0$. Now, replace the membership value of each nodes with $[\mu_v(v_i)-\epsilon, \mu_v(v_i)+\epsilon]$ and each edges by $[\mu_e(e_{ij})-\epsilon, \mu_e(e_{ij})+\epsilon]$. As fuzzy magic graph admits all distinct membership values for each nodes and edges so, when we choose ϵ, satisfying the conditions of definition 2.5 and above additional conditions, we always get disjoint interval because intervals are symmetric about ϵ. In this way the obtained graph becomes interval-valued fuzzy magic graph. □

Example 3.4. In this example we are obtaining interval-valued fuzzy magic labeled graph G^* from a fuzzy magic graph $\tilde{G}$. In Figure 3, $\tilde{G}$ is a fuzzy magic labeled graph whose magic value is 2.1. Now, on taking $\epsilon = .02$ we get ϵ-neighborhood intervals for G^*. We observed that this interval-valued fuzzy graph satisfies all the conditions of magic labeling of interval-valued fuzzy graph.

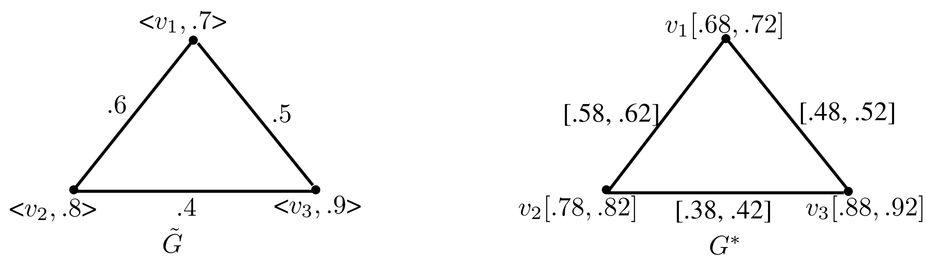

FIGURE 3. Fuzzy magic labeled graph and corresponding interval-valued fuzzy magic labeled graph

Theorem 3.5. *Any fuzzy labeled graph can be converted into interval-valued fuzzy magic graph but the interval membership never be mutually disjoint.*

Proof. We know that fuzzy labeled graph assigns some membership value for its nodes and edges which is bijective. Thus we are able to find some ϵ which on adding corresponding to each nodes and edges we get the magic sum for each pair of nodes and related edges (see [7]), we assume it as the upper limit for the intervals. Now for the lower limit of the interval we just multiply all the obtained upper limits by 0.1 as one is an identity for multiplication and place of decimal provides the length for the interval shown in FIGURE 4. Thus in this process the obtained interval-valued fuzzy graph satisfies the condition of magic labeling. But the resulted interval need not to be disjoint because we are choosing the length of the interval arbitrarily. □

Example 3.6. A path fuzzy labeled graph with four vertices which is converted into interval-valued fuzzy magic graph. In this example, for node v_1 and edge v_1v_2 we

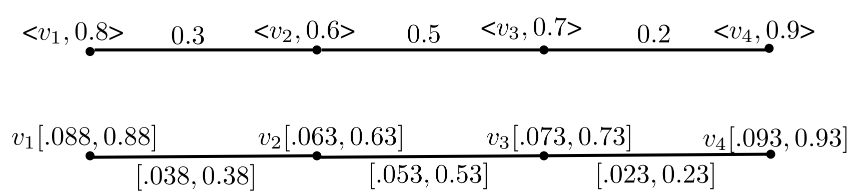

FIGURE 4. A path fuzzy labeled graph and corresponding interval-valued fuzzy magic path

assign $\epsilon = .08$ and for the rest of the nodes and edges we assign $\epsilon = .03$. After adding these ϵ to concern nodes or edges we assume those values as an upper limit for the intervals, thus we get $\mu_A^+(v_i)+\mu_B^+(v_i,v_j)+\mu_A^+(v_j) = 1.89$ for all $v_1, v_2, v_3 \in V$ now on multiplying each of the upper limit by 0.1 we get $\mu_A^-(v_i)+\mu_B^-(v_i,v_j)+\mu_A^-(v_j) = 0.189$, for all $v_1, v_2, v_3 \in V$ which satisfy the condition of magic labeling of an interval-valued fuzzy graph.

Definition 3.7. (see [7]) A star in a fuzzy graph consist of two node sets V and U with $|V| = 1$ and $|U| > 1$, such that $\mu_e(v, u_i) > 0$ and $\mu_e(u_i, u_{i+1}) = 0$, $1 \le i \le n$. It is denoted by $S_{1,n}$.

Theorem 3.8. *A fuzzy labeled star graph need not be an interval-valued fuzzy magic graph.*

Proof. As it is possible that in a fuzzy labeled star graph there may be an infinite number of pendent nodes and for those nodes we need mutually disjoint membership for edges. So, in this case it is impossible to find an ϵ-neighborhood interval for each pendent nodes and edges, so that the lower limit of membership interval for each pair of nodes and edges remains equal and also remain same for its upper limit of membership. Hence, we say that a fuzzy star graph need not be an interval-valued fuzzy magic graph. □

Example 3.9. A fuzzy labeled star graph with four pendent nodes which is not an interval-valued fuzzy magic graph. Here all the nodes and edges have distinct

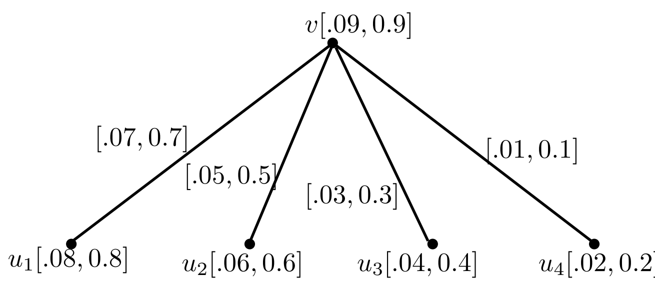

FIGURE 5. Interval-valued labeled star $S_{1,4}$ which is not magic labeled

membership interval, but here $\mu_A^-(v)+\mu_B^-(v,u_j)+\mu_A^-(u_j)$ for each pair of nodes and edges are 0.24, 0.20, 0.16 and 0.12 and similarly $\mu_A^+(v)+\mu_B^+(v,u_j)+\mu_A^+(u_j)$ are 2.4, 2, 1.6 and 1.2 which do not satisfy the condition of magic labeling.

Theorem 3.10. *Every cycle with odd number of vertices are always an interval-valued fuzzy magic graph.*

Proof. Let G be a cycle with odd number of nodes $v_1, v_2, v_3, ..., v_n$ and $v_1v_2, v_2v_3, ..., v_nv_1$ be the edges corresponding to it. Let $\epsilon \in [0, 1]$ such that one can choose $\epsilon_1 = 0.01$ for lower limit and $\epsilon_2 = 0.1$ for upper limit for $n \leq 3$ and for $n \geq 4$ we can choose $\epsilon_1 = 0.001$ for lower limit and $\epsilon_2 = 0.01$ for upper limit and set the membership interval as follows

$$\mu_A^-(v_{2i}) = (2n+i)\epsilon_1, 1 \le i \le \frac{(n-1)}{2}$$
$$\mu_A^+(v_{2i}) = (2n+i)\epsilon_2, 1 \le i \le \frac{(n-1)}{2}$$
$$and\ \mu_A^-(v_{2i-1}) = min\{\mu_A^-(v_{2i})/1 \le i \le \frac{(n-1)}{2}\} - i\epsilon_1, 1 \le i \le \frac{(n-1)}{2}$$
$$\mu_A^+(v_{2i-1}) = min\{\mu_A^+(v_{2i})/1 \le i \le \frac{(n-1)}{2}\} - i\epsilon_2, 1 \le i \le \frac{(n-1)}{2}$$
$$Similarly, \mu_B^-(v_1, v_n) = \frac{1}{2}max\{\mu_A^-(v_i)/1 \le i \le n\}$$
$$\mu_B^+(v_1, v_n) = \frac{1}{2}max\{\mu_A^+(v_i)/1 \le i \le n\}$$
$$and\ \mu_B^-(v_{n-i+1}, v_{n-i}) = \mu_B^-(v_1, v_n) - i\epsilon_1, 1 \le i \le n-1$$
$$\mu_B^+(v_{n-i+1}, v_{n-i}) = \mu_B^+(v_1, v_n) - i\epsilon_2, 1 \le i \le n-1$$

Case(i) When i is even, let $i = 2k$ for any positive integer k. For each edge v_i, v_{i+1}

$$\begin{aligned}
m_0^-(C_n) &= \mu_A^-(v_i) + \mu_B^-(v_i, v_{i+1}) + \mu_A^-(v_{i+1}) \\
&= \mu_A^-(v_{2k}) + \mu_B^-(v_{2k}, v_{2k+1}) + \mu_A^-(v_{2k+1}) \\
&= (2n+1-k)\epsilon_1 + \tfrac{1}{2}max\{\mu_A^-(v_i)/1 \le i \le n\} - (n-2k)\epsilon_1 \\
&\quad + min\{\mu_A^-(v_{2i})/1 \le i \le \tfrac{(n-1)}{2} - (k+1)\epsilon_1 \\
&= \tfrac{1}{2}max\{\mu_A^-(v_i)/1 \le i \le n\} + min\{\mu_A^-(v_{2i})/1 \le i \le \tfrac{(n-1)}{2}\} + n(\epsilon_1)
\end{aligned}$$

and

$$\begin{aligned}
m_0^+(C_n) &= \mu_A^+(v_i) + \mu_B^+(v_i, v_{i+1}) + \mu_A^+(v_{i+1}) \\
&= \mu_A^+(v_{2k}) + \mu_B^+(v_{2k}, v_{2k+1}) + \mu_A^+(v_{2k+1}) \\
&= (2n+1-k)\epsilon_2 + \tfrac{1}{2}max\{\mu_A^+(v_i)/1 \le i \le n\} - (n-2k)\epsilon_2 \\
&\quad + min\{\mu_A^+(v_{2i})/1 \le i \le \tfrac{(n-1)}{2} - (k+1)\epsilon_2 \\
&= \tfrac{1}{2}max\{\mu_A^+(v_i)/1 \le i \le n\} + min\{\mu_A^+(v_{2i})/1 \le i \le \tfrac{(n-1)}{2}\} + n(\epsilon_2)
\end{aligned}$$

case(ii) When i is odd, let $i = 2k+1$ for any positive integer k. For each edge v_i, v_{i+1}

$$\begin{aligned}
m_0^-(C_n) &= \mu_A^-(v_i) + \mu_B^-(v_i, v_{i+1}) + \mu_A^-(v_{i+1}) \\
&= \mu_A^-(v_{2k+1}) + \mu_B^-(v_{2k+1}, v_{2k+2}) + \mu_A^-(v_{2k+2}) \\
&= min\{\mu_A^-(v_{2i})/1 \le i \le \tfrac{(n-1)}{2}\} - (k+1)\epsilon_1 \\
&\quad + \tfrac{1}{2}max\{\mu_A^-(v_i)/1 \le i \le n\} - (n-2k-1)\epsilon_1 + (2n-k)\epsilon_1 \\
&= \tfrac{1}{2}max\{\mu_A^-(v_i)/1 \le i \le n\} + min\{\mu_A^-(v_{2i})/1 \le i \le \tfrac{(n-1)}{2}\} + n(\epsilon_1)
\end{aligned}$$

again,

$$m_0^+(C_n) = \mu_A^+(v_i) + \mu_B^+(v_i, v_{i+1}) + \mu_A^+(v_{i+1})$$
$$= \mu_A^+(v_{2k+1}) + \mu_B^+(v_{2k+1}, v_{2k+2}) + \mu_A^+(v_{2k+2})$$
$$= \frac{1}{2}max\{\mu_A^+(v_i)/1 \leq i \leq n\} + min\{\mu_A^+(v_{2i})/1 \leq i \leq \frac{(n-1)}{2}\} + n(\epsilon_2)$$

Hence from here we can say that the odd cycle is always an interval-valued fuzzy magic graph. It is not always true for even cycle because when we apply this process for even cycle then some nodes receive such interval membership which violets the condition of magic labeling. □

Theorem 3.11. *Every interval-valued fuzzy magic graph whose interval memberships are ϵ-neighborhood of a fuzzy graph, always contains at least one fuzzy bridge.*

Proof. Any fuzzy graph $\tilde{G}$ can be labeled by many ways as discussed above but, when we label it by taking ϵ-neighborhood interval membership then it contains at least one fuzzy bridge. Because, if an ϵ-neighborhood interval-valued fuzzy graph is magic labeled then it is necessarily be obtained from such fuzzy graph whose every nodes and edges admits distinct membership values otherwise it is not possible that aforesaid interval-valued fuzzy graph is magic labeled. Hence it is clear that ϵ-neighborhood interval-valued fuzzy magic graph have all distinct interval memberships for nodes and edges. Thus on taking interval membership for every nodes and edges there must exist at least one interval for an edge whose lower limit is the greatest lower limit and upper limit of the interval is also the greatest upper limit among all interval memberships of edges because, intervals are symmetric about ϵ. Thus we get an edge for which the condition $\mu'^{\infty}(u,v) < \mu(u,v)$ would satisfy. Hence that edge must be the fuzzy bridge. For example in Figure 3 edge v_1v_2 is a fuzzy bridge of G^*. □

4. Conclusion

Interval-valued fuzzy graph have numerous application in the modeling of real life systems where the level of information inherited in the system varies with respect to time and have different level of precision. Most of the actions in real life are time dependent, symbolic models used in expert system are more effective than traditional one. In this paper, we introduced the concept of interval-valued fuzzy labeling and interval-valued fuzzy magic labeling graphs. In future we extend this concept to bipolar fuzzy graphs, hypergraphs and in some more areas of graph theory.

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