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Model operator $\boxcircle_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}$ in intuitionistic fuzzy BG-algebras

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Abstract. In this paper, we study the effect of model operator $\boxcircle_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}$ in intuitionistic fuzzy BG-algebras and the the effect of model operator $\boxcircle_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}$ in homomorphism of intuitionistic fuzzy BG-algebras and obtained some interesting properties.

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1. Introduction

BCK and BCI-algebras are two important classes of logical algebras introduced by Imai and Iseki [15] and Iseki [16] respectively in the year 1966. It is known that the class of BCK-algebra is a proper subclass of the class of BCI-algebras. B-algebra is introduced in [19] by Neggers and Kim and which is related to several classes of algebras such as BCI/BCK-algebras. The generalization of B-algebra is BG-algebra and was introduced by Kim and Kim [17]. After the introduction of fuzzy sets in 1965 by Zadeh [23], researchers have been trying to fuzzify all the usual mathematical concepts in almost every branch of Mathematics. The concept of intuitionistic fuzzy subsets (IFS) was introduced by Atanassov [3] in 1983, which is a generalization of the notion of fuzzy sets. The intuitionistic fuzzy model operators $\Box$ and $\Diamond$ was introduced by Atanassov [3] in 1983. The extension on both the operators $\Box$ and $\Diamond$ is the new operator D_α which represents both of them. Further, the extension of all the operators is the operator $F_{\alpha,\beta}$ called (α,β)-model operator. The extended model operator $F_{\alpha,\beta}$ is not the final generalised model operator, the other generalisations are $\boxplus, \boxtimes, \boxplus_\alpha, \boxtimes_\alpha, \boxplus_{\alpha,\beta}, \boxtimes_{\alpha,\beta}, E_{\alpha,\beta}, \boxplus_{\alpha,\beta,\gamma}, \boxtimes_{\alpha,\beta,\gamma}, \boxdot_{\alpha,\beta,\gamma,\delta}$ and $\boxcircle_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}$. The operator $\boxcircle_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}$ was introduced by Atanassov [10] in 2008. The effect of all the model operators on IFSs is again an IFSs. The model operators play an important role in the study of IFSs. A lot of operators are defined over IFS. Some of these operators were studied in [4, 7, 8, 9, 11, 12, 13, 14, 18, 21]. The concept of fuzzy subalgebras of BG-algebras was introduced by Ahn and Lee [1] and the concept of intuitionistic fuzzy subalgebras and intuitionistic fuzzy ideals were introduced by Zarandi and Saeid [22]. Here in this paper, we study the effect of these generalised model operators as discussed above on intuitionistic fuzzy BG-algebras.

2. Preliminaries

Definition 2.1 ([17])**.** A BG-algebra is a non-empty set X with a constant 0 and a binary operation $*$ satisfying the following axioms:

(i) $x * x = 0$,
(ii) $x * 0 = x$,
(iii) $(x * y) * (0 * y) = x, \ \forall\, x, y \in X$.

For brevity, we also call X a BG-algebra.

Example 2.2. Let X = {0, 1, 2, 3, 4} with the following cayley table

*	0	1	2	3	4
0	0	4	3	2	1
1	1	0	4	3	2
2	2	1	0	4	3
3	3	2	1	0	4
4	4	3	2	1	0

Then (X, $*$, 0) is a BG algebra .

Definition 2.3 ([17])**.** A non-empty subset S of a BG-algebra X is called a subalgebra of X if x $*$ y ∈ S, for all x, y ∈ S.

Definition 2.4 ([1])**.** A fuzzy subset μ of a BG-algebra X is called a fuzzy subalgebra of X if $\mu(x * y) \geq min\{\mu(x), \mu(y)\}$, for all x, y ∈ X.

Definition 2.5 ([3, 4, 5])**.** An intuitionistic fuzzy set (IFS) A of a BG-algebra X is an object of the form $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$, where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$. The numbers $\mu_A(x)$ and $\nu_A(x)$ denote respectively the degree of membership and the degree of non-membership of the element x in set A. For the sake of simplicity, we shall use the symbol $A = (\mu_A, \nu_A)$ for the intuitionistic fuzzy set $A = \{< x, \mu_A(x), \nu_A(x) > | \ x \in X\}$. The function $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ for all $x \in X$. is called the degree of uncertainty of $x \in A$. The class of IFSs on a universe X is denoted by $IFS(X)$.

Definition 2.6 ([3, 4, 5])**.** If $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ and $B = \{< x, \mu_B(x), \nu_B(x) > | x \in X\}$ are any two IFS of a set X, then

$A \subseteq B$ if and only if for all $x \in X, \mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$,
$A = B$ if and only if for all $x \in X, \mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$,
$A \cap B = \{< x, (\mu_A \cap \mu_B)(x), (\nu_A \cap \nu_B)(x) > | x \in X\}$,

where $(\mu_A \cap \mu_B)(x) = \min\{\mu_A(x), \mu_B(x)\}$ and $(\nu_A \cap \nu_B)(x) = \max\{\nu_A(x), \nu_B(x)\}$,

$A \cup B = \{< x, (\mu_A \cup \mu_B)(x), (\nu_A \cup \nu_B)(x) > | x \in X\}$,

where $(\mu_A \cup \mu_B)(x) = \max\{\mu_A(x), \mu_B(x)\}$ and $(\nu_A \cap \nu_B)(x) = \min\{\nu_A(x), \nu_B(x)\}$.

Definition 2.7 ([2]). If $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ and $B = \{< x, \mu_B(x), \nu_B(x) > | x \in X\}$ are any two IFS of a set X, then their cartesian product
$A \times B = \{< (x, y), (\mu_A \times \mu_B)(x, y), (\nu_A \times \nu_B)(x, y) > | x, y \in X\}$,
where $(\mu_A \times \mu_B)(x, y) = \min\{\mu_A(x), \mu_B(y)\}$ and $(\nu_A \times \nu_B)(x, y) = \max\{\nu_A(x), \nu_B(y)\}$,

Definition 2.8 ([5, 6]). For any Intuitionistic fuzzy set $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ of X and $\alpha \in [0\ 1]$, the operator $\square : IFS(X) \to IFS(X), \diamondsuit : IFS(X) \to IFS(X), D_\alpha : IFS(X) \to IFS(X)$ are defined as
(i) $\square(A) = \{< x, \mu_A(x), 1 - \mu_A(x) > | x \in X\}$ is called necessity operator,
(ii) $\diamondsuit(A) = \{< x, 1 - \nu_A(x), \nu_A(x) > | x \in X\}$ is called possibility operator,
(iii) $D_\alpha(A) = \{< x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + (1 - \alpha)\pi_A(x) > | x \in X\}$ is called $\alpha-$ model operator.
Clearly $\square(A) \subseteq A \subseteq \diamondsuit(A)$ and the equality hold, when A is a fuzzy set also $D_0(A) = \square(A)$ and $D_1(A) = \diamondsuit(A)$. Therefore the $\alpha-$ Model operator $D_\alpha(A)$ is an extension of necessity operator $\square$ and possibility operator $\diamondsuit$.

Definition 2.9 ([2, 6]). For any IFS $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ of X and for any $\alpha, \beta \in [0\ 1]$ such that $\alpha + \beta \leq 1$, the $(\alpha, \beta)-$ model operator $F_{\alpha,\beta} : IFS(X) \to IFS(X)$ is defined as $F_{\alpha,\beta}(A) = \{< x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + \beta\pi_A(x) > | x \in X\}$, where $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ for all $x \in X$. Therefore we can write $F_{\alpha,\beta}(A)$ as $F_{\alpha,\beta}(A)(x) = (\mu_{F_{\alpha,\beta}(A)}(x), \nu_{F_{\alpha,\beta}(A)}(x))$ where $\mu_{F_{\alpha,\beta}}(x) = \mu_A(x) + \alpha\pi_A(x)$ and $\nu_{F_{\alpha,\beta}(A)}(x)) = \nu_A(x) + \beta\pi_A(x)$. Clearly, $F_{0,1}(A) = \square(A), F_{1,0}(A) = \diamondsuit(A)$ and $F_{\alpha,1-\alpha}(A) = D_\alpha(A)$

Definition 2.10 ([2, 11, 13]). For any IFS $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ of X and $\alpha \in [01]$, the operators $\boxplus, \boxtimes, \boxplus_\alpha, \boxtimes_\alpha : IFS(X) \to IFS(X)$ are defined as
(i) $\boxplus(A) = \{< x, \frac{\mu_A(x)}{2}, \frac{\nu_A(x)+1}{2} > | x \in X\}$,
(ii) $\boxtimes(A) = \{< x, \frac{\mu_A(x)+1}{2}, \frac{\nu_A(x)}{2} > | x \in X\}$,
(iii) $\boxplus_\alpha A = \{< x, \alpha\mu_A(x), \alpha\nu_A(x) + 1 - \alpha > | x \in X\}$,
(iv) $\boxtimes_\alpha A = \{< x, \alpha\mu_A(x) + 1 - \alpha, \alpha\nu_A(x) > | x \in X\}$.
Therefore $\boxplus_{0.5}A = \boxplus A$ and $\boxtimes_{0.5}A = \boxtimes A$ Hence $\boxplus_\alpha, \boxtimes_\alpha$ are the generalisation of $\boxplus, \boxtimes$.

Definition 2.11 ([2, 11, 13]). For any IFS $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ of X and $\alpha, \beta, \alpha + \beta \in [0\ 1]$, the operator $\boxplus_{\alpha,\beta}, \boxtimes_{\alpha,\beta}, E_{\alpha,\beta} : IFS(X) \to IFS(X)$ are defined as
(i) $\boxplus_{\alpha,\beta}(A) = \{< x, \alpha\mu_A(x), \alpha\nu_A(x) + \beta > | x \in X\}$,
(ii) $\boxtimes_{\alpha,\beta}(A) = \{< x, \alpha\mu_A(x) + \beta, \alpha\nu_A(x) > | x \in X\}$,
(iii) $E_{\alpha,\beta}A = \{< x, \beta(\alpha\mu_A(x) + 1 - \alpha), \alpha(\beta\nu_A(x) + 1 - \beta) > | x \in X\}$.
Therefore $\boxplus_{0.5,0.5}A = \boxplus A$ and $\boxtimes_{0.5,0.5}A = \boxtimes A, \boxplus_{\alpha,1-\alpha}(A) = \boxplus_\alpha(A), \boxtimes_{\alpha,1-\alpha}(A) = \boxtimes_\alpha(A)$ Hence $\boxplus_{\alpha,\beta}$, $\boxtimes_{\alpha,\beta}$ are the generalisation of $\boxplus$, $\boxtimes$ and also $\boxplus_\alpha$, $\boxtimes_\alpha$. The operators $\boxplus_{\alpha,\beta}, \boxtimes_{\alpha,\beta}$ was introduced by Katerina Dencheva[13]. Which is further extended to operators as defined below.

Definition 2.12 ([2, 11, 13]). For any IFS $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ of X and $\alpha, \beta, \gamma \in [0\ 1]$, and $max(\alpha, \beta) + \gamma \leq 1$ the operator $\boxplus_{\alpha,\beta,\gamma}, \boxtimes_{\alpha,\beta,\gamma} : IFS(X) \to IFS(X)$ are defined as
(i) $\boxplus_{\alpha,\beta,\gamma}(A) = \{< x, \alpha\mu_A(x), \beta\nu_A(x) + \gamma > | x \in X\}$.

(ii) $\boxtimes_{\alpha,\beta,\gamma}(A) = \{< x, \alpha\mu_A(x) + \gamma, \beta\nu_A(x) > | x \in X\}$
Therefore $\boxplus_{0.5,0.5,0.5}A = \boxplus A$, $\boxtimes_{0.5,0.5,0.5}A = \boxtimes A, \boxplus_{\alpha,\alpha,1-\alpha}(A) = \boxplus_{\alpha}(A), \boxtimes_{\alpha,1-\alpha}(A) = \boxtimes_{\alpha}(A), \boxplus_{\alpha,\alpha,\beta,}(A) = \boxplus_{\alpha,\beta,}(A),\ \boxtimes_{\alpha,\alpha,\beta}(A) = \boxtimes_{\alpha,\beta}(A)$ Hence $\boxplus_{\alpha,\beta,\gamma},\ \boxtimes_{\alpha,\beta,\gamma}$ are the generalisation of all operators $\boxplus, \boxtimes, \boxplus_{\alpha}, \boxtimes_{\alpha}, \boxplus_{\alpha,\beta}, \boxtimes_{\alpha,\beta}$. The extension of operators $\boxplus_{\alpha,\beta,\gamma}$, and $\boxtimes_{\alpha,\beta,\gamma}$ is the operator $\boxdot_{\alpha,\beta,\gamma,\delta}$ and is defined as follows:

Definition 2.13 ([2, 11, 13]). For any IFS $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ of X and $\alpha, \beta, \gamma.\delta \in [0\ 1]$, and $max(\alpha, \beta) + \gamma + \delta \le 1$ the operator $\boxdot_{\alpha,\beta,\gamma,\delta} : IFS(X) \to IFS(X)$ are defined as

$$\boxdot_{\alpha,\beta,\gamma,\delta}(A) = \{< x, \alpha\mu_A(x) + \gamma, \beta\nu_A(x) + \delta > | x \in X\}.$$

Therefore $\boxdot_{0.5,0.5,0.5,0.5}A = \boxplus A$, $\boxdot_{0.5,0.5,0.5,0.5}A = \boxtimes A$,
$\boxdot_{\alpha,\alpha,0,1-\alpha}(A) = \boxplus_{\alpha}(A), \boxtimes_{\alpha,\alpha,1-\alpha,0}(A) = \boxtimes_{\alpha}(A)$,
$\boxdot_{\alpha,\alpha,0,\beta}(A) = \boxplus_{\alpha,\beta}(A),\ \boxdot_{\alpha,\alpha,\beta,0}(A) = \boxtimes_{\alpha,\beta,}(A)$,
$\boxdot_{\alpha,\beta,0,\gamma,}(A) = \boxplus_{\alpha,\beta,\gamma}(A),\ \boxdot_{\alpha,\alpha,\gamma,0}(A) = \boxtimes_{\alpha,\beta,\gamma}(A)$.
Also $E_{\alpha,\beta}(A) = \boxdot_{\alpha\beta,\alpha\beta,(1-\alpha)\beta,(1-\beta)\alpha}(A)$.
Hence $\boxdot_{\alpha,\beta,\gamma,\delta}$ is the generalisation of all operators as discussed above. Now final extension above operators are the operator $\circledcirc_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}$, which is defined as follows:

Definition 2.14 ([2, 10, 11]). For any IFS $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ of X and $\alpha, \beta, \gamma.\delta, \epsilon, \zeta \in [0\ 1]$, and $max(\alpha-\zeta, \beta-\epsilon)+\gamma+\delta \le 1, min(\alpha-\zeta, \beta-\epsilon)+\gamma+\delta \ge 0$ the operator $\circledcirc_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta} : IFS(X) \to IFS(X)$ are defined as

$$\circledcirc_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A) = \{< x, \alpha\mu_A(x) - \epsilon\nu_A(x) + \gamma, \beta\nu_A(x) - \zeta\mu_A(x) + \delta > | x \in X\}.$$

Therefore $\circledcirc_{0.5,0.5,0,0.5,0,0}A = \boxplus A$ and $\circledcirc_{0.5,0.5,0.5,0,0,0}A = \boxtimes A$,
$\circledcirc_{\alpha,\alpha,0,1-\alpha,0,0}(A) = \boxplus_{\alpha}(A), \circledcirc_{\alpha,\alpha,1-\alpha,0,0,0}(A) = \boxtimes_{\alpha}(A)$,
$\circledcirc_{\alpha,\alpha,0,\beta,0,0}(A) = \boxplus_{\alpha,\beta,}(A),\ \circledcirc_{\alpha,\alpha,\beta,0,0,0}(A) = \boxtimes_{\alpha,\beta,}(A)$.
$\circledcirc_{\alpha,\beta,0,\gamma,0,0}(A) = \boxplus_{\alpha,\beta,\gamma}(A),\ \circledcirc_{\alpha,\alpha,\gamma,0}(A) = \boxtimes_{\alpha,\beta,\gamma,0,0,0}(A)$
Also $\circledcirc_{\alpha,\beta,\gamma,\delta,0,0}(A) = \boxdot_{\alpha,\beta,\gamma,\delta}(A)$.
Hence $\circledcirc_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)$ is the final generalisation of all operators discussed above.

Definition 2.15 ([21]). Let X and Y be two non empty sets and $f : X \longrightarrow Y$ be a mapping. Let A and B be IFS's of X and Y respectively . Then

(i) the image of A under f, denoted by f(A), is defined by $f(A)(y) = (\mu_{f(A)}(y), \nu_{f(A)}(y))$, where

$$\mu_{f(A)}(y) = \begin{cases} \vee\{\mu_A(x) : x \in f^{-1}(y)\} \\ 0 \qquad \text{otherwise} \end{cases}$$

and

$$\nu_{f(A)}(y) = \begin{cases} \wedge\{\nu_A(x) : x \in f^{-1}(y)\} \\ 1 \qquad \text{otherwise .} \end{cases}$$

(ii) the pre image of B under f denoted by $f^{-1}(B)$, a is defined as $\forall x \in X$,

$$f^{-1}(B)(x) = (\mu_{f^{-1}(B)}, \nu_{f^{-1}(B)}) = (\mu_B(f(x)), \nu_B(f(x))).$$

Remark 2.16. $\mu_A(x) \le \mu_{f(A)}(f(x))$ and $\nu_A(x) \ge \nu_{f(A)}(f(x)) \quad \forall x \in X$ however equality hold when the map f is bijective.

Definition 2.17 ([22]). An intuitionistic fuzzy set A of a BG-algebra X is said to be an intuitionistic fuzzy BG-subalgebra of X if

(i) $\mu_A(x * y) \ge \min\{\mu_A(x), \mu_A(y)\}$,

(ii) $\nu_A(x * y) \leq \max\{\nu_A(x), \nu_A(y)\} \;\forall\, x, y \in X$.

Example 2.18. Consider a BG-algebra $X = \{0, 1, 2\}$ with the following cayley table:

TABLE 1. Example of intuitionistic fuzzy BG-subalgebra.

*	0	1	2
0	0	1	2
1	1	0	1
2	2	2	0

The intuitionistic fuzzy subset $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ given by $\mu_A(0) = \mu_A(1) = 0.6, \mu_A(2) = 0.2$ and $\nu_A(0) = \nu_A(1) = 0.3, \nu_A(2) = 0.5$ is an intuitionistic fuzzy BG-subalgebra of X.

Definition 2.19. ([17]) An intuitionistic fuzzy set A of a BG-algebra X is said to be an intuitionistic fuzzy normal subalgebra of X if
(i) $\mu_A((x * a) * (y * b)) \geq min\{\mu_A(x * y), \mu_A(a * b)\}$,
(ii) $\nu_A((x * a) * (y * b)) \leq max\{\nu_A(x * y), \nu_A(a * b)\}$, $\forall\, x, y \in X$.

3. Model operator $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}$ in intuitionistic fuzzy subalgebras

In this section, we study the effect of model operator $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}$ in intuitionistic fuzzy subalgebra of BG-algebra X.

Theorem 3.1. *If A is an IF subalgebra of BG-algebra X , then $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)$ is also an IF subalgebra of BG-algebra X.*

Proof. Let $x, y \in X$, then

$$\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(x * y) = (\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x * y), \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x * y)),$$

where

$$\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x * y) = \alpha\mu_A(x * y) - \epsilon\nu_A(x * y) + \gamma$$

and

$$\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x * y) = \beta\nu_A(x * y) - \zeta\mu_A(x * y) + \delta.$$

Now

$$\begin{aligned}
&\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x * y) \\
&= \alpha\mu_A(x * y) - \epsilon\nu_A(x * y) + \gamma \\
&\geq \alpha \min\{\mu_A(x), \mu_A(y)\} - \epsilon \max\{\nu_A(x), \nu_A(y)\} + \gamma \\
&= \alpha \min\{\mu_A(x), \mu_A(y)\} + \epsilon \min\{1 - \nu_A(x), 1 - \nu_A(y)\} + \gamma - \epsilon \\
&= \min\{\alpha\mu_A(x) + \epsilon(1 - \nu_A(x)), \alpha\mu_A(y) + \epsilon(1 - \nu_A(y))\} + \gamma - \epsilon \\
&= \min\{\alpha\mu_A(x) + \epsilon(1 - \nu_A(x)) + \gamma - \epsilon, \alpha\mu_A(y) + \epsilon(1 - \nu_A(y)) + \gamma - \epsilon\} \\
&= \min\{\alpha\mu_A(x) - \epsilon\nu_A(x) + \gamma, \alpha\mu_A(y) - \epsilon\nu_A(y) + \gamma\} \\
&= \min\{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x), \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(y)\}.
\end{aligned}$$

Which implies

$$\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x*y) \geq \min\{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x), \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(y)\}.$$

Similarly we can prove

$$\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x*y) \leq \max\{\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x), \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(y)\}.$$

Hence $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)$ is an IF subalgebra of BG-algebra X. □

Remark 3.2. The converse of above Theorem need not be true as shown in following Example.

Example 3.3. Consider a BG-algebra $X = \{0, 1, 2\}$ with the following cayley table:

Table 2. Illustration of converse of Theorem 3.1.

*	0	1	2
0	0	1	2
1	1	0	1
2	2	2	0

The intuitionistic fuzzy subset $A = \{< x, \mu_A(x), \nu_A(x) > |x \in X\}$ given by $\mu_A(0) = 0.48, \mu_A(1) = 0.5, \mu_A(2) = 0.3$ and $\nu_A(0) = 0.3, \nu_A(1) = 0.4, \nu_A(2) = 0.5$ is not an intuitionistic fuzzy BG-subalgebra of X. Since $\mu_A(0) = 0.48 \ngeq min\{\mu_A(1), \mu_A(1)\} = \mu_A(1) = 0.5$. Now take $\alpha = 0.6, \zeta = 0.2, \beta = 0.4, \epsilon = 0.3, \gamma = 0.3, \delta = 0.2$ Then $\alpha - \zeta = 0.4, \beta - \epsilon = 0.1$ and $\max(\alpha - \zeta, \beta - \epsilon) + \gamma + \delta = 0.9 \leq 1, \min(\alpha - \zeta, \beta - \epsilon) + \gamma + \delta = 0.6 \geq 0$, then $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A) = \{< x, \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x), \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x) > |x \in X\}$ is $\mu_{\boxdot_{0.6,0.4,0.3,0.2,0.3,0.2}(A)}(0) = 0.49, \mu_{\boxdot_{0.6,0.4,0.3,0.2,0.3,0.2}(A)}(1) = 0.48, \mu_{\boxdot_{0.6,0.4,0.3,0.2,0.3,0.2}(A)}(2) = 0.33$ and $\nu_{\boxdot_{0.6,0.4,0.3,0.2,0.3,0.2}(A)}(0) = 0.224, \nu_{\boxdot_{0.6,0.4,0.3,0.2,0.3,0.2}(A)}(1) = 0.26$, $\nu_{\boxdot_{0.6,0.4,0.3,0.2,0.3,0.2}(A)}(2) = 0.34$. It can easily verified that $\boxdot_{0.6,0.4,0.3,0.2,0.3,0.2}(A)$ is an IF BG-subalgebra of X.

Corollary 3.4. *If A is an IF subalgebra of BG-algebra X, then*

(i) $\boxplus(A)$ *is also an IF subalgebra of BG-algebra X.*
(ii) $\boxtimes(A)$ *is also an IF subalgebra of BG-algebra X.*
(iii) $\boxplus_\alpha(A)$ *is also an IF subalgebra of BG-algebra X.*
(iv) $\boxtimes_\alpha(A)$ *is also an IF subalgebra of BG-algebra X.*
(v) $\boxplus_{\alpha,\beta}(A)$ *is also an IF subalgebra of BG-algebra X.*
(vi) $\boxtimes_{\alpha,\beta}(A)$ *is also an IF subalgebra of BG-algebra X.*
(vii) $\boxplus_{\alpha,\beta,\gamma}(A)$ *is also an IF subalgebra of BG-algebra X.*
(viii) $\boxtimes_{\alpha,\beta,\gamma}(A)$ *is also an IF subalgebra of BG-algebra X.*
(ix) $\boxdot_{\alpha,\beta,\gamma,\delta}(A)$ *is also an IF subalgebra of BG-algebra X.*

Theorem 3.5. *If A is an IF subalgebra of BG-algebra X, then*

(a) $\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(0) \geq \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x)$.
(b) $\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(0) \leq \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x), \quad \forall x \in X$.

Proof. Since A is an IF subalgebra of X, $\boxcircle_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)$ is also an IF subalgebra of BG-algebra X.
Now

$$\begin{aligned}\mu_{\boxcircle_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(0) &= \mu_{\boxcircle_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x*x)\\ &\geq min\{\mu_{\boxcircle_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x), \mu_{\boxcircle_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x)\}\\ &= \mu_{\boxcircle_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x).\end{aligned}$$

And

$$\begin{aligned}\nu_{\boxcircle_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(0) &= \nu_{\boxcircle_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x*x)\\ &\leq max\{\nu_{\boxcircle_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x), \nu_{\boxcircle_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x)\}\\ &= \nu_{\boxcircle_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x) \quad \forall x \in X.\end{aligned}$$

□

Corollary 3.6. *If A is an IF subalgebra of BG-algebra X, then*

(i) *(a)* $\mu_{\boxplus(A)}(0) \geq \mu_{\boxplus(A)}(x)$,
(b) $\nu_{\boxplus(A)}(0) \leq \nu_{\boxplus(A)}(x) \quad \forall x \in X$.
(ii) *(a)* $\mu_{\boxtimes(A)}(0) \geq \mu_{\boxtimes(A)}(x)$,
(b) $\nu_{\boxtimes(A)}(0) \leq \nu_{\boxtimes(A)}(x) \quad \forall x \in X$.
(iii) *(a)* $\mu_{\boxplus_\alpha(A)}(0) \geq \mu_{\boxplus_\alpha(A)}(x)$,
(b) $\nu_{\boxplus_\alpha(A)}(0) \leq \nu_{\boxplus_\alpha(A)}(x) \quad \forall x \in X$.
(iv) *(a)* $\mu_{\boxtimes_\alpha(A)}(0) \geq \mu_{\boxtimes_\alpha(A)}(x)$,
(b) $\nu_{\boxtimes_\alpha(A)}(0) \leq \nu_{\boxtimes_\alpha(A)}(x) \quad \forall x \in X$.
(v) *(a)* $\mu_{\boxplus_{\alpha,\beta}(A)}(0) \geq \mu_{\boxplus_{\alpha,\beta}(A)}(x)$,
(b) $\nu_{\boxplus_{\alpha,\beta}(A)}(0) \leq \nu_{\boxplus_{\alpha,\beta}(A)}(x) \quad \forall x \in X$.
(vi) *(a)* $\mu_{\boxtimes_{\alpha,\beta}(A)}(0) \geq \mu_{\boxtimes_{\alpha,\beta}(A)}(x)$,
(b) $\nu_{\boxtimes_{\alpha,\beta}(A)}(0) \leq \nu_{\boxtimes_{\alpha,\beta}(A)}(x) \quad \forall x \in X$.
(vii) *(a)* $\mu_{\boxplus_{\alpha,\beta,\gamma}(A)}(0) \geq \mu_{\boxplus_{\alpha,\beta,\gamma}(A)}(x)$,
(b) $\nu_{\boxplus_{\alpha,\beta,\gamma}(A)}(0) \leq \nu_{\boxplus_{\alpha,\beta,\gamma}(A)}(x) \quad \forall x \in X$.
(viii) *(a)* $\mu_{\boxtimes_{\alpha,\beta,\gamma}(A)}(0) \geq \mu_{\boxtimes_{\alpha,\beta,\gamma}(A)}(x)$,
(b) $\nu_{\boxtimes_{\alpha,\beta,\gamma}(A)}(0) \leq \nu_{\boxtimes_{\alpha,\beta,\gamma}(A)}(x) \quad \forall x \in X$.
(ix) *(a)* $\mu_{\boxdot_{\alpha,\beta,\gamma,\delta}(A)}(0) \geq \mu_{\boxdot_{\alpha,\beta,\gamma,\delta}(A)}(x)$,
(b) $\nu_{\boxdot_{\alpha,\beta,\gamma,\delta}(A)}(0) \leq \nu_{\boxdot_{\alpha,\beta,\gamma,\delta}(A)}(x) \quad \forall x \in X$.

Theorem 3.7. *If A and B are two IF subalgebras of BG-algebra X, then*
(i) $A \cap B$ *is also an IF subalgebra of BG-algebra X.*
(ii) $A \times B$ *is also an IF subalgebra of BG-algebra* $X \times X$.

Proof. (i) We have $A \cap B = \{< x, \mu_{(A\cap B)}(x), \nu_{(A\cap B)}(x) > | x \in X\}$,
where

$$\mu_{(A\cap B)}(x) = (\mu_A \cap \mu_B)(x) = \min\{\mu_A(x), \mu_B(x)\}$$

and

$$\nu_{(A\cap B)}(x) = (\nu_A \cap \nu_B)(x) = \max\{\nu_A(x), \nu_B(x)\}.$$

Let $x, y \in X$. Since both A, B are subalgebras of X,

$$\mu_A(x*y) \geq min\{\mu_A(x), \mu_A(y)\} \text{ and } \nu_A(x*y) \leq max\{\nu_A(x), \nu_A(y)\}.$$

Also

$$\mu_B(x*y) \geq min\{\mu_B(x), \mu_B(y)\} \text{ and } \nu_B(x*y) \leq max\{\nu_B(x), \nu_B(y)\} .$$

Now

$$\begin{aligned}
\mu_{(A\cap B)}(x*y) \quad &= min\{\mu_A(x*y), \mu_A(x*y)\} \\
&\geq min\{min\{\mu_A(x), \mu_A(y)\}, min\{\mu_B(x), \mu_B(y)\}\} \\
&= min\{min\{\mu_A(x), \mu_B(x)\}, min\{\mu_A(y), \mu_B(y)\}\} \\
&= min\{\mu_{(A\cap B)}(x), \mu_{(A\cap B)}(y)\} \\
&\geq min\{\mu_{(A\cap B)}(x), \mu_{(A\cap B)}(y)\}.
\end{aligned}$$

Similarly we can prove

$$\nu_{(A\cap B)}(x*y) \quad \leq max\{\nu_{(A\cap B)}(x), \nu_{(A\cap B)}(y)\}.$$

Hence $A \cap B$ is also an IF subalgebra of BG-algebra X.
(ii) Proof same as (i). □

Theorem 3.8. *If A and B are two IF subalgebras of BG-algebra X, then*
(i) $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A \cap B)$ *is also an IF subalgebra of BG-algebra X.*
(ii) $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A \times B)$ *is also an IF subalgebra of BG-algebra* $X \times X$*.*

Proof. (i) We have

$$\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\cap B)(x) = \{< x, \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\cap B)}(x), \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\cap B)}(x) > | x \in X\},$$

where

$$\mu_{(A\cap B)}(x) = (\mu_A \cap \mu_B)(x) = \min\{\mu_A(x), \mu_B(x)\}$$

and

$$\nu_{(A\cap B)}(x) = (\nu_A \cap \nu_B)(x) = \max\{\nu_A(x), \nu_B(x)\}.$$

Let $x, y \in X$. Since both A, B are subalgebras of X,

$$\mu_A(x*y) \geq \min\{\mu_A(x), \mu_A(y)\} \text{ and } \nu_A(x*y) \leq \max\{\nu_A(x), \nu_A(y)\}.$$

Also

$$\mu_B(x*y) \geq \min\{\mu_B(x), \mu_B(y)\} \text{ and } \nu_B(x*y) \leq \max\{\nu_B(x), \nu_B(y)\}.$$

Now

$$\begin{aligned}
&\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\cap B)}(x*y)\\
&=\alpha\mu_{(A\cap B)}(x*y)-\epsilon\nu_{(A\cap B)}(x*y)+\gamma\\
&=\alpha\min(\mu_A(x*y),\mu_B(x*y))-\epsilon max(\nu_A(x*y),\nu_B(y*y))+\gamma\\
&\geq\alpha\min\{\min(\mu_A(x),\mu_A(y)),min(\mu_B(x),\mu_B(y))\}-\epsilon\max\{\max(\nu_A(x),\nu_A(y)),\\
&\quad\max(\nu_B(x),\nu_B(y))\}+\gamma\\
&=\alpha\min\{min(\mu_A(x),\mu_B(x)),min(\mu_A(y),\mu_B(y))\}-\epsilon\max\{max(\nu_A(x),\nu_B(x)),\\
&\quad\max(\nu_A(y),\nu_B(y))\}+\gamma\\
&=\alpha\min\{\mu_{(A\cap B)}(x),\mu_{(A\cap B)}(y)\}-\epsilon\max\{\nu_{(A\cap B)}(x),\nu_{(A\cap B)}(y)\}+\gamma\\
&=\alpha\min\{\mu_{(A\cap B)}(x),\mu_{(A\cap B)}(y)\}+\epsilon-\epsilon\max\{\nu_{(A\cap B)}(x),\nu_{(A\cap B)}(y)\}+\gamma-\epsilon\\
&=\alpha\min\{\mu_{(A\cap B)}(x),\mu_{(A\cap B)}(y)\}+\epsilon\min\{1-\nu_{(A\cap B)}(x),1-\nu_{(A\cap B)}(y)\}+\gamma-\epsilon\\
&=\min\{\alpha\mu_{(A\cap B)}(x)+\epsilon(1-\nu_{(A\cap B)}(x))+\gamma-\epsilon,\alpha\mu_{(A\cap B)}(y)+\epsilon(1-\nu_{(A\cap B)}(y))\\
&\quad+\gamma-\epsilon\}\\
&=\min\{\alpha\mu_{(A\cap B)}(x)-\epsilon\nu_{(A\cap B)}(x)+\gamma,\alpha\mu_{(A\cap B)}(y)-\epsilon\nu_{(A\cap B)}(y)+\gamma\}\\
&=\min\{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\cap B)}(x),\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\cap B)}(y)\}.
\end{aligned}$$

Thus

$$\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\cap B)}(x*y)\geq min\{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\cap B)}(x),\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\cap B)}(y)\}.$$

Similarly, we can prove

$$\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\cap B)}(x*y)\leq max\{\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\cap B)}(x),\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\cap B)}(y)\}.$$

Hence $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\cap B)$ is also an IF subalgebra of BG-algebra X.

(ii) We have

$$\begin{aligned}
&\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\times B)(x,y)\\
&=\{<(x,y),\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\times B)}(x,y),\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\times B)}(x,y)>|x,y\in X\},
\end{aligned}$$

where

$$\mu_{(A\times B)}(x,y)=(\mu_A\times\mu_B)(x,y)=\min\{\mu_A(x),\mu_B(y)\}$$

and

$$\nu_{(A\times B)}(x,y)=(\nu_A\times\nu_B)(x,y)=\max\{\nu_A(x),\nu_B(y)\}.$$

Let $x_1,x_2,y_1,y_2\in X$. Since both A, B are subalgebras of X,

$$\mu_A(x_1*x_2)\geq\min\{\mu_A(x_1),\mu_A(x_2)\}\text{ and }\nu_A(x_1*x_2)\leq\max\{\nu_A(x_1),\nu_A(x_2)\}.$$

Also

$$\mu_B(y_1*y_2)\geq\min\{\mu_B(y_1),\mu_B(y_2)\}\text{ and }\nu_B(y_1*y_2)\leq\max\{\nu_B(y_1),\nu_B(y_2)\}.$$

Now

$$
\begin{aligned}
&\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\times B)}((x_1,y_1)*(x_2,y_2))\\
&\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\times B)}(x_1*x_2,y_1*y_2)\\
&=\alpha\mu_{(A\times B)}(x_1*x_2,y_1*y_2)-\epsilon\nu_{(A\times B)}(x_1*x_2,y_1*y_2)+\gamma\\
&=\alpha\min(\mu_A(x_1*x_2),\mu_B(y_1*y_2))-\epsilon max(\nu_A(x_1*x_2),\nu_B(y_1*y_2))+\gamma\\
&\geq\alpha\min\{\min(\mu_A(x_1),\mu_A(x_2)),min(\mu_B(y_1),\mu_B(y_2))\}-\\
&\qquad\epsilon\max\{\max(\nu_A(x_1),\nu_A(x_2)),max(\nu_B(y_1),\nu_B(y_2))\}+\gamma\\
&=\alpha\min\{\min(\mu_A(x_1),\mu_B(y_1)),min(\mu_A(x_2),\mu_B(y_2))\}-\\
&\qquad\epsilon\max\{\max(\nu_A(x_1),\nu_B(y_1)),max(\nu_A(x_2),\nu_B(y_2))\}+\gamma\\
&=\alpha\min\{\mu_{(A\times B)}(x_1,y_1),\mu_{(A\times B)}(x_2,y_2)\}-\\
&\qquad\epsilon\max\{\nu_{(A\times B)}(x_1,y_1),\nu_{(A\times B)}(x_2,y_2)\}+\gamma\\
&=\alpha\min\{\mu_{(A\times B)}(x_1,y_1),\mu_{(A\times B)}(x_2,y_2)\}+\epsilon-\\
&\qquad\epsilon\max\{\nu_{(A\times B)}(x_1,y_1),\nu_{(A\times B)}(x_2,y_2)\}+\gamma-\epsilon\\
&=\alpha\min\{\mu_{(A\times B)}(x_1,y_1),\mu_{(A\times B)}(x_2,y_2)\}+\epsilon\min\{1-\nu_{(A\times B)}(x_1,y_1),\\
&\qquad 1-\nu_{(A\times B)}(x_2,y_2)\}\\
&\qquad +\gamma-\epsilon\\
&=\min\{\alpha\mu_{(A\times B)}(x_1,y_1)+\epsilon(1-\nu_{(A\times B)}(x_1,y_1))+\gamma-\epsilon,\alpha\mu_{(A\times B)}(x_2,y_2)\\
&\qquad +\epsilon(1-\nu_{(A\times B)}(x_2,y_2))+\gamma-\epsilon\}\\
&=\min\{\alpha\mu_{(A\times B)}(x_1,y_1)-\epsilon\nu_{(A\times B)}(x_1,y_1)+\gamma,\alpha\mu_{(A\times B)}(x_2,y_2)-\\
&\qquad\epsilon\nu_{(A\times B)}(x_2,y_2)+\gamma\}\\
&=\min\{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\times B)}(x_1,y_1),\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\times B)}(x_2,y_2)\}.
\end{aligned}
$$

Thus

$$
\begin{aligned}
&\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\times B)}((x_1,y_1)*(x_2,y_2))\\
&\geq min\{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\times B)}(x_1,y_1),\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\times B)}(x_2,y_2)\}.
\end{aligned}
$$

Similarly, we can prove

$$
\begin{aligned}
&\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\times B)}((x_1,y_1)*(x_2,y_2))\\
&\leq max\{\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\times B)}(x_1,y_1),\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\times B)}(x_2,y_2)\}.
\end{aligned}
$$

Hence $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A\times B)$ is also an IF subalgebra of BG-algebra $X\times X$. □

Theorem 3.9. *If $\{A_i : i = 1, 2, ..., n\}$ be n IF subalgebras of X, then*
(i) $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(\cap_{i=1}^n A_i : i = 1, 2, ..., n\}$ is also an IF subalgebra of X.
(ii) $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(\times_{i=1}^n A_i : i = 1, 2, ..., n\}$ is also an IF subalgebra of $\times_{i=1}^n X_i$.

Theorem 3.10. *If $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A) =< \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}A}, \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}A} >$ is an IF subalgebra of X. Then the sets*

$$X_{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}}} = \{x\in X|\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x)=\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(0)\}$$

and

$$X_{\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}}} = \{x\in X|\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x)=\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(0)\}$$

are subalgebras of X.

Proof. Let $x, y \in X_{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}}}$. Then

$$\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x) = \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(y) = \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(0).$$

Now

$$\begin{aligned}\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x*y) \quad & \geq min\{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x), \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(y)\} \\ & = min\{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(0), \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(0)\} \\ & = \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(0).\end{aligned}$$

By Theorem 3.5,

$$\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(0) \geq \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}}(x*y).$$

Thus $\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x*y) = \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(0)$. So $x*y \in X_{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}}}$.
Again let $x, y \in X_{\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}}}$. Then

$$\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x) = \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(y) = \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(0).$$

Now

$$\begin{aligned}\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x*y) \quad & \leq min\{\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x), \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(y)\} \\ & = \min\{\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(0), \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(0)\} \\ & = \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(0).\end{aligned}$$

By Theorem 3.5,

$$\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(0) \leq \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x*y).$$

Thus $\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x*y) = \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(0)$. So $x*y \in X_{\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}}}$. Hence $X_{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}}}$ and $X_{\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}}}$ are subalgebras of X. □

Proposition 3.11. *If A and B be two IFS sets of X and Y respectively and* $f : X \longrightarrow Y$ *be a mapping, then*
(i) $f^{-1}(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(B)) = \boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f^{-1}(B))$.
(ii) $f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)) \subseteq \boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f(A))$.

Proof. (i) Let $x \in X$. Then

$$\begin{aligned}f^{-1}(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(B))(x) \quad & = (\mu_{f^{-1}(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(B))}(x), \nu_{f^{-1}(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(B))}(x)) \\ & = (\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(B)}(f(x)), \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(B)}(f(x))).\end{aligned}$$

But,

$$\begin{aligned}\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(B)}(f(x)) \quad & = \alpha\mu_B(f(x) - \epsilon\nu_B(f(x) + \gamma \\ & = \alpha\mu_{f^{-1}(B)}(x) - \epsilon\nu_{f^{-1}(B)}(x) + \gamma \\ & = \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f^{-1}(B))}(x).\end{aligned}$$

Thus

$$\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(B)}(f(x)) = \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f^{-1}(B))}(x).$$

Similarly, we can prove

$$\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(B)}(f(x)) = \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f^{-1}(B))}(x).$$

Hence

$$\begin{aligned} f^{-1}(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(B)) &= (\mu_{f^{-1}(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(B))}(x), \nu_{f^{-1}(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(B))}(x)) \\ &= \boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f^{-1}(B)). \end{aligned}$$

(ii) Here $f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))(y) = (\mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}(y), \nu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}(y))$.
Now

$$\begin{aligned} \mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}(y) &= \sup_{f(x)=y} \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x) \\ &= \sup_{f(x)=y} \alpha\mu_A(x) - \epsilon\nu_A(x) + \gamma \\ &\leq \sup_{f(x)=y} \alpha\mu_{f(A)}(f(x)) - \epsilon\nu_{f(A)}(f(x)) + \gamma \\ &\quad \text{By Remarks 2.16} \\ &= \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f(A))}(f(x)) \\ &= \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f(A))}(y). \end{aligned}$$

Thus $\mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}(y) \leq \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f(A))}(y)$.
Similarly, we can prove

$$\nu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}(y) \geq \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f(A))}(y),\ \forall\, y \in Y.$$

Hence

$$\begin{aligned} f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))(y) &= (\mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}(y), \nu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}(y)) \\ &\subseteq (\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f(A))}(y), \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f(A))}(y)) \\ &= \boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f(A)). \end{aligned}$$

□

Theorem 3.12. *If A is an IF normal subalgebra of BG-algebra X, then $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)$ is also an IF normal subalgebra of BG-algebra X.*

Proof. Let $x, y, a, b \in X$. Then

$$\begin{aligned} &\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}((x*a)*(y*b)) \\ &= (\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}((x*a)*(y*b)), \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}((x*a)*(y*b))), \end{aligned}$$

where

$$\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}((x*a)*(y*b)) = \alpha\mu_A((x*a)*(y*b)) - \epsilon\nu_A((x*a)*(y*b)) + \gamma$$

and

$$\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}((x*a)*(y*b)) = \beta\nu_A((x*a)*(y*b)) - \zeta\mu_A((x*a)*(y*b)) + \delta.$$

Now

$$\begin{aligned}
&\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}((x*a)*(y*b))\\
&= \alpha\mu_A((x*a)*(y*b)) - \epsilon\nu_A((x*a)*(y*b)) + \gamma\\
&\geq \alpha min\{\mu_A(x*y), \mu_A(a*b)\} - \epsilon\max\{\nu_A(x*y), \nu_A(a*b)\} + \gamma\\
&\qquad\qquad \text{Since } A =< \mu_A, \nu_A > \text{ is IF normal subalgebra of X}\\
&= \alpha min\{\mu_A(x*y), \mu_A(a*b)\} + \epsilon\min\{1-\nu_A(x*y), 1-\nu_A(a*b)\} + \gamma - \epsilon\\
&= \min\{\alpha\mu_A(x*y) + \epsilon(1-\nu_A(x*y)), \alpha\mu_A(a*b) + \epsilon(1-\nu_A(a*b))\} + \gamma - \epsilon\\
&= \min\{\alpha\mu_A(x*y) + \epsilon(1-\nu_A(x*y)) + \gamma - \epsilon, \alpha\mu_A(a*b) + \epsilon(1-\nu_A(y)) + \gamma - \epsilon\}\\
&= \min\{\alpha\mu_A(x*y) - \epsilon\nu_A(x*y) + \gamma, \alpha\mu_A(a*b) - \epsilon\nu_A(y) + \gamma\}\\
&= \min\{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x*y), \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(a*b)\}.
\end{aligned}$$

Which implies

$$\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}((x*a)*(y*b)) \geq \min\{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x*y), \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(a*b)\}.$$

Similarly, we can prove

$$\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}((x*a)*(y*b)) \leq \max\{\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x*y), \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(a*b)\}.$$

Hence $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)$ is an IF normal subalgebra of BG-algebra X. □

Theorem 3.13. *If A and B are two IF subalgebras of BG-algebra X, then*
(i) $A \cap B$ is also an IF normal subalgebra of BG-algebra X.
(ii) $A \times B$ is also an IF normal subalgebra of BG-algebra $X \times X$.

Theorem 3.14. *If $\{A_i : i = 1, 2, ..., n\}$ be n IF normal subalgebras of X, then*
(i) $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(\cap_{i=1}^n A_i : i = 1, 2, ..., n\}$ is also an IF normal subalgebra of X.
(ii) $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(\times_{i=1}^n A_i : i = 1, 2, ..., n\}$ is also an IF normal subalgebra of $\times_{i=1}^n X_i$.

4. Homomorphism of model operators in intuitionistic fuzzy BG-algebras

Definition 4.1. Let X and Y be two BG-algebras, then a mapping $f : X \longrightarrow Y$ is said to be homomorphism if $f(x*y) = f(x)*f(y)$, $\forall x, y \in X$.

Theorem 4.2. *Let $f : X \longrightarrow Y$ be a homomorphism of BG-algebras. If $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)$ is an IF BG-subalgebra of Y, then $f^{-1}(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))$ is also an IF BG-subalgebra of X.*

Proof. Since $f^{-1}(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)) = \boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f^{-1}(A)$, by Proposition 3.11, it is enough to show that $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f^{-1}(A))$ is IF BG-subalgebra of X.
Let $x, y \in X$. Then

$$\begin{aligned}
\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f^{-1}(A))}(x*y) &= \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}((A))}f(x*y)\\
&= \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(f(x)*f(y))\\
&\geq min\{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(f(x), \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(f(y)\}\\
&= min\{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f^{-1}(A))}(x), \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f^{-1}(A))}(y)\}\\
\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f^{-1}(A))}(x*y) &\geq min\{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f^{-1}(A))}(x), \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f^{-1}(A))}(y)\}.
\end{aligned}$$

Similarly, we can show

$$\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f^{-1}(A))}(x*y) \leq max\{\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f^{-1}(A))}(x), \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(f^{-1}(A))}(y)\}.$$

Hence $f^{-1}(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))$ is also an IF BG-subalgebra of X. □

Theorem 4.3. *Let $f : X \longrightarrow Y$ be a homomorphism of BG-algebras. If $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)$ is an IF normal subalgebra of Y, then $f^{-1}(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))$ is also an IF normal subalgebra of X.*

Theorem 4.4. *Let $f : X \longrightarrow Y$ be an onto homomorphism of BG-algebras. If $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)$ is an IF fuzzy subalgebra of X, then $f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))$ is an IF fuzzy BG-subalgebra of Y.*

Proof. Let $y_1, y_2 \in Y$. Since f is onto, there exists $x_1, x_2 \in X$ such that $f(x_1) = y_1, f(x_2) = y_2$. Then
$f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))(y_1 * y_2) = (\mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}(y_1 * y_2), \nu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}(y_1 * y_2))$.
Now $\mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}(y_1*y_2) = \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x_1*x_2)$, where $y_1*y_2 = f(x_1)*f(x_2) = f(x_1 * x_2)$. Thus

$$\begin{aligned}
&\mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}(y_1 * y_2)\\
&= \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x_1 * x_2)\\
&= \alpha\mu_A(x_1 * x_2) - \epsilon\mu_A(x_1 * x_2) + \gamma\\
&= \alpha\mu_A(x_1 * x_2) + \epsilon - \epsilon\mu_A(x_1 * x_2) + \gamma - \epsilon\\
&= \alpha\mu_A(x_1 * x_2) + \epsilon(1 - \mu_A(x_1 * x_2)) + \gamma - \epsilon\\
&\geq \alpha min\{\mu_A(x_1), \mu_A(x_2)\} + \epsilon(1 - max\{\mu_A(x_1), \mu_A(x_2)\} + \gamma - \epsilon\\
&= \alpha min\{\mu_A(x_1), \mu_A(x_2)\} + \epsilon min\{(1 - \mu_A(x_1), 1 - \mu_A(x_2)\} + \gamma - \epsilon\\
&= min\{\alpha\mu_A(x_1) + \epsilon(1 - \mu_A(x_1)) + \gamma - \epsilon, \alpha\mu_A(x_2) + \epsilon(1 - \mu_A(x_2)) + \gamma - \epsilon\}\\
&= min\{\alpha\mu_A(x_1) - \epsilon\mu_A(x_1) + \gamma, \alpha\mu_A(x_2) - \epsilon\mu_A(x_2) + \gamma\}\\
&= min\{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x_1), \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x_2)\}\\
&= min\{\mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}(f(x_1)), \mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}(f(x_2)\}\\
&= min\{\mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}(y_1), \mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}(y_2)\}.
\end{aligned}$$

So

$$\mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}(y_1 * y_2) \geq min\{\mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(y_1), \mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(y_2)\}.$$

Similarly, we can show

$$\nu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(y_1 * y_2) \leq max\{\nu_{f(\boxdot_{\alpha,\beta\gamma,\delta,\epsilon,\zeta}(A)}(y_1), \nu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(y_2)\}.$$

Hence $f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))$ is an IF fuzzy BG-subalgebra of Y. □

Theorem 4.5. *Let $f : X \longrightarrow Y$ be an onto homomorphism of BG-algebras. If $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)$ is an IF normal subalgebra of X, then $f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))$ is also an IF normal subalgebra of Y.*

Proof. Let $y_1, y_2, y_3, y_4 \in Y$. Since f is onto, therefore there exists $x_1, x_2, x_3, x_4 \in X$ such that $f(x_i) = y_i : i = 1, 2, 3, 4$. Now
$f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))((y_1 * y_3) * (y_2 * y_4))$
$= (\mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}((y_1 * y_3) * (y_2 * y_4)), \nu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}((y_1 * y_3) * (y_2 * y_4)))$.
Again,

$$\mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}((y_1 * y_3) * (y_2 * y_4)) = \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}((x_1 * x_3) * (x_2 * x_4)),$$

where

$$\begin{aligned}&(y_1 * y_3) * (y_2 * y_4) = (f(x_1) * f(x_3)) * (f(x_2) * f(x_4))\\&= (f(x_1 * x_3)) * (f(x_2 * x_4)) = f((x_1 * x_3) * (x_2 * x_4)).\end{aligned}$$

$$\begin{aligned}&\mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}((y_1 * y_3) * (y_2 * y_4))\\&= \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}((x_1 * x_3) * (x_2 * x_4))\\&= \alpha\mu_A((x_1 * x_3) * (x_2 * x_4)) - \epsilon\mu_A((x_1 * x_3) * (x_2 * x_4)) + \gamma\\&= \alpha\mu_A((x_1 * x_3) * (x_2 * x_4)) + \epsilon - \epsilon\mu_A((x_1 * x_3) * (x_2 * x_4)) + \gamma - \epsilon\\&= \alpha\mu_A((x_1 * x_3) * (x_2 * x_4)) + \epsilon(1 - \mu_A(x_1 * x_3) * (x_2 * x_4))) + \gamma - \epsilon\\&\geq \alpha min\{\mu_A(x_1 * x_2), \mu_A(x_3 * x_4)\} + \epsilon(1 - max\{\mu_A(x_1 * x_2), \mu_A(x_3 * x_4)\}\\&\quad +\gamma - \epsilon\\&= \alpha min\{\mu_A(x_1 * x_2), \mu_A(x_3 * x_4)\} + \epsilon min\{(1 - \mu_A(x_1 * x_2), 1 - \mu_A(x_3 * x_4)\}\\&\quad +\gamma - \epsilon\\&= min\{\alpha\mu_A(x_1 * x_2) + \epsilon(1 - \mu_A(x_1 * x_2)) + \gamma - \epsilon, \alpha\mu_A(x_3 * x_4)\\&\quad +\epsilon(1 - \mu_A(x_3 * x_4)) + \gamma - \epsilon\}\\&= min\{\alpha\mu_A(x_1 * x_2) - \epsilon\mu_A(x_1 * x_2) + \gamma, \alpha\mu_A(x_3 * x_4) - \epsilon\mu_A(x_3 * x_4) + \gamma\}\\&= min\{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x_1 * x_2), \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(x_3 * x_4)\}\\&= min\{\mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}f(x_1 * x_2), \mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}f(x_3 * x_4)\}\\&= min\{\mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}(y_1 * y_2), \mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}(y_3 * y_4)\}.\end{aligned}$$

Thus

$$\begin{aligned}&\mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))}((y_1 * y_3)) * (y_2 * y_4))\\&\geq min\{\mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(y_1 * y_2), \mu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(y_3 * y_4)\}.\end{aligned}$$

Similarly, we can show

$$\begin{aligned}&\nu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}((y_1 * y_3)) * (y_2 * y_4))\\&\leq max\{\nu_{f(\boxdot_{\alpha,\beta\gamma,\delta,\epsilon,\zeta}(A)}(y_1 * y_2), \nu_{f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A)}(y_3 * y_4)\}\end{aligned}$$

Hence $f(\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}(A))$ is an IF normal subalgebra of Y. □

5. Conclusions

In this paper, we study the effect of model operators $D_\alpha, F_{\alpha,\beta}, \boxplus, \boxtimes, \boxplus_\alpha, \boxtimes_\alpha$, $\boxplus_{\alpha,\beta}, \boxtimes_{\alpha,\beta}, E_{\alpha,\beta}, \boxplus_{\alpha,\beta,\gamma}, \boxtimes_{\alpha,\beta,\gamma}, \boxdot_{\alpha,\beta,\gamma,\delta}$ and $\boxdot_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}$ in intuitionistic fuzzy BG-algebras. We observed that the effect of these operators is similar to intuitionistic fuzzy translations and intuitionistic fuzzy multiplications as in [20]. Further each of these operators can be represented by a matrix may be called as matrix of the operator.

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