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Error estimates for fixed point theorems in fuzzy normed spaces

S. A. M. Mohsenialhosseini

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Abstract. We define fixed point error estimates for two important fixed point theorems involving quasi contractive type operators in fuzzy normed spaces. We also obtain complete statements for the fixed point theorems of Zamfirescu, including both a priori and a posteriori error estimates, when the Picard iteration is used to approximate the fixed points on fuzzy normed spaces.

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Corresponding Author: S. A. M. Mohsenialhosseini (`amah@vru.ac.ir`; `mohsenhosseini@yazd.ac.ir`; `mohsenialhosseini@gmail.com`)

1. Intrductin

Chitra and Mordeson [6] introduce a definition of norm fuzzy and thereafter the concept of fuzzy normed space has been introduced and generalized in different ways by Bag and Samanta in [2, 3]. Furthermore, the fixed point theory in this kind of spaces has been intensively studied. For details, one can refer to, Bag and Samanta, [4] George [8] Mohsenalhosseini et al. [9, 10].

Nowadays, fixed point play an important role in different areas of mathematics, and its applications, Particularly in mathematical physics, differential equation, game theory, and dynamic programming (See [1, 7, 12]). Since fuzzy mathematics along with the classical ones are constantly developing, the fuzzy type of the fixed point can also play an important role in the new fuzzy area.

In this paper, starting from the article of Berinde [5], we study some well-known contractive type mappings in Rhoades classification [13] on fuzzy normed spaces, and we give some fuzzy approximate fixed points of such mappings.

2. Preliminaries

We begin by recalling some needed definitions and results.

Definition 2.1 ([2])**.** Let U be a linear space on $\mathbf{R}$. A function $N : U\times\mathbf{R} \to [0,1]$ is called fuzzy norm if and only if for every $x, u \in U$ and for every $c \in \mathbf{R}$ the following properties are satisfied :

(F_{N1}) : $N(x,t) = 0$ for every $t \in \mathbf{R}^{-} \cup \{0\}$,

(F_{N2}) : $N(x,t) = 1$ if and only if $x = 0$ for every $t \in \mathbf{R}^{+}$,

(F_{N3}) : $N(cx,t) = N(x, \frac{t}{|c|})$ for every $c \neq 0$ and t $\in \mathbf{R}^{+}$,

(F_{N4}) : $N(x+u, s+t) \geq min\{N(x,s), N(u,t)\}$ for every $s,t \in \mathbf{R}^{+}$,

(F_{N5}) : the function $N(x,.)$ is nondecreasing on $\mathbf{R}$, and $\lim_{t\to\infty} N(x,t) = 1$.

The pair (U,N) is called a fuzzy normed space. Sometimes, We need two additional conditions as follows:
(F_{N6}) : $\forall t \in \mathbf{R}^{+}$ $N(x,t) > 0 \;\Rightarrow x = 0$.
(F_{N7}) : function $N(x,.)$ is continuous for every $x \neq 0$, and on subset

$$\{t :\; 0 < N(x,t) < 1\}$$

is strictly increasing.
Let (U,N) be a fuzzy normed space. For all $\alpha \in (0,1)$, we define α norm on U as follows :

$$\|x\|_{\alpha} \quad = \quad \wedge\{t > 0 : N(x,t) \geq \alpha\}\; for\; every\; x \in U. \tag{1}$$

Then $\{\|x\|_{\alpha} :\; \alpha \in (0,1]\}$ is an ascending family of normed on U and they are called α-norm on U corresponding to the fuzzy norm N on U. Some definitions, lemmas and example needed for this paper are given below.

Lemma 2.2 ([2])**.** *Let (U,N) be a fuzzy normed space such that satisfy conditions F_{N6} and F_{N7}. Define the function $N' : U \times \boldsymbol{R} \to [0,1]$ as follows:*

$$N'(x,t) = \begin{cases} \vee\{\alpha \in (0,1) : \|x\|_{\alpha} \leq t\} & (x,t) \neq (0,0) \\ \\ 0 & (x,t) = (0,0) \end{cases}$$

Then
a)N' is a fuzzy norm on U.
b) $N = N'$.

Lemma 2.3 ([2])**.** *Let (U,N) be a fuzzy normed space such that satisfy conditions F_{N6} and F_{N7}. and $\{x_n\} \subseteq U$, Then $\lim_{n\to\infty} N(x_n - x, t) = 1$ if and only if*

$$\lim_{n\to\infty} \|x_n - x\|_{\alpha} = 0$$

for every $\alpha \in (0,1)$.

Note that the sequence $\{x_n\} \subseteq U$ converges if there exists $x \in U$ such that

$$\lim_{n\to\infty} N(x_n - x, t) = 1 \; for\; every\; t \in \boldsymbol{R}^{+}.$$

In this case x is called the limit of $\{x_n\}$.

Example 2.4 ([2]). Let V be the Real or Complex vector space and N be defined on $V \times R$ as follows :

$$N(x,t) = \begin{cases} 1 & t > |x| \\ \\ 0 & t \leq |x| \end{cases}$$

for all $x \in V$ and $t \in R$. Then (V, N) is a fuzzy normed space and the function N satisfy conditions F_{N6} and $\|x\|_\alpha = |x|$ for every $\alpha \in (0,1)$.

Definition 2.5 ([9]). Let (U, N) be a fuzzy normed space, $T : U \to U$, $\epsilon > 0$ and $u_0 \in U$. Then u_0 is a F-approximate fixed point (fuzzy approximate fixed point) of T if for some $\alpha \in (0,1)$

$$\wedge\{t > 0 : N(u_0 - Tu_0, t) \geq \alpha\} \leq \epsilon.$$

In 2013, Mohsenialhosseini [9] defined F-Kannan, F -Chatterjea and F -Zamfirescu operators on fuzzy normed space as following:

Definition 2.6 ([9]). A mapping $T : U \to U$ is a F-Kannan operator if there exists $a \in (0, \frac{1}{2})$ such that

$$\begin{aligned} &\wedge\{t > 0 : N(Tu - Tv, t) \geq \alpha\} \\ &\leq a[\wedge\{t > 0 : N(u - Tu, t) \geq \alpha\} + \wedge\{t > 0 : N(v - Tv, t) \geq \alpha\}], \end{aligned}$$

for all $u, v \in U$.

Definition 2.7 ([9]). A mapping $T : U \to U$ is a F-Chatterjea operator if there exists $a \in (0, \frac{1}{2})$ such that

$$\begin{aligned} &\wedge\{t > 0 : N(Tu - Tv, t) \geq \alpha\} \\ &\leq a[\wedge\{t > 0 : N(u - Tv, t) \geq \alpha\} + \wedge\{t > 0 : N(v - Tu, t) \geq \alpha\}], \end{aligned} \tag{2}$$

for all $u, v \in U$.

Definition 2.8 ([9]). A mapping $T : U \to U$ is a F-Zamfirescu operator if there exist $\alpha_1, \beta, \gamma \in \mathbf{R}$, $\alpha_1 \in [0, 1[, \beta \in [0, \frac{1}{2}[, \gamma \in [0, \frac{1}{2}[$ such that for all $x, y \in U$ at least one of the following is true:

$$(F_{Z1}): \quad \wedge\{t > 0 : N(Tu - Tv, t) \geq \alpha\} \leq \alpha_1 \wedge \{t > 0 : N(u - v, t) \geq \alpha\},$$

$$\begin{aligned} (F_{Z2}): \quad &\wedge\{t > 0 : N(Tu - Tv, t) \geq \alpha\} \\ &\leq \beta[\wedge\{t > 0 : N(u - Tu, t) \geq \alpha\} + \wedge\{t > 0 : N(v - Tv, t) \geq \alpha\}], \end{aligned}$$

$$\begin{aligned} (F_{Z2}): \quad &\wedge\{t > 0 : N(Tu - Tv, t) \geq \alpha\} \\ &\leq \gamma[\wedge\{t > 0 : N(u - Tv, t) \geq \alpha\} + \wedge\{t > 0 : N(v - Tu, t) \geq \alpha\}]. \end{aligned}$$

Definition 2.9 ([11]). A mapping $T : U \to U$, is called, F^z-weak contraction or (δ_1, L)-contraction if there exist a constant $\delta_1 \in (0,1)$ and some $L \geq 0$ such that

$$\begin{aligned} &\wedge\{t > 0 : N(Tu - Tv, t) \geq \alpha\} \\ &\leq \delta_1 \wedge \{t > 0 : N(u - v, t) \geq \alpha\} + L \wedge \{t > 0 : N(u - Tu, t) \geq \alpha\}, \end{aligned} \tag{3}$$

for all $u, v \in U$.

Proposition 2.10 ([11]). *Let (U, N) be a fuzzy normed space and $T : U \to U$, is an F^z-weak contraction Then*

1) $F(T) \neq \varnothing$.

2) The Picard iteration $\{u_n\}_{n=0}^{\infty}$ associated to T, given by

$$u_{n+1} = Tu_n \qquad n = 0, 1, 2, ...$$

converges to u^* for any $u_0 \in U$.

3) The following estimates

$$i) \wedge \{t > 0 : N(u_n - u^*, t) \geq \alpha\} \leq \frac{\delta^n}{1-\delta} \wedge \{t > 0 : N(u_0 - u_1, t) \geq \alpha\}$$

for all n=0,1,2,...

$$ii) \wedge \{t > 0 : N(u_n - u^*, t) \geq \alpha\} \leq \frac{\delta}{1-\delta} \wedge \{t > 0 : N(u_{n-1} - u^*, t) \geq \alpha\}$$

for all n=0,1,2,... hold, where $\delta_1 \in (0, 1)$ is constant.

Remark 2.11 ([11]). Due to the symmetry of the distance, the weak contraction condition (3) implicity includes the following dual one

$$\begin{aligned} &\wedge\{t > 0 : N(Tu - Tv, t) \geq \alpha\} \\ &\leq \delta_1 \wedge \{t > 0 : N(u - v, t) \geq \alpha\} + L \wedge \{t > 0 : N(u - Tv, t) \geq \alpha\}, \end{aligned} \tag{4}$$

for all $u, v \in U$.

Remark 2.12 ([9]). In the rest of the paper we will denote the set of all F^z-approximate fixed points of T, for a given $\epsilon > 0$, by

$$F^z_\epsilon(T) = \{u \in U : \wedge\{t > 0 : N(u - Tu, t) \geq \alpha\} \leq \epsilon \; for \; some \; \alpha \in (0, 1)\}. \tag{5}$$

3. Fuzzy error estimates for fixed point theorems

In this section, we give error estimates for two important fixed point theorems in fuzzy normed spaces.

Remark 3.1. In the rest of the paper we will denote the set of all $Fuzzy$- fixed points of T, by

$$F(T) = \{t > 0 : N(u - Tu, t) = 1 \quad for \quad all \quad u \in U\}.$$

Proposition 3.2. *Let (U, N) be a fuzzy normed space and T be a self map of satisfying condition for any $u_0 \in U$, consider the Picard iteration $\{u_n\}_{n=0}^{\infty}$ associated to T, given by*

$$u_{n+1} = Tu_n, n = 0, 1, 2, ... \tag{6}$$

Then

1) T has a unique fixed point , i.e. $F(T) \neq \varnothing$.

2) The Picard iteration converges to u^* for any $u_0 \in U$.

3) The following estimates

$$i) \wedge \{t > 0 : N(u_n - u^*, t) \geq \alpha\} \leq \frac{\delta^n}{1-\delta} \wedge \{t > 0 : N(u_0 - u_1, t) \geq \alpha\} \tag{7}$$

for all n=0,1,2,...

$$ii) \wedge \{t>0 : N(u_n - u^*, t) \geq \alpha\} \quad \leq \quad \frac{\delta}{1-\delta} \wedge \{t>0 : N(u_{n-1} - u^*, t) \geq \alpha\} \quad (8)$$

for all n=0,1,2,... hold, with

$$\delta = \frac{\alpha_1}{1-\alpha_1}$$

where α_1 is the constant involved in (F_{Z1}).

4) The rate of convergence of the Picard iteration is given by

$$\wedge\{t>0 : N(u_n - u^*, t) \geq \alpha\} \quad \leq \quad \delta \wedge \{t>0 : N(u_{n-1} - u^*, t) \geq \alpha\} \quad (9)$$

for all n=0,1,2,...

Proof. First note that if T satisfies (F_{Z2}), then T has at most one fixed point.

$$\begin{aligned} \wedge\{t>0 : N(u_n - u_{n+1}, t) \geq \alpha\} &= \wedge\{t>0 : N(Tu_{n-1} - Tu_n, t) \geq \alpha\} \\ &\leq \alpha_1[\wedge\{t>0 : N(u_{n-1} - u_n, t) \geq \alpha\} \\ &+ \wedge\{t>0 : N(u_n - u_{n+1}, t) \geq \alpha\}] \end{aligned}$$

which yields

$$\wedge\{t>0 : N(u_n - u_{n+1}, t) \geq \alpha\} \quad \leq \quad \frac{\alpha_1}{1-\alpha_1} \wedge \{t>0 : N(u_{n-1} - u_n, t) \geq \alpha\} \quad (10)$$

for all n=0,1,2,... Since $\alpha_1 \in [0, 1[$, we have $\frac{\alpha_1}{1-\alpha_1} < 1$. Therefore, for $\delta = \frac{\alpha_1}{1-\alpha_1}$ and induction, we have

$$\wedge\{t>0 : N(u_{n+k} - u_{n+k+1}, t) \geq \alpha\} \quad \leq \quad \delta^k \wedge \{t>0 : N(u_n - u_{n-1}, t) \geq \alpha\} \quad (11)$$

for all $k \in N^*$. Hence

$$\begin{aligned} &\wedge\{t>0 : N(u_{n+p} - u_n, t) \geq \alpha\} \\ &\qquad \leq (\delta + \delta^2 + ... + \delta^p) \\ &\qquad \times \wedge\{t>0 : N(u_n - u_{n-1}, t) \geq \alpha\} \end{aligned}$$

which implies

$$\begin{aligned} &\wedge\{t>0 : N(u_{n+p} - u_n, t) \geq \alpha\} \\ &\qquad \leq \frac{\delta^n(1-\delta^p)}{1-\delta} \wedge \{t>0 : N(u_n - u_{n-1}, t) \geq \alpha\} (12) \end{aligned}$$

$$\begin{aligned} &\wedge\{t>0 : N(u_n - u_{n-1}, t) \geq \alpha\} \\ &\qquad \leq \delta^{n-1} \wedge \{t>0 : N(u_0 - u_1, t) \geq \alpha\}, \quad n \geq 1 \end{aligned}$$

from (9) we obtain

$$\wedge\{t>0 : N(u_{n+p} - u_n, t) \geq \alpha\} \quad \leq \quad \frac{\delta^n(1-\delta^p)}{1-\delta} \wedge \{t>0 : N(u_0 - u_1, t) \geq \alpha\}, (13)$$

for all $n, p \in N^*$.

Now by letting $p \to \infty$ in (13) and (12) we get the estimates (7) and (8), respectively.

Again, by (12) we have

$$\begin{aligned}\wedge\{t>0:N(Tu-Tv,t)\geq\alpha\} & \\ \leq\ & \alpha_1[\wedge\{t>0:N(u-Tu,t)\geq\alpha\} \\ +\ & \wedge\{t>0:N(v-Tv,t)\geq\alpha\}] \\ \leq\ & \alpha_1\{\wedge\{t>0:N(u-Tu,t)\geq\alpha\} \\ +\ & [\wedge\{t>0:N(v-u,t)\geq\alpha\} \\ +\ & \wedge\{t>0:N(u-Tu,t)\geq\alpha\} \\ +\ & \wedge\{t>0:N(Tu-Tv,t)\geq\alpha\}]\}\end{aligned}$$

which yields

$$\begin{aligned}\wedge\{t>0:N(Tu-Tv,t)\geq\alpha\} \quad \leq\quad & \frac{\alpha_1}{1-\alpha_1}\wedge\{t>0:N(u-v,t)\geq\alpha\} \\ +\quad & \frac{2\alpha_1}{1-\alpha_1}\wedge\{t>0:N(u-Tu,t)\geq\alpha\} \qquad (14)\end{aligned}$$

Take $u:=u^*$, $\quad v:=u_{n-1}$ in (14) to obtain

$$\wedge\{t>0:N(u_n-u^*,t)\geq\alpha\} \quad\leq\quad \frac{\alpha_1}{1-\alpha_1}\wedge\{t>0:N(u_{n-1}-u^*,t)\geq\alpha\}$$

for all n=0,1,2,... that is, the estimate (9). □

Remark 3.3. 1) The estimates (7) and (8) show that the Picard iteration converges to u^*, the unique fixed point of T, at least as fast as a geometric progression;

2) The estimate (9) shows that the rate of convergence of the Picard iteration is linear.

Proposition 3.4. *Let (U,N) be a fuzzy normed space and $T:U\to U$ be a F-Zamfirescu operator. For any $u_0\in U$, consider the Picard iteration $\{u_n\}_{n=0}^{\infty}$ associated to T, given by (6). Then*

1) T has a unique fixed point , i.e. $F(T)\neq\varnothing$.

2) The Picard iteration converges to u^* for any $u_0\in U$.

3) The following estimates

$$i)\wedge\{t>0:N(u_n-u^*,t)\geq\alpha\} \quad\leq\quad \frac{\delta^n}{1-\delta}\wedge\{t>0:N(u_0-u_1,t)\geq\alpha\} \quad (15)$$

for n=0,1,2,...

$$ii)\wedge\{t>0:N(u_n-u^*,t)\geq\alpha\} \quad\leq\quad \frac{\delta}{1-\delta}\wedge\{t>0:N(u_{n-1}-u^*,t)\geq\alpha\} (16)$$

for all n=0,1,2,... hold, with

$$\delta:=\eta=max\{\alpha_1,\frac{\beta_1}{1-\beta_1},\frac{\gamma_1}{1-\gamma_1}\}$$

where $\alpha_1,\beta_1,\gamma_1$ are the constants involved in $(F_{Z1}),(F_{Z2}),(F_{Z3})$, respectively.

4) The rate of convergence of the Picard iteration is given by

$$\wedge\{t>0:N(u_n-u^*,t)\geq\alpha\} \quad\leq\quad \delta\wedge\{t>0:N(u_{n-1}-u^*,t)\geq\alpha\} \qquad (17)$$

for all n=0,1,2,....

Proof. We first fix $u, v \in U$. At least one of (F_{Z1}), (F_{Z2}) or (F_{Z3}) is true. If (F_{Z2}) holds, then as we have seen in proving Proposition 3.2, the following inequality

$$\begin{aligned}&\wedge\{t>0 : N(Tu-TV,t)\geq\alpha\}\\ &\leq \frac{\beta}{1-\beta}\wedge\{t>0 : N(u-v,t)\geq\alpha\}+\frac{2\beta}{1-\beta}\wedge\{t>0 : N(u-Tu,t)\geq\alpha\}, \quad (18)\end{aligned}$$

hold. If (F_{Z3}) holds, then similarly we get

$$\begin{aligned}&\wedge\{t>0 : N(Tu-TV_n,t)\geq\alpha\}\\ &\leq \frac{\gamma}{1-\gamma}\wedge\{t>0 : N(u-v,t)\geq\alpha\}+\frac{2\gamma}{1-\gamma}\{t>0 : N(u-Tu,t)\geq\alpha\}, \quad (19)\end{aligned}$$

Therefore by denoting

$$\eta = max\{\alpha_1, \frac{\beta_1}{1-\beta_1}, \frac{\gamma_1}{1-\gamma_1}\}$$

we have $0 \leq \eta < 1$ and, for all $u, v \in U$, the following inequality

$$\begin{aligned}&\wedge\{t>0 : N(Tu-TV_n,t)\geq\alpha\}\\ &\leq \eta\wedge\{t>0 : N(u-v,t)\geq\alpha\}+2\eta\wedge\{t>0 : N(u-Tu,t)\geq\alpha\}, \quad (20)\end{aligned}$$

holds. In a similar manner we obtain

$$\begin{aligned}&\wedge\{t>0 : N(Tu-TV_n,t)\geq\alpha\}\\ &\leq \eta\wedge\{t>0 : N(u-v,t)\geq\alpha\}+2\eta\wedge\{t>0 : N(u-Tv,t)\geq\alpha\}, \quad (21)\end{aligned}$$

valid for all $u, v \in U$. But by (20) it follows that T has at most one fixed point and remains to show that there exists a fixed point.
If we take $u := u_n, v := u_{n-1}$ in (20), then we get

$$\wedge\{t>0 : N(u_n-u_{n+1},t)\geq\alpha\} \quad \leq \quad \eta\wedge\{t>0 : N(u_{n-1}-u_n,t)\geq\alpha\}$$

and the rest of the proof is similar to that of Proposition 3.2 □

Remark 3.5. The priori and posteriori error estimates as well as the rate of convergence given here for both Kannan's and Zamfirescu's fixed point theorems are formally the same as in Banach's fixed point theorem.

Proposition 3.6. *Let (U, N) be a fuzzy normed space and $T : U \to U$ be a F-Chatterjea operator. Then*
1) T has a unique fixed point , i.e. $F(T) \neq \varnothing$.
2) The Picard iteration converges to u^* for any $u_0 \in U$.
3) The following estimates

$$i)\wedge\{t>0 : N(u_n-u^*,t)\geq\alpha\} \quad \leq \quad \frac{\delta^n}{1-\delta}\wedge\{t>0 : N(u_0-u_1,t)\geq\alpha\} \quad (22)$$

for all n=0,1,2,...

$$ii)\wedge\{t>0 : N(u_n-u^*,t)\geq\alpha\} \quad \leq \quad \frac{\delta}{1-\delta}\wedge\{t>0 : N(u_{n-1}-u^*,t)\geq\alpha\} \quad (23)$$

for all n=0,1,2,... hold, where

$$\delta = \frac{a}{1-a}.$$

4) The rate of convergence of the Picard iteration is given by

$$\wedge\{t>0 : N(u_n - u^*, t) \geq \alpha\} \quad \leq \quad \delta \wedge \{t>0 : N(u_{n-1} - u^*, t) \geq \alpha\} \tag{24}$$

for all n=0,1,2,....

Proof. By Definition 2.7 and

$$\begin{aligned} \wedge\{t>0 : N(u-TV, t) \geq \alpha\} \quad &\leq \quad \wedge\{t>0 : N(u-v, t) \geq \alpha\} \\ &+ \quad \wedge\{t>0 : N(v-Tu, t) \geq \alpha\} \\ &+ \quad \wedge\{t>0 : N(Tu-Tv, t) \geq \alpha\}, \end{aligned} \tag{25}$$

we get after simple computations,

$$\begin{aligned} \wedge\{t>0 : N(Tu-TV, t) \geq \alpha\} \quad &\leq \quad \frac{a}{1-a} \wedge \{t>0 : N(u-v, t) \geq \alpha\} \\ &+ \quad \frac{2a}{1-a}\{t>0 : N(u-Tv, t) \geq \alpha\}, \end{aligned} \tag{26}$$

which is (3), with

$$\delta_1 = \frac{a}{1-a}, \quad L = \frac{2a}{1-a}.$$

The symmetry of (2) also implies (3). In a similar way we prove that (3) and (4) are also satisfied. The conclusion now follows by Proposition 2.10. □

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S. A. M. Mohsenialhosseini (amah@vru.ac.ir; mohsenialhosseini@gmail.com; mohsenhosseini@yazd.ac.ir)

Faculty of Mathematics, Vali-e-Asr University, Rafsanjan, Iran—Faculty of Mathematics, Yazd University, Yazd, Iran