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Fuzzy e-irresolute mappings on fuzzy topological spaces

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Abstract. In this paper, we study the characteristic properties of fuzzy e-irresolute mappings on fuzzy topological spaces.

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1. Introduction

Weaker forms of fuzzy continuity on fuzzy topological spaces have been considered by many workers using the concepts of fuzzy semiopen sets and fuzzy preopen sets. Seenivasan [10] defined the concept of fuzzy e-open set and studied fuzzy e-continuous mappings on fuzzy topological spaces. Fuzzy e-open sets are weaker then fuzzy δ-preopen set, fuzzy δ-semiopen set and stronger then the fuzzy β-open sets. Using this notion, he studied fuzzzy e-continuous (e-open, e-closed) mappings on fuzzy topological spaces. In this paper, using of fuzzy e-interiors and fuzzy e-closures we investigate the characteristic properties of fuzzy e-irresolute mappings on fuzzy topological spaces.

2. Preliminaries

Throughout this paper (X, T_1) and (Y, T_2) (or simply X and Y) represent non-empty fuzzy topological spaces. Let μ be a fuzzy subset of a space X. The fuzzy closure of μ and fuzzy interior of μ are denoted by $Cl(\mu)$ and $Int(\mu)$ respectively. A fuzzy subset μ of space X is called fuzzy regular open [2] (resp. fuzzy regular closed) if $\mu = Int(Cl(\mu))$ (resp. $\mu = Cl(Int(\mu))$. The fuzzy δ-interior of fuzzy subset μ of X is the union of all fuzzy regular open sets contained in A. A fuzzy subset μ is called fuzzy δ-open [15] if $\mu = Int_\delta(\mu)$. The complement of fuzzy δ-open set is called fuzzy

δ-closed (i.e, $\mu = Cl_\delta(\mu)$). The fuzzy δ-closure of μ and the fuzzy δ-interior of μ are denoted by $Cl_\delta(\mu)$ and $Int_\delta(\mu)$.

A fuzzy subset μ of a space X is called fuzzy semi open [2] (resp. fuzzy β-open set [3], fuzzy pre-open set[4], fuzzy δ- preopen [1], fuzzy δ-semi open)[8] if $\mu \leq ClInt\mu$ (resp. $\mu \leq Cl(Int(Cl(\mu)))$; $\mu \leq Int(Cl(\mu))$; $\mu \leq Int(Cl_\delta(\mu))$, $\mu \leq Cl(Int_\delta(\mu))$). The complement of a fuzzy semiopen (resp. fuzzy preopen , fuzzy δ- semiopen, fuzzy δ-preopen set) is called fuzzy semiclosed (resp. fuzzy preclosed , fuzzy δ-semiclosed, fuzzy δ-preclosed). The union of all fuzzy δ-semi open (resp. fuzzy δ-preopen) sets contained in a fuzzy set μ in a fuzzy topological space X is called the fuzzy δ-semi interior [8] (resp. fuzzy δ-pre interior [1]) of μ and it is denoted by $sInt_\delta(\mu)$ (resp. $pInt_\delta(\mu)$). The intersection of all fuzzy δ-semi closed (resp. fuzzy δ-preclosed) sets containing a fuzzy set μ in a fuzzy topological space X is called the fuzzy δ-semiclosure [8] (resp. fuzzy δ-preclosure [1]) of μ and it is denoted by $sCl_\delta(\mu)$ (resp. $pCl_\delta(\mu)$).

A fuzzy point in X with support $x \in X$ and value $(0 < \alpha \leq 1)$ is denoted by x_α. A fuzzy set λ in X is said to be q-coincident with a fuzzy set μ, denoted by $\lambda q \mu$, if there exists $x \in X$ such that $\lambda(x) + \mu(x) > 1$ [9]. It is known [9] that $\lambda \leq \mu$ if and only if λ and $1 - \mu$ are not q-coincident, denoted by $\lambda \overline{q}(1 - \mu)$. The words 'neighborhood' and 'fuzzy topological space' will be abbreviated as 'nbd' and 'fts', respectively.

A mapping $f : X \to Y$ is said to be fuzzy continuous if $f^{-1}(\nu)$ is a fuzzy open set in X for any fuzzy open set ν in Y.

Definition 2.1 ([12])**.** Let μ ba a fuzzy set of a fuzzy topological space X. Then μ is said to be fuzzy semi δ-preopen set of X if $\mu \leq ClIntCl_\delta\mu$.

Definition 2.2 ([10])**.** Let μ be a fuzzy set of a topological space X. Then μ is called:

(i) a fuzzy e-open set of X if $\mu \leq Cl(Int_\delta\mu) \vee Int(Cl_\delta\mu)$,
(ii) a fuzzy e-closed set of X if $Cl(Int_\delta\mu) \wedge Int(Cl_\delta\mu) \leq \mu$.

Lemma 2.3 ([10])**.** (i) *Any union of fuzzy e-open sets is a fuzzy e-open set.*
(ii) *Any intersection of fuzzy e-closed sets is a fuzzy e-closed set.*

Theorem 2.4. *Let X and Y be fuzzy topological spaces such that X is product related to Y. Then the product $\mu \times \nu$ of a fuzzy e-open set μ in X and a fuzzy e-open set ν in Y is a fuzzy e-open set of the fuzzy product topological space $X \times Y$.*

Definition 2.5 ([10])**.** Let μ be a fuzzy set of a fuzzy topological space X.
(1) The fuzzy e-interior of μ is

$$eInt\mu = \vee\{\nu|\nu \leq \mu, \nu \text{ is a fuzzy } e\text{-open set }\},$$

(2) The fuzzy e-closure of μ is

$$eCl\mu = \wedge\{\nu|\nu \geq \mu, \nu \text{ is a fuzzy } e\text{-closed set }\}.$$

Obviously, $eCl\mu$ is the smallest fuzzy e-closed set which contains μ, and $eInt\mu$ is the largest fuzzy e-open set which is contained in μ. Also, $eCl\mu = \mu$ for any fuzzy e-closed set μ and $eInt\mu = \mu$ for any fuzzy e-open set μ.

Theorem 2.6 ([10])**.** *Let μ be a fuzzy set of a fuzzy topological space X. Then $eInt\mu^c = (eCl\mu)^c$ and $eCl\mu^c = (eInt\mu)^c$.*

Definition 2.7 ([1, 2, 4, 8, 12, 10]). Let X and Y be fuzzy topological spaces and $f : X \to Y$ be a mapping. Then f is called:

(i) a fuzzy semicontinuous mapping if $f^{-1}(\nu)$ is a fuzzy semiopen set in X for any fuzzy open set ν in Y,
(ii) a fuzzy precontinuous mapping if $f^{-1}(\nu)$ is a fuzzy preopen set in X for any fuzzy open set ν in Y,
(iii) a fuzzy δ-semicontinuous mapping if $f^{-1}(\nu)$ is a fuzzy δ-semiopen set in X for any fuzzy open set ν in Y,
(iv) a fuzzy δ-precontinuous mapping if $f^{-1}(\nu)$ is a fuzzy δ-preopen set in X for any fuzzy open set ν in Y,
(v) a fuzzy semi δ-precontinuous mapping if $f^{-1}(\nu)$ is a fuzzy semi δ-preopen set in X for any fuzzy open set ν in Y,
(vi) a fuzzy e-continuous mapping if $f^{-1}(\nu)$ is a fuzzy e-open set in X for any fuzzy open set ν in Y.

Definition 2.8. Let $f : X \to Y$ be a mapping. Then f is called a fuzzy δ-irresolute mapping if $f^{-1}(\nu)$ is a fuzzy δ-open set in X for each fuzzy δ-open set ν in Y.

Definition 2.9 ([13]). Let $f : X \to Y$ be a mapping. Then f is called a fuzzy semi δ-preirresolute mapping if $f^{-1}(\nu)$ is a fuzzy semi δ-preopen set in X for each fuzzy semi δ-preopen set ν in Y.

Definition 2.10 ([14]). Let (X, τ) and (Y, σ) be fuzzy topological spaces. A fuzzy function $f : X \to Y$ is said to be fuzzy slightly e-continuous if for each fuzzy point $x_\alpha \in X$ and each fuzzy Clopen set λ in Y containing $f(x_\alpha)$, there exists a fuzzy e-open set μ in X containing x_α such that $f(\mu) \leq \lambda$.

Definition 2.11 ([10]). A fuzzy topological space (X, τ) is said to be fuzzy e-T_1 if for each pair of distinct points x and y of X, there exists fuzzy e-open sets U_1 and U_2 such that $x \in U_1$ and $y \in U_2$, $x \notin U_2$ and $y \notin U_1$.

Definition 2.12 ([10]). A fuzzy topological space (X, τ) is said to be fuzzy e-T_2 (i.e., fuzzy e-Hausdorff) if for each pair of distinct points x and y of X, there exists disjoint fuzzy e-open sets U and V such that $x \in U$ and $y \in V$.

Definition 2.13 ([6]). A fuzzy space X is said to be fuzzy co-T_1 if for each pair of distinct fuzzy points x_α and y_β of X there exist fuzzy Clopen sets λ and μ containing x_α and y_β, respectively such that $y_\beta \notin \lambda$ and $x_\alpha \notin \mu$.

Definition 2.14 ([6]). A fuzzy space X is said to be fuzzy co-T_2 (fuzzy co-Hausdorff) if for each pair of distinct fuzzy points x_α and y_β in X, there exist disjoint fuzzy Clopen sets λ and μ in X such that $x_\alpha \in \lambda$ and $y_\beta \in \mu$.

Theorem 2.15 (). *Let X and Y be fuzzy topological spaces and $f : X \to Y$ be a mapping. Then the following are equivalent:*

(1) *f is fuzzy e-continuous.*
(2) *The inverse image of each fuzzy closed set in Y is a fuzzy e-closed set in X.*
(3) *$f(eCl\mu) \leq Cl(f(\mu))$ for each fuzzy set μ in X.*
(4) *$eCl(f^{-1}(\nu)) \leq f^{-1}(Cl\nu)$ for each fuzzy set ν in Y.*
(5) *$f^{-1}(Int\nu) \leq eInt(f^{-1}(\nu))$ for each fuzzy set ν in Y.*

Proof. Obvious □

Theorem 2.16. *Let X and Y be fuzzy topological spaces and $f : X \to Y$ be a mapping. Then for each fuzzy set ν in Y, $f^{-1}(Int\nu) \leq ClInt_\delta(f^{-1}(\nu)) \vee IntCl_\delta(f^{-1}(\nu))$.*

Proof. Let ν be a fuzzy set in Y. Then $Int\nu$ is a fuzzy open set in Y and so $f^{-1}(Int\nu)$ is a fuzzy e-open set in X. Hence

$$\begin{aligned} f^{-1}(Int\nu) &\leq ClInt_\delta(f^{-1}(Int\nu)) \vee IntCl_\delta(f^{-1}(Int\nu)) \\ &\leq ClInt_\delta(f^{-1}(\nu)) \vee IntCl_\delta(f^{-1}(\nu)). \end{aligned}$$

□

Theorem 2.17. *Let X and Y be fuzzy topological spaces and $f : X \to Y$ be a bijection. Then f is fuzzy e-continuous if and only if $Int(f(\mu)) \leq f(eInt\mu)$ for each fuzzy set μ in X.*

Proof. Let μ be a fuzzy set in X. Then by Theorem 2.15,

$$f^{-1}(Int(f(\mu))) \leq eInt(f^{-1}(f(\mu))).$$

Since f is a bijection,

$$Int(f(\mu)) = f(f^{-1}(Int(f(\mu)))) \leq f(eInt\mu).$$

Conversely, let ν be a fuzzy set in Y. Then

$$Int(f(f^{-1}(\nu))) \leq f(eInt(f^{-1}(\nu))).$$

Since f is a bijection,

$$Int\nu = Int(f(f^{-1}(\nu))) \leq f(eInt(f^{-1}(\nu)))$$

and

$$f^{-1}(Int\nu) \leq f^{-1}(f(eInt(f^{-1}(\nu)))) = eInt(f^{-1}(\nu)).$$

Therefore, by Theorem 2.15, f is fuzzy e-continuous. □

3. Fuzzy e-irresolute mappings

Definition 3.1 ([10])**.** Let $f : X \to Y$ be a mapping. Then f is called a fuzzy e-irresolute mapping if $f^{-1}(\nu)$ is a fuzzy e-open set in X for each fuzzy e-open set ν in Y.

From the above definition, Every fuzzy e-irresolute mapping is a fuzzy e-continuous mapping. But the converse is not true in general. A fuzzy semicontinuous mapping and a fuzzy e-irresolute mapping do not have any specific relations. Also, fuzzy precontinuous mapping and fuzzy e-irresolute mapping are independent.

Example 3.2. Let $\mu_1,\ \mu_2,\ \mu_3,\ \mu_4$ and η_1 be fuzzy sets of $X = \{a,\ b,\ c\}$, defined as follows. $\mu_1 = \frac{0.3}{a}+\frac{0.4}{b}+\frac{0.5}{c}$, $\mu_2 = \frac{0.6}{a}+\frac{0.5}{b}+\frac{0.5}{c}$, $\mu_3 = \frac{0.6}{a}+\frac{0.5}{b}+\frac{0.4}{c}$, $\mu_4 = \frac{0.3}{a}+\frac{0.4}{b}+\frac{0.4}{c}$ and $\eta_1 = \frac{0.2}{a}+\frac{0.2}{b}+\frac{0.2}{c}$. Consider fuzzy topologies $T_1 = \{0_X,\ 1_X,\ \mu_1,\ \mu_2,\ \mu_3,\ \mu_4\}$ and $T_2 = \{0_X,\ 1_X,\ \eta_1\}$ and the identity mapping $i_X : (X, T_1) \to (X, T_2)$. Then i_X is a fuzzy e-continuous mapping, but i_X is not a fuzzy e-irresolute mapping.

Example 3.3. Let $\mu_1,\ \mu_2,\ \mu_3,\ \mu_4,\ \mu_5$ and η_1 be fuzzy sets of $X = \{a,\ b,\ c\}$, defined as follows. $\mu_1 = \frac{0.2}{a} + \frac{0.2}{b} + \frac{0.2}{c}$, $\mu_2 = \frac{0.3}{a} + \frac{0.4}{b} + \frac{0.5}{c}$, $\mu_3 = \frac{0.6}{a} + \frac{0.5}{b} + \frac{0.5}{c}$, $\mu_4 = \frac{0.6}{a} + \frac{0.5}{b} + \frac{0.4}{c}$, $\mu_5 = \frac{0.3}{a} + \frac{0.4}{b} + \frac{0.4}{c}$ and $\eta_1 = \frac{0.2}{a} + \frac{0.2}{b} + \frac{0.2}{c}$. Consider fuzzy topologies $T_1 = \{0_X,\ 1_X,\ \mu_1,\ \mu_2,\ \mu_3,\ \mu_4,\ \mu_5\}$ and $T_2 = \{0_X,\ 1_X,\ \eta_1\}$ and the identity mapping $i_X : (X, T_1) \to (X, T_2)$. Then i_X are fuzzy semicontinuous and fuzzy precontinuous, but i_X is not a e-irresolute mapping

Example 3.4. Let μ_1 and η_1 be fuzzy sets of $X = \{a,\ b,\ c\}$, defined as follows. $\mu_1 = \frac{0.9}{a} + \frac{0.9}{b} + \frac{0.9}{c}$, $\eta_1 = \frac{0.8}{a} + \frac{0.8}{b} + \frac{0.8}{c}$. Consider fuzzy topologies $T_1 = \{0_X,\ 1_X,\ \mu_1\}$ and $T_2 = \{0_X,\ 1_X,\ \eta_1\}$ and the identity mapping $i_X : (X, T_1) \to (X, T_2)$. Then i_X is a fuzzy e-irresolute mapping, but i_X is not a fuzzy semicontinuous mapping and also i_X is not a fuzzy δ-semicontinuous mapping.

Example 3.5. Let μ_1 and η_1 be fuzzy sets of $X = \{a,\ b,\ c\}$, defined as follows. $\mu_1 = \frac{0.1}{a} + \frac{0.1}{b} + \frac{0.1}{c}$, $\eta_1 = \frac{0.8}{a} + \frac{0.8}{b} + \frac{0.8}{c}$. Consider fuzzy topologies $T_1 = \{0_X,\ 1_X,\ \mu_1\}$ and $T_2 = \{0_X,\ 1_X,\ \eta_1\}$ and the identity mapping $i_X : (X, T_1) \to (X, T_2)$. Then i_X is a fuzzy e-irresolute mapping, but i_X is not a fuzzy precontinuous mapping and also i_X is not a fuzzy δ-precontinuous mapping.

Example 3.6. Let μ and η be fuzzy sets of $X = \{a,\ b,\ c\}$, defined as follows. $\mu = \frac{0.8}{a} + \frac{0.8}{b} + \frac{0.8}{c}$, $\eta = \frac{0.5}{a} + \frac{0.5}{b} + \frac{0.5}{c}$. Consider fuzzy topologies $T_1 = \{0_X,\ 1_X,\ \mu\}$ and $T_2 = \{0_X,\ 1_X,\ \eta\}$ and the identity mapping $i_X : (X, T_1) \to (X, T_2)$. Then i_X is a fuzzy e-irresolute mapping, but i_X is not a fuzzy δ-irresolute.

Example 3.7. In Example 3.2, the mapping i_X is a fuzzy semi-δ-precontinuous mapping, but i_X is not a fuzzy δ-semicontinuous mapping and also i_X is not a fuzzy semi-δ-preirresolute.

Remark 3.8. From the above discussions and known results we have the following implications.

Theorem 3.9. *Let X and Y be fuzzy topological spaces and $f : X \to Y$ be a mapping. Then the following are equivalent:*

(i) *f is fuzzy e-irresolute.*
(ii) *The inverse image of each fuzzy e-closed set in Y is a fuzzy e-closed set in X.*
(iii) *$eCl(f^{-1}(\nu)) \leq f^{-1}(eCl\nu)$ for each fuzzy set ν in Y.*
(iv) *$f(eCl\mu) \leq eCl(f(\mu))$ for each fuzzy set μ in X.*

Proof. (i) implies (ii): Let ν be a fuzzy e-closed set in Y. Then ν^c is a fuzzy e-open set. Since f is fuzzy e-irresolute, $f^{-1}(\nu^c) = (f^{-1}(\nu))^c$ is a fuzzy e-open set in X. Therefore, $f^{-1}(\nu)$ is a fuzzy e-closed set in X.

(ii) implies (i): Let ν be a fuzzy e-open set in Y. Then ν^c is a fuzzy e-closed set and $f^{-1}(\nu^c) = (f^{-1}(\nu))^c$ is a fuzzy e-closed set in X. Since $f^{-1}(\nu)$ is a fuzzy e-open set in X, f is fuzzy e-irresolute.

(ii) implies (iii): Let ν be a fuzzy set in Y. Then $\nu \leq eCl\nu$ and $f^{-1}(\nu) \leq f^{-1}(eCl\nu)$. Since $f^{-1}(eCl\nu)$ is a fuzzy e-closed set in X,

$$eCl(f^{-1}(\nu)) \leq eCl(f^{-1}(eCl\nu)) = f^{-1}(eCl\nu).$$

(iii) implies (iv): Let μ be a fuzzy set in X. Then $f(\mu) \leq eCl(f(\mu))$ and

$$eCl\mu \leq eCl(f^{-1}(f(\mu))) \leq f^{-1}(eCl(f(\mu))).$$

This implies that

$$f(eCl\mu) \leq f(f^{-1}(eCl(f(\mu)))) \leq eCl(f(\mu)).$$

(iv) implies (ii): Let ν be a fuzzy e-closed set in Y. Then

$$f(eCl(f^{-1}(\nu))) \leq eCl(f(f^{-1}(\nu))) \leq eCl\nu = \nu.$$

That implies that

$$eCl(f^{-1}(\nu))) \leq f^{-1}(f(eCl(f^{-1}(\nu)))) \leq f^{-1}(\nu).$$

Therefore, $f^{-1}(\nu)$ is a fuzzy e-closed set in X. □

Theorem 3.10 (). *A mapping $f : X \to Y$ is fuzzy e-irresolute if and only if $f^{-1}(eInt\nu) \leq eInt(f^{-1}(\nu))$ for each fuzzy set ν in Y.*

Proof. Let ν be a fuzzy set in Y. Then $eInt\nu \leq \nu$. Since f is fuzzy e-irresolute, $f^{-1}(eInt\nu)$ is a fuzzy e-open set in X. Hence

$$f^{-1}(eInt\nu) = eInt(f^{-1}(eInt\nu)) \leq eInt(f^{-1}(\nu)).$$

Conversely, let ν be a fuzzy e-open set in Y. Then

$$f^{-1}(\nu) = f^{-1}(eInt\nu) \leq eInt(f^{-1}(\nu)).$$

Therefore, $f^{-1}(\nu)$ is a fuzzy e-open set in X and consequently f is a fuzzy e-irresolute mapping. □

Theorem 3.11. *Let X and Y be fuzzy topological spaces and $f : X \to Y$ be a bijection. Then, f is fuzzy e-irresolute if and only if $eInt(f(\mu)) \leq f(eInt\mu)$ for each fuzzy set μ in X.*

Proof. Let μ be a fuzzy set in X. Then by Theorem 3.10,

$$f^{-1}(eInt(f(\mu))) \leq eInt(f^{-1}(f(\mu))).$$

Since f is a bijection,

$$eInt(f(\mu)) = f(f^{-1}(eInt(f(\mu)))) \leq f(eInt(f^{-1}(f(\mu)))) = f(eInt\mu).$$

Conversely, let ν be a fuzzy e-open set in Y. Then

$$eInt(f(f^{-1}(\nu))) \leq f(eInt(f^{-1}(\nu))).$$

Since f is a bijection,

$$eInt\nu \leq f(eInt(f^{-1}(\nu))).$$

This implies that

$$f^{-1}(eInt\nu) \leq f^{-1}(f(eInt(f^{-1}(\nu)))) = eInt(f^{-1}(\nu)).$$

Therefore, by Theorem 3.10, f is a fuzzy e-irresolute mapping. □

4. Applications

Theorem 4.1. *If $f : X \to Y$ is a fuzzy slightly e-continuous injection and Y is fuzzy co-T_1, then X is fuzzy e-T_1.*

Proof. Suppose that Y is fuzzy co-T_1. For any distinct fuzzy points x_α and y_β in X, there exist fuzzy Clopen sets λ, μ in Y such that $f(x_\alpha) \in \lambda$, $f(y_\beta) \notin \lambda$, $f(x_\alpha) \notin \mu$ and $f(y_\beta) \in \mu$. Since f is fuzzy slightly e-continuous, $f^{-1}(\lambda)$ and $f^{-1}(\mu)$ are fuzzy e-open sets in X such that $x_\alpha \in f^{-1}(\lambda)$, $y_\beta \notin f^{-1}(\lambda)$, $x_\alpha \notin f^{-1}(\mu)$ and $y_\beta \in f^{-1}(\mu)$. This shows that X is fuzzy e-T_1. □

Theorem 4.2. *If $f : X \to Y$ is a fuzzy slightly e-continuous injection and Y is fuzzy co-T_2, then X is fuzzy e-T_2.*

Proof. For any pair of distinct fuzzy points x_α and y_β in X, there exist disjoint fuzzy Clopen sets λ and μ in Y such that $f(x_\alpha) \in \lambda$ and $f(y_\beta) \in \mu$. Since f is fuzzy slightly e-continuous, $f^{-1}(\lambda)$ and $f^{-1}(\mu)$ are fuzzy e-open sets in X containing x_α and y_β respectively. We have $f^{-1}(\lambda) \wedge f^{-1}(\mu) = \phi$. This shows that X is fuzzy e-T_2. □

Definition 4.3. A fuzzy space is called fuzzy co-regular [6] (respectively fuzzy strongly e-regular) if for each fuzzy Clopen (respectively fuzzy e-closed) set η and each fuzzy point $x_\alpha \notin \eta$, there exist disjoint fuzzy open sets λ and μ such that $\eta \leq \lambda$ and $x_\alpha \in \mu$.

Definition 4.4. A fuzzy space is called fuzzy co-normal [6] (respectively fuzzy strongly e-normal) if for every pair of disjoint fuzzy Clopen (respectively fuzzy e-closed) set η_1 and η_2 in X, there exist disjoint fuzzy open sets λ and μ such that $\eta_1 \leq \lambda$ and $\eta_2 \leq \mu$.

Theorem 4.5. *If f is fuzzy slightly e-continuous injective fuzzy open function from a fuzzy strongly e-regular space X onto a fuzzy space Y, then Y is fuzzy co-regular.*

Proof. Let η be fuzzy Clopen set in Y and be $y_\beta \notin \eta$. Take $y_\beta = f(x_\alpha)$. Since f is fuzzy slightly e-continuous, $f^{-1}(\eta)$ is a fuzzy e-closed set. Take $\gamma = f^{-1}(\eta)$. We have $x_\alpha \notin \gamma$. Since X is fuzzy strongly e-regular, there exist disjoint fuzzy open sets λ and μ such that $\gamma \leq \lambda$ and $x_\lambda \in \mu$. We obtain that $\eta = f(\gamma) \leq f(\lambda)$ and $y_\beta = f(x_\alpha) \in f(\mu)$ such that $f(\lambda)$ and $f(\mu)$ are disjoint fuzzy open sets. This shows that Y is fuzzy co-regular. □

Theorem 4.6. *If f is fuzzy slightly e-continuous injective fuzzy open function from a fuzzy strongly e-normal space X onto a fuzzy space Y, then Y is fuzzy co-normal.*

Proof. Let η_1 and η_2 be disjoint fuzzy Clopen sets in Y. Since f is fuzzy slightly e-continuous, $f^{-1}(\eta_1)$ and $f^{-1}(\eta_2)$ are fuzzy e-closed sets. Take $\lambda = f^{-1}(\eta_1)$ and $\mu = f^{-1}(\eta_2)$. We have $\lambda \wedge \mu = \phi$. Since X is fuzzy strongly e-normal, there exist disjoint fuzzy open sets γ and β such that $\lambda \leq \gamma$ and $\mu \leq \beta$. We obtain that $\eta_1 = f(\lambda) \leq f(\gamma)$ and $\eta_2 = f(\mu) \leq f(\beta)$ such that $f(\gamma)$ and $f(\beta)$ are disjoint fuzzy open sets. Thus, Y is fuzzy co-normal. □

Definition 4.7 ([10])**.** A fuzzy set λ in a topological space (X, τ) is said to be fuzzy e-connected if λ cannot be expressed as the union of two fuzzy e-open sets.

Equivalently, a fuzzy topological space (X, τ) is said to be fuzzy e-connected if fuzzy sets which are both fuzzy e-open and fuzzy e-closed sets are 0_X and 1_X.

Theorem 4.8. *A fuzzy topological space (X, τ) is e-connected iff X has no non-zero e-open sets λ and μ such that $\lambda + \mu = 1_X$.*

Proof. (Necessity) Suppose (X, τ) is fuzzy e-connected. If X has two non-zero fuzzy e-open sets λ and μ such that $\lambda + \mu = 1_X$, then λ is proper fuzzy e-open and fuzzy e-closed set of X. Hence, X is not fuzzy e-connected, a contradiction.

(Sufficiency) If (X, τ) is not fuzzy e-connected then it has a proper fuzzy set λ of X which is both fuzzy e-open and fuzzy e-closed. So $\mu = 1 - \lambda$, is a fuzzy e-open set of X such that $\lambda + \mu = 1_X$, which is a contradiction. □

Theorem 4.9. *If $f : (X, \tau) \to (Y, \sigma)$ is fuzzy e-continuous surjection and (X, τ) is fuzzy e-connected, then (Y, σ) is fuzzy connected.*

Proof. Let X be a fuzzy e-connected space and Y is not fuzzy connected. As Y is not fuzzy connected, then there exists a proper fuzzy set λ of Y such that $\lambda \neq 0_Y, \lambda \neq 1_Y$ and λ is both fuzzy open and fuzzy closed set. Since, f is fuzzy e-continuous, $f^{-1}(\lambda)$ is both fuzzy e-open and fuzzy e-closed set in X such that $f^{-1}(\lambda) \neq 0_X$ and $f^{-1}(\lambda) \neq 1_X$. Hence, X is not fuzzy e-connected, a contradiction. □

Theorem 4.10. *If $f : (X, \tau) \to (Y, \sigma)$ is fuzzy e-irresolute surjection and X is fuzzy e-connected, then Y is so.*

Proof. Similar to the proof of the above Theorem 4.9. □

Definition 4.11 ([7])**.** A collection μ of fuzzy sets in a fuzzy space X is said to be cover of a fuzzy set η of X if $(\bigvee_{A \in \mu} A)(x) = 1$, for every $x \in s(\eta)$. A fuzzy cover μ of a fuzzy set η in a fuzzy space X is said to be have a finite subcover if there exists a finite subcollection $\rho = \{A_1, A_2, \ldots, A_n\}$ of μ such that $(\bigvee_{j=1}^{n} A_j)(x) \geq \eta(x)$, for every $x \in s(\eta)$, where $s(\eta)$ denotes the support of a fuzzy set η.

Definition 4.12 ([5]). A fuzzy topological space (X, τ) is fuzzy compact space if every fuzzy open cover of X has a finite subcover.

Definition 4.13 ([11]). Let (X, τ) be a fts a family μ of fuzzy sets is e-open cover of a fuzzy set λ if $\lambda \leq \bigvee\{G : G \in \mu\}$ and each member of μ is e-open cover of a fuzzy set. A subcover of μ is subfamily which is also cover.

Definition 4.14 ([11]). A fuzzy topological space (X, τ) is fuzzy e-compact space if every fuzzy e-open cover of X has a finite subcover.

In this proposition we will explain that the union of any two fuzzy e-compact is also fuzzy e-compact

Proposition 4.15. *Let (X, τ) be a fts, if A and B are two fuzzy e-compact subsets of X, then $A \cup B$ is also fuzzy e-compact.*

Proof. Let $\{G_\lambda : \lambda \in \Lambda\}$ be a fuzzy e-open cover of $A \cup B$.

Then $\mu_{A\cup B}(x) \leq \sup_{\lambda\in\Lambda}\{\mu_{G_\lambda}(x)\}$, i.e., $Max\{\mu_A(x), \mu_B(x)\} \leq \sup_{\lambda\in\Lambda}\{\mu_{G_\lambda}(x)\}$

Hence, $A \cup B \subseteq \bigcup_{\lambda\in\Lambda} G_\lambda$.

Since, $\mu_A(x) \leq \mu_{A\cup B}(x)$. Then $A \subseteq A \cup B$.

Also $\mu_B(x) \leq \mu_{A\cup B}(x)$. Then $B \subseteq A \cup B$.

It is follows that $\{G_\lambda : \lambda \in \Lambda\}$ is a fuzzy e-open cover of A and a fuzzy e-open cover of B.

Since A and B are two fuzzy e-compact sets, then there exists a finite subcover $\{G_{\lambda_1}, G_{\lambda_2}, ..., G_{\lambda_n}\}$, which covering A belong to $\{G_\lambda : \lambda \in \Lambda\}$.

Then $\mu_A(x) \leq Max\{\mu_{G_{\lambda_i}}(x)\}$

Hence $A \subseteq \bigcup_{i=1}^{n} G_{\lambda_i}$ and there exists a finite subcover $\{G_{\lambda_1}, G_{\lambda_2}, ..., G_{\lambda_m}\}$ which covering B belongs to $\{G_\lambda : \lambda \in \Lambda\}$

Then, $\mu_B(x) \leq Max\{\mu_{G_{\lambda_j}}(x)\}$

Hence, $B \subseteq \bigcup_{j=1}^{m} G_{\lambda_j}$

It is follows that $\mu_{A\cup B}(x) \leq Max\{G_{\lambda_K}(x)\}$, $k = 1, 2, ..., n + m$

Then $A \cup B \subseteq \bigcup_{k=1}^{n+m} G_{\lambda_k}$

Thus, $A \cup B$ is fuzzy e-compact. □

Proposition 4.16. *Let (X, τ) be a fts, if A and B are two fuzzy e-compact subsets of X, then $A \cap B$ need not be fuzzy e-compact.*

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