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Characterizations of near-rings by interval valued (α, β)-Fuzzy ideals

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Abstract. In this paper, we introduce the concept of interval valued (α, β)-fuzzy subnear-rings and ideal of near-rings, where α, β any two of the $\{\in, q, \in \vee q, \in \wedge q\}$ with $\alpha \neq \in \wedge q$, by using belongs to relation $\in$ and quasi-coincidence with relation q between interval valued fuzzy points and interval valued fuzzy sets. We also discussed some characterizations of interval valued $(\alpha, \in \vee q)$-fuzzy ideals(subnear-rings), mainly discuss interval valued $(\in, \in \vee q)$-fuzzy ideals(subnear-rings) of near-rings.

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1. Introduction

A near-ring satisfies all axioms of an associative ring, except commutative of addition and one of the two distributive laws. In 1965, the fundamental concept of a fuzzy set was first initiated by Zadeh[24]. Then the fuzzy sets have been used in the reconsideration of classical mathematics. Ten years later Zadeh[25] introduced the concept of interval valued fuzzy subsets, where the values of the membership functions are the intervals instead of numbers. Rosenfeld[19] introduced the concept of fuzzy subgroup and give some of its properties. The concept of the interval valued fuzzy subgroup was first discussed by Biswas[6] in 1994. Abou-zaid [1] proposed the notion of fuzzy subnear-rings and ideals of near-rings. A new type of fuzzy subgroup, namely, (α, β)-fuzzy subgroup was introduced by Bhakat and Das[3, 4, 5] using the relation “belongs to” $(\in)$ and “quasi-coincidence” (q) of fuzzy points and fuzzy sets initiated by Pu Pao-Ming and Liu-Ming[18].The $(\in, \in \vee q)$-fuzzy subgroup is an important and useful generalization of Rosenfeld’s fuzzy subgroup. In [12], Dudek et al. introduced the concept of (α, β)-fuzzy ideals and (α, β)-fuzzy h-ideals in hemirings. Davvaz[7, 8] used this concept in the theory of near-rings and introduced $(\in, \in \vee q)$-fuzzy subnear-rings (ideals, R-subgroups) of near-rings. Young Bae Jun[22, 23], gave some results on (α, β)-fuzzy h-ideals in hemirings and discussed some properties of $(\in, \in \vee q_k)$-fuzzy subalgebras in BCK/BCI- algebras. Narayanan and Manikandan[16] introduced the notion of an $(\in, \in \vee q)$-fuzzy quasi-ideals in near-rings. Asger Khan[2] introduced the notion of generalized fuzzy ideals of ordered semigroups. Muhammad Shabir[15], initiated the concept of interval valued generalized fuzzy ideals of regular LA-semigroups. Deena and Coumaressane[11] proposed the notion of $(\in, \in \vee q_k)$-fuzzy subnear-rings and ideals of near-rings which is a generalization of $(\in, \in \vee q)$-fuzzy subnear-rings and ideals. In [13, 14], Zhan et al. have considered the idea of interval valued (α, β)-fuzzy hyperideals of hypernear-rings and a new view of fuzzy hypernear-rings. Davvaz[9, 10], discussed few concepts of fuzzy ideals of near-rings and generalized fuzzy H_v-submodules endowed with interval valued membership functions.

2. Preliminaries

In this section, we present some elementary definitions that we use in the sequal.

Definition 2.1 ([8, 17])**.** A near-ring is an algebraic system $(R, +, \cdot)$ consisting of a non empty set R together with two binary operations called $+$ and $\cdot$ such that $(R, +)$ is a group not necessarily abelian and $(R, \cdot)$ is a semigroup connected by the following distributive law: $x \cdot (y + z) = x \cdot y + x \cdot z$ valid for all $x, y, z \in R$. We will use the word ‘near-ring ’to mean ‘left near-ring ’. We denote xy instead of $x \cdot y$. An ideal I of a near-ring R is the subset of R such that (i) $(I, +)$ is a normal subgroup of $(R, +)$, (ii) $RI \subseteq I$, (iii) $(x + a)y - xy \in I$, for any $a \in I$ and $x, y \in R$.

Note that I is a left ideal of R if I satisfies (i) and (ii), and right ideal of R if I satisfies (i) and (iii).

Definition 2.2 ([7])**.** A fuzzy subset μ of R is said to be an $(\in, \in \vee q)$-fuzzy subnear-ring of R if for all $x, y \in R$ and $t, r \in (0, 1]$:

(1) $x_t, y_r \in \mu$ implies $(x + y)_{\min\{t,r\}} \in \vee q\mu$,

(2) $x_t \in \mu$ implies $(-x)_t \in \vee q\mu$,

(3) $x_t, y_r \in \mu$ implies $(xy)_{\min\{t,r\}} \in \vee q\mu$.

μ is called an $(\in, \in \vee q)$-fuzzy ideal of R if μ is a $(\in, \in \vee q)$-fuzzy subnear-ring of R and

(4) $x_t \in \mu$ implies $(y + x - y)_t \in \vee q\mu$,

(5) $y_r \in \mu$ and $x \in R$ implies $(xy)_r \in \vee q\mu$,

(6) $a_t \in \mu$ and $x, y \in R$ implies $((x + a)y - xy)_t \in \vee q\mu$, for any $x, y, a \in R$.

Definition 2.3. A fuzzy subset μ of R is a map $\mu : R \to [0, 1]$. A fuzzy subset of the form

$$\mu(y) = \begin{cases} t \in (0, 1], & \text{if } y = x, \\ 0, & \text{if } y \neq x. \end{cases}$$

is called a fuzzy point with support x and value t and is denoted by x_t.

For a fuzzy point x_t and a fuzzy subset μ of the same set R, Pu Ming and Liu Ming[18] introduced the symbol $x_t\alpha\mu$, where $\alpha \in \{\in, q, \in \wedge q, \in \vee q\}$. A fuzzy point x_t is said to belong to (resp.quasi-coincident with) a fuzzy subset μ, written as $x_t \in \mu$(resp. $x_t q\mu$) if $\mu(x) \geq t$(resp. $\mu(x) + t > 1$). The symbol $x_t \in \vee q\mu$ means that $x_t \in \mu$ or $x_t q\mu$. Similarly, $x_t \in \wedge q\mu$ denotes that $x_t \in \mu$ and $x_t q\mu$. $x_t \overline{\in} \mu$ and $x_t \overline{\in \vee q}\mu$ means that $x_t \in \mu$ and $x_t \in \vee q\mu$ do not hold, respectively.

Notation 2.4 ([10, 20])**.** By an interval number $\widetilde{a}$, we mean an interval $[a^-, a^+]$ such that $0 \leq a^- \leq a^+ \leq 1$ where a^- and a^+ are the lower and upper limits of $\widetilde{a}$ respectively. The set of all cosed subintervals of $[0,1]$ is denoted by $D[0,1]$. We also identify the interval $[a, a]$ by the number $a \in [0,1]$. For any interval numbers $\widetilde{a}_i = [a_i^-, a_i^+], \widetilde{b}_i = [b_i^-, b_i^+] \in D[0,1], i \in I$ we define

$$\max{}^i\{\widetilde{a}_i, \widetilde{b}_i\} = [\max\{a_i^-, b_i^-\}, \max\{a_i^+, b_i^+\}],$$
$$\min{}^i\{\widetilde{a}_i, \widetilde{b}_i\} = [\min\{a_i^-, b_i^-\}, \min\{a_i^+, b_i^+\}],$$
$$\inf \widetilde{a}_i = \left[\bigcap_{i\in I} a_i^-, \bigcap_{i\in I} a_i^+\right], \sup \widetilde{a}_i = \left[\bigcup_{i\in I} a_i^-, \bigcup_{i\in I} a_i^+\right]$$

and let

(1) $\widetilde{a} \leq \widetilde{b} \Longleftrightarrow a^- \leq b^-$ and $a^+ \leq b^+$,
(2) $\widetilde{a} = \widetilde{b} \Longleftrightarrow a^- = b^-$ and $a^+ = b^+$,
(3) $\widetilde{a} < \widetilde{b} \Longleftrightarrow \widetilde{a} \leq \widetilde{b}$ and $\widetilde{a} \neq \widetilde{b}$,
(4) $k\widetilde{a} = [ka^-, ka^+]$, whenever $0 \leq k \leq 1$.

Definition 2.5 ([20])**.** Let X be a non-empty set. A mapping $\widetilde{\mu} : X \to D[0,1]$ is called an interval valued fuzzy subset of X. For any $x \in X$, $\widetilde{\mu}(x) = [\mu^-(x), \mu^+(x)]$, where μ^- and μ^+ are fuzzy subsets of X such that $\mu^-(x) \leq \mu^+(x)$. Thus $\widetilde{\mu}(x)$ is an interval(a closed subset of $[0,1]$) and not a number from the interval $[0,1]$ as in the case of a fuzzy set.

Let $\widetilde{\mu}, \widetilde{\nu}$ be interval valued fuzzy subsets of X. The following are defined by
(1) $\widetilde{\mu} \leq \widetilde{\nu} \Leftrightarrow \widetilde{\mu}(x) \leq \widetilde{\nu}(x)$. (2) $\widetilde{\mu} = \widetilde{\nu} \Leftrightarrow \widetilde{\mu}(x) = \widetilde{\nu}(x)$.
(3) $(\widetilde{\mu} \cup \widetilde{\nu}) = \max^i\{\widetilde{\mu}(x), \widetilde{\nu}(x)\}$. (4) $(\widetilde{\mu} \cap \widetilde{\nu}) = \min^i\{\widetilde{\mu}(x), \widetilde{\nu}(x)\}$.

Definition 2.6 ([20])**.** Let $\widetilde{\mu}$ be an interval valued fuzzy subset of X and $[t_1, t_2] \in D[0,1]$. Then the set $\widetilde{U}(\widetilde{\mu} : [t_1, t_2]) = \{x \in X \mid \widetilde{\mu}(x) \geq [t_1, t_2]\}$ is called the upper level set of $\widetilde{\mu}$.

Definition 2.7 ([20])**.** Let I be a subset of a near-ring R. Define a function $\widetilde{f_I} : R \to D[0,1]$ by

$$\widetilde{f_I}(x) = \begin{cases} \widetilde{1} & \text{if } x \in I \\ \widetilde{0} & \text{otherwise} \end{cases}$$

for all $x \in R$. Clearly $\widetilde{f_I}$ is an interval valued fuzzy subset of R and $\widetilde{f_I}$ is called the interval valued characteristic function of I.

3. Interval valued (α, β)-fuzzy ideals

We now extend the idea of quasi-coincident of fuzzy point with a fuzzy set to the concept of quasi-coincidence of a interval value fuzzy point with an interval valued fuzzy set as follows.

Definition 3.1. An interval valued fuzzy set $\widetilde{\mu}$ of a near-ring R of the form

$$\widetilde{\mu}(y) = \begin{cases} \widetilde{t} \neq [0,0], & \text{if } y = x, \\ [0,0], & \text{if}, y \neq x, \end{cases}$$

is said to be an interval value fuzzy point with support x and interval value $\widetilde{t}$ and is denoted by $x_{\widetilde{t}}$. An interval value fuzzy point $x_{\widetilde{t}}$ is said to belong to (resp. be quasi-coincidence with) an interval valued fuzzy set $\widetilde{\mu}$, written as $x_{\widetilde{t}} \in \widetilde{\mu}$ (resp. $x_{\widetilde{t}} q \widetilde{\mu}$) if $\widetilde{\mu}(x) \geq \widetilde{t}$ (resp. $\widetilde{\mu}(x) + \widetilde{t} > [1,1]$). If $x_{\widetilde{t}} \in \widetilde{\mu}$ or $x_{\widetilde{t}} q \widetilde{\mu}$, then we write $x_{\widetilde{t}} \in \vee q \widetilde{\mu}$ and if $x_{\widetilde{t}} \in \widetilde{\mu}$ and $x_{\widetilde{t}} q \widetilde{\mu}$, then we write $x_{\widetilde{t}} \in \wedge q \widetilde{\mu}$. The symbol $\overline{\in \vee q}$ means $\in \vee q$ does not hold.

Throughout this paper R will denote a left near-ring and α and β denote any one of $\{\in, q, \in \vee q, \in \wedge q\}$ unless otherwise specified. Also $\widetilde{\mu}(x) = [\mu^-(x), \mu^+(x)]$ satisfies the following conditions:

(1) Any two elements of $D[0,1]$ are comparable.

(2) $[\mu^-(x), \mu^+(x)] \geq [0.5, 0.5]$ or $[\mu^-(x), \mu^+(x)] < [0.5, 0.5]$, for all $x \in R$.

In this section, we present some fundamental concepts and characterizations of interval valued (α, β)-fuzzy ideals in which the central role is played by $(\alpha, \in \vee q)$-fuzzy ideals, especially $(\in, \in \vee q)$-fuzzy ideals.

We first extend the idea of fuzzy ideals to interval valued (α, β)-fuzzy ideals of near-rings.

Definition 3.2. An interval valued fuzzy set $\widetilde{\mu}$ of R is said to be an interval valued (α, β)- fuzzy subnear-ring of R with $\alpha \neq \in \wedge q$ if it satisfies the following conditions:

(1) $x_{\widetilde{t}} \alpha \widetilde{\mu}$ and $y_{\widetilde{r}} \alpha \widetilde{\mu}$ implies $(x+y)_{\min^i\{\widetilde{t}, \widetilde{r}\}} \beta \widetilde{\mu}$,

(2) $x_{\widetilde{t}} \alpha \widetilde{\mu}$ implies $(-x)_{\widetilde{t}} \beta \widetilde{\mu}$,

(3) $x_{\widetilde{t}} \alpha \widetilde{\mu}$ and $y_{\widetilde{r}} \alpha \widetilde{\mu}$ implies $(xy)_{\min^i\{\widetilde{t}, \widetilde{r}\}} \beta \widetilde{\mu}$, for all $t, r \in (0,1]$ and $x, y \in R$.

Definition 3.3. An interval valued fuzzy set $\widetilde{\mu}$ of R is said to be an interval valued (α, β)- fuzzy ideals of R with $\alpha \neq \in \wedge q$ if the following conditions hold:

(4) $\widetilde{\mu}$ is an interval valued (α, β)- fuzzy subnear-ring of R,

(5) $x_{\widetilde{t}} \alpha \widetilde{\mu}$ and $y \in R$ implies $(y + x - y)_{\widetilde{t}} \beta \widetilde{\mu}$

(6) $y_{\widetilde{t}} \alpha \widetilde{\mu}$ and $x \in R$ implies $(xy)_{\widetilde{t}} \beta \widetilde{\mu}$,

(7) $z_{\widetilde{t}} \alpha \widetilde{\mu}$ and $x, y \in R$ implies $((x+z)y - xy)_{\widetilde{t}} \beta \widetilde{\mu}$, for all $t, r, \in (0,1]$ and $x, y, z \in R$.

The conditions (1) and (2) in Definition 3.2 is equivalent to the following condition:

(1') $x_{\widetilde{t}} \alpha \widetilde{\mu}$, and $y_{\widetilde{r}} \alpha \widetilde{\mu}$ implies $(x-y)_{\min^i\{\widetilde{t}, \widetilde{r}\}} \beta \widetilde{\mu}$.

Let $\widetilde{\mu}$ be an interval valued fuzzy subset of R such that $\widetilde{\mu}(x) \leq [0.5, 0.5]$ for all $x \in R$. Suppose that $x \in R$ and $t \in (0,1]$ such that $x_{\widetilde{t}} \in \wedge q \widetilde{\mu}$. Then $\widetilde{\mu}(x) \geq \widetilde{t}$ and $\widetilde{\mu}(x) + \widetilde{t} > [1,1]$. It follows that $[1,1] < \widetilde{\mu}(x) + \widetilde{t} \leq \widetilde{\mu}(x) + \widetilde{\mu}(x) = 2\widetilde{\mu}(x)$. This means that $\widetilde{\mu}(x) > [0.5, 0.5]$, and so $\{x_{\widetilde{t}} \mid x_{\widetilde{t}} \in \wedge q \widetilde{\mu}\} = \varnothing$. Therefore the case $\alpha = \in \wedge q$ in Definitions 3.2 and 3.3 are omitted.

Example 3.4. Let $R = \{a, b, c, d\}$ be a set with two binary operations defined as follows:

$+$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

$\cdot$	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	b	c	d

Then $(R, +, \cdot)$ is a near-ring and $I = \{a, b\}$ is its ideal. Let $\widetilde{\mu} : R \longrightarrow D[0,1]$ be an interval valued fuzzy subset of R defined by $\widetilde{\mu}(a) = [0.8, 0.9], \widetilde{\mu}(b) = [0.6, 0.7]$ and $\widetilde{\mu}(c) = [0.5, 0.5] = \widetilde{\mu}(d)$. Then, clearly, $\widetilde{\mu}$ is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R. But

(1) $\widetilde{\mu}$ is not an interval valued $(\in, \in)$-fuzzy ideal of R, since $b_{[0.58,0.68]} \in \widetilde{\mu}$ but $((c+b)d - cd)_{[0.58,0.68]} = d_{[0.58,0.68]} \overline{\in}\, \widetilde{\mu}$.
(2) $\widetilde{\mu}$ is not an interval valued (q, q)-fuzzy ideal of R, since $a_{[0.2,0.3]} q \widetilde{\mu}$ and $b_{[0.48,0.58]} q \widetilde{\mu}$ but $(a - b)_{[0.2,0.3]} = b_{[0.2,0.3]} \overline{q}\, \widetilde{\mu}$.
(3) $\widetilde{\mu}$ is not an interval valued $(q, \in \wedge q)$-fuzzy ideal of R, since $a_{[0.2,0.3]} q \widetilde{\mu}$ and $c_{[0.58,0.59]} q \widetilde{\mu}$ but $(a - c)_{[0.2,0.3]} = d_{[0.2,0.3]} \overline{q}\, \widetilde{\mu}$.
(4) $\widetilde{\mu}$ is not an interval valued $(\in, \in \wedge q)$-fuzzy ideal of R, since $b_{[0.58,0.68]} \in \widetilde{\mu}$ and $c_{[0.48,0.49]} \in \widetilde{\mu}$ but $(b - c)_{[0.48,0.49]} = c_{[0.48,0.49]} \overline{\in \wedge q}\, \widetilde{\mu}$.
(5) $\widetilde{\mu}$ is not an interval valued $(\in \vee q, \in \wedge q)$-fuzzy ideal of R, since $b_{[0.58,0.68]} \in \vee q \widetilde{\mu}$ and $c_{[0.48,0.49]} \in \vee q \widetilde{\mu}$ but $(b-c)_{[0.48,0.49]} = c_{[0.48,0.49]} \overline{\in \wedge q}\, \widetilde{\mu}$.
(6) $\widetilde{\mu}$ is not an interval valued $(\in, q)$-fuzzy ideal of R, since $b_{[0.58,0.68]} \in \widetilde{\mu}$ and $c_{[0.48,0.49]} \in \widetilde{\mu}$ but $(b - c)_{[0.48,0.49]} = c_{[0.48,0.49]} \overline{q}\, \widetilde{\mu}$.
(7) $\widetilde{\mu}$ is not an interval valued $(q, \in)$-fuzzy ideal of R, since $b_{[0.58,0.68]} q \widetilde{\mu}$ and $c_{[0.52,0.54]} q \widetilde{\mu}$ but $(b - c)_{[0.52,0.54]} = c_{[0.52,0.54]} \overline{\in}\, \widetilde{\mu}$.
(8) $\widetilde{\mu}$ is not an interval valued $(\in \vee q, \in)$-fuzzy ideal of R, since $b_{[0.58,0.68]} \in \vee q \widetilde{\mu}$ and $c_{[0.52,0.54]} \in \vee q \widetilde{\mu}$ but $(b - c)_{[0.52,0.54]} = c_{[0.52,0.54]} \overline{\in}\, \widetilde{\mu}$.
(9) $\widetilde{\mu}$ is not an interval valued $(\in \vee q, q)$-fuzzy ideal of R, since $a_{[0.2,0.2]} \in \vee q \widetilde{\mu}$ and $b_{[0.3,0.4]} \in \vee q \widetilde{\mu}$ but $(a - b)_{[0.2,0.2]} = b_{[0.2,0.2]} \overline{q}\, \widetilde{\mu}$.

In the next theorem, using an interval valued (α, β)-fuzzy ideal of R, we present a method of constructing an ideal of R.

Theorem 3.5. *Let $\widetilde{\mu}$ be an interval valued (α, β)-fuzzy ideal of R. Then the set $S_{\widetilde{\mu}} = \{x \in R \,|\, \widetilde{\mu}(x) > [0, 0]\}$ is an ideal of R.*

Proof. $S_{\widetilde{\mu}} = \{x \in R \mid \widetilde{\mu}(x) > [0, 0]\}$. Let $x, y \in S_{\widetilde{\mu}}$ be such that $\widetilde{\mu}(x) > [0, 0]$ and $\widetilde{\mu}(y) > [0, 0]$. Let $\widetilde{\mu}(x - y) = [0, 0]$. If $\alpha \in \{\in, \in \vee q\}$, then $x_{\widetilde{\mu}(x)} \alpha \widetilde{\mu}$ and $y_{\widetilde{\mu}(y)} \alpha \widetilde{\mu}$ but $\widetilde{\mu}(x - y) = [0, 0] < \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}$ and $\widetilde{\mu}(x - y) + \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \leq [0, 0] + [1, 1] = [1, 1]$. So, $(x - y)_{\min^i\{\widetilde{\mu}(x),\widetilde{\mu}(y)\}} \overline{\beta} \widetilde{\mu}$ for every $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$, which is a contradiction. Hence $\widetilde{\mu}(x - y) > [0, 0]$, that is, $x - y \in S_{\widetilde{\mu}}$. Also, $x_{[1,1]} q \widetilde{\mu}$ and $y_{[1,1]} q \widetilde{\mu}$ but $(x - y)_{[1,1]} \overline{\beta} \widetilde{\mu}$ for every $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$, a contradiction. Hence $\widetilde{\mu}(x - y) > [0, 0]$, that is, $x - y \in S_{\widetilde{\mu}}$. Now, let $x \in S_{\widetilde{\mu}},\ y \in R$ implies $\widetilde{\mu}(x) > [0, 0]$ and we assume that $\widetilde{\mu}(y + x - y) = [0, 0]$. If $\alpha \in \{\in, \in \vee q\}$ then $x_{\widetilde{\mu}(x)} \alpha \widetilde{\mu}$ but $(y + x - y)_{\widetilde{\mu}(x)} \overline{\beta} \widetilde{\mu}$, for every $\beta \in \{\in, q, \in \wedge q, \in \vee q\}$, a contradiction, this means that $(y+x-y) \in S_{\widetilde{\mu}}$. Also, $x_{[1,1]} q \widetilde{\mu}$ but $(y+x-y)_{[1,1]} \overline{\beta} \widetilde{\mu}$ for every $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$. This leads to a contradiction and so $\widetilde{\mu}(y + x - y) > [0, 0]$, that is, $y + x - y \in S_{\widetilde{\mu}}$. Again, let $y \in S_{\widetilde{\mu}},\ x \in R$ implies $\widetilde{\mu}(y) > [0, 0]$. Let $\widetilde{\mu}(xy) = [0, 0]$. If $\alpha \in \{\in, \in \vee q\}$ then $y_{\widetilde{\mu}(y)} \alpha \widetilde{\mu}$ but $(xy)_{\widetilde{\mu}(y)} \overline{\beta} \widetilde{\mu}$ for every $\beta \in \{\in, q, \in \wedge q, \in \vee q\}$, a contradiction, this implies that $xy \in S_{\widetilde{\mu}}$. Also, $y_{[1,1]} q \widetilde{\mu}$ but $(xy)_{[1,1]} \overline{\beta} \widetilde{\mu}$ for every $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$. This leads to a contradiction and so $\mu(xy) > [0, 0]$, that is, $xy \in S_{\widetilde{\mu}}$. Let $z \in S_{\widetilde{\mu}}$ and $x, y \in R$. Then $\widetilde{\mu}(z) > [0, 0]$. Suppose that $\widetilde{\mu}((x + z)y - xy) = [0, 0]$. If $\alpha \in \{\in, \in \vee q\}$, then $z_{\widetilde{\mu}(z)} \alpha \widetilde{\mu}$, but $((x + z)y - xy)_{\mu(z)} \overline{\beta} \widetilde{\mu}$, for every $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$, a contradiction. Thus $\widetilde{\mu}((x + z)y - xy) > [0, 0]$, that is $((x + z)y - xy) \in S_{\widetilde{\mu}}$. Also,

$z_{[1,1]}q\widetilde{\mu}$ but $((x+z)y-xy)_{[1,1]}\overline{\beta}\widetilde{\mu}$ for every $\beta \in \{\in, q, \in\vee q, \in\wedge q\}$, a contradiction. Thus $\widetilde{\mu}((x+z)y-xy) > [0,0]$, implies, $(x+z)y-xy \in S_{\widetilde{\mu}}$. This shows that $S_{\widetilde{\mu}}$ is an ideal of R. □

Theorem 3.6. *If I is an ideal of R, then an interval valued fuzzy subset $\widetilde{\mu}$ of R such that*

$$\widetilde{\mu}(x) = \begin{cases} \geq [0.5, 0.5] & \text{if } x \in I \\ [0,0] & \text{otherwise} \end{cases}$$

is an interval valued $(\alpha, \in\vee q)$-fuzzy ideal of R.

Proof. (a) Let $x, y \in R$ and $\widetilde{t}, \widetilde{r} \in D[0,1]$ with $\widetilde{t}, \widetilde{r} \neq [0,0]$ be such that $x_{\widetilde{t}} \in \widetilde{\mu}$ and $y_{\widetilde{r}} \in \widetilde{\mu}$. Then $\widetilde{\mu}(x) \geq \widetilde{t}$ and $\widetilde{\mu}(y) \geq \widetilde{r}$. Thus $x, y \in I$ and so $x - y \in I$, that is, $\widetilde{\mu}(x-y) \geq [0.5, 0.5]$. If $\min^i\{\widetilde{t}, \widetilde{r}\} \leq [0.5, 0.5]$, then $\widetilde{\mu}(x-y) \geq [0.5, 0.5] \geq \min^i\{\widetilde{t}, \widetilde{r}\}$. Hence $(x-y)_{\min^i\{\widetilde{t},\widetilde{r}\}} \in \widetilde{\mu}$. If $\min^i\{\widetilde{t}, \widetilde{r}\} > [0.5, 0.5]$, then $\widetilde{\mu}(x-y) + \min^i\{\widetilde{t}, \widetilde{r}\} > [0.5, 0.5] + [0.5, 0.5] = [1,1]$ and so $(x-y)_{\min^i\{\widetilde{t},\widetilde{r}\}} q\widetilde{\mu}$. Therefore $(x-y)_{\min^i\{\widetilde{t},\widetilde{r}\}} \in\vee q\widetilde{\mu}$. Now, let $x, y \in R$ and $\widetilde{t} \in D[0,1]$ with $\widetilde{t} \neq [0,0]$ be such that $x_{\widetilde{t}} \in \widetilde{\mu}$. Then $\widetilde{\mu}(x) \geq \widetilde{t}$, which implies $x \in I$ and so $y + x - y \in I$. Consequently $\widetilde{\mu}(y + x - y) \geq [0.5, 0.5]$. If $\widetilde{t} \leq [0.5, 0.5]$, then $\widetilde{\mu}(y+x-y) \geq [0.5, 0.5] \geq \widetilde{t}$. Hence $(y+x-y)_{\widetilde{t}} \in \widetilde{\mu}$. If $\widetilde{t} > [0.5, 0.5]$, then $\widetilde{\mu}(y + x - y) + \widetilde{t} > [0.5, 0.5] + [0.5, 0.5] = [1,1]$ and so $(y + x - y)_{\widetilde{t}} q\widetilde{\mu}$. Thus $(y + x - y)_{\widetilde{t}} \in \vee q\widetilde{\mu}$. Also, let $x, y \in R$ and $\widetilde{t} \in D[0,1]$ with $\widetilde{t} \neq [0,0]$ be such that $y_{\widetilde{t}} \in \widetilde{\mu}$. Then $\widetilde{\mu}(y) \geq \widetilde{t}$. Thus $y \in I$ and so $xy \in I$, that is $\widetilde{\mu}(xy) \geq [0.5, 0.5]$. If $\widetilde{t} \leq [0.5, 0.5]$, then $\widetilde{\mu}(xy) \geq [0.5, 0.5] \geq \widetilde{t}$. Hence $(xy)_{\widetilde{t}} \in \widetilde{\mu}$. If $\widetilde{t} > [0.5, 0.5]$, then $\widetilde{\mu}(xy) + \widetilde{t} > [0.5, 0.5] + [0.5, 0.5] = [1,1]$ and so $(xy)_{\widetilde{t}} q\widetilde{\mu}$. This implies that $(xy)_{\widetilde{t}} \in \vee q\widetilde{\mu}$. Similarly, we can prove that $((x+z)y-xy)_{\widetilde{t}} \in \vee q\widetilde{\mu}$. Therefore $\widetilde{\mu}$ is an interval valued $(\in, \in\vee q)$-fuzzy ideal of R.

(b) Let $x, y \in R$ and $\widetilde{t}, \widetilde{r} \in D[0,1]$ with $\widetilde{t}, \widetilde{r} \neq [0,0]$ be such that $x_{\widetilde{t}} q\widetilde{\mu}$ and $y_{\widetilde{t}} q\widetilde{\mu}$. Then $x, y \in I, \widetilde{\mu}(x) + \widetilde{t} > [1,1]$ and $\widetilde{\mu}(y) + \widetilde{r} > [1,1]$. Since $x - y \in I$, we have $\widetilde{\mu}(x-y) \geq [0.5, 0.5]$. If $\min^i\{\widetilde{t}, \widetilde{r}\} \leq [0.5, 0.5]$, then $\widetilde{\mu}(x-y) \geq [0.5, 0.5] \geq \min^i\{\widetilde{t}, \widetilde{r}\}$. Hence $(x-y)_{\min^i\{\widetilde{t},\widetilde{r}\}} \in \widetilde{\mu}$. If $\min^i\{\widetilde{t}, \widetilde{r}\} > [0.5, 0.5]$, then $\widetilde{\mu}(x-y) + \min^i\{\widetilde{t}, \widetilde{r}\} > [0.5, 0.5] + [0.5, 0.5] = [1,1]$ and so $(x-y)_{\min^i\{\widetilde{t},\widetilde{r}\}} q\widetilde{\mu}$. Thus $(x-y)_{\min^i\{\widetilde{t},\widetilde{r}\}} \in \vee q\widetilde{\mu}$. Now let $x, y \in R$ and $\widetilde{t} \in D[0,1]$ with $\widetilde{t} \neq [0,0]$ be such that $x_{\widetilde{t}} q\widetilde{\mu}$. This means that $\widetilde{\mu}(x) + \widetilde{t} > [1,1]$. Thus $x \in I$ and so $y + x - y \in I$. This implies that $\widetilde{\mu}(y + x - y) \geq [0.5, 0.5]$. If $\widetilde{t} \leq [0.5, 0.5]$, then $\widetilde{\mu}(y+x-y) \geq [0.5, 0.5] \geq t$. Hence $(y+x-y)_{\widetilde{t}} \in \widetilde{\mu}$. If $\widetilde{t} > [0.5, 0.5]$, then $\widetilde{\mu}(y+x-y) + \widetilde{t} > [0.5, 0.5] + [0.5, 0.5] = [1,1]$ and so $(y+x-y)_{\widetilde{t}} q\widetilde{\mu}$. Thus $(y + x - y)_{\widetilde{t}} \in \vee q\widetilde{\mu}$. Let $x, y \in R$ and $\widetilde{t} \in D[0,1]$ with $\widetilde{t} \neq [0,0]$ be such that $y_{\widetilde{t}} q\widetilde{\mu}$ implies $\widetilde{\mu}(y) + \widetilde{t} > [1,1]$. Then $y \in I$ and so $xy \in I$. This implies that $\widetilde{\mu}(xy) \geq [0.5, 0.5]$. If $\widetilde{t} \leq [0.5, 0.5]$, then $\widetilde{\mu}(xy) \geq [0.5, 0.5] \geq t$. Hence $(xy)_{\widetilde{t}} \in \widetilde{\mu}$. If $\widetilde{t} > [0.5, 0.5]$, then $\widetilde{\mu}(xy) + \widetilde{t} > [0.5, 0.5] + [0.5, 0.5] = [1,1]$ and so $(xy)_{\widetilde{t}} q\widetilde{\mu}$. Hence $(xy)_{\widetilde{t}} \in \vee q\widetilde{\mu}$. Let $x, y, z \in R$ and $\widetilde{t} \in D[0,1]$ with $\widetilde{t} \neq [0,0]$ be such that $z_{\widetilde{t}} q\widetilde{\mu}$. Then $\widetilde{\mu}(z) + \widetilde{t} > [1,1]$ and it follows that $z \in I$. Then $(x+z)y - xy \in I$ and so $\widetilde{\mu}((x+z)y-xy) \geq [0.5, 0.5]$. If $\widetilde{t} \leq [0.5, 0.5]$, then $\widetilde{\mu}((x+z)y-xy) \geq [0.5, 0.5] \geq \widetilde{t}$. Hence $((x+z)y-xy)_{\widetilde{t}} \in \widetilde{\mu}$. If $\widetilde{t} > [0.5, 0.5]$, then $\widetilde{\mu}((x+z)y-xy) + \widetilde{t} > [0.5, 0.5] + [0.5, 0.5] = [1,1]$ and so $((x+z)y-xy)_{\widetilde{t}} q\widetilde{\mu}$. Thus $((x+z)y-xy)_{\widetilde{t}} \in \vee q\widetilde{\mu}$. Hence $\widetilde{\mu}$ is an interval valued $(q, \in\vee q)$-fuzzy ideal of R.

(c) Similar consequence of (a) and (b), we have to prove that $\widetilde{\mu}$ is an interval valued $(\in\vee q, \in\vee q)$-fuzzy ideal of R. □

Remark 3.7. The following example proves that every interval valued fuzzy set $\widetilde{\mu}$ defined in Theorem 3.6 is an interval valued $(\alpha, \in\vee q)$-fuzzy ideal of R but $\widetilde{\mu}$ is not an interval valued (α, β)-fuzzy ideal of R, for every $\beta \in \{\in, q, \in\vee q, \in\wedge q\}$.

Example 3.8. Let $R = \{a, b, c, d\}$ be a set with two binary operations defined as follows:

$+$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

$\cdot$	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	a	b	b

Then $(R, +, \cdot)$ is a near-ring and $I = \{a, b\}$ is its ideal. Let $\widetilde{\mu} : R \longrightarrow D[0, 1]$ be an interval valued fuzzy subset of R defined by $\widetilde{\mu}(a) = [0.6, 0.7], \widetilde{\mu}(b) = [0.5, 0.6]$ and $\widetilde{\mu}(c) = [0, 0] = \widetilde{\mu}(d)$. Then, clearly, $\widetilde{\mu}$ is an interval valued $(\in, \in\vee q)$-fuzzy ideal of R. Since, $a_{[0.26,0.28]} \in \widetilde{\mu}$. Then, $(a - a)_{[0.26,0.28]} = a_{[0.26,0.28]} \in \vee q\widetilde{\mu}$ but $(a-a)_{[0.26,0.28]} = a_{[0.26,0.28]}\overline{q}\,\widetilde{\mu}$, which implies that $(a-a)_{[0.26,0.28]} = a_{[0.26,0.28]}\overline{\in\wedge q}\,\widetilde{\mu}$. Then, $\widetilde{\mu}$ is not an $(\alpha, \in\wedge q)$-fuzzy ideal of R.

4. Interval valued $(\in, \in\vee q)$-fuzzy ideal of near-rings

In this section, we introduce the notion of interval valued $(\in, \in\vee q)$-fuzzy ideal of near-ring and investigate some of its properties.

Definition 4.1 ([21])**.** An interval valued fuzzy subset μ of a near-ring R is said to be an i-v $(\in, \in\vee q)$-fuzzy subnear-ring of R if for all $x, y \in R$ and $t, r \in (0, 1]$:

(1) $x_{\widetilde{t}} \in \widetilde{\mu}$ and $y_{\widetilde{r}} \in \widetilde{\mu}$ implies $(x + y)_{\min^i\{\widetilde{t},\widetilde{r}\}} \in \vee q\widetilde{\mu}$,
(2) $x_{\widetilde{t}} \in \widetilde{\mu}$ implies $(-x)_{\widetilde{t}} \in \vee q\widetilde{\mu}$,
(3) $x_{\widetilde{t}} \in \widetilde{\mu}$ and $y_{\widetilde{r}} \in \widetilde{\mu}$ implies $(xy)_{\min^i\{\widetilde{t},\widetilde{r}\}} \in \vee q\widetilde{\mu}$,

The conditions (1) and (2) in Definition 4.1 is equivalent to
(1') $x_{\widetilde{t}}, y_{\widetilde{r}} \in \widetilde{\mu}$ implies $(x - y)_{\min^i\{\widetilde{t},\widetilde{r}\}} \in \vee q\widetilde{\mu}$.

Definition 4.2. An interval valued fuzzy subset $\widetilde{\mu}$ of R is said to be an interval valued $(\in, \in\vee q)$-fuzzy ideal of R if it satisfies the following conditions for all $t, r \in (0, 1]$ and $x, y, z \in R$,
(1) $\widetilde{\mu}$ is an interval valued $(\in, \in\vee q)$-fuzzy subnear-ring of R,
(2) $x_{\widetilde{t}} \in \widetilde{\mu}$ and $y \in R$ implies $(y + x - y)_{\widetilde{t}} \in \vee q\widetilde{\mu}$,
(3) $y_{\widetilde{t}} \in \widetilde{\mu}$ and $x \in R$ implies $(xy)_{\widetilde{t}} \in \vee q\widetilde{\mu}$.
(4) $z_{\widetilde{t}} \in \widetilde{\mu}$ and $x, y \in R$ implies $((x + z)y - xy)_{\widetilde{t}} \in \vee q\widetilde{\mu}$.

Theorem 4.3 ([21])**.** *An interval valued fuzzy subset $\widetilde{\mu}$ of R is an interval valued $(\in, \in\vee q)$-fuzzy subnear-ring of R if and only if*

(1) $\widetilde{\mu}(x - y) \geq \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\}$,
(2) $\widetilde{\mu}(xy) \geq \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\}$, *for all* $x, y \in R$.

Lemma 4.4. *Let $\widetilde{\mu}$ be an interval valued fuzzy subset of R and $\widetilde{t}, \widetilde{r} \in D[0, 1]$ with $\widetilde{t}, \widetilde{r} \neq [0, 0]$. Then*

(1) *(a) μ is an $(\in, \in \vee q)$-fuzzy subnear-ring of R and*
(b) $\widetilde{\mu}(x-y) \geq \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\}, \widetilde{\mu}(xy) \geq \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\}$ for all $x, y \in R$ are equivalent.
(2) *(c) $x_{\widetilde{t}} \in \widetilde{\mu}$ and $y \in R$ implies $(y + x - y)_{\widetilde{t}} \in \vee q \widetilde{\mu}$, and*
(d) $\widetilde{\mu}(y + x - y) \geq \min^i\{\widetilde{\mu}(x), [0.5, 0.5]\}$, for all $x, y \in R$ are equivalent.
(3) *(e) $y_{\widetilde{t}} \in \widetilde{\mu}$ and $x \in R$ implies $(xy)_{\widetilde{t}} \in \vee q \widetilde{\mu}$ and*
(f) $\widetilde{\mu}(xy) \geq \min^i\{\widetilde{\mu}(y), [0.5, 0.5]\}$, for all $x, y \in R$ are equivalent.
(4) *(g) $z_{\widetilde{t}} \in \widetilde{\mu}$ and $x, y \in R$ implies $((x + z)y - xy)_{\widetilde{t}} \in \vee q \widetilde{\mu}$ and*
(h) $\widetilde{\mu}((x+z)y-xy) \geq \min^i\{\widetilde{\mu}(z), [0.5, 0.5]\}$. for all $x, y, z \in R$ are equivalent.

Proof. Let $\widetilde{\mu}$ be an interval fuzzy subset of R.
(1) $(a) \iff (b)$, Theorem 4.3.
(2) $(c) \implies (d)$: Suppose that (d) is not valid, then there exists $x, y \in R$ such that $\widetilde{\mu}(y + x - y) < \min^i\{\widetilde{\mu}(x), [0.5, 0.5]\}$. Now, we aries the following two cases: (i) $\widetilde{\mu}(x) \leq [0.5, 0.5]$ (ii) $\widetilde{\mu}(x) > [0.5, 0.5]$.
Case (i): We have $\widetilde{\mu}(y+x-y) < \widetilde{\mu}(x)$. Choose an interval $\widetilde{t}$ such that $\widetilde{\mu}(y+x-y) < \widetilde{t} < \widetilde{\mu}(x)$. This implies $x_{\widetilde{t}} \in \widetilde{\mu}$ and $(y + x - y)_{\widetilde{t}} \overline{\in \vee q}\, \widetilde{\mu}$, which contradicts (c). So, $\widetilde{\mu}(y + x - y) \geq \widetilde{\mu}(x) = \min^i\{\widetilde{\mu}(x), [0.5, 0.5]\}$.
Case (ii): We have $\widetilde{\mu}(y + x - y) \leq [0.5, 0.5]$. Then $x_{[0.5,0.5]} \in \widetilde{\mu}$ and $(y + x - y)_{[0.5,0.5]} \overline{\in \vee q}\, \widetilde{\mu}$, which is a contradiction to (c). Hence $\widetilde{\mu}(y + x - y) \geq [0.5, 0.5] = \min^i\{\widetilde{\mu}(x), [0.5, 0.5]\}$.
$(d) \implies (c)$: Let $x_{\widetilde{t}} \in \widetilde{\mu}$ and $y \in R$. Then $\widetilde{\mu}(x) \geq \widetilde{t}$. Now (d), we have $\widetilde{\mu}(y+x-y) \geq \min^i\{\mu(x), [0.5, 0.5]\} \geq \min^i\{\widetilde{t}, [0.5, 0.5]\}$. If $\widetilde{t} \leq [0.5, 0.5]$ then $\widetilde{\mu}(y + x - y) \geq \widetilde{t}$ and so $(y+x-y)_{\widetilde{t}} \in \widetilde{\mu}$. If $\widetilde{t} > [0.5, 0.5]$, then $\widetilde{\mu}(y+x-y)+\widetilde{t} > [1, 1]$ and so $(y+x-y)_{\widetilde{t}} q \widetilde{\mu}$. This implies that $(y + x - y)_{\widetilde{t}} \in \vee q \widetilde{\mu}$.
(3) $(e) \implies (f)$: Let us assume that (f) is not valid. Then $x, y \in R$, we can write $\widetilde{\mu}(xy) < \min^i\{\widetilde{\mu}(y), [0.5, 0.5]\}$. We consider the following two cases: (i) $\widetilde{\mu}(y) \leq [0.5, 0.5]$ (ii) $\widetilde{\mu}(y) > [0.5, 0.5]$.
Case (i): We have $\widetilde{\mu}(xy) < \widetilde{\mu}(y)$. Choose $\widetilde{t}$ such that $\widetilde{\mu}(xy) < \widetilde{t} < \widetilde{\mu}(y)$. Then $y_{\widetilde{t}} \in \widetilde{\mu}$, but $(xy)_{\widetilde{t}} \overline{\in \vee q}\, \widetilde{\mu}$, which contradicts (e).
Case (ii): We have $\widetilde{\mu}(xy) < [0.5, 0.5] \leq \widetilde{\mu}(y)$. This implies that $y_{[0.5,0.5]} \in \widetilde{\mu}$, but $(xy)_{[0.5,0.5]} \overline{\in \vee q}\, \widetilde{\mu}$, which contradicts (e). Therefore $(xy)_{\widetilde{t}} \overline{\in \vee q}\, \widetilde{\mu}$.
$(f) \implies (e)$: Let $y_{\widetilde{t}} \in \widetilde{\mu}$ and $x \in R$ be such that $\widetilde{\mu}(y) \geq \widetilde{t}$. We have $\mu(xy) \geq \min^i\{\widetilde{\mu}(y), [0.5, 0.5]\} \geq \min^i\{\widetilde{t}, [0.5, 0.5]\}$, which implies that $\widetilde{\mu}(xy) \geq \widetilde{t}$ or $\widetilde{\mu}(xy) \geq [0.5, 0.5]$ according to $\widetilde{t} \leq [0.5, 0.5]$ or $\widetilde{t} > [0.5, 0.5]$. Therefore $(xy)_{\widetilde{t}} \in \vee q \widetilde{\mu}$.
Similarly, we can prove $(4)(g) \implies (h)$ and $(h) \implies (g)$. This completes the proof. □

By Definition 4.2 and Lemma 4.4, we obtain the following theorem.

Theorem 4.5. *An interval valued fuzzy subset $\widetilde{\mu}$ of R is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R if and only if*

(1) *$\widetilde{\mu}$ is an interval valued $(\in, \in \vee q)$-fuzzy subnear-ring of R,*
(2) $\widetilde{\mu}(y + x - y) \geq \min^i\{\mu(x), [0.5, 0.5]\}$,
(3) $\widetilde{\mu}(xy) \geq \min^i\{\widetilde{\mu}(y), [0.5, 0.5]\}$,
(4) $\widetilde{\mu}((x + z)y - xy) \geq \min^i\{\widetilde{\mu}(z), [0.5, 0.5]\}$, *for all $x, y, z \in R$.*

In the following theorem, we explain the construction of an interval valued generalized fuzzy ideal form an ideal.

Theorem 4.6. *Let I be an ideal of R. For every $\widetilde{t} \in D[0, 0.5]$ with $\widetilde{t} \neq [0, 0]$ there exists an interval valued $(\in, \in \vee q)$-fuzzy ideal $\widetilde{\mu}$ of R such that $\widetilde{U}(\widetilde{\mu} : \widetilde{t}) = I$.*

Proof. Let $\widetilde{\mu}$ be an interval valued fuzzy subset in R defined by

$$\widetilde{\mu}(x) = \begin{cases} \widetilde{t} & \text{if } x \in I \\ [0, 0] & \text{otherwise} \end{cases}$$

for all $x \in R$, where $\widetilde{t} \in D[0, 0.5]$ with $\widetilde{t} \neq [0, 0]$. Obviously, $\widetilde{U}(\widetilde{\mu} : \widetilde{t}) = I$. Assume that $\widetilde{\mu}(x - y) < \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\}$, for some $x, y \in R$. Since $|Im(\widetilde{\mu})| = 2$, it follows that $\widetilde{\mu}(x - y) = [0, 0]$ and $\min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\} = \widetilde{t}$. Hence $\widetilde{\mu}(x) = \widetilde{t} = \widetilde{\mu}(y)$ and so $x, y \in I$. Thus $x-y \in I$, since I is an ideal of R and so $\widetilde{\mu}(x-y) = \widetilde{t}$, which is a contradiction. Therefore $\widetilde{\mu}(x - y) \geq \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\}$. Let us suppose that $\widetilde{\mu}(y+x-y) < \min^i\{\widetilde{\mu}(x), [0.5, 0.5]\}$, for some $x, y \in R$. It follows that $\widetilde{\mu}(y+x-y) = [0, 0]$ and $\min^i\{\widetilde{\mu}(x), [0.5, 0.5]\} = \widetilde{t}$. Hence $\widetilde{\mu}(x) = \widetilde{t}$ and so $x \in I$. Since I is an ideal of R, then $y+x-y \in I$. Thus $\widetilde{\mu}(y+x-y) = \widetilde{t}$, which is a contradiction and hence $\widetilde{\mu}(y+x-y) \geq \min^i\{\widetilde{\mu}(x), [0.5, 0.5]\}$. Assume that $\widetilde{\mu}(xy) < \min^i\{\widetilde{\mu}(y), [0.5, 0.5]\}$, for some $x, y \in R$. Then $\widetilde{\mu}(xy) = [0, 0]$ and $\min^i\{\widetilde{\mu}(y), [0.5, 0.5]\} = \widetilde{t}$. Hence $\widetilde{\mu}(y) = \widetilde{t}$ and so $y \in I$. Since I is an ideal of R, then $xy \in I$. Thus $\widetilde{\mu}(xy) = \widetilde{t}$, which is a contradiction and therefore $\widetilde{\mu}(xy) \geq \min^i\{\widetilde{\mu}(y), [0.5, 0.5]\}$. Similarly, the same procedure we have $\widetilde{\mu}((x + z)y - xy) \geq \min^i\{\widetilde{\mu}(z), [0.5, 0.5]\}$. □

The next theorem brings out the relationship between interval valued $(\in, \in \vee q)$-fuzzy ideals of R and the crisp ideals of R.

Theorem 4.7. *A nonempty subset I of R is an ideal of R if and only if $\widetilde{f_I}$ is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R.*

Proof. Let I be an ideal of R. Then $\widetilde{f_I}$ is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R by Theorem 4.6.

Conversely, assume that $\widetilde{f_I}$ is an interval valued $(\in, \in \vee q)$ fuzzy ideal of R. Then clearly, $\widetilde{f_I}(x-y) \geq \min^i\{\widetilde{f_I}(x), \widetilde{f_I}(y), [0.5, 0.5]\} = \min\{[1, 1], [0.5, 0.5]\} = [0.5, 0.5] \neq [0, 0]$, which implies $\widetilde{f_I}(x - y) = [1, 1]$ and so $x - y \in I$. Let $x \in I$ and $y \in R$. Then, $\widetilde{f_I}(y + x - y) \geq \min^i\{\widetilde{f_I}(x), [0.5, 0.5]\} = \min^i\{[1, 1], [0.5, 0.5]\} = [0.5, 0.5] \neq [0, 0]$. This implies that $\widetilde{f_I}(y+x-y) = [1, 1]$ and so $y+x-y \in I$. Let $y \in I$ and $x \in R$ be such that $f_I(y) = [1, 1]$. Then, $\widetilde{f_I}(xy) \geq \min^i\{\widetilde{f_I}(y), [0.5, 0.5]\} = [0.5, 0.5] \neq [0, 0]$. This implies that $\widetilde{f_I}(xy) = [1, 1]$ and so $xy \in I$. Similarly, we proceed like this $(x + z)y - xy \in I$. □

Now, we characterize the interval valued $(\in, \in \vee q)$-fuzzy ideals using their level ideals.

Theorem 4.8. *An interval valued fuzzy subset $\widetilde{\mu}$ of R is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R if and only if the level subset $\widetilde{U}(\widetilde{\mu} : \widetilde{t})$ is an ideal of R for all $[0, 0] < \widetilde{t} \leq [0.5, 0.5]$.*

Proof. Let $\widetilde{\mu}$ be an interval valued $(\in, \in \vee q)$-fuzzy ideal of R and $[0,0] < \widetilde{t} \leq [0.5, 0.5]$. Let $x, y \in \widetilde{U}(\widetilde{\mu} : \widetilde{t})$ then $\widetilde{\mu}(x) \geq \widetilde{t}$ and $\widetilde{\mu}(y) \geq \widetilde{t}$. Now by Theorem 4.5, we have $\widetilde{\mu}(x-y) \geq \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\} \geq \min^i\{\widetilde{t}, \widetilde{t}, [0.5, 0.5]\} = \widetilde{t}$. So $x - y \in \widetilde{U}(\widetilde{\mu} : \widetilde{t})$. If $x \in \widetilde{U}(\widetilde{\mu} : \widetilde{t})$ and $y \in R$. Then $\widetilde{\mu}(x) \geq \widetilde{t}$. Consequently by Theorem 4.5, we have $\widetilde{\mu}(y + x - y) \geq \min^i\{\widetilde{\mu}(x), [0.5, 0.5]\} \geq \min^i\{\widetilde{t}, [0.5, 0.5]\} = \widetilde{t}$. So $y + x - y \in \widetilde{U}(\widetilde{\mu} : \widetilde{t})$. Let $y \in \widetilde{U}(\widetilde{\mu} : \widetilde{t})$ and $x \in R$. Then $\widetilde{\mu}(y) \geq \widetilde{t}$. Since $\widetilde{\mu}$ is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R, we have $\widetilde{\mu}(xy) \geq \min^i\{\widetilde{\mu}(y), [0.5, 0.5]\} \geq \min^i\{\widetilde{t}, [0.5, 0.5]\} = \widetilde{t}$. Thus $xy \in \widetilde{U}(\widetilde{\mu}, \widetilde{t})$ and so $\widetilde{U}(\widetilde{\mu} : \widetilde{t})$ is a left ideal of R. Also, for every $z \in \widetilde{U}(\widetilde{\mu} : \widetilde{t})$ and $x, y \in R$ such that $\widetilde{\mu}(z) \geq \widetilde{t}$. Then $\widetilde{\mu}((x+z)y - xy) \geq \min^i\{\widetilde{\mu}(z), [0.5, 0.5]\} \geq \min^i\{\widetilde{t}, [0.5, 0.5]\} = \widetilde{t}$ and so $(x+z)y - xy \in \widetilde{U}(\widetilde{\mu} : \widetilde{t})$. Therefore $\widetilde{U}(\widetilde{\mu} : \widetilde{t})$ is an ideal of R.

Conversely, assume that $\widetilde{\mu}$ is an interval valued fuzzy subset of R such that $\widetilde{U}(\widetilde{\mu} : \widetilde{t})(\neq \varnothing)$ become an ideal of R, for all $[0,0] < \widetilde{t} \leq [0.5, 0.5]$. Let $x, y \in R$. Suppose that $\widetilde{\mu}(x-y) < \min^i\{\mu(x), \mu(y), [0.5, 0.5]\}$. Then we can choose $\widetilde{t}$ such that $\widetilde{\mu}(x-y) < \widetilde{t} < \min^i\{\mu(x), \mu(y), [0.5, 0.5]\}$. This implies that $x, y \in \widetilde{U}(\widetilde{\mu} : \widetilde{t})$. Since $\widetilde{U}(\widetilde{\mu} : \widetilde{t})$ is an additive subgroup of R, then $(x - y) \in \widetilde{U}(\widetilde{\mu} : \widetilde{t})$ and so $\widetilde{\mu}(x-y) \geq \widetilde{t}$, which is a contradiction. Thus $\widetilde{\mu}(x - y) \geq \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\}$. Let us assume that $\widetilde{\mu}(y + x - y) < \min^i\{\widetilde{\mu}(x), [0.5, 0.5]\}$. Choose $\widetilde{t}$ such that $\widetilde{\mu}(y + x - y) < \widetilde{t} < \min^i\{\widetilde{\mu}(x), [0.5, 0.5]\}$. Then $x \in \widetilde{U}(\widetilde{\mu} : \widetilde{t})$ and so $y + x - y \in \widetilde{U}(\widetilde{\mu} : \widetilde{t})$, since $\widetilde{U}(\widetilde{\mu} : \widetilde{t})$ is a ideal of R. This implies that $\widetilde{\mu}(y + x - y) \geq \widetilde{t}$, which contradicts to our hypothesis. Hence $\widetilde{\mu}(y + x - y) \geq \min^i\{\widetilde{\mu}(x), [0.5, 0.5]\}$. Suppose that $\mu(xy) < \min^i\{\widetilde{\mu}(y), [0.5, 0.5]\}$, for all $x, y \in R$. Then there exist $\widetilde{t}$ such that $\widetilde{\mu}(xy) < \widetilde{t} < \min^i\{\widetilde{\mu}(y), [0.5, 0.5]\}$. Thus $y \in \widetilde{U}(\widetilde{\mu} : \widetilde{t})$ and so $xy \in \widetilde{U}(\widetilde{\mu} : \widetilde{t})$, since $\widetilde{U}(\widetilde{\mu} : \widetilde{t})$ is an ideal of R. Hence $\widetilde{\mu}(xy) \geq \widetilde{t}$, which contradicts to our hypothesis. Hence $\widetilde{\mu}(xy) \geq \min^i\{\widetilde{\mu}(y), [0.5, 0.5]\}$. Similarly, we can prove that $\widetilde{\mu}((x+z)y - xy) \geq \min^i\{\widetilde{\mu}(z), [0.5, 0.5]\}$. Therefore $\widetilde{\mu}$ is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R. □

Next, we discuss the relationship between these generalized interval valued fuzzy ideals.

Theorem 4.9. *Every interval valued $(\in \vee q, \in \vee q)$-fuzzy ideal of R is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R.*

Proof. Let $\widetilde{\mu}$ be an interval valued $(\in \vee q, \in \vee q)$-fuzzy ideal of R. Suppose that $x, y \in R$ and $\widetilde{t}, \widetilde{r} \in D[0,1]$ with $\widetilde{t}, \widetilde{r} \neq [0,0]$ such that $x_{\widetilde{t}} \in \widetilde{\mu}$ and $y_{\widetilde{r}} \in \widetilde{\mu}$. Then $x_{\widetilde{t}} \in \vee q\widetilde{\mu}$ and $y_{\widetilde{r}} \in \vee q\widetilde{\mu}$. By the hypothesis $(x-y)_{\min^i\{\widetilde{t}, \widetilde{r}\}} \in \vee q\widetilde{\mu}$. Now $x, y \in R$ and $\widetilde{t} \in D[0,1]$ with $\widetilde{t} \neq [0,0]$ such that $x_{\widetilde{t}} \in \widetilde{\mu}$. Then $x_{\widetilde{t}} \in \vee q\widetilde{\mu}$, so by hypothesis $(y + x - y)_{\widetilde{t}} \in \vee q\widetilde{\mu}$. Similarly, we prove $(xy)_{\widetilde{t}} \in \vee q\widetilde{\mu}$ and $((x+z)y - xy)_{\widetilde{t}} \in \vee q\widetilde{\mu}$. Therefore $\widetilde{\mu}$ is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R. □

The following theorem gives the connection between interval valued $(\in, \in)$-fuzzy ideal and interval valued fuzzy ideal.

Theorem 4.10. *An interval valued fuzzy subset $\widetilde{\mu}$ of R is an interval valued $(\in, \in)$-fuzzy ideal of R if and only if it is an interval valued fuzzy ideal of R.*

Proof. Assume that $\widetilde{\mu}$ is an interval valued fuzzy ideal of R. Let $x, y \in R$ and $\widetilde{t}, \widetilde{r} \in D[0,1]$ with $\widetilde{t}, \widetilde{r} \neq [0,0]$ be such that $x_{\widetilde{t}}, y_{\widetilde{r}} \in \widetilde{\mu}$. Then $\widetilde{\mu}(x) \geq \widetilde{t}$ and $\widetilde{\mu}(y) \geq \widetilde{r}$.

Since $\widetilde{\mu}$ is an interval valued fuzzy ideal of R, we have $\widetilde{\mu}(x-y) \geq \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \geq \min^i\{\widetilde{t}, \widetilde{r}\}$, it follows that $(x-y)_{\min\{\widetilde{t},\widetilde{r}\}} \in \widetilde{\mu}$. Now let $x, y \in R$ and $\widetilde{t} \in D[0,1]$ with $\widetilde{t} \neq [0,0]$. Then $x_{\widetilde{t}} \in \widetilde{\mu}$ and so $\widetilde{\mu}(x) \geq \widetilde{t}$. Since $\widetilde{\mu}$ is an interval valued fuzzy ideal of R, we have $\widetilde{\mu}(y+x-y) \geq \widetilde{\mu}(x) \geq \widetilde{t}$. Hence $(y+x-y)_{\widetilde{t}} \in \widetilde{\mu}$. Let $x, y \in R$ and $\widetilde{t} \in D[0,1]$ with $\widetilde{t} \neq [0,0]$. Then $y_{\widetilde{t}} \in \widetilde{\mu}$ and so $\widetilde{\mu}(y) \geq \widetilde{t}$. Hence $\widetilde{\mu}(xy) \geq \widetilde{\mu}(y) \geq \widetilde{t}$, because $\widetilde{\mu}$ is an interval valued fuzzy ideal of R. Thus $(xy)_{\widetilde{t}} \in \widetilde{\mu}$. Again let $x, z, y \in R$ and $\widetilde{t} \in D[0,1]$ with $\widetilde{t} \neq [0,0]$ be such that $\widetilde{\mu}(z) \geq \widetilde{t}$. Since $\widetilde{\mu}$ is an interval valued fuzzy ideal of R, then $\widetilde{\mu}((x+z)y-xy) \geq \widetilde{\mu}(z) \geq \widetilde{t}$. Thus $((x+z)y-xy)_{\widetilde{t}} \in \widetilde{\mu}$ and therefore $\widetilde{\mu}$ is an interval valued $(\in, \in)$-fuzzy ideal of R.

Conversely, assume that $\widetilde{\mu}$ is an interval valued $(\in, \in)$-fuzzy ideal of R. On the contrary assume that there exist $x, y \in R$ such that $\widetilde{\mu}(x-y) < \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}$. Choose $\widetilde{t}$ such that $\widetilde{\mu}(x-y) < \widetilde{t} < \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}$. Then $x_{\widetilde{t}}, y_{\widetilde{t}} \in \widetilde{\mu}$ and $(x-y)_{\widetilde{t}} \overline{\in} \widetilde{\mu}$. This is a contradiction to our assumption that $\widetilde{\mu}$ is an interval valued $(\in, \in)$-fuzzy ideal of R. Thus $\widetilde{\mu}(x-y) \geq \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}$. Suppose that $\widetilde{\mu}(y+x-y) < \widetilde{\mu}(x)$, for some $x, y \in R$. Choose $\widetilde{t}$ such that $\widetilde{\mu}(y+x-y) < \widetilde{t} < \widetilde{\mu}(x)$. Then $x_{\widetilde{t}} \in \widetilde{\mu}$ and $(y+x-y)_{\widetilde{t}} \overline{\in} \widetilde{\mu}$, which is a contradiction and hence $\widetilde{\mu}(y+x-y) \geq \widetilde{\mu}(x)$. Let us assume that $\widetilde{\mu}(xy) < \widetilde{\mu}(y)$, for some $x, y \in R$. Then there exist $\widetilde{t}$ such that $\widetilde{\mu}(xy) < \widetilde{t} < \widetilde{\mu}(y)$. This implies that $y_{\widetilde{t}} \in \widetilde{\mu}$ but $(xy)_{\widetilde{t}} \overline{\in} \widetilde{\mu}$. This contradicts our hypothesis. Hence $\widetilde{\mu}(xy) \geq \widetilde{\mu}(y)$. Again the contrary assume that there exist $x, y, z \in R$ such that $\widetilde{\mu}((x+z)y-xy) < \widetilde{\mu}(z)$. Let $\widetilde{t}$ be such that $\widetilde{\mu}((x+z)y-xy) < \widetilde{t} < \widetilde{\mu}(z)$. Then $z_{\widetilde{t}} \in \widetilde{\mu}$ but $((x+z)y-xy)_{\widetilde{t}} \overline{\in} \widetilde{\mu}$, which is a contradiction and so $\widetilde{\mu}((x+z)y-xy) \geq \widetilde{\mu}(z)$. Therefore $\widetilde{\mu}$ is an interval valued fuzzy ideal of R. □

Theorem 4.11. *Every interval valued $(\in, q)$-fuzzy ideal of R is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R.*

Proof. The proof is straightforward. □

The converse part of the above Theorem 4.11 is not true is general as shown in Example 3.4(6).

Theorem 4.12. *Every interval valued $(\in, \in)$-fuzzy ideal of R is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R.*

Proof. The proof is straightforward. □

The converse part of the above Theorem 4.12 is not true is general as shown in Example 3.4(1).

In the following theorem, we give a condition for an interval valued $(\in, \in \vee q)$-fuzzy ideal of R to be an interval valued $(\in, \in)$-fuzzy ideal of R.

Theorem 4.13. *Let $\widetilde{\mu}$ be an interval valued $(\in, \in \vee q)$-fuzzy ideal of R such that $\widetilde{\mu}(x) < [0.5, 0.5]$ for all $x \in R$. Then $\widetilde{\mu}$ is an interval valued $(\in, \in)$-fuzzy ideal of R.*

Proof. Let $x, y \in R$ and $\widetilde{t}, \widetilde{r} \in D[0,1]$ with $\widetilde{t}, \widetilde{r} \neq [0,0]$ be such that $x_{\widetilde{t}}, y_{\widetilde{r}} \in \widetilde{\mu}$. Then $\widetilde{\mu}(x) \geq \widetilde{t}, \widetilde{\mu}(y) \geq \widetilde{r}$. Since $\widetilde{\mu}$ is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R, then $\widetilde{\mu}(x-y) \geq \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\} \geq \min^i\{\widetilde{t}, \widetilde{r}, [0.5, 0.5]\} = \min^i\{\widetilde{t}, \widetilde{r}\}$ and so $(x-y)_{\min^i\{\widetilde{t},\widetilde{r}\}} \in \widetilde{\mu}$. Let $x, y \in R$ and $\widetilde{t} \in D[0,1]$ with $\widetilde{t} \neq [0,0]$ be such that $x_{\widetilde{t}} \in \widetilde{\mu}$. Then $\widetilde{\mu}(x) \geq \widetilde{t}$. Thus $\widetilde{\mu}(y+x-y) \geq \min^i\{\widetilde{\mu}(x), [0.5, 0.5]\} \geq \widetilde{t}$, since $\widetilde{\mu}$ is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R. Hence $(y+x-y)_{\widetilde{t}} \in \widetilde{\mu}$. Let $x, y \in R$ and $\widetilde{t} \in D[0,1]$ with $\widetilde{t} \neq [0,0]$. Then $y_{\widetilde{t}} \in \widetilde{\mu}$ implies $\widetilde{\mu}(y) \geq \widetilde{t}$. So $\widetilde{\mu}(xy) \geq \min^i\{\widetilde{\mu}(y), [0.5, 0.5]\} \geq \widetilde{t}$, since $\widetilde{\mu}$ is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R. Thus $(xy)_{\widetilde{t}} \in \widetilde{\mu}$. Similarly, we can prove that $((x+z)y-xy)_{\widetilde{t}} \in \widetilde{\mu}$. Therefore $\widetilde{\mu}$ is an interval valued $(\in, \in)$-fuzzy ideal of R. □

Theorem 4.14 ([21])**.** *If $\{\widetilde{\mu}_i | i \in \Omega\}$ is a family of interval valued $(\in, \in \vee q)$-fuzzy subnear-ring of a near-ring R, then $\widetilde{\mu} = \bigcap_{i \in \Omega} \widetilde{\mu}_i$ is an interval valued $(\in, \in \vee q)$-fuzzy subnear-ring of a near-ring R, where Ω is any index set.*

Theorem 4.15. *If $\{\widetilde{\mu}_i | i \in \Omega\}$ is a family of interval valued $(\in, \in \vee q)$-fuzzy ideal of R, then $\widetilde{\mu} = \bigcap_{i \in \Omega} \widetilde{\mu}_i$ is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R, where Ω is any index set.*

Proof. Let $x, y, z \in R$. Then, clearly, $\widetilde{\mu} = \bigcap_{i \in \Omega} \widetilde{\mu}_i$ is an interval valued $(\in, \in \vee q)$ fuzzy subnear-ring of R from Theorem 4.15. Then,

$$\begin{aligned}
\widetilde{\mu}(y+x-y) = \bigcap_{i\in\Omega} \widetilde{\mu}_i(y+x-y) &= \inf{}^i\{\widetilde{\mu}_i(y+x-y) : i \in \Omega\} \\
&\geq \inf{}^i\{\min{}^i\{\widetilde{\mu}_i(x), [0.5, 0.5]\} : i \in \Omega\} \\
&= \min{}^i\{\inf{}^i\{\widetilde{\mu}_i(x) : i \in \Omega\}, [0.5, 0.5]\} \\
&= \min{}^i\left\{\bigcap_{i\in\Omega} \widetilde{\mu}_i(x), [0.5, 0.5]\right\} \\
&= \min{}^i\{\widetilde{\mu}(x), [0.5, 0.5]\}.
\end{aligned}$$

$$\begin{aligned}
\widetilde{\mu}(xy) = \bigcap_{i\in\Omega} \widetilde{\mu}_i(xy) &= \inf{}^i\{\widetilde{\mu}_i(xy) : i \in \Omega\} \\
&\geq \inf{}^i\{\min{}^i\{\widetilde{\mu}_i(y), [0.5, 0.5]\} : i \in \Omega\} \\
&= \min{}^i\{\inf{}^i\{\widetilde{\mu}_i(y) : i \in \Omega\}, [0.5, 0.5]\} \\
&= \min{}^i\left\{\bigcap_{i\in\Omega} \widetilde{\mu}_i(y), [0.5, 0.5]\right\} \\
&= \min{}^i\{\widetilde{\mu}(y), [0.5, 0.5]\}
\end{aligned}$$

Similarly, $\widetilde{\mu}((x+z)y-xy) \geq \min^i\{\widetilde{\mu}(z), [0.5, 0.5]\}$. Therefore $\widetilde{\mu} = \bigcap_{i\in\Omega} \widetilde{\mu}_i$ is an interval valued $(\in, \in \vee q)$ fuzzy ideal of R. □

Theorem 4.16 ([21])**.** *Let $\widetilde{\mu}$ be an interval valued fuzzy subset of R. $\widetilde{\mu} = [\mu^-, \mu^+]$ is an interval valued $(\in, \in \vee q)$-fuzzy subnear-ring of R if and only if μ^-, μ^+ are $(\in, \in \vee q)$-fuzzy subnear-ring of R.*

The following theorem establishes the connection between interval valued $(\in, \in \vee q)$-fuzzy ideal of R and $(\in, \in \vee q)$-fuzzy ideal of R.

Theorem 4.17. *Let $\widetilde{\mu}$ be an interval valued fuzzy subset of R. $\widetilde{\mu} = [\mu^-, \mu^+]$ is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R if and only if μ^-, μ^+ are $(\in, \in \vee q)$-fuzzy ideal of R.*

Proof. Let $\widetilde{\mu}$ be an interval valued $(\in, \in \vee q)$-fuzzy ideal of R. For any $x, y, z \in R$.

$$\begin{aligned} [\mu^-(x-y), \mu^+(x-y)] &= \widetilde{\mu}(x-y) \\ &\geq \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\} \\ &= \min^i\{[\mu^-(x), \mu^+(x)], [\mu^-(y), \mu^+(y)], [0.5, 0.5]\} \\ &= [\min\{\mu^-(x), \mu^-(y), 0.5\}, \min\{\mu^+(x), \mu^+(y), 0.5\}]. \end{aligned}$$

It follows that $\mu^-(x-y) \geq \min\{\mu^-(x), \mu^-(y), 0.5\}$ and $\mu^+(x-y) \geq \min\{\mu^+(x), \mu^+(y), 0.5\}$. And

$$\begin{aligned} [\mu^-(y+x-y), \mu^+(y+x-y)] &= \widetilde{\mu}(y+x-y) \\ &\geq \min^i\{\widetilde{\mu}(x), [0.5, 0.5]\} \\ &= \min^i\{[\mu^-(x), \mu^+(x)], [0.5, 0.5]\} \\ &= [\min\{\mu^-(x), 0.5\}, \min\{\mu^+(x), 0.5\}]. \end{aligned}$$

It follows that $\mu^-(y+x-y) \geq \min\{\mu^-(x), 0.5\}$ and $\mu^+(y+x-y) \geq \min\{\mu^+(x), 0.5\}$. Further,

$$\begin{aligned} [\mu^-(xy), \mu^+(xy)] = \widetilde{\mu}(xy) &\geq \min^i\{\widetilde{\mu}(y), [0.5, 0.5]\} \\ &= \min^i\{[\mu^-(y), \mu^+(y)], [0.5, 0.5]\} \\ &= [\min\{\mu^-(y), 0.5\}, \min\{\mu^+(y), 0.5\}]. \end{aligned}$$

It follows that $\mu^-(xy) \geq \min\{\mu^-(y), 0.5\}$ and $\mu^+(xy) \geq \min\{\mu^+(y), 0.5\}$. Similarly, $\mu^-((x+z)y-xy) \geq \min\{\mu^-(z), 0.5\}$, $\mu^+((x+z)y-xy) \geq \min\{\mu^+(z), 0.5\}$. Therefore μ^+ and μ^- are $(\in, \in \vee q)$-fuzzy ideal of R.
Conversely, assume that μ^+ and μ^- are $(\in, \in \vee q)$- fuzzy ideal of R. Let $x, y, z \in R$. Then,

$$\begin{aligned} \widetilde{\mu}(x-y) &= [\mu^-(x-y), \mu^+(x-y)] \\ &\geq [\min\{\mu^-(x), \mu^-(y), 0.5\}, \min\{\mu^+(x), \mu^+(y), 0.5\}] \\ &= \min^i\{[\mu^-(x), \mu^+(x)], [\mu^-(y), \mu^+(y)], [0.5, 0.5]\} \\ &= \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\}. \end{aligned}$$

Further,

$$\begin{aligned} \widetilde{\mu}(y+x-y) &= [\mu^-(y+x-y), \mu^+(y+x-y)] \\ &\geq [\min\{\mu^-(x), 0.5\}, \min\{\mu^+(x), 0.5\}] \\ &= \min^i\{[\mu^-(x), \mu^+(x)], [0.5, 0.5]\} \\ &= \min^i\{\widetilde{\mu}(x), [0.5, 0.5]\}. \end{aligned}$$

And

$$\begin{aligned}\widetilde{\mu}(xy) &= [\mu^-(xy), \mu^+(xy)] \\ &\geq [\min\{\mu^-(y), 0.5\}, \min\{\mu^+(y), 0.5\}] \\ &= \min{}^i\{[\mu^-(y), \mu^+(y)], [0.5, 0.5]\} \\ &= \min{}^i\{\widetilde{\mu}(y), [0.5, 0.5]\}.\end{aligned}$$

Similarly, $\widetilde{\mu}((x+z)y - xy) \geq \min^i\{\widetilde{\mu}(z), [0.5, 0.5]\}$. □

Definition 4.18. For any interval valued fuzzy subset $\widetilde{\mu}$ of R and $\widetilde{t} \in D[0,1]$ with $\widetilde{t} \neq [0,0]$ we consider two subsets: $\widetilde{Q}(\widetilde{\mu};\widetilde{t}) = \{x \in R | x_{\widetilde{t}} q \widetilde{\mu}\}$ and $[\widetilde{\mu}]_{\widetilde{t}} = \{x \in R | x_{\widetilde{t}} \in \vee q \widetilde{\mu}\}$. Obviously, $[\widetilde{\mu}]_{\widetilde{t}} = \widetilde{U}(\widetilde{\mu} : \widetilde{t}) \cup \widetilde{Q}(\widetilde{\mu};\widetilde{t})$.

We call $[\widetilde{\mu}]_{\widetilde{t}}$ as an $\in \vee q$-level ideal and $\widetilde{Q}(\widetilde{\mu};\widetilde{t})$ a q-level ideal of $\widetilde{\mu}$.

Lemma 4.19. *Every interval valued fuzzy subset $\widetilde{\mu}$ of R satisfies the following assertion $\widetilde{t} \in D[0, 0.5]$ with $\widetilde{t} \neq [0,0]$ implies $[\widetilde{\mu}]_{\widetilde{t}} = \widetilde{U}(\widetilde{\mu} : \widetilde{t})$.*

Proof. Let $\widetilde{t} \in D[0, 0.5]$ with $\widetilde{t} \neq [0,0]$. Clearly, $\widetilde{U}(\widetilde{\mu} : \widetilde{t}) \subseteq [\widetilde{\mu}]_{\widetilde{t}}$. Let $x \in [\widetilde{\mu}]_{\widetilde{t}}$. If $x \notin \widetilde{U}(\widetilde{\mu} : \widetilde{t})$, then $\widetilde{\mu}(x) < \widetilde{t}$ and so $\widetilde{\mu}(x) + \widetilde{t} \leq \widetilde{t} + \widetilde{t} = 2\widetilde{t} \leq [1,1]$. This implies that $x_{\widetilde{t}} \overline{q} \widetilde{\mu}$, that is $x \notin \widetilde{Q}(\widetilde{\mu};\widetilde{t})$. Thus $x \notin \widetilde{U}(\widetilde{\mu} : \widetilde{t}) \cup \widetilde{Q}(\widetilde{\mu}; t) = [\widetilde{\mu}]_{\widetilde{t}}$. This leads to a contradiction and so $x \in \widetilde{U}(\widetilde{\mu} : \widetilde{t})$. Thus $[\widetilde{\mu}]_t \subseteq \widetilde{U}(\widetilde{\mu} : \widetilde{t})$. Therefore $[\widetilde{\mu}]_{\widetilde{t}} = \widetilde{U}(\widetilde{\mu} : \widetilde{t})$. □

Using the $(\in \vee q)$-level ideals of near-rings, we characterize the interval valued $(\in, \in \vee q)$-fuzzy ideals of near-rings.

Theorem 4.20. *An interval valued fuzzy subset $\widetilde{\mu}$ of R is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R if and only if $[\widetilde{\mu}]_{\widetilde{t}} (\neq \varnothing)$ is an ideal of R.*

Proof. Assume that $\widetilde{\mu}$ is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R and let $\widetilde{t} \in D[0, 0.5]$ with $\widetilde{t} \neq [0,0]$ be such that $[\widetilde{\mu}]_{\widetilde{t}} (\neq \varnothing)$. Let $x, y \in [\widetilde{\mu}]_{\widetilde{t}}$ such that $\widetilde{\mu}(x) \geq \widetilde{t}$ or $\widetilde{\mu}(x) + \widetilde{t} > [1,1]$ and $\widetilde{\mu}(y) \geq \widetilde{t}$ or $\widetilde{\mu}(y) + \widetilde{t} > [1,1]$. We can consider four cases:
(i) $\widetilde{\mu}(x) \geq \widetilde{t}$ and $\widetilde{\mu}(y) \geq \widetilde{t}$,
(ii) $\widetilde{\mu}(x) \geq \widetilde{t}$ and $\widetilde{\mu}(y) + \widetilde{t} > [1,1]$,
(iii) $\widetilde{\mu}(x) + \widetilde{t} > [1,1]$ and $\widetilde{\mu}(y) \geq \widetilde{t}$,
(iv) $\widetilde{\mu}(x) + \widetilde{t} > [1,1]$ and $\widetilde{\mu}(y) + \widetilde{t} > [1,1]$.
Consider Case (i): $\widetilde{\mu}(x) \geq \widetilde{t}$ and $\widetilde{\mu}(y) \geq \widetilde{t}$. This implies that

$$\begin{aligned}\widetilde{\mu}(x-y) \geq \min{}^i\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\} &\geq \min{}^i\{\widetilde{t}, [0.5, 0.5]\} \\ &= \begin{cases}[0.5, 0.5] & \text{if } \widetilde{t} > [0.5, 0.5] \\ \widetilde{t} & \text{if } \widetilde{t} \leq [0.5, 0.5]\end{cases}\end{aligned}$$

If $\widetilde{t} > [0.5, 0.5]$, then $\widetilde{\mu}(x-y) \geq [0.5, 0.5]$ and so $\widetilde{\mu}(x-y) + \widetilde{t} > [0.5, 0.5] + [0.5, 0.5] = [1,1]$, that is, $(x-y)_{\widetilde{t}} q \widetilde{\mu}$. If $\widetilde{t} \leq [0.5, 0.5]$, then $\widetilde{\mu}(x-y) \geq \widetilde{t}$ and thus $(x-y)_{\widetilde{t}} \in \widetilde{\mu}$. Therefore, $(x-y)_{\widetilde{t}} \in \vee q \widetilde{\mu}$, that is, $(x-y) \in [\widetilde{\mu}]_{\widetilde{t}}$. Case(ii): $\widetilde{\mu}(x) \geq \widetilde{t}$ and $\widetilde{\mu}(y) + \widetilde{t} > [1,1]$. If $\widetilde{t} > [0.5, 0.5]$, then $\widetilde{\mu}(x-y) \geq \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\} \geq \min^i\{\widetilde{t}, [1,1] - \widetilde{t}, [0.5, 0.5]\} = [1,1] - \widetilde{t}$, that is, $\widetilde{\mu}(x-y) + \widetilde{t} > [1,1]$ and thus $(x-y)_{\widetilde{t}} q \widetilde{\mu}$. If $\widetilde{t} \leq [0.5, 0.5]$, then $\widetilde{\mu}(x-y) \geq \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\} \geq \min^i\{\widetilde{t}, [1,1] - \widetilde{t}, [0.5, 0.5]\} = \widetilde{t}$, that is, $(x-y)_{\widetilde{t}} \in \widetilde{\mu}$ and thus $(x-y)_{\widetilde{t}} \in \vee q \widetilde{\mu}$. This means that $x - y \in [\widetilde{\mu}]_{\widetilde{t}}$. Similarly, we

can prove the result for the case(iii). Next we consider the case(iv): $\widetilde{\mu}(x)+\widetilde{t}>[1,1]$ and $\widetilde{\mu}(y)+\widetilde{t}>[1,1]$. If $\widetilde{t}>[0.5,0.5]$, then $\widetilde{\mu}(x-y)\geq \min^i\{\widetilde{\mu}(x),\widetilde{\mu}(y),[0.5,0.5]\}\geq \min^i\{[1,1]-\widetilde{t},[1,1]-\widetilde{t},[0.5,0.5]\}=[1,1]-\widetilde{t}$. So, $\widetilde{\mu}(x-y)+\widetilde{t}>[1,1]$, that is, $(x-y)_{\widetilde{t}}q\widetilde{\mu}$. If $\widetilde{t}\leq[0.5,0.5]$, then $\widetilde{\mu}(x-y)\geq \min^i\{\widetilde{\mu}(x),\widetilde{\mu}(y),[0.5,0.5]\}\geq \min\{[1,1]-\widetilde{t},[1,1]-\widetilde{t},[0.5,0.5]\}=[0.5,0.5]\geq\widetilde{t}$, that is, $(x-y)_{\widetilde{t}}\in\widetilde{\mu}$ and hence $(x-y)_{\widetilde{t}}\in\vee q\widetilde{\mu}$. This means that $x-y\in[\widetilde{\mu}]_{\widetilde{t}}$. Consequently, $[\widetilde{\mu}]_{\widetilde{t}}$ is a subnear-ring of $(R,+)$. Let $x\in[\widetilde{\mu}]_{\widetilde{t}}$ and $y\in R$ such that $\widetilde{\mu}(x)\geq\widetilde{t}$ and $\widetilde{\mu}(x)+\widetilde{t}>[1,1]$ and we consider two cases:
Case(i): $\widetilde{\mu}(x)\geq\widetilde{t}$. Since $\widetilde{\mu}$ is an $(\in,\in\vee q)$-fuzzy ideal of R, we have $\widetilde{\mu}(y+x-y)\geq \min^i\{\widetilde{\mu}(x),[0.5,0.5]\}\geq \min^i\{\widetilde{t},[0.5,0.5]\}$. If $\widetilde{t}>[0.5,0.5]$, then $\widetilde{\mu}(y+x-y)\geq [0.5,0.5]$ and so $\widetilde{\mu}(y+x-y)+\widetilde{t}>[0.5,0.5]+[0.5,0.5]=[1,1]$, that is, $\widetilde{\mu}(y+x-y)+\widetilde{t}>[1,1]$. Thus $(y+x-y)_{\widetilde{t}}q\widetilde{\mu}$. If $\widetilde{t}\leq[0.5,0.5]$, then $\widetilde{\mu}(y+x-y)\geq\widetilde{t}$. Hence $(y+x-y)_{\widetilde{t}}\in\widetilde{\mu}$.
Case(ii): $\widetilde{\mu}(x)+\widetilde{t}>[1,1]$. Since $\widetilde{\mu}$ is an interval valued $(\in,\in\vee q)$-fuzzy ideal of R, we have $\widetilde{\mu}(y+x-y)\geq \min^i\{\widetilde{\mu}(x),[0.5,0.5]\}>\min^i\{[1,1]-\widetilde{t},[0.5,0.5]\}$. If $\widetilde{t}>[0.5,0.5]$, then $\widetilde{\mu}(y+x-y)>[1,1]-\widetilde{t}$. Thus $(y+x-y)_{\widetilde{t}}q\widetilde{\mu}$. If $\widetilde{t}\leq[0.5,0.5]$, then $\widetilde{\mu}(y+x-y)>[0.5,0.5]\geq\widetilde{t}$. Hence $(y+x-y)_{\widetilde{t}}\in\widetilde{\mu}$. This means that $(y+x-y)_{\widetilde{t}}\in\vee q\widetilde{\mu}$, that is, $y+x-y\in[\widetilde{\mu}]_{\widetilde{t}}$. Let $y\in[\widetilde{\mu}]_{\widetilde{t}}$ and $x\in R$. Then $\widetilde{\mu}(y)\geq\widetilde{t}$ or $\widetilde{\mu}(y)+\widetilde{t}>[1,1]$. Assume that $\widetilde{\mu}(y)\geq\widetilde{t}$. Since $\widetilde{\mu}$ is an $(\in,\in\vee q)$-fuzzy ideal of R, we have $\widetilde{\mu}(xy)\geq \min^i\{\widetilde{\mu}(y),[0.5,0.5]\}\geq \min^i\{\widetilde{t},[0.5,0.5]\}$. If $\widetilde{t}>[0.5,0.5]$, then $\widetilde{\mu}(xy)\geq[0.5,0.5]$ implies $\widetilde{\mu}(xy)+\widetilde{t}>[0.5,0.5]+[0.5,0.5]=[1,1]$. So, $\widetilde{\mu}(xy)+\widetilde{t}>[1,1]$ and thus $(xy)_{\widetilde{t}}q\widetilde{\mu}$. If $\widetilde{t}\leq[0.5,0.5]$, then $\widetilde{\mu}(xy)\geq\widetilde{t}$. Hence $(xy)_{\widetilde{t}}\in\widetilde{\mu}$.
Let us assume that $\widetilde{\mu}(y)+\widetilde{t}>[1,1]$. Since $\widetilde{\mu}$ is an interval valued $(\in,\in\vee q)$-fuzzy ideal of R, we have $\widetilde{\mu}(xy)\geq \min^i\{\widetilde{\mu}(y),[0.5,0.5]\}>\min^i\{[1,1]-\widetilde{t},[0.5,0.5]\}$. If $\widetilde{t}>[0.5,0.5]$, then $\widetilde{\mu}(xy)>[1,1]-\widetilde{t}$. So, $(xy)_{\widetilde{t}}q\widetilde{\mu}$. If $\widetilde{t}\leq[0.5,0.5]$, then $\widetilde{\mu}(xy)>[0.5,0.5]\geq\widetilde{t}$. Thus $(xy)_{\widetilde{t}}\in\widetilde{\mu}$. This means that $(xy)_{\widetilde{t}}\in\vee q\widetilde{\mu}$, that is, $xy\in[\widetilde{\mu}]_{\widetilde{t}}$ and $[\widetilde{\mu}]_{\widetilde{t}}$ is a left ideal of R. Again, let $x,y\in R$ and $z\in[\widetilde{\mu}]_{\widetilde{t}}$ for $[0,0]<\widetilde{t}\leq[1,1]$. Then $z_{\widetilde{t}}\in\vee q\widetilde{\mu}$, that is, $\widetilde{\mu}(z)\geq\widetilde{t}$ and $\widetilde{\mu}(z)+\widetilde{t}>[1,1]$. Since $\widetilde{\mu}$ is an interval valued $(\in,\in\vee q)$-fuzzy ideal of R, then we have $\widetilde{\mu}((x+z)y-xy)\geq \min^i\{\widetilde{\mu}(z),[0.5,0.5]\}$. Similarly, we can prove that $(x+z)y-xy\in[\widetilde{\mu}]_{\widetilde{t}}$ and $[\widetilde{\mu}]_{\widetilde{t}}$ the ideal of R. Therefore, $[\widetilde{\mu}]_{\widetilde{t}}$ is a right ideal of R.

Conversely, assume that $\widetilde{\mu}$ be an interval valued fuzzy subset in R and let $[0,0]<\widetilde{t}\leq[1,1]$ be such that $[\widetilde{\mu}]_{\widetilde{t}}$ is an ideal of R.
Suppose that $\widetilde{\mu}(x-y)<\min^i\{\widetilde{\mu}(x),\widetilde{\mu}(y),[0.5,0.5]\}$. Choose $\widetilde{t}$ such that $\widetilde{\mu}(x-y)<\widetilde{t}<\min^i\{\widetilde{\mu}(x),\widetilde{\mu}(y),[0.5,0.5]\}$. Then $[0,0]<\widetilde{t}\leq[0.5,0.5]$ and $x,y\in\widetilde{U}(\widetilde{\mu}:\widetilde{t})\subseteq[\widetilde{\mu}]_{\widetilde{t}}$. Since $[\widetilde{\mu}]_{\widetilde{t}}$ is an ideal of R, then $x-y\in[\widetilde{\mu}]_{\widetilde{t}}$ and we have $\widetilde{\mu}(x-y)\geq\widetilde{t}$ or $\widetilde{\mu}(x-y)+\widetilde{t}>[1,1]$, which is a contradiction. Thus $\widetilde{\mu}(x-y)\geq \min^i\{\widetilde{\mu}(x),\widetilde{\mu}(y),[0.5,0.5]\}$, for all $x,y\in R$. Now, let $x,y\in R$ be such that $\widetilde{\mu}(y+x-y)<\widetilde{t}<\min^i\{\widetilde{\mu}(x),[0.5,0.5]\}$. Then $[0,0]<\widetilde{t}\leq[0.5,0.5]$ and $x\in\widetilde{U}(\widetilde{\mu}:\widetilde{t})\subseteq[\widetilde{\mu}]_{\widetilde{t}}$. Since $[\widetilde{\mu}]_{\widetilde{t}}$ is an ideal of R, then $y+x-y\in[\widetilde{\mu}]_{\widetilde{t}}$ and so $\widetilde{\mu}(y+x-y)\geq\widetilde{t}$ or $\widetilde{\mu}(y+x-y)+\widetilde{t}>[1,1]$. This is a contradiction to our assumption. Hence $\widetilde{\mu}(y+x-y)\geq \min^i\{\widetilde{\mu}(x),[0.5,0.5]\}$, for all $x,y\in R$. For, let $x,y\in R$ be such that $\widetilde{\mu}(xy)<\widetilde{t}<\min^i\{\widetilde{\mu}(y),[0.5,0.5]\}$. Then $[0,0]<\widetilde{t}\leq[0.5,0.5]$ and $y\in\widetilde{U}(\widetilde{\mu}:\widetilde{t})\subseteq[\widetilde{\mu}]_{\widetilde{t}}$. Since $[\widetilde{\mu}]_{\widetilde{t}}$ is an ideal of R, then $xy\in[\widetilde{\mu}]_{\widetilde{t}}$ and so $\widetilde{\mu}(xy)\geq\widetilde{t}$ or $\widetilde{\mu}(xy)+\widetilde{t}>[1,1]$ which is a contradiction to

our assumption. Hence $\widetilde{\mu}(xy) \geq \min^i\{\widetilde{\mu}(y), [0.5, 0.5]\}$, for all $x, y \in R$. Similarly we have to prove $\widetilde{\mu}((x+z)y - xy) \geq \min^i\{\widetilde{\mu}(z), [0.5, 0.5]\}$ and therefore $\widetilde{\mu}$ is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R. □

Theorem 4.21. *If $\widetilde{\mu}$ is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R, then the set $\widetilde{Q}(\widetilde{\mu}; \widetilde{t})(\neq \varnothing)$ is an ideal of R for all $[0.5, 0.5] < \widetilde{t} \leq [1, 1]$.*

Proof. Assume that $\widetilde{\mu}$ is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R and let $[0.5, 0.5] < \widetilde{t} \leq [1, 1]$ be such that $\widetilde{Q}(\widetilde{\mu}; \widetilde{t}) \neq \varnothing$. Let $x, y \in \widetilde{Q}(\widetilde{\mu}; \widetilde{t})$ be such that $\widetilde{\mu}(x) + \widetilde{t} > [1, 1]$ and $\widetilde{\mu}(y) + \widetilde{t} > [1, 1]$ and we have $\widetilde{\mu}(x - y) \geq \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\}$. If $\min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \geq [0.5, 0.5]$, then $\widetilde{\mu}(x-y) \geq [0.5, 0.5] > [1, 1] - \widetilde{t}$. If $\min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} < [0.5, 0.5]$, then $\widetilde{\mu}(x - y) \geq \min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} > [1, 1] - \widetilde{t}$. This implies that $x - y \in \widetilde{Q}(\widetilde{\mu}; t)$. Now, let $x \in \widetilde{Q}(\widetilde{\mu}; \widetilde{t})$ and $y \in R$ be such that $\widetilde{\mu}(x) + \widetilde{t} > [1, 1]$. Since $\widetilde{\mu}$ is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R, then we have $\widetilde{\mu}(y + x - y) \geq \min^i\{\widetilde{\mu}(x), [0.5, 0.5]\}$. If $\widetilde{\mu}(x) \geq [0.5, 0.5]$, then $\widetilde{\mu}(y + x - y) \geq [0.5, 0.5] > [1, 1] - \widetilde{t}$. If $\widetilde{\mu}(x) < [0.5, 0.5]$, then $\widetilde{\mu}(y + x - y) \geq \widetilde{\mu}(x) > [1, 1] - \widetilde{t}$. Thus $y + x - y \in \widetilde{Q}(\widetilde{\mu}; \widetilde{t})$. Similarly, let $y \in \widetilde{Q}(\widetilde{\mu}; \widetilde{t})$ and $x \in R$, then $xy \in \widetilde{Q}(\widetilde{\mu}; \widetilde{t})$. Again let $x, y \in R$ and $z \in \widetilde{Q}(\widetilde{\mu}; \widetilde{t})$ be such that $\widetilde{\mu}(z) + \widetilde{t} > [1, 1]$. Since $\widetilde{\mu}$ is an interval valued $(\in, \in \vee q)$-fuzzy ideal of R, then we have $\widetilde{\mu}((x + z)y - xy) \geq \min^i\{\widetilde{\mu}(z), [0.5, 0.5]\}$. If $\widetilde{\mu}(z) \geq [0.5, 0.5]$, then $\widetilde{\mu}((x+z)y - xy) \geq [0.5, 0.5] > [1, 1] - \widetilde{t}$ and if $\widetilde{\mu}(z) < [0.5, 0.5]$, then $\widetilde{\mu}((x+z)y - xy) \geq \widetilde{\mu}(z) > [1, 1] - \widetilde{t}$ and thus $(x+z)y - xy \in \widetilde{Q}(\widetilde{\mu}; \widetilde{t})$. Therefore $\widetilde{Q}(\widetilde{\mu}; \widetilde{t})$ is an ideal of R. □

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