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Rough approximations induced by a soft relation

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Abstract. In the paper, motivated by Yao's idea, two kinds of rough approximation defined by a soft relation are introduced, some basic properties of these rough soft sets are studied.

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1. Introduction

Z. Pawlak initiated rough set theory to study incomplete and insufficient information, and his two rough operators first defined by means of a given indiscernibility relation [14]. Usually indiscernibility relations are supposed to be equivalences. As a generalization, Yao studied two rough operators induced by an arbitrary binary relation [15].

In 1999, D. Molodtsov introduced the concept of soft set [13] to solve complicated problems and various types of uncertainties. P. K. Maji, M. I. Ali, F. Feng, et al. studied some operations on soft sets in [10, 4], fuzzy soft set in [1, 2, 9, 5], and provided a common framework to combine soft sets, rough sets and fuzzy sets together, which gives rise to several interesting new concepts and problems, such as soft rough set, rough soft set, fuzzy rough soft set, etc., see [6, 8, 12, 3].

In [16], the notion of soft relation was introduced. As a application of Yao's method, we define two kinds of rough operators in soft set, and discuss some of their properties.

The above contents are arranged into three parts, Section 3: Soft Relation; Section 4: Rough Soft Sets induced by a Soft Relation. In Section 2, we give an overview of rough sets and soft sets, which surveys Preliminaries

2. Preliminaries

The section is devoted to some main notions for each area, i.e., rough sets [14, 15] and soft sets [10, 13, 4].

2.1. Rough sets.

In rough set theory, the approximation of an arbitrary subset of a universe by two definable subsets are called lower and upper approximations, which correspond to two rough operators. The two rough operators were first defined by means of a given indiscernibility relation in [14]. Usually indiscernibility relations are supposed to be equivalences.

Let (X, R) be an approximation space, and $R \subseteq X \times X$ be an equivalence relation, then for $A \subseteq X$, two subsets $\underline{R}(A)$ and $\overline{R}(A)$ of X are defined:

$$\underline{R}(A) = \{x \in X \mid [x]_R \subseteq A\}, \qquad \overline{R}(A) = \{x \in X \mid [x]_R \cap A \neq \varnothing\},$$

where $[x]_R = \{y \in X \mid xRy\}$.

If $\underline{R}(A) = \overline{R}(A)$, A is called a definable set; if $\underline{R}(A) \neq \overline{R}(A)$, A is called an undefinable set, and $(\underline{R}(A), \overline{R}(A))$ is referred to as a pair of rough set. Therefore, $\underline{R}$ and $\overline{R}$ are called two rough operators.

Furthermore, as a generalization, in [15], Yao defined the two rough operators by an arbitrary binary relation. Suppose R is a binary relation on X, for $x \in X$, let $r(x) = \{y \mid xRy\}$, then a pair of lower and upper approximations is defined: for $A \subseteq X$,

$$\underline{appr}A = \{x \mid r(x) \subseteq A\}, \qquad \overline{appr}A = \{x \mid r(x) \cap A \neq \varnothing\}.$$

Furthermore, there are many generalizations of the theory of rough sets.

2.2. Soft sets.

In the section, we recall the notion of soft set, and some operations on soft sets accompanied with examples.

Let X be an initial universe set and E_X (simple E) be a collection of all possible parameters with respect to X. Then the pair (X, E) will be called a soft universe.

In [13], D.Molodtsov introduced the notion of soft set as follows.

Definition 2.1. A pair (F, A) is called a soft set over X if $A \subseteq E$, and $F : A \to 2^X$, where 2^X is the power set of X.

Example 2.2. Consider a soft universe (X, E), $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2, e_3\}$. Let $A_1 = \{e_1, e_2\}$, $F_1 : A_1 \to 2^X$, where $F_1(e_1) = \{x_1, x_2\}$, $F_1(e_2) = \{x_3, x_4\}$, clearly (F_1, A_1) is a soft set.

Let SF(X) be the set of all soft sets over X. On which, there exist two kinds of special elements: one is called a absolute soft set $\Gamma_A = (F, A)$, $\forall e \in A$, $F(e) = X$; the other is called a null soft set $\Phi_A = (F, A)$, $\forall e \in A$, $F(e) = \varnothing$.

In [13], the equality of two soft sets was introduced. For $(F, A), (G, B) \in$SF(X), $(F, A)\tilde{\subseteq}(G, B)$ if $A \subseteq B$, and for every $e \in A$, $F(e) \subseteq G(e)$. $(F, A) = (G, B)$ if $(F, A)\tilde{\subseteq}(G, B)$ and $(G, B)\tilde{\subseteq}(F, A)$.

Example 2.3. Follows Example 2.2, let $A_2 = \{e_1, e_2, e_3\}$, $F_2 : A_2 \to 2^X$, where $F_2(e_1) = \{x_1, x_2\}$, $F_2(e_2) = \{x_2, x_3, x_4\}$, $F_2(e_3) = \{x_1, x_4\}$. Thus (F_2, A_2) is also a soft set. Clearly $(F_1, A_1)\tilde{\subseteq}(F_2, A_2)$ holds.

In [10], P. K. Maji et al. defined the union of soft sets as follows.

Definition 2.4. Suppose $(F, A), (G, B) \in$SF(X) are two soft sets, the union of (F, A) and (G, B) is a soft set (H, C), where $C = A \cup B$, and for $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } \ e \in A - B \\ G(e) & \text{if } \ e \in B - A \\ F(e) \cup G(e) & \text{if } \ e \in A \cap B \end{cases}$$

and written as $(F, A)\tilde{\bigcup}(G, B) = (H, C)$.

Example 2.5. Follows Example 2.3, let $A_3 = \{e_2, e_3\}$, $F_3 : A_3 \to 2^X$,

where $F_3(e_2) = \{x_1, x_2, x_4\}$, $F_3(e_3) = \{x_1\}$. Clearly (F_3, A_3) is a soft set.

Then we have

$(H, C) = (F_1, A_1)\tilde{\bigcup}(F_3, A_3)$, where $C = A_1 \cup A_3 = \{e_1, e_2, e_3\}$;

$H(e_1) = F_1(e_1)$, $H(e_2) = \{x_1, x_2, x_3, x_4\}$, $H(e_3) = F_3(e_3)$.

In [4], M. I. Ali et al. defined the extended intersection of soft sets.

Definition 2.6. Suppose $(F, A), (G, B) \in$SF(X) are two soft sets, the extended intersection of (F, A) and (G, B) is also a soft set (J, C), where $C = A \cup B$, and for $e \in C$,

$$J(e) = \begin{cases} F(e) & \text{if } \ e \in A - B \\ G(e) & \text{if } \ e \in B - A \\ F(e) \cap G(e) & \text{if } \ e \in A \cap B \end{cases}$$

and written as $(F, A) \sqcap (G, B) = (J, C)$.

In Example 2.5, clearly, we have

$(J, C) = (F_1, A_1) \sqcap (F_3, A_3)$, where $C = A_1 \cup A_3 = \{e_1, e_2, e_3\}$;

$J(e_1) = F_1(e_1)$, $J(e_2) = \{x_4\}$, $J(e_3) = F_3(e_3)$.

In [10], P. K. Maji et al. introduced the notion of NOT set of a set of parameters, that is Definition 2.7.

Definition 2.7. Let $E = \{e_1, e_2, \cdots, e_n\}$ be the set of parameters. The NOT set of E denoted by $\rceil E$ is defined by $\rceil E = \{\neg e_1, \neg e_2, \cdots, \neg e_n\}$, where $\neg e_i = not \ e_i$, $\forall i$. (It may be noted that $\rceil$ and $\neg$ are different operators).

About the NOT set of E, the following proposition holds, see [10].

Proposition 2.8.
(1) $\rceil(\rceil A) = A$, *(2)* $\rceil(A \cup B) = \rceil A \cup \rceil B$, *(3)* $\rceil(A \cap B) = \rceil A \cap \rceil B$.

Definition 2.9. The complement of a soft set (F, A) is denoted by $(F, A)^c$, and is defined by $(F, A)^c = (F^c, \rceil A)$, where $F^c : \rceil A \to L^X$, for every $\neg e \in \rceil A$, $F^c(\neg e) = X - F(e)$.

In Example 2.2, the complement of (F_1, A_1) is $(F_1^c, \rceil A_1)$, where $\rceil A_1 = \{\neg e_1, \neg e_2\}$, $F_1^c(\neg e_1) = \{x_3, x_4\}$, $F_1^c(\neg e_2) = \{x_1, x_2\}$.

3. Soft relation

In [11], Majumdar et al. introduced the notion of soft mapping. Based on that mapping and relation are intimately connected, Zhang et.al proposed the concept of soft relation and studied some properties in [16].

Definition 3.1. Let X be a set, E be a parameter set, then the mapping

$$\varphi : E \to 2^{X \times X}$$

is called a soft relation on X under E.

Example 3.2. Follows Example 2.2, put $R_1 = \{(x_1, x_1), (x_1, x_2), (x_1, x_3)\}$, $R_2 = \{(x_2, x_4), (x_3, x_4), (x_4, x_4)\}$, $R_3 = \{(x_1, x_1), (x_2, x_2), (x_3, x_3)\}$.

We define a soft relation φ: $\varphi(e_1) = R_1, \varphi(e_2) = R_2$, $\varphi(e_3) = R_3$.

Suppose φ is a soft relation, then we say that

φ is (soft) reflexive $\Leftrightarrow$ for every $e \in E$, $\varphi(e)$ is reflexive,

φ is (soft) symmetric $\Leftrightarrow$ for every $e \in E$, $\varphi(e)$ is symmetric,

φ is (soft) transitive $\Leftrightarrow$ for every $e \in E$, $\varphi(e)$ is transitive.

A soft relation φ is called a soft equivalence relation if it is reflexive, symmetric, and transitive.

Example 3.3. Follows Example 2.2, put

$S_1 = \{(x_1, x_1), (x_2, x_2), (x_3, x_3), (x_4, x_4), (x_1, x_2), (x_2, x_1)\}$,

$S_2 = \{(x_1, x_1), (x_2, x_2), (x_3, x_3), (x_4, x_4), (x_3, x_4), (x_4, x_3)\}$,

$S_3 = \{(x_1, x_1), (x_2, x_2), (x_3, x_3), (x_4, x_4)\}$.

Let $\varphi_1(e_1) = S_1, \varphi_1(e_2) = S_2$, $\varphi_1(e_3) = S_3$, then φ_1 is a soft relation on X under E, and it is reflexive, symmetric.

4. Rough soft sets induced by a soft relation

In the section, we introduce two kinds of rough operators in soft set, investigate some of their properties.

4.1. **Method I.** We introduce a method to define rough soft sets.

Suppose φ is a soft relation, $\bigcup \varphi = \bigcup_{e \in E} \varphi(e)$. In Example 3.2,

$$\begin{aligned}\bigcup \varphi &= R_1 \cup R_2 \cup R_3 \\ &= \{(x_1, x_1), (x_1, x_2), (x_1, x_3), (x_2, x_2), (x_2, x_4), (x_3, x_3), (x_3, x_4), (x_4, x_4)\}.\end{aligned}$$

So $\bigcup\varphi$ is a classical binary relation on the universe set X. Certainly, if φ is reflexive, symmetric, $\bigcup\varphi$ is also reflexive, symmetric; but not vice versa. In general, $\bigcup\varphi$ is not an equivalence relation.

Consider $\bigcup\varphi$ is a binary relation on X, by Yao's method[15], we introduce rough operators in soft set. Let

$$r(x) = \{y \mid y \in X, (x, y) \in \bigcup\varphi\}$$

Definition 4.1. Suppose φ is a soft relation on X under E, then for a soft set (F, A), two soft sets are defined as follows, for $e \in A$,

$$F_*(e) = \{x \mid r(x) \subseteq F(e)\},$$
$$F^*(e) = \{x \mid r(x) \cap F(e) \neq \varnothing\}.$$

If $(F_*, A) = (F^*, A)$, then (F, A) is called a definable soft set; if $(F_*, A) \neq (F^*, A)$, (F, A) is called an undefinable soft set, and $((F_*, A), (F^*, A))$ is referred to as a pair of rough soft set.

Example 4.2. Follows Example 3.2, φ is a soft relation on X under E, we have $r(x_1) = \{x_1, x_2, x_3\}$, $r(x_2) = \{x_2, x_4\}$, $r(x_3) = \{x_3, x_4\}$, $r(x_4) = \{x_4\}$. Then for $(F, A) = (F_2, A_2)$ (see Example 2.3), we obtain

(F_*, A): $F_*(e_1) = \varnothing$, $F_*(e_2) = \{x_2, x_4\}$, $F_*(e_3) = \{x_4\}$;

(F^*, A): $F^*(e_1) = \{x_1, x_2\}$, $F^*(e_2) = \{x_1, x_2, x_3, x_4\}$, $F^*(e_3) = \{x_1, x_2, x_3, x_4\}$.

We investigate some properties.

Proposition 4.3. *Suppose* $(F, A)\tilde{\subseteq}(G, B)$, *then*

(1) $(F_*, A)\tilde{\subseteq}(G_*, B)$,

(2) $(F^*, A)\tilde{\subseteq}(G^*, B)$,

(3) $(\Phi^*, A) = (\Phi, A)$,

(4) $(\Gamma_*, A) = (\Gamma, A)$.

Proof. (1) Since $(F, A)\tilde{\subseteq}(G, B)$, so $A \subseteq B$, and for every $e \in A$, $F(e) \subseteq G(e)$. Thus $F_*(e) = \{x \mid r(x) \subseteq F(e)\} \subseteq \{x \mid r(x) \subseteq G(e)\} = G_*(e)$.

(2) Similarly to (1).

(3) and (4) are obvious. □

Proposition 4.4. *Suppose* $(H, C), (J, C)$ *are the union and the extended intersection of* (F, A),(G, B), *i.e.,* $(H, C) = (F, A)\tilde{\cup}(G, B)$, $(J, C) = (F, A) \sqcap (G, B)$, *then*

(1) $(H^*, C) = (F^*, A)\tilde{\cup}(G^*, B)$,

(2) $(J^*, C)\tilde{\subseteq}(F^*, A) \sqcap (G^*, B)$,

(3) $(F_*, A)\tilde{\cup}(G_*, B)\tilde{\subseteq}(H_*, C)$,

(4) $(J_*, C) = (F_*, A) \sqcap (G_*, B)$.

Proof. (1) Since $(H,C) = (F,A)\tilde{\cup}(G,B)$, so $C = A \cup B$. For every $e \in C$, if $e \in A - B$, $H(e) = F(e)$, we obtain $H^*(e) = F^*(e)$; if $e \in B - A$, $H(e) = G(e)$, we obtain $H^*(e) = G^*(e)$; if $e \in A \cap B$, $H(e) = F(e) \cup G(e)$, we obtain

$$\begin{aligned} H^*(e) &= \{x \mid r(x) \cap H(e) \neq \varnothing\} \\ &= \{x \mid r(x) \cap (F(e) \cup G(e)) \neq \varnothing\} \\ &= \{x \mid r(x) \cap F(e) \neq \varnothing\} \cup \{x \mid r(x) \cap G(e) \neq \varnothing\} \\ &= F^*(e) \cup G^*(e). \end{aligned}$$

By the above proof, $(H^*, C) = (F^*, A)\tilde{\cup}(G^*, B)$ holds.

(2),(3) and (4) similar to (1). □

Proposition 4.5. *Suppose (F,A) is a soft set, then*

(1) $(F^{c*}, \rceil A) = ((F_*)^c, \rceil A)$,

(2) $((F^c)_*, \rceil A) = (F^{*c}, \rceil A)$.

Proof. (1) For every $\neg e \in \rceil A$,

$$\begin{aligned} F^{c*}(\neg e) &= \{x \mid r(x) \cap F^c(\neg e) \neq \varnothing\} \\ &= \{x \mid r(x) \cap (X - F(e)) \neq \varnothing\}. \end{aligned}$$

Thus

$$\begin{aligned} x \in F^{c*}(\neg e) &\Leftrightarrow r(x) \cap (X - F(e)) \neq \varnothing \\ &\Leftrightarrow r(x) \nsubseteq F(e) \\ &\Leftrightarrow x \notin F_*(e) \\ &\Leftrightarrow x \in (F_*)^c(\neg e). \end{aligned}$$

(2) For every $\neg e \in \rceil A$,

$$\begin{aligned} (F^c)_*(\neg e) &= \{x \mid r(x) \subseteq F^c(\neg e)\} \\ &= \{x \mid r(x) \subseteq X - F(e)\}. \end{aligned}$$

Thus

$$\begin{aligned} x \in F^c_*(\neg e) &\Leftrightarrow r(x) \subseteq X - F(e) \\ &\Leftrightarrow r(x) \cap F(e) = \varnothing \\ &\Leftrightarrow x \notin F^*(e) \\ &\Leftrightarrow x \in F^{*c}(\neg e). \end{aligned}$$

□

Proposition 4.6. *Suppose $(F,A), (G,B) \in SF(X)$, $(H,C) = (F,A)\tilde{\cup}(G,B)$, then*

(1) $(H^{c*}, \rceil C)\tilde{\subseteq}(F^{c*}, \rceil A) \sqcap (G^{c*}, \rceil B)$,

(2) $((H^c)_*, \rceil C) = ((F^c)_*, \rceil A) \sqcap ((G^c)_*, \rceil B)$,

(3) $(H^{*c}, \rceil C) = (F^{*c}, \rceil A) \sqcap (G^{*c}, \rceil B)$,

(4) $((H_*)^c, \rceil C)\tilde{\subseteq}((F_*)^c, \rceil A) \sqcap ((G_*)^c, \rceil B)$.

Proof. (1) Since $(H,C) = (F,A)\tilde{\cup}(G,B)$, so $C = A \cup B$, we obtain $\rceil C = \rceil A \cup \rceil B$. For every $\neg e \in \rceil C$, if $\neg e \in \rceil A - \rceil B$, we have $e \in A - B$, $H(e) = F(e)$, then $H^c(\neg e) =$

$F^c(\neg e)$, which implies $H^{c*}(\neg e) = F^{c*}(\neg e)$; if $\neg e \in \rceil B - \rceil A$, in the same way, we also obtain $H^{c*}(\neg e) = G^{c*}(\neg e)$; if $\neg e \in \rceil A \cap \rceil B$, we have $e \in A \cap B$, $H(e) = F(e) \cup G(e)$, then

$$\begin{aligned} H^{c*}(\neg e) &= \{x \mid r(x) \cap H^c(\neg e) \neq \varnothing\} \\ &= \{x \mid r(x) \cap (X - H(e)) \neq \varnothing\} \\ &= \{x \mid r(x) \cap (X - (F(e) \cup G(e))) \neq \varnothing\} \\ &= \{x \mid r(x) \cap (X - (F(e)) \cap (X - G(e))) \neq \varnothing\} \\ &= \{x \mid r(x) \cap F^c(\neg e) \cap G^c(\neg e) \neq \varnothing\} \\ &\subseteq \{x \mid r(x) \cap F^c(\neg e) \neq \varnothing\} \cap \{x \mid r(x) \cap G^c(\neg e) \neq \varnothing\}, \end{aligned}$$

So we obtain $(H^{c*}, \rceil C) \tilde{\subseteq} (F^{c*}, \rceil A) \sqcap (G^{c*}, \rceil B)$.

(2) Since $(H, C) = (F, A) \tilde{\cup} (G, B)$, so $C = A \cup B$, we obtain $\rceil C = \rceil A \cup \rceil B$. For every $\neg e \in \rceil C$, if $\neg e \in \rceil A - \rceil B$, we have $e \in A - B$, $H(e) = F(e)$, then $H^c(\neg e) = F^c(\neg e)$, which implies $(H^c)_*(\neg e) = (F^c)_*(\neg e)$; if $\neg e \in \rceil B - \rceil A$, in the same way, we also obtain $(H^c)_*(\neg e) = (G^c)_*(\neg e)$; if $\neg e \in \rceil A \cap \rceil B$, we have $e \in A \cap B$, $H(e) = F(e) \cup G(e)$, then

$$\begin{aligned} (H^c)_*(\neg e) &= \{x \mid r(x) \subseteq H^c(\neg e)\} \\ &= \{x \mid r(x) \subseteq X - H(e)\} \\ &= \{x \mid r(x) \subseteq X - (F(e) \cup G(e))\} \\ &= \{x \mid r(x) \subseteq (X - (F(e)) \cap (X - G(e))\} \\ &= \{x \mid r(x) \subseteq X - F(e)\} \cap \{x \mid r(x) \subseteq X - G(e)\} \\ &= (F^c)_*(e) \cap (G^c)_*(e). \end{aligned}$$

So $((H^c)_*, \rceil C) = ((F^c)_*, \rceil A) \sqcap ((G^c)_*, \rceil B)$ holds.

(3) By Proposition 4.4(1).

(4) By Proposition 4.4(3). □

Proposition 4.7. *Suppose* $(F, A), (G, B) \in SF(X)$, $(J, C) = (F, A) \sqcap (G, B)$, *then*

(1) $(J^{c*}, \rceil C) = (F^{c*}, \rceil A) \tilde{\cup} (G^{c*}, \rceil B)$,

(2) $((F^c)_*, \rceil A) \tilde{\cup} ((G^c)_*, \rceil B) \tilde{\subseteq} ((J^c)_*, \rceil C)$,

(3) $(F^{*c}, \rceil A) \tilde{\cup} (G^{*c}, \rceil B) \tilde{\subseteq} (J^{*c}, \rceil C)$,

(4) $((J_*)^c, \rceil C) = ((F_*)^c, \rceil A) \tilde{\cup} ((G_*)^c, \rceil B)$.

Proof. Similarity to Proposition 4.6.

□

4.2. **Method II.** We introduce the other method to define rough soft sets.

Suppose φ is a soft relation, then for $e \in E$, $\varphi(e)$ is a binary relation on X, according to Yao's definition, for $x \in X$, we obtain

$$r_e(x) = \{y \mid y \in X, (x, y) \in \varphi(e)\}$$

Thus for a soft set (F, A), $F(e)$ is a subset of X, we define

$$\underline{F}(e) = \{x \mid r_e(x) \subseteq F(e)\},$$
$$\overline{F}(e) = \{x \mid r_e(x) \cap F(e) \neq \varnothing\}.$$

So we obtain two soft sets $(\underline{F}, A)$ and $(\overline{F}, A)$.

If $(\underline{F}, A) = (\overline{F}, A)$, then (F, A) is called a definable soft set; if $(\underline{F}, A) \neq (\overline{F}, A)$, (F, A) is called an undefinable soft set, and $(\underline{F}, A), (\overline{F}, A)$ is referred to as a pair of rough soft set.

Example 4.8. Follows Example 3.2, φ is a soft relation on X under E, we have
$r_{e_1}(x_1) = \{x_1, x_2, x_3\}$, $r_{e_1}(x_2) = \varnothing$, $r_{e_1}(x_3) = \varnothing$, $r_{e_1}(x_4) = \varnothing$;
$r_{e_2}(x_1) = \varnothing$, $r_{e_2}(x_2) = \{x_4\}$, $r_{e_2}(x_3) = \{x_4\}$, $r_{e_2}(x_4) = \{x_4\}$;
$r_{e_3}(x_1) = \{x_1\}$, $r_{e_3}(x_2) = \{x_2\}$, $r_{e_3}(x_3) = \{x_3\}$, $r_{e_3}(x_4) = \varnothing$.

Thus, for $(F, A) = (F_2, A_2)$ (see Example 2.3), we obtain
$(\underline{F}, A)$: $\underline{F}(e_1) = \{x_2, x_3, x_4\}$, $\underline{F}(e_2) = \{x_2, x_3, x_4\}$, $\underline{F}(e_3) = \{x_1, x_4\}$;
$(\overline{F}, A)$: $\overline{F}(e_1) = \{x_1\}$, $\overline{F}(e_2) = \{x_2, x_3, x_4\}$, $\overline{F}(e_3) = \{x_1\}$.

Corresponding the method I, the following propositions hold.

Proposition 4.9. *Suppose* $(F, A)\tilde{\subseteq}(G, B)$, *then*

(1) $(\underline{F}, A)\tilde{\subseteq}(\underline{G}, B)$,

(2) $(\overline{F}, A)\tilde{\subseteq}(\overline{G}, B)$,

(3) $(\overline{\Phi}, A) = (\Phi, A)$,

(4) $(\underline{\Gamma}, A) = (\Gamma, A)$.

Proposition 4.10. *Suppose* $(H, C), (J, C)$ *are the union and the extended intersection of* (F, A),(G, B), *i.e.,* $(H, C) = (F, A)\tilde{\cup}(G, B)$, $(J, C) = (F, A) \sqcap (G, B)$, *then*

(1) $(\overline{H}, C) = (\overline{F}, A)\tilde{\cup}(\overline{G}, B)$,

(2) $(\overline{J}, C)\tilde{\subseteq}(\overline{F}, A) \sqcap (\overline{G}, B)$,

(3) $(\underline{F}, A)\tilde{\cup}(\underline{G}, B)\tilde{\subseteq}(\underline{J}, C)$,

(4) $((\underline{J}, C) = (\underline{F}, A) \sqcap (\underline{G}, B)$.

Proposition 4.11. *Suppose* (F, A) *is a soft set, then*

(1) $((\overline{F^c}), \rceil A) = ((\underline{F})^c, \rceil A)$,

(2) $((\underline{F^c}), \rceil A) = ((\overline{F})^c, \rceil A)$.

Proposition 4.12. *Suppose* $(F, A), (G, B) \in$*SF(X),* $(H, C) = (F, A)\tilde{\cup}(G, B)$, *then*

(1) $((\overline{H^c}), \rceil C)\tilde{\subseteq}((\overline{F^c}), \rceil A) \sqcap ((\overline{G^c}), \rceil B)$,

(2) $((\underline{H^c}), \rceil C) = ((\underline{F^c}), \rceil A) \sqcap ((\underline{G^c}), \rceil B)$,

(3) $((\overline{H})^c, \rceil C) = ((\overline{F})^c, \rceil A) \sqcap ((\overline{G})^c, \rceil B)$,

(4) $((\underline{H})^c, \rceil C)\tilde{\subseteq}((\underline{F})^c, \rceil A) \sqcap ((\underline{G})^c, \rceil B)$.

Proposition 4.13. *Suppose* $(F, A), (G, B) \in$*SF(X),* $(J, C) = (F, A) \sqcap (G, B)$*, then*

(1) $((\overline{J^c}), \rceil C) = ((\overline{F^c}), \rceil A)\tilde{\cup}((\overline{G^c}), \rceil B)$,

(2) $((\underline{F^c}), \rceil A)\tilde{\cup}((\underline{G^c}), \rceil B)\tilde{\subseteq}((\underline{J^c}), \rceil C)$,

(3) $((\overline{F})^c, \rceil A)\tilde{\cup}((\overline{G})^c, \rceil B)\tilde{\subseteq}((\overline{J})^c, \rceil C)$,

(4) $((\underline{J})^c, \rceil C) = ((\underline{F})^c, \rceil A)\tilde{\cup}((\underline{G})^c, \rceil B)$.

Example 4.14. In [7], suppose X is a universal set, E a parameter set,$\theta = (K, D)$ is a soft set, then (X, θ) may be viewed as a approximation space, for $U \subseteq X$,

$$\underline{apr}(U) = \{y \mid y \in X, \exists e \in D, y \in K(e) \subseteq U\},$$
$$\overline{apr}(U) = \{y \mid y \in X, \exists e \in D, y \in K(e), K(e) \cap U \neq \varnothing\}.$$

In fact, given a soft set $\theta = (K, D)$, we obtain a soft relation $\varphi = X \times K$, i.e., for each $e \in E$, $\varphi(e) = X \times K(e)$, thus for $x \in X$,

$$\begin{aligned} r_e(x) &= \{y \mid y \in X, (x, y) \in \varphi_(e)\} \\ &= \{y \mid y \in X, (x, y) \in X \times K(e)\} \\ &= K(e). \end{aligned}$$

So for (F, A), for $e \in A$, we have

$$\underline{F}(e) = \{x \mid r_e(x) \subseteq F(e)\} = \{x \mid x \in K(e), K(e) \subseteq F(e)\},$$
$$\overline{F}(e) = \{x \mid r_e(x) \cap F(e) \neq \varnothing\} = \{x \mid x \in K(e), K(e) \cap F(e) \neq \varnothing\}.$$

In the end, we compare the two methods and obtain

Proposition 4.15. *Suppose* φ *is a soft relation on* X *under* E*, then*

(1) *For every* $e \in E$, $r_e(x) \subseteq r(x)$,

(2) *for a soft set* (F, A), $(F_*, A)\tilde{\subseteq}(\underline{F}, e)$ *and* $(\overline{F}, A)\tilde{\subseteq}(F^*, A)$.

If φ *is reflexive, then* $(F_*, A)\tilde{\subseteq}(\underline{F}, e)\tilde{\subseteq}(\overline{F}, A)\tilde{\subseteq}(F^*, A)$.

Remark 4.16. Suppose $E = X$, and for every soft set (F, A), for all $e \in A$, $F(e) = e$, then (F, A) is the same with the subset A, the above two kinds method coincide with Yao's definition [15].

Remark 4.17. In [12], suppose X is a universal set, E is a parameter set. R is a equivalence relation on X, then (X, R) is a Pawlak approximation space, R generates a partition on $X/R = \{[x]_R \mid x \in X\}$.

For a soft set (F, A), the lower and upper approximations $(F_*, A), (F^*, A)$ of (F, A) were defined in [12], for every $t \in A$,

$$F_*(t) = \{x \mid [x]_R \subseteq F(t)\}, \qquad F^*(t) = \{x \mid [x]_R \cap F(t) \neq \varnothing\}.$$

In fact, given a binary relation $R \subseteq X \times X$, we think it is a special soft relation,

$$\varphi : E \to 2^{X \times X}$$

for every $t \in E$, $\varphi(t) = R$. By Method I, or II, we also obtain the above formula.

5. Conclusions

In the paper, the two rough operators on SF(X) were defined by a soft relation φ and its union $\bigcup \varphi$, respectively. Some of their basic properties were investigated, and some related works in the literatures were discussed.

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References

[1] Abhijit Saha, Anjan Mukherjee, Soft interval-valued intuitionistic fuzzy rough sets, Ann. Fuzzy math. Inform. 9 (2) (2015) 279–292.
[2] B. Ahmad and A. Kharal, On Fuzzy Soft Sets, Advances in Fuzzy Systems 2009 (2009) 1–6.
[3] M.I. Ali, A note on soft sets, rough sets and fuzzy sets, Appl. Soft Comput. 11 (2011) 3329–3332.
[4] M. I. Ali, Feng Feng, Xiaoyan Liu, Won Keun Min and M. Shabir, On some new operations in soft set theory, Comput. Math. Appl. 57 (2009) 1547–1553.
[5] M. I. Ali and M. Shabir, Comments on De Morgan's law in fuzzy soft sets, J. Fuzzy Math. 18 (3) (2010) 679–686.
[6] Xueyou Chen, Rough soft sets in fuzzy setting, Lecture notes in computer science 7982 (2013) 530–539.
[7] F. Feng, C. Li, B. Davvaz and M. I. Ali, Soft sets combined with fuzzy sets and rough sets a tentative approach, Soft Comput. 14 (6) (2010) 899–911.
[8] F. Feng, X. Y Liu, Violeta Leoreanu-Fotea and Young Bae Jun, Soft sets and soft rough sets, Inform. Sci. 181 (2011) 1125–1137.
[9] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, J. Fuzzy Math. 9 (3) (2001) 589–602.
[10] P. K. Maji, R. Biswas and A. R. Roy, Soft Set Theory, Comput. Math. Appl. 45 (2003), 555–562.
[11] Majumdar, Pinaki and S. K. Samanta, On soft mappings, Comput. Math. Appl. 60 (2010) 2666–2672.
[12] Dan Meng, Xiaohong Zhao and Keyun Qin, Soft rough fuzzy sets and soft fuzzy rough sets, Comput. Math. Appl. 62 (2011) 4635–4645.
[13] D. Molodtsov, Soft Set Theory First Results, Comput. Math. Appl. 37 (1999) 19–31.
[14] Z. Pawlak, Rough sets, International Journal of Computer and Information Sciences 11 (1982) 341–356.
[15] Y. Y. Yao, Constructive and algebraic methods of the theory of rough sets, Inform. Sci. 109 (1998) 21–47.
[16] Y. Zhang and X. Yuan, Soft relations and fuzzy soft relation, Advances in Intelligent systems and computing 211 (2014) 205–213.

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