Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 10, No. 6, (December 2015), pp. 971–978
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

On soft open and closed mapping

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Received 4 March 2015; Revised 30 March 2015; Accepted 18 May 2015

Abstract. In this paper, we define soft compact metric space in terms of soft totally bounded and soft complete space and discussed some results of soft compact space. We define soft open and soft closed maps and drive some necessary and sufficient conditions for soft metric to be soft open or soft closed map. We also prove that every soft continuous map of a soft compact space is a soft closed map.

2010 AMS Classification: 06D72, 54A40

Keywords: Soft set, Soft bounded, Soft metric, Soft open, Soft continuous, Soft Lipschitz, Soft complete.

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1. INTRODUCTION

In real life, there are many complicated problems in economics, engineering, environmental, social science, medical science etc that involve uncertainties, vagueness that can not be solved by classical methods. In order to overcome these, Molodstov [7], in the year 1999, introduced the soft set theory as a new mathematical tool. The concept of soft set theory has a rich potential for applications in several directions. There is no need of membership function in soft set theory and hence very convenient and easy to apply in practise. As this area is new, interesting and has many applications, many researchers started working in this area. Researchers like P. K. Maji et al [6], F. Feng, C. X. Li, B. Davvaz and M. I. Ali [5], K. V. Babitha, J. Sunil, [1], M. Shabbir et al.[8], Sujao Das and Samanta [3, 4], Sadi Bayrramov and CigdemGunduz [2], B. Surendranath Reddy and Sayyed Jalil[9] etc. have contributed to the development of soft set theory.

In this paper, section 2 gives the required preliminaries. In section 3, we define soft compact metric space in terms of soft complete and soft totally bounded metrics and drive necessary and sufficient conditions for soft compact metric space. We also prove that every soft continuous image of soft compact metric space is soft compact and every soft closed subset of soft compact set is soft compact. In section 4 we introduce the concept of soft open mapping and soft closed mapping and derive some necessary and sufficient conditions for a soft open or soft closed mapping. We also prove that soft continuous map on a soft compact set is soft closed.

2. Preliminaries

In this section we recall basic definitions and results about soft sets.

Definition 2.1 ([7]). Let U be the universe and E be the set of parameters. Let $P(U)$ denote the power set of U and A be a non empty subset of E. A pair (F, A) is called a soft set over U where F is given by $F : A \to P(U)$.

In other words, the soft set is a parameterized family of subsets of the set U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-elements of the set (F, A), or as the set of ε-approximate elements of the soft set.
i.e. (F, A) is given as consisting of collection of approximations: $(F, A) = \{F(\varepsilon) | \, \varepsilon \in A\}$.

Definition 2.2 ([5]). For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is soft subset of (G, B) if

(1) $A \subseteq B$ and
(2) for all $e \in A$, $\quad F(e) \subseteq G(e)$

and it is denoted by $(F, A)\tilde{\subset}(G, B)$.

Definition 2.3 ([5]). Two soft sets (F, A) and (G, B) over a common universe U are said to be equal if (F, A) is soft subset of (G, B) and (G, B) is soft subset of (F, A).

Definition 2.4 ([5]). The complement of a soft set (F, A) over U is denoted by $(F, A)^c$ and is defined as $(F, A)^c = (F^c, A)$, where $F^c : A \to P(U)$ is a mapping given by $F^c(\lambda) = U - F(\lambda) = F(\lambda)^c$, for all $\lambda \in A$
i.e.$(F, A)^c = \{F(e_i)^c$, for all, $e_i \in A\}$.

Definition 2.5 ([6]). A soft set (F, E) over U is said to be a absolute soft set denoted by $\tilde{U}$ if $F(e) = U$ for all $e \in E$.

Definition 2.6 ([3]). Let X be a non empty set and E be a non empty parameter set then the function $\varepsilon : E \to X$ is said to be soft element of X.
A soft element ε is said to belongs to a soft set (F, A) of X if $\varepsilon(e) \in F(e)$, for all $e \in A$ and is denoted by $\varepsilon\tilde{\in}(F, A)$.

Definition 2.7 ([3]). Let $\mathbb{R}$ be the set of real numbers and $\mathfrak{B}(\mathbb{R})$ be the collection of all non empty bounded subsets of $\mathbb{R}$ and A be a set of parameters. Then the mapping $F : A \to \mathfrak{B}(\mathbb{R})$ is called a soft real set. It is denoted by (F, A). In particular, if (F, A) is singleton soft set then identifying (F, A) with the corresponding soft element, it will be called a soft real number.
We denote soft real numbers by $\tilde{r}, \tilde{s}, \tilde{t}$ and $\bar{r}, \bar{s}, \bar{t}$ will denote a particular type of soft numbers such that $\bar{r}(\lambda) = r$, for all $\lambda \in A$ etc.
For example $\bar{0}(\lambda) = 0$ and $\bar{1}(\lambda) = 1$, for all $\lambda \in A$.

Definition 2.8 ([3]). A soft set (P, A) over X is said to be a soft point if there is exactly one $\lambda \in A$, such that $P(\lambda) = x$, for some $x \in X$ and $P(\mu) = \varnothing$, for all $\mu \in A \backslash \{\lambda\}$. It is denoted by P^x_λ.

Definition 2.9 ([3]). A soft point P_λ^x is said to belong to a soft set (F, A) if $\lambda \in A$ and $P(\lambda) = \{x\} \subset F(\lambda)$ and we write $P_\lambda^x \tilde{\in} (F, A)$.

Definition 2.10 ([3]). Two soft points P_λ^x and P_μ^y are said to be equal if $\lambda = \mu$ and $P(\lambda) = P(\mu)$ i.e $x = y$. Thus $P_\lambda^x \neq P_\mu^y$ if and only if $x \neq y$ or $\lambda \neq \mu$.
Let X be an initial universal set and A be a non empty set of parameters. Let $\tilde{X}$ be the absolute soft set. Let $SP(\tilde{X})$ be the collection of all soft points of $\tilde{X}$. Let $\mathbb{R}(A^*)$ denote the set of all non negative soft real numbers.

Definition 2.11 ([4]). A mapping $\tilde{d} : SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(A^*)$ is said to be a soft metric on the soft set $\tilde{X}$ if

(1) $\tilde{d}(P_\lambda^x, P_\mu^y) \tilde{\geq} \tilde{0}$ for all $P_\lambda^x, P_\mu^y \tilde{\in} \tilde{X}$,
(2) $\tilde{d}(P_\lambda^x, P_\mu^y) = \tilde{0}$ if and only if $P_\lambda^x = P_\mu^y$,
(3) $\tilde{d}(P_\lambda^x, P_\mu^y) = \tilde{d}(P_\mu^y, P_\lambda^x)$ for all $P_\lambda^x, P_\mu^y \tilde{\in} \tilde{X}$,
(4) $\tilde{d}(P_\lambda^x, P_\mu^y) \tilde{\leq} \tilde{d}(P_\lambda^x, P_\gamma^z) + \tilde{d}(P_\gamma^z, P_\mu^y)$, for all $P_\lambda^x, P_\mu^y, P_\gamma^z \tilde{\in} \tilde{X}$.

The soft set $\tilde{X}$ with the soft metric $\tilde{d}$ on $\tilde{X}$ is called a soft metric space and denoted by $(\tilde{X}, \tilde{d}, E)$ *or* $(\tilde{X}, \tilde{d})$.

Definition 2.12 ([4]). Let $(\tilde{X}, \tilde{d}, E)$ be a soft metric space and $\tilde{r}$ be a non negative soft real number. Then the soft set $B(P_\lambda^x, \tilde{r}) = \{P_\mu^y \in SP(\tilde{X}) : \tilde{d}(P_\lambda^x, P_\mu^y) \tilde{<} \tilde{r}\}$ is called soft open ball with center P_λ^x and of radius $\tilde{r}$.

Definition 2.13 ([4]). Let (Y, A) be a soft subset of metric space $(\tilde{X}, \tilde{d}, E)$. Then the interior of a soft set is denoted as $(Y, A)^\circ$ and is given by

$$(Y, A^\circ) = \{P_\lambda^x \tilde{\in} (Y, A) | P_\lambda^x \tilde{\in} B(P_\lambda^x, \tilde{r}\} \tilde{\subset} (Y, A), \text{for some } \tilde{r} \tilde{>} \tilde{0}\}.$$

Definition 2.14 ([4]). Let $(\tilde{X}, \tilde{d}, E)$ be a soft metric space and $(Y, A) \tilde{\subset} \tilde{X}$. Then the soft set generated by the collection of all soft points of (Y, A) and soft limit points of (Y, A) in $(\tilde{X}, d)$ is said to be the soft closure of (Y, A) in $(\tilde{X}, \tilde{d}, E)$. It is denoted by $\overline{(Y, A)}$.

Definition 2.15 ([9]). Let $(\tilde{X}, \tilde{d}, E)$ be a soft metric space and (Y, A) be a non null soft subset of $\tilde{X}$. Then we say that (Y, A) is soft bounded if there exists $P_\lambda^x \tilde{\in} \tilde{X}$ and a soft real number $\tilde{\epsilon} \tilde{>} \tilde{0}$ such that $(Y, A) \tilde{\subset} B(P_\lambda^x, \tilde{\epsilon})$.

Definition 2.16 ([9]). Let $(\tilde{X}, \tilde{d}, E)$ be a soft metric space and $(Y, A) \tilde{\subset} (\tilde{X}, E)$. We say that (Y, A) is soft totally bounded if for a given $\tilde{\epsilon} \tilde{>} \tilde{0}$, there exists an $\tilde{\epsilon}$-net for (Y, A) i.e., there exist finitely many soft points $P_{\lambda_i}^{x_i} \tilde{\in} \tilde{X}$ such that $(Y, A) \tilde{\subset} \tilde{\bigcup}_{i=1}^{n} B(P_{\lambda_i}^{x_i}, \tilde{\epsilon})$.

Definition 2.17 ([1]). Let $(\tilde{X}, \tilde{d}_1, E_1)$ and $(\tilde{Y}, \tilde{d}_2, E_2)$ be two soft metric spaces. The mapping $(f, \phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Y}, \tilde{d}_2, E_2)$ is called a soft mapping, where $f : X \to Y, \phi : E_1 \to E_2$ are two mappings.
The soft mapping $(f, \phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Y}, \tilde{d}_2, E_2)$ is one-one soft mapping if $(f, \phi)(P_\lambda^x) = (f, \phi)(P_\mu^y)$ then $P_\lambda^x = P_\mu^y$.
The soft mapping $(f, \phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Y}, \tilde{d}_2, E_2)$ is onto soft mapping if $(f, \phi)(X, E_1) = (Y, E_2)$.

Definition 2.18. Let $(\tilde{X}, \tilde{d}_1, E_1)$ and $(\tilde{Y}, \tilde{d}_2, E_2)$ be two soft metric spaces. The soft mapping $(f, \phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Y}, \tilde{d}_2, E_2)$ is said to be soft continuous at the soft point $P^x_\lambda \tilde{\in} SP(\tilde{X})$, if for every $\tilde{\epsilon} \tilde{>} \tilde{0}$, there exists a $\tilde{\delta} \tilde{>} \tilde{0}$ such that for any soft points $P^x_\lambda, P^y_\mu \tilde{\in} \tilde{X}$ with $\tilde{d}_1(P^x_\lambda, P^y_\mu) \tilde{\leq} \tilde{\delta}$, then $\tilde{d}_2\big((f,\phi)(P^x_\lambda), (f,\phi)(P^y_\mu)\big) \tilde{\leq} \tilde{\epsilon}$.

Definition 2.19. Let $(\tilde{X}, \tilde{d}_1, E_1)$ and $(\tilde{Y}, \tilde{d}_2, E_2)$ be two soft metric spaces. The soft mapping $(f, \phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Y}, \tilde{d}_2, E_2)$ is said to be soft uniformly continuous mapping if given any $\tilde{\epsilon} \tilde{>} \tilde{0}$, there exists a $\tilde{\delta} \tilde{>} \tilde{0}$ ($\tilde{\delta}$ depends only on $\tilde{\epsilon}$) for any soft points $P^x_\lambda, P^y_\mu \tilde{\in} \tilde{X}$ with $\tilde{d}_1(P^x_\lambda, P^y_\mu) \tilde{\leq} \tilde{\delta}$, then $\tilde{d}_2\big((f,\phi)(P^x_\lambda), (f,\phi)(P^y_\mu)\big) \tilde{\leq} \tilde{\epsilon}$.

Definition 2.20 ([4])**.** A soft metric space $(\tilde{X}, \tilde{d}, E)$ is called soft complete if every soft Cauchy sequence in $\tilde{X}$ converges to some soft point of $\tilde{X}$.

3. Soft compact space

Definition 3.1. A soft metric space $(\tilde{X}, \tilde{d}, E)$ is called soft compact if it is soft complete and soft totally bounded.

Theorem 3.2. *A soft metric space $(\tilde{X}, \tilde{d}, E)$ is soft compact if and only if every soft sequence in $\tilde{X}$ has a soft subsequence that converges to a soft point in $\tilde{X}$.*

Proof. Suppose $(\tilde{X}, \tilde{d}, E)$ is soft compact. This implies that $\tilde{X}$ is soft totally bounded and soft complete. Let $\{P^{x_n}_{\lambda_n}\}$ be a soft sequence in $\tilde{X}$. Since $\tilde{X}$ is soft totally bounded, the sequence $\{P^{x_n}_{\lambda_n}\}$ has a soft cauchy subsequence $\{P^{x_{n_k}}_{\lambda_{n_k}}\}$.
As $\tilde{X}$ is soft complete, $\{P^{x_{n_k}}_{\lambda_{n_k}}\}$ converges in $\tilde{X}$. Therefore, every soft sequence in $\tilde{X}$ has a convergent soft subsequence in $\tilde{X}$.
Conversely, suppose every soft sequence in $\tilde{X}$ has a convergent soft subsequence in $\tilde{X}$. Since convergent implies cauchy, $\tilde{X}$ is soft totally bounded.
Now to show that $\tilde{X}$ is soft complete, let $\{P^{x_n}_{\lambda_n}\}$ be a soft cauchy sequence in $\tilde{X}$. Since every soft sequence in $\tilde{X}$ has a cauchy subsequence, $\{P^{x_n}_{\lambda_n}\}$ has a soft subsequence $\{P^{x_{n_k}}_{\lambda_{n_k}}\}$ which converges in $\tilde{X}$, and therefore $\{P^{x_n}_{\lambda_n}\}$ also converges.
Therefore, $\tilde{X}$ is soft complete and hence $\tilde{X}$ is soft compact. □

Theorem 3.3. *If $(f, \phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Y}, \tilde{d}_2, E_2)$ is soft continuous and (H, A) is soft compact in $\tilde{X}$ then $(f, \phi)(H, A)$ is soft compact in $\tilde{Y}$.*

Proof. To show that $(f, \phi)(H, A)$ is soft compact. Let $\{P^{y_n}_{\mu_n}\}$ be a soft sequence in $(f, \phi)(H, A). Then (P^{y_n}_{\mu_n}) = (f, \phi)(P^{x_n}_{\lambda_n})$, where $P^{x_n}_{\lambda_n} \tilde{\in} (H, A)$. Thus $\{P^{x_n}_{\lambda_n}\}$ is a soft sequence in the soft compact space (H, A). So $\{P^{x_n}_{\lambda_n}\}$ has a soft subsequence $\{P^{x_{n_m}}_{\lambda_{n_m}}\}$ such that $P^{x_{n_m}}_{\lambda_{n_m}} \to P^x_\lambda$, for some $P^x_\lambda \tilde{\in} (H, A)$.
Now, since (f, ϕ) is soft continuous,

$$(f, \phi)(P^{x_{n_m}}_{\lambda_{n_m}}) \to (f, \phi)(P^x_\lambda) \text{ where } (f, \phi)(P^x_\lambda) \tilde{\in} (f, \phi)(H, A).$$

Thus $P^{y_n}_{\mu_n}$ has a soft sequence $\{P^{y_{n_m}}_{\mu_{n_m}}\}$ which converges in $(f, \phi)(H, A)$.
This implies $(f, \phi)(H, A)$ is soft compact. □

Theorem 3.4. *Let (H, A) be a soft subset of a soft metric space $(\tilde{X}, \tilde{d}, E)$. If (H, A) is soft compact, then (H, A) is soft closed in $\tilde{X}$. If $\tilde{X}$ is soft compact and (H, A) is soft closed, then (H, A) is soft compact.*

Proof. Suppose that (H, A) is soft compact. Let $\{P^{x_n}_{\lambda_n}\}$ be a soft sequence in (H, A) that converges to a soft point in P^x_λ in $\tilde{X}$. Then, by Theorem 3.2, $\{P^{x_n}_{\lambda_n}\}$ has a soft subsequence that converges in (H, A), and hence $P^x_\lambda \in (H, A)$. Thus, (H, A) is soft closed.
Next, suppose that $\tilde{X}$ is soft compact and (H, A) is soft closed in $\tilde{X}$. Let $\{P^{x_n}_{\lambda_n}\}$ be any soft sequence in (H, A). Then by Theorem 3.2, there is a soft subsequence of $\{P^{x_n}_{\lambda_n}\}$ that converges to a soft point $P^x_\lambda \tilde{\in} \tilde{X}$. As (H, A) is soft closed, we must have $P^x_\lambda \tilde{\in} (H, A)$. Thus (H, A) is soft compact. □

4. Soft open mapping

Definition 4.1. Let $(\tilde{X}, \tilde{d}_1, E_1)$ and $(\tilde{Y}, \tilde{d}_2, E_2)$ be two soft metric spaces, and $(f, \phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Y}, \tilde{d}_2, E_2)$ be a soft mapping then (f, ϕ) is said to be a soft open mapping if for each soft open set (H, A) in $(\tilde{X}, \tilde{d}_1, E_1)$, the image set $(f, \phi)(H, A)$ is soft open in $(\tilde{Y}, \tilde{d}_2, E_2)$.
Similarly, (f, ϕ) is said to be a soft closed mapping if for each soft closed set (H', A') in $(\tilde{X}, \tilde{d}_1, E_1)$, the image set $(f, \phi)(H', A')$ is soft closed in $(\tilde{Y}, \tilde{d}_2, E_2)$.

Theorem 4.2. *Let $(\tilde{X}, \tilde{d}_1, E_1)$ and $(\tilde{Y}, \tilde{d}_2, E_2)$ be two soft metric spaces.*
(1) If (f, ϕ) is soft open, then for each soft subset (H, A_2) of $(\tilde{Y}, \tilde{d}_2, E_2)$, and any soft closed set (K, A_1) containing $(f, \phi)^{-1}(H, A_2)$, there exists a soft closed set (H^*, A_2^*) in $(\tilde{Y}, \tilde{d}_2, E_2)$ containing (H, A_2), such that $(f, \phi)^{-1}(H^*, A_2^*) \tilde{\subseteq} (K, A_1)$.
(2) If (f, ϕ) is soft closed, then for each soft subset (H, A_2) of $(\tilde{Y}, \tilde{d}_2, E_2)$, and for any soft open set (K, A_1) containing $(f, \phi)^{-1}(H, A_2)$, there exists a soft open set (H^*, A_2^*) in $(\tilde{Y}, \tilde{d}_2, E_2)$ containing (H, A_2), such that $(f, \phi)^{-1}(H^*, A_2^*) \tilde{\subseteq} (K, A_1)$.

Proof.
(1) Let (K_1, A_1) be a soft closed and (f, ϕ) be soft open. Let

$$(H^*, A_2^*) = \big((f, \phi)(K, A_1)^c\big)^c = (Y, E_2) - (f, \phi)\big((X, E_1) - (K, A_1)\big).$$

Since $(K, A_1)^c$ is soft open, and (f, ϕ) is soft open, we have $(f, \phi)(K, A_1)^c$ is soft open. Thus $(H^*, A_2^*) = \big((f, \phi)(K, A_1)^c\big)^c$ is soft closed.
Since $(f, \phi)^{-1}(H, A_2) \tilde{\subseteq} (K, A_1)$, $\big((f, \phi)^{-1}(H, A_2)\big)^c \tilde{\supseteq} (K, A_1)^c$. So
$(f, \phi)^{-1}(H, A_2)^c \tilde{\supseteq} (K, A_1)^c$
$\Rightarrow (H, A_2)^c \tilde{\supseteq} (f, \phi)(K, A_1)^c$
$\Rightarrow (H, A_2) \tilde{\subseteq} \big((f, \phi)(K, A_1)^c\big)^c$
$i.e. (H, A_2) \tilde{\subseteq} (H^*, A_2^*)$.
Hence,

$$\begin{aligned}(f, \phi)^{-1}(H^*, A_2^*) &= (f, \phi)^{-1}(Y, E_2) - (f, \phi)^{-1}(f, \phi)\big((X, E_1) - (K, A_1)\big) \\ &\tilde{\subseteq} (X, E_1) - \big((X, E_1) - (K, A_1)\big) = (K, A_1).\end{aligned}$$

Therefore, $(f, \phi)^{-1}(H^*, A_2^*) \tilde{\subseteq} (K, A_1)$.

(2) Let (K, A_1) be a soft open and (f, ϕ) be soft closed. Let $(H^*, A_2^*) = (Y, E_2) - (f, \phi)\big((X, E_1) - (K, A_1)\big)$. Since $(K, A_1)^c$ is soft closed and (f, ϕ) is soft closed, $(f, \phi)(K, A_1)^c$ is soft closed. Thus $(H^*, A_2^*) = \big((f, \phi)(K, A_1)^c\big)^c$ is soft open.
Since $(f, \phi)^{-1}(H, A_2)\tilde{\subseteq}(K, A_1)$, $\big((f, \phi)^{-1}(H, A_2)\big)^c\tilde{\supseteq}(K, A_1)^c$. So
$(f, \phi)^{-1}(H, A_2)^c\tilde{\supseteq}(K, A_1)^c$
$\Rightarrow (H, A_2)^c\tilde{\supseteq}(f, \phi)(K, A_1)^c$
$\Rightarrow (H, A_2)\tilde{\subseteq}\big((f, \phi)(K, A_1)^c\big)^c i.e.(H, A_2)\tilde{\subseteq}(H^*, A_2^*)$.
Hence,

$$\begin{aligned}(f, \phi)^{-1}(H^*, A_2^*) &= (f, \phi)^{-1}(Y, E_2) - (f, \phi)^{-1}(f, \phi)\big((X, E_1) - (K, A_1)\big)\\ &\tilde{\subseteq}(X, E_1) - \big((X, E_1) - (K, A_1)\big) = (K, A_1).\end{aligned}$$

Therefore, $(f, \phi)^{-1}(H^*, A_2^*)\tilde{\subseteq}(K, A_1)$.

□

Theorem 4.3. *Let $(\tilde{X}, \tilde{d}_1, E_1)$ and $(\tilde{Y}, \tilde{d}_2, E_2)$ be soft metric spaces, and $(f, \phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Y}, \tilde{d}_2, E_2)$ be a soft mapping then the following are equivalent.*
(1) (f, ϕ) is soft open mapping.
(2) $(f, \phi)[(H, A)^\circ]\tilde{\subseteq}\big((f, \phi)(H, A)\big)^\circ$ for each soft subset (H, A) of $\tilde{X}$
(3) For each soft point $P_\lambda^x \in \tilde{X}$ and a soft neighborhood (N, A_1) of P_λ^x, there exists a soft neighborhood (M, A_2) in (Y, E_2) of $(f, \phi)(P_\lambda^x)$ such that $(M, A_1)\tilde{\subseteq}(f, \phi)(N, A_2)$.

Proof. (1) ⇒ (2) Since $(H, A)^\circ\tilde{\subseteq}(H, A)$, we get
$(f, \phi)(H, A)^\circ\tilde{\subseteq}(f, \phi)(H, A)$. Since $(H, A)^\circ$ is soft open, $(f, \phi)(H, A)^\circ$ is soft open set. Thus $(f, \phi)(H, A)^\circ = \big((f, \phi)(H, A)^\circ\big)^\circ\tilde{\subseteq}\big((f, \phi)(H, A)\big)^\circ$.
(2) ⇒ (3) Let (N, A_1) be a soft neighborhood of a soft point P_λ^x in $\tilde{X}$. Then there exists a soft open ball $B(P_\lambda^x, \tilde{\epsilon})$ contained in (N, A_1).
From (2), we have $(f, \phi)\big(B(P_\lambda^x, \tilde{\epsilon})\big)^\circ\tilde{\subseteq}\big((f, \phi)B(P_\lambda^x, \tilde{\epsilon})\big)^\circ$.*Thus*
$(f, \phi)(P_\lambda^x)\tilde{\in}(f, \phi)\big(B(P_\lambda^x, \tilde{\tilde{\epsilon}})\big)\tilde{\subseteq}\big((f, \phi)B(P_\lambda^x, \tilde{\epsilon})\big)^\circ\tilde{\subseteq}(f, \phi)(N, A_1)$.
So $(M, A_2) = \big((f, \phi)B(P_\lambda^x, \tilde{\epsilon})\big)^\circ$ is a soft neighborhood of $(f, \phi)(P_\lambda^x)$ in $(\tilde{Y}, \tilde{d}_2, E_2)$.
(3) ⇒ (1) Let (H, A) be a soft open in $(\tilde{X}, \tilde{d}_1, E_1)$ and P_μ^y be a soft point in $(f, \phi)(H, A)$. Then there exists a soft neighborhood $(M_{P_\mu^y}, A_2)$ in (Y, E_2) of P_μ^y such that $(M_{P_\mu^y}, A_2)\tilde{\subseteq}(f, \phi)(H, A)$.
$(f, \phi)(H, A) = \bigcup_{P_\mu^y \in (f, \phi)(H, A)} (M_{P_\mu^y}, A_2)$. Therefore $(f, \phi)(H, A)$ is soft open in $(\tilde{Y}, \tilde{d}_2, E_2)$.

□

Theorem 4.4. *Let $(\tilde{X}, \tilde{d}_1, E_1)$ and $(\tilde{Y}, \tilde{d}_2, E_2)$ be soft metric spaces. A soft mapping $(f, \phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Y}, \tilde{d}_2, E_2)$ is soft closed if and only if $\overline{(f, \phi)(H, A)}\tilde{\subseteq}(f, \phi)\overline{(H, A)}$ for every soft subset (H, A) of $(\tilde{X}, \tilde{d}_1, E_1)$.*

Proof. Let (f, ϕ) be soft closed and (H, A) be a soft subset of $\tilde{X}$.
Then $(f, \phi)\overline{(H, A)}$ is soft closed in $(\tilde{Y}, \tilde{d}_2, E_2)$. Since $(f, \phi)(H, A)\tilde{\subseteq}(f, \phi)\overline{(H, A)}$, we get
$\overline{(f, \phi)(H, A)}\tilde{\subseteq}\overline{(f, \phi)\overline{(H, A)}} = (f, \phi)\overline{(H, A)}$.
Conversely, assume that $\overline{(f, \phi)(H, A)}\tilde{\subseteq}(f, \phi)\overline{(H, A)}$ holds for each soft subset (H, A)

of $(\tilde{X}, \tilde{d}_1, E_1)$. Let (H_1, A_1) be a soft closed set in $(\tilde{X}, \tilde{d}_1, E_1)$. Then
$\overline{(f,\phi)(H_1,A_1)} \tilde{\subseteq} (f,\phi)\overline{(H_1,A_1)} = (f,\phi)(H_1,A_1)$
And we know $(f,\phi)(H_1,A_1) \tilde{\subseteq} \overline{(f,\phi)(H_1,A_1)}$ Therefore, $\overline{(f,\phi)(H_1,A_1)} = (f,\phi)(H_1,A_1)$
This means, $(f,\phi)(H_1,A_1)$ is soft closed set. Therefore, (f,ϕ) is a soft closed map. □

Theorem 4.5. *Let $(\tilde{X}, \tilde{d}_1, E_1)$ and $(\tilde{Y}, \tilde{d}_2, E_2)$ be two soft metric spaces, and $(f,\phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Y}, \tilde{d}_2, E_2)$ be a soft mapping. Then (f,ϕ) is soft continuous and soft closed map if and only if $(f,\phi)\overline{(H,A)} = \overline{(f,\phi)(H,A)}$ for every soft subset (H,A) of $(\tilde{X}, \tilde{d}_1, E_1)$.*

Proof. Let (f,ϕ) be soft closed and soft continuous. Since (f,ϕ) is soft continuous we know that,
$(f,\phi)\overline{(H,A)} \tilde{\subseteq} \overline{(f,\phi)(H,A)}$.
As (f,ϕ) is soft closed by Theorem 4.4, we get
$\overline{(f,\phi)(H,A)} \tilde{\subseteq} (f,\phi)\overline{(H,A)}. Thus (f,\phi)\overline{(H,A)} = \overline{(f,\phi)(H,A)}$.
Conversely, let $(f,\phi)\overline{(H,A)} = \overline{(f,\phi)(H,A)}$.
In particularly, let $(f,\phi)\overline{(H,A)} \tilde{\subseteq} \overline{(f,\phi)(H,A)}. Then (f,\phi)$ is soft continuous. So, by Theorem 4.4, it is soft closed. □

Theorem 4.6. *Let $(\tilde{X}, \tilde{d}_1, E_1)$, $(\tilde{Y}, \tilde{d}_2, E_2)$ and $(\tilde{Z}, \tilde{d}_3, E_3)$ be soft metric spaces and $(f,\phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Y}, \tilde{d}_2, E_2)$ and $(g,\psi) : (\tilde{Y}, \tilde{d}_2, E_2) \to (\tilde{Z}, \tilde{d}_3, E_3)$ are soft mappings.*
(1) If (f,ϕ) is soft continuous and onto, and $(g,\psi) \circ (f,\phi)$ is soft open, then (g,ψ) is soft open.
(2) If (g,ψ) is soft continuous and one-one, $(g,\psi) \circ (f,\phi)$ is soft open, then (f,ϕ) is soft open.

Proof.
(1)Let (H,A) be soft open in $(\tilde{Y}, \tilde{d}_2, E_2)$. Since (f,ϕ) is soft continuous, $(f,\phi)^{-1}(H,A)$ is soft open in $(\tilde{X}, \tilde{d}_1, E_1)$.
As $(g,\psi) \circ (f,\phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Z}, \tilde{d}_3, E_3)$ is soft open map,
$\big((g,\psi) \circ (f,\phi)\big)\big((f,\phi)^{-1}(H,A)\big)$ is soft open in $(\tilde{Z}, \tilde{d}_3, E_3)$. Thus
$(g,\psi)\big((f,\phi) \circ (f,\phi)^{-1}(H,A)\big)$ is soft open in $(\tilde{Z}, \tilde{d}_3, E_3)$.
Since (f,ϕ) is onto, $\big((f,\phi) \circ (f,\phi)^{-1}\big)(H,A) = (H,A)$. So $(g,\psi)(H,A)$ is soft open in $(\tilde{Z}, \tilde{d}_3, E_3)$. Hence (g,ψ) is a soft open map.

(2) Let (K,B) be soft open in $(\tilde{X}, \tilde{d}_1, E_1)$. Since $(g,\psi) \circ (f,\phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Z}, \tilde{d}_3, E_3)$ is soft open map, $\big((g,\psi) \circ (f,\phi)\big)(K,B)$ is soft open in $(\tilde{Z}, \tilde{d}_3, E_3)$.
Since (g,ψ) is soft continuous, $(g,\psi)^{-1}\big((g,\psi)\circ(f,\phi)(K,B)\big)$ is soft open in $(\tilde{Y}, \tilde{d}_2, E_2)$.
Thus $\big((g,\psi)^{-1} \circ (g,\psi)\big)\big((f,\phi)(K,B)\big)$ is soft open in $(\tilde{Y}, \tilde{d}_2, E_2)$.
Since (g,ψ) is soft injective, $\big((g,\psi)^{-1} \circ (g,\psi)\big)\big((f,\phi)(K,B)\big) = (f,\phi)(K,B)$.
So $(f,\phi)(K,B)$ is soft open in $(\tilde{Y}, \tilde{d}_2, E_2)$. Hence (f,ϕ) is a soft open map.

□

Remark 4.7. The above theorem holds for soft closed map as proof goes in similar lines.

Theorem 4.8. *If* $(\tilde{X}, \tilde{d}_1, E_1)$ *is soft compact and* $(f, \phi) : (\tilde{X}, \tilde{d}_1, E_1) \rightarrow (\tilde{Y}, \tilde{d}_2, E_2)$ *is soft continuous, then* (f, ϕ) *is a soft closed map.*

Proof. Let (H, A) be soft closed in $(\tilde{X}, \tilde{d}_1, E_1)$. Since $(\tilde{X}, \tilde{d}_1, E_1)$ is soft compact, (H, A) is soft compact. Also as (f, ϕ) is soft continuous therefore $(f, \phi)(H, A)$ is soft compact. Then by theorem 3.4, $(f, \phi)(H, A)$ is soft closed.
Therefore, (f, ϕ) is a soft closed map. □

5. Acknowledgements

One of the author is thankful to University Grants Commission(UGC), India, for providing financial support thorough MAULANA AZAD NATIONAL FELLOWSHIP.

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