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# Weakly generalized connectedness in intuitionistic fuzzy topological spaces

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ABSTRACT. The aim of this paper is to introduce weakly generalized connected spaces in intuitionistic fuzzy topological spaces and study some of their properties. We also investigate their characterizations and basic properties.

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## 1. INTRODUCTION

Connectedness is one of the basic notions in topology. Turanli et.al [10] introduced several types of fuzzy connectedness in intuitionistic fuzzy topological spaces defined by Coker and investigated some interrelations between them together with the preservation properties under fuzzy continuous functions in 2000. Thakur et.al [8] introduced and studied the concept of intuitionistic fuzzy GO-connected space in 2006. In [9], Thakur et.al introduced and discussed the concept of intuitionistic fuzzy GO-connectedness between sets in 2010. Recently the concept of fuzzy connectedness had been generalized in intuitionistic fuzzy topological spaces.

In this paper, we introduce intuitionistic fuzzy weakly generalized connected spaces and investigate some of their basic properties. Also, we study intuitionistic fuzzy weakly generalized connectedness between two intuitionistic fuzzy sets.

### 2. Preliminaries

**Definition 2.1** ([1]). Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form  $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$  where the functions  $\mu_A(x) : X \to [0,1]$  and  $\nu_A(x) : X \to [0,1]$  denote the degree of membership (namely  $\mu_A(x)$ ) and the degree of non-membership (namely  $\nu_A(x)$ ) of each element  $x \in X$  to the set A, respectively, and  $0 \le \mu_A(x) + \nu_A(x) \le 1$  for each  $x \in X$ .

**Definition 2.2** ([1]). Let A and B be IFSs of the forms  $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$ and  $B = \{\langle x, \mu_B(x), \nu_B(x) \rangle : x \in X\}$ . Then

(a)  $A \subseteq B$  if and only if  $\mu_A(x) \leq \mu_B(x)$  and  $\nu_A(x) \geq \nu_B(x)$  for all  $x \in X$ , (b) A = B if and only if  $A \subseteq B$  and  $B \subseteq A$ , (c)  $A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle : x \in X\}$ , (d)  $A \cap B = \{\langle x, \mu_A(x) \land \mu_B(x), \nu_A(x) \lor \nu_B(x) \rangle : x \in X\}$ , (e)  $A \cup B = \{\langle x, \mu_A(x) \lor \mu_B(x), \nu_A(x) \land \nu_B(x) \rangle : x \in X\}$ .

For the sake of simplicity, the notation  $A = \langle x, \mu_A, \nu_A \rangle$  shall be used instead of  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ . Also for the sake of simplicity, we shall use the notation  $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle$  instead of  $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$ .

The intuitionistic fuzzy sets  $0_{\sim} = \{\langle x, 0, 1 \rangle : x \in X\}$  and  $1_{\sim} = \{\langle x, 1, 0 \rangle : x \in X\}$  are the empty set and the whole set of X, respectively.

**Definition 2.3** ([3]). An intuitionistic fuzzy topology (IFT in short) on a non empty set X is a family  $\tau$  of IFSs in X satisfying the following axioms: (a) $0_{\sim}, 1_{\sim} \in \tau$ , (b) $G_1 \cap G_2 \in \tau$  for any  $G_1, G_2 \in \tau$ , (c) $\cup G_i \in \tau$  for any arbitrary family  $\{G_i : i \in J\} \subseteq \tau$ .

In this case, the pair  $(X, \tau)$  is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in  $\tau$  is known as an intuitionistic fuzzy open set (IFOS in short) in X.

The complement  $A^c$  of an IFOS A in an IFTS  $(X, \tau)$  is called an intuitionistic fuzzy closed set (IFCS in short) in X.

**Definition 2.4** ([3]). Let  $(X, \tau)$  be an IFTS and  $A = \langle x, \mu_A, \nu_A \rangle$  be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

 $int(A) = \bigcup \{G : G \text{ is an IFOS in } X \text{ and } G \subseteq A\},\\ cl(A) = \cap \{K : K \text{ is an IFCS in } X \text{ and } A \subseteq K\}.$ 

Note that for any IFS A in  $(X, \tau)$ , we have  $cl(A^c) = (int(A))^c$  and  $int(A^c) = (cl(A))^c$ .

**Definition 2.5** ([4]). An IFS  $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$  in an IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy weakly generalized closed set (IFWGCS in short)

if  $cl(int(A)) \subseteq U$  whenever  $A \subseteq U$  and U is an IFOS in X.

The family of all IFWGCSs of an IFTS  $(X, \tau)$  is denoted by IFWGC(X).

**Definition 2.6** ([4]). An IFS  $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$  is said to be an intuitionistic fuzzy weakly generalized open set (IFWGOS in short) in an IFTS  $(X, \tau)$  if the complement  $A^c$  is an IFWGCS in X.

The family of all IFWGOSs of an IFTS  $(X, \tau)$  is denoted by IFWGO(X).

**Remark 2.7** ([4]). Every IFCS, IF $\alpha$ CS, IFGCS, IFRCS, IF $\alpha$ GCS is an IFWGCS but the converses need not be true in general.

**Definition 2.8** ([5]). Let  $(X, \tau)$  be an IFTS and  $A = \langle x, \mu_A, \nu_A \rangle$  be an IFS in X. Then the intuitionistic fuzzy weakly generalized interior and an intuitionistic fuzzy weakly generalized closure are defined by

wgint(A)=  $\cup$  { $G : G is an IFWGOS in X and G \subseteq A$ }, wgcl(A)=  $\cap$  { $K : K is an IFWGCS in X and A \subseteq K$ }.

**Definition 2.9.** Let  $f: (X, \tau) \to (Y, \sigma)$  be a mapping from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$ . Then f is said to be

(a)([6]) intuitionistic fuzzy weakly generalized continuous (IFWG continuous in short) if  $f^{-1}(B)$  is an IFWGCS in X for every IFCS B in Y,

(b) ([5]) intuitionistic fuzzy weakly generalized irresolute (IFWG irresolute in short) if  $f^{-1}(B)$  is an IFWGCS in X for every IFWGCS B in Y.

**Definition 2.10** ([2]). Two IFSs A and B in X are said to be q-coincident (AqB in short) if and only if there exists an element  $x \in X$  such that  $\mu_A(x) > \nu_B(x)$  or  $\nu_A(x) < \mu_B(x)$ .

**Definition 2.11** ([2]). Two IFSs A and B in X are said to be not q-coincident  $(Aq^cB \text{ in short})$  if and only if  $A \subseteq B^c$ .

**Definition 2.12** ([10]). An IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy  $C_5$ connected space if the only IFSs which are both intuitionistic fuzzy open and intuitionistic fuzzy closed are  $0_{\sim}$  and  $1_{\sim}$ .

**Definition 2.13** ([8]). An IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy GOconnected space if the only IFSs which are both intuitionistic fuzzy generalized open and intuitionistic fuzzy generalized closed are  $0_{\sim}$  and  $1_{\sim}$ .

**Definition 2.14** ([7]). An IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy alpha generalized connected space if the only IFSs which are both intuitionistic fuzzy alpha generalized open and intuitionistic fuzzy alpha generalized closed are  $0_{\sim}$  and  $1_{\sim}$ .

**Definition 2.15** ([4]). An IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy  ${}_{wg}T_{1/2}$  space (IF ${}_{wg}T_{1/2}$  space in short) if every IFWGCS in X is an IFCS in X.

**Definition 2.16** ([4]). An IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy  ${}_{wg}T_p$  space  $(IF_{wg}T_p$  space in short) if every IFWGCS in X is an IFPCS in X.

#### 3. Intuitionistic fuzzy weakly generalized connected space

In this section, we introduce intuitionistic fuzzy weakly generalized connected space and study some of their properties. Also we provide characterization theorems for intuitionistic fuzzy weakly generalized connected space.

**Definition 3.1.** An IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy weakly generalized connected space if the only IFSs which are both intuitionistic fuzzy weakly generalized open and intuitionistic fuzzy weakly generalized closed are  $0_{\sim}$  and  $1_{\sim}$ .

**Example 3.2.** Let  $X = \{a, b\}$  and  $\tau = \{0_{\sim}, T, 1_{\sim}\}$  be an IFT on X where  $T = \langle x, (0.5, 0.6), (0.5, 0.4) \rangle$ . Then  $(X, \tau)$  is an intuitionistic fuzzy weakly generalized connected space.

**Theorem 3.3.** Every intuitionistic fuzzy weakly generalized connected space is an intuitionistic fuzzy  $C_5$ -connected space but not conversely.

*Proof.* Let  $(X, \tau)$  be an intuitionistic fuzzy weakly generalized connected space. Suppose  $(X, \tau)$  is not an intuitionistic fuzzy  $C_5$ -connected space, then there exists a proper IFS A which is both intuitionistic fuzzy open and intuitionistic fuzzy closed in X. That is, A is both intuitionistic fuzzy weakly generalized open and intuitionistic fuzzy weakly generalized closed in X. This implies that  $(X, \tau)$  is not an intuitionistic fuzzy weakly generalized connected space. Therefore we get contradiction. Hence  $(X, \tau)$  is an intuitionistic fuzzy  $C_5$ -connected space.

**Example 3.4.** Let  $X = \{a, b\}$  and  $\tau = \{0_{\sim}, T_1, T_2, 1_{\sim}\}$  be an *IFT* on X where  $T_1 = \langle x, (0.2, 0.2), (0.7, 0.8) \rangle$ ,  $T_2 = \langle x, (0.6, 0.5), (0.4, 0.4) \rangle$ . Then  $(X, \tau)$  is an intuitionistic fuzzy  $C_5$ -connected space but not an intuitionistic fuzzy weakly generalized connected space, since the IFS  $A = \langle x, (0.6, 0.1), (0.2, 0.4) \rangle$  is both intuitionistic fuzzy weakly generalized open and intuitionistic fuzzy weakly generalized closed in X.

**Theorem 3.5.** Every intuitionistic fuzzy weakly generalized connected space is an intuitionistic fuzzy GO-connected space.

*Proof.* Let  $(X, \tau)$  be an intuitionistic fuzzy weakly generalized connected space. Suppose  $(X, \tau)$  is not an intuitionistic fuzzy GO-connected space, then there exists a proper IFS A which is both intuitionistic fuzzy generalized open and intuitionistic fuzzy generalized closed in X. That is, A is both intuitionistic fuzzy weakly generalized closed in X. This implies that  $(X, \tau)$  is not an intuitionistic fuzzy weakly generalized connected space. Therefore we get contradiction. Hence  $(X, \tau)$  is an intuitionistic fuzzy GO-connected space.  $\Box$ 

**Theorem 3.6.** Every intuitionistic fuzzy weakly generalized connected space is an intuitionistic fuzzy alpha generalized connected space.

*Proof.* Let  $(X, \tau)$  be an intuitionistic fuzzy weakly generalized connected space. Suppose  $(X, \tau)$  is not an intuitionistic fuzzy alpha generalized connected space, then there exists a proper IFS A which is both intuitionistic fuzzy alpha generalized open and intuitionistic fuzzy alpha generalized closed in X. That is, A is both intuitionistic fuzzy weakly generalized open and intuitionistic fuzzy weakly generalized closed

in X. This implies that  $(X, \tau)$  is not an intuitionistic fuzzy weakly generalized connected space. Therefore we get contradiction. Hence  $(X, \tau)$  is an intuitionistic fuzzy alpha generalized connected space.

**Theorem 3.7.** An IFTS  $(X, \tau)$  is an intuitionistic fuzzy weakly generalized connected space if and only if there exists no non-zero IFWGOSs A and B in X such that  $A = B^c$ .

## *Proof.* Necessity:

Suppose A and B are IFWGOSs in X such that  $A \neq 0_{\sim} \neq B$  and  $A = B^c$ . Since  $A = B^c$ , B is an IFWGOS which implies that  $B^c = A$  is an IFWGCS and  $B \neq 0_{\sim}$ . This implies that  $B^c \neq 1_{\sim}$  (i.e)  $A \neq 1_{\sim}$ . Hence there exists a proper IFS A ( $A \neq 0_{\sim}, A \neq 1_{\sim}$ ) which is both intuitionistic fuzzy weakly generalized open and intuitionistic fuzzy weakly generalized closed in X. Hence  $(X, \tau)$  is not an intuitionistic fuzzy weakly generalized connected space. But this is contradictory to our hypothesis. Thus there exists no non-zero IFWGOSs A and B in X such that  $A = B^c$ . Sufficiency:

Let  $(X, \tau)$  be an IFTS and A is both intuitionistic fuzzy weakly generalized open and intuitionistic fuzzy weakly generalized closed in X such that  $1_{\sim} \neq A \neq 0_{\sim}$ . Now take  $B = A^c$ . In this case, B is an IFWGOS and  $A \neq 1_{\sim}$ , this implies that  $B = A^c \neq 0_{\sim}$ . Hence  $B \neq 0_{\sim}$ , which is a contradictory to our hypothesis. Therefore there is no proper IFS of X which is both intuitionistic fuzzy weakly generalized open and intuitionistic fuzzy weakly generalized closed in X. Hence  $(X, \tau)$  is an intuitionistic fuzzy weakly generalized connected space.

**Theorem 3.8.** An IFTS  $(X, \tau)$  is an intuitionistic fuzzy weakly generalized connected space if and only if there exists no non-zero IFWGOSs A and B in X such that  $A = B^c$ ,  $B = (wgcl(A))^c$  and  $A = (wgcl(B))^c$ .

## *Proof.* Necessity:

Assume that there exists IFSs A and B in X such that  $A \neq 0_{\sim} \neq B$ ,  $B = A^c$ ,  $B = (wgcl(A))^c$  and  $A = (wgcl(B))^c$ . Since  $(wgcl(A))^c$  and  $(wgcl(B))^c$  are IFWGOSs in X, A and B are IFWGOSs in X. This implies  $(X, \tau)$  is not an intuitionistic fuzzy weakly generalized connected space, which is a contradiction. Therefore there exists no non-zero IFWGOSs A and B in X such that  $A = B^c$ ,  $B = (wgcl(A))^c$  and  $A = (wgcl(B))^c$ .

#### Sufficiency:

Let A be both intuitionistic fuzzy weakly generalized open and intuitionistic fuzzy weakly generalized closed in X such that  $1 \neq A \neq 0_{\sim}$ . Now by taking  $B = A^c$ , we obtain contradictory to our hypothesis. Hence  $(X, \tau)$  is an intuitionistic fuzzy weakly generalized connected space.

**Theorem 3.9.** Let  $(X, \tau)$  be an  $IF_{wg}T_{1/2}$  space. Then the following statements are equivalent.

(i)  $(X, \tau)$  is an intuitionistic fuzzy weakly generalized connected space.

- (ii)  $(X, \tau)$  is an intuitionistic fuzzy GO-connected space.
- (iii)  $(X, \tau)$  is an intuitionistic fuzzy  $C_5$ -connected space.

*Proof.* (i)  $\Rightarrow$  (ii): It is obvious from the Theorem 3.5.

(ii)  $\Rightarrow$  (*iii*): It is obvious.

(iii)  $\Rightarrow$  (i): Let  $(X, \tau)$  be an intuitionistic fuzzy  $C_5$ -connected space. Suppose  $(X, \tau)$  is not an intuitionistic fuzzy weakly generalized connected space, then there exists a proper IFS A in X which is both intuitionistic fuzzy weakly generalized open and intuitionistic fuzzy weakly generalized closed in X. But since  $(X, \tau)$  is an  $IF_{wg}T_{1/2}$  space, A is both intuitionistic fuzzy open and intuitionistic fuzzy closed in X. This implies  $(X, \tau)$  is not an intuitionistic fuzzy  $C_5$ -connected space which is contradictory to our hypothesis. Therefore  $(X, \tau)$  is an intuitionistic fuzzy weakly generalized connected space.

**Theorem 3.10.** If  $f : (X, \tau) \to (Y, \sigma)$  is an intuitionistic fuzzy weakly generalized continuous surjection and  $(X, \tau)$  is an intuitionistic fuzzy weakly generalized connected space then  $(Y, \sigma)$  is an intuitionistic fuzzy  $C_5$ -connected space.

Proof. Let  $(X, \tau)$  be an intuitionistic fuzzy weakly generalized connected space. Suppose  $(Y, \sigma)$  is not an intuitionistic fuzzy  $C_5$ -connected space, then there exists a proper IFS A which is both intuitionistic fuzzy open and intuitionistic fuzzy closed in Y. Since f is an intuitionistic fuzzy weakly generalized continuous surjection,  $f^{-1}(A)$  is a proper IFS of X which is both intuitionistic fuzzy weakly generalized open and intuitionistic fuzzy weakly generalized open and intuitionistic fuzzy weakly generalized closed in X. But this is contradictory to our hypothesis. Hence  $(Y, \sigma)$  is an intuitionistic fuzzy  $C_5$ -connected space.

**Theorem 3.11.** If  $f : (X, \tau) \to (Y, \sigma)$  is intuitionistic fuzzy weakly generalized irresolute surjection and  $(X, \tau)$  is an intuitionistic fuzzy weakly generalized connected space then  $(Y, \sigma)$  is an intuitionistic fuzzy weakly generalized connected space.

*Proof.* Let  $(X, \tau)$  be an intuitionistic fuzzy weakly generalized connected space. Suppose  $(Y, \sigma)$  is not an intuitionistic fuzzy weakly generalized connected space, then there exists a proper IFS A which is both intuitionistic fuzzy weakly generalized open and intuitionistic fuzzy weakly generalized closed in Y. Since f is an intuitionistic fuzzy weakly generalized irresolute surjection,  $f^{-1}(A)$  is a proper IFS of X which is both intuitionistic fuzzy weakly generalized closed in X. Hence  $(X, \tau)$  is not an intuitionistic fuzzy weakly generalized closed in X. Hence  $(X, \tau)$  is not an intuitionistic fuzzy weakly generalized connected space, which is contradictory to our hypothesis. Hence  $(Y, \sigma)$  is an intuitionistic fuzzy weakly generalized connected space.

**Definition 3.12** ([10]). An IFTS  $(X, \tau)$  is called an intuitionistic fuzzy  $C_5$ -connected between two IFSs A and B if there is no intuitionistic fuzzy open set E in X such that  $A \subseteq E$  and  $E q^c B$ .

**Definition 3.13.** An IFTS  $(X, \tau)$  is called an intuitionistic fuzzy weakly generalized connected between two IFSs A and B if there is no intuitionistic fuzzy weakly generalized open set E in X such that  $A \subseteq E$  and  $E q^c B$ .

**Example 3.14.** Let  $X = \{a, b\}$  and  $\tau = \{0_{\sim}, T, 1_{\sim}\}$  be an IFT on X where  $T = \langle x, (0.5, 0.3), (0.5, 0.1) \rangle$ . Then the IFTS  $(X, \tau)$  is an intuitionistic fuzzy weakly generalized connected between the two IFSs  $A = \langle x, (0.4, 0.3), (0.5, 0.6) \rangle$  and  $B = \langle x, (0.5, 0.3), (0.1, 0.0) \rangle$ .

**Theorem 3.15.** If an IFTS  $(X, \tau)$  is intuitionistic fuzzy weakly generalized connected between two IFSs A and B, then it is intuitionistic fuzzy  $C_5$ -connected between two IFSs A and B but the converse may not be true in general.

*Proof.* Suppose  $(X, \tau)$  is not an intuitionistic fuzzy  $C_5$ -connected between IFSs A and B, then there exists an IFOS E in X such that  $A \subseteq E$  and E  $q^c$  B. Since every IFOS is an IFWGOS, there exists an IFWGOS E in X such that  $A \subseteq E$  and E  $q^c$  B. This implies  $(X, \tau)$  is not an intuitionistic fuzzy weakly generalized connected between A and B, a contradiction to our hypothesis. Therefore  $(X, \tau)$  is an intuitionistic fuzzy  $C_5$ -connected between A and B.  $\Box$ 

**Example 3.16.** Let  $X = \{a, b\}$  and  $\tau = \{0_{\sim}, T, 1_{\sim}\}$  be an IFT on X where  $T = \langle x, (0.3, 0.3), (0.2, 0.3) \rangle$ . Then the IFTS  $(X, \tau)$  is an intuitionistic fuzzy  $C_5$ -connected between the two IFSs  $A = \langle x, (0.3, 0.4), (0.6, 0.6) \rangle$  and  $B = \langle x, (0.5, 0.4), (0.5, 0.6) \rangle$ . But  $(X, \tau)$  is not an intuitionistic fuzzy weakly generalized connected between A and B, since the IFS  $E = \langle x, (0.4, 0.4), (0.6, 0.5) \rangle$  is an IFWGOS in X such that  $A \subseteq E$  and  $E \subseteq B^c$ .

**Theorem 3.17.** If an IFTS  $(X, \tau)$  is intuitionistic fuzzy weakly generalized connected between two IFSs A and B,  $A \subseteq A_1$  and  $B \subseteq B_1$ , then  $(X, \tau)$  is intuitionistic fuzzy weakly generalized connected between  $A_1$  and  $B_1$ .

Proof. Suppose that  $(X, \tau)$  is not an intuitionistic fuzzy weakly generalized connected between  $A_1$  and  $B_1$ , then there exists an IFWGOS E in X such that  $A_1 \subseteq E$  and E  $q^c B_1$ . This implies  $E \subseteq B_1^c$  and  $A_1 \subseteq E$ . That is,  $A \subseteq A_1 \subseteq E$ . Hence  $A \subseteq E$ . Now let us prove that  $E \subseteq B^c$ , that is let us prove E  $q^c$  B. Suppose that E q B, then there exists an element x in X such that  $\mu_E(x) > \nu_B(x)$  and  $\nu_E(x) < \mu_B(x)$ . Therefore  $\mu_E(x) > \nu_B(x) > \nu_{B_1}(x)$  and  $\nu_E(x) < \mu_B(x) < \mu_{B_1}(x)$ , since  $B \subseteq B_1$ . Thus E  $q B_1$ . But  $E \subseteq B_1$ . That is, E  $q^c B_1$  which is a contradiction. Therefore  $E q^c B$ . That is,  $E \subseteq B^c$ . Hence  $(X, \tau)$  is not an intuitionistic fuzzy weakly generalized connected between IFSs A and B, which is contradictory to our hypothesis. Thus  $(X, \tau)$  is an intuitionistic fuzzy weakly generalized connected between  $A_1$  and  $B_1$ .

**Theorem 3.18.** Let  $(X, \tau)$  be an IFTS and A, B be IFSs in X. If A q B then  $(X, \tau)$  is intuitionistic fuzzy weakly generalized connected between A and B.

*Proof.* Suppose  $(X, \tau)$  is not an intuitionistic fuzzy weakly generalized connected between A and B, then there exists an IFWGOS E in X such that  $A \subseteq E$  and  $E \subseteq B^c$ . This implies that  $A \subseteq B^c$ . That is, A  $q^c$  B. But this is contradiction to our hypothesis. Therefore  $(X, \tau)$  is an intuitionistic fuzzy weakly generalized connected between A and B.

**Theorem 3.19.** Let  $(Y, \tau_y)$  be a subspace of an IFTS  $(X, \tau)$  and A, B be two intuitionistic fuzzy subsets of Y. If  $(Y, \tau_y)$  is intuitionistic fuzzy weakly generalized connected between A and B, then so is  $(X, \tau)$ .

*Proof.* Suppose that  $(X, \tau)$  is not an intuitionistic fuzzy weakly generalized connected between IFSs A and B, then there exists an intuitionistic fuzzy weakly generalized open and intuitionistic fuzzy weakly generalized closed set E in X such that

 $A \subseteq E$  and E  $q^c$  B. Put  $E_y = E \cap Y$ . Then  $E_y$  is intuitionistic fuzzy weakly generalized open and intuitionistic fuzzy weakly generalized closed set in  $(Y, \tau_y)$  such that  $A \subseteq E_y$  and  $E_y q^c$  B. This implies  $(Y, \tau_y)$  is not an intuitionistic fuzzy weakly generalized connected between A and B, a contradiction to our hypothesis. Therefore  $(X, \tau)$  is an intuitionistic fuzzy weakly generalized connected between A and B.

**Theorem 3.20.** Let  $(Y, \tau_y)$  be an intuitionistic fuzzy closed open subspace of an IFTS  $(X, \tau)$  and A, B be intuitionistic fuzzy subsets of Y. If  $(X, \tau)$  is intuitionistic fuzzy weakly generalized connected between IFSs A and B, then so is  $(Y, \tau_y)$ .

*Proof.* If  $(Y, \tau_y)$  is not an intuitionistic fuzzy weakly generalized connected between IFSs A and B, then there exists an intuitionistic fuzzy weakly generalized open and intuitionistic fuzzy weakly generalized closed set E in Y such that  $A \subseteq E$  and E  $q^c$  B. Since  $(Y, \tau_y)$  is intuitionistic fuzzy closed open in  $(X, \tau)$ , E is intuitionistic fuzzy weakly generalized open and intuitionistic fuzzy weakly generalized closed set in  $(X, \tau)$ . Hence  $(X, \tau)$  is not an intuitionistic fuzzy weakly generalized closed set in  $(X, \tau)$ . Hence  $(X, \tau)$  is not an intuitionistic fuzzy weakly generalized connected between IFSs A and B, a contradiction to our hypothesis. Hence  $(Y, \tau_y)$  is an intuitionistic fuzzy weakly generalized connected between IFSs A and B.

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