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# Urysohn's lemma and Tietze's extension theorem in soft topology

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Abstract. The aim of this paper is to introduce a new type of soft mapping, continuous soft mapping and to establish Urysohn's lemma and Tietze's extension theorem in soft topological spaces using this type of continuous soft mappings.

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## 1. Introduction

In 1999, Molodtsov [18] introduced the concept of soft sets, which is a new mathematical tool to deal with uncertainties. Soft set theory has rich potential for practical applications in different fields such as physical science, biological science, engineering, economics, social science, medical science etc. Maji et al. [15, 16] formulated some set theoretic operations on soft sets, and described an application of soft set theory to a decision making problem. Later on theoretical studies on soft sets have been done in different directions. To mention some of them, Aktas and Cagman [1] have introduced soft groups; Jun [11, 12] applied soft sets to the theory of BCK/BCI algebras and introduced the concept of soft BCK/BCI-algebras; Feng et al. [8] defined soft semi-rings; Shabir and Ali [25] studied soft semi-groups and soft ideals; Kharal and Ahmed [13] as well as Majumdar and Samanta [17] studied soft mappings etc. Shabir and Naz [26] came up with an idea of soft topological spaces. Afterward Zorlutuna et al. [28], Cagman et al. [3], Hussain and Ahmed [10], Hazra et al. [9], Aygunoglu et al. [2], Varol et al. [27], Sahin et al. [24] studied on soft topological spaces. In [6], Das & Samanta introduced a concept of soft metric. Recently investigations are going on in developing algebraico topological structures in soft setting [4, 7, 19, 20, 21]. In 2014, Mrudula Ravindran and Remya P. B. [22, 23] tried to extend Urysohn's lemma and Tietze's extention theorem in soft topological spaces. But some inconsistencies (apart from printing errors) in this papers are noticed. For example, the statements of Urysohn's lemma and Tietze's extension theorem as stated in [22, 23] are inconsistent in the sense that the authors have referred to [20] for the definition of soft continuous function but considered a different one, because the codomain of the mapping as per Nazmul and Samanta [20] is a soft topological space whereas that of [22, 23] are crisp topological spaces. In this paper we have introduced new type of continuity of a soft mapping over soft topological spaces and call it a continuous soft mapping. With this notion of soft continuous mapping we have been able to extend the celebrated Urysohn's lemma and Tietze's extension theorem in soft topological spaces.

The organization of the paper is as follows: Section 2 contains the Preliminaries. In section 3, we define soft mapping and continuous soft mapping and study some properties of those mappings. In soft topological space setting, Urysohn's lemma is dealt with in section 4 and Tietze's extension theorem in section 5. Section 6 concludes the paper.

## 2. Preliminaries

**Definition 2.1** ([18])**.** Let $X$ be a universal set and $E$ be a set of parameters. Let $P(X)$ denote the power set of $X$ and $A$ be a subset of $E$. A pair $(F, A)$ is called a soft set over $X$, where $F$ is a mapping given by $F : A \to P(X)$. In other words, a soft set over $X$ is a parametrized family of subsets of the universe $X$. For $\alpha \in A$, $F(\alpha)$ may be considered as the set of $\alpha$ - approximate elements of the soft set $(F, A)$.

In [14] the soft sets are redefined as follows:

Let $E$ be the set of parameters and $A \subseteq E$. Then for each soft set $(F, A)$ over $X$ a soft set $(H, E)$ is constructed over $X$, where $\forall \alpha \in E$,

$$H(\alpha) = \begin{cases} F(\alpha) & if\ \alpha \in A \\ \phi & if\ \alpha \in E \setminus A, \end{cases}$$

Thus the soft sets $(F, A)$ and $(H, E)$ are equivalent to each other and the usual set operations of the soft sets $(F_i, A_i), i \in \Delta$ is the same as those of the soft sets $(H_i, E), i \in \Delta$. For this reason, in this paper, we have considered our soft sets over the same parameter set $E$.

Following Molodtsov and Maji et al. [15, 16, 18] definitions of soft subset, absolute soft set, null soft set, arbitrary union of soft sets etc. are presented in [19] considering the same parameter set. For arbitrary intersection of soft sets we follow [8] considering the same parameter set.

Unless otherwise stated, $X$ will be assumed to be an initial universal set, $E$ will be taken to be a set of parameters and $SS(X, E)$ denote the set of all soft sets over $X$.

**Definition 2.2** ([20])**.** A soft set $(F, E)$ over $X$ is said to be a soft element if $\exists e \in E$ such that $F(e)$ is a singleton, say,$\{x\}$ and $F(e') = \phi$ , $\forall e'(\neq e) \in E$. Such a soft element is denoted by $F_e^x$. For simplicity of notation we denote such soft element as $e_x$. Let $SE(X, E)$ be the set of all soft elements of the universal set $X$.

**Definition 2.3** ([20]). The soft element $e_x$ is said to be in the soft set $(F, E)$, denoted by $e_x \tilde{\in} (F, E)$, if $x \in F(e)$ for some $e \in E$.

**Definition 2.4** ([21]). Let $SS(X, E)$ denote the set of all soft sets over $X$ under the parameter set $E$. A soft set $(F, E) \in SS(X, E)$ is said to be pseudo constant soft set if $F(e) = X$ or $\phi$, $\forall e \in E$. Let $CS(X, E)$ denote the set of all pseudo constant soft sets over $X$ under the parameter set $E$.

**Definition 2.5** ([26]). Let $\tau$ be the collection of soft sets over $X$. Then $\tau$ is said to be a soft topology on $X$ if

$(i)$ $(\tilde{\Phi}, E)$, $(\tilde{X}, E) \in \tau$, where $\tilde{\Phi}(e) = \phi$ and $\tilde{X}(e) = X$, $\forall e \in E$.

$(ii)$ the intersection of any two soft sets in $\tau$ belongs to $\tau$.

$(iii)$ the union of any number of soft sets in $\tau$ belongs to $\tau$.

The triplet $(X, \tau, E)$ is called a soft topological space over $X$.

**Definition 2.6** ([21]). A soft topology $\tau$ on $X$ is said to be an enriched soft topology if $(i)$ of Definition 2.5 is replaced by $(i)'$ $(F, E) \in \tau$, $\forall (F, E) \in CS(X, E)$.
The triplet $(X, \tau, E)$ is called an enriched soft topological space over $X$.

**Definition 2.7** ([26]). Let $(X, \tau, E)$ be a soft topological space over X. Then the collection $\tau^e = \{F(e) : (F, E) \in \tau\}$ for each $e \in E$, defines a topology on $X$.

**Definition 2.8** ([4]). Let $X$ be a non-empty set, $E$ be the set of parameters and for each $e \in E$, $\tau^e$ is a crisp topology on $X$. Then $\tau^* = \{(G, E) \in SS(X, E) : G(e) \in \tau^e, \forall e \in E\}$ is an enriched soft topology on $X$.

**Definition 2.9** ([20]). If $(X, \tau, E)$ be a soft topological space and if $\tau^* = \{(G, E) \in S(X, E) : G(e) \in \tau^e, \forall e \in E\}$, then $\tau^*$ is an enriched soft topology on X such that $\tau \subseteq \tau^*$and $[\tau^*]^e = \tau^e, \forall e \in E$.

**Definition 2.10** ([20]). Let $X$ and $Y$ be two non-empty sets and $f : X \to Y$ be a mapping. Then
$(i)$ the image of a soft set $(F, E) \in SS(X, E)$ under the mapping $f$ is defined by $f\left[(F, E)\right] = (f(F), E)$, where $\left[f(F)\right](e) = f\left[F(e)\right], \forall e \in E$.
$(ii)$ the inverse image of a soft set $(G, A) \in SS(Y, E)$ under the mapping $f$ is defined by $f^{-1}\left[(G, E)\right] = (f^{-1}(G), E)$, where $\left[f^{-1}(G)\right](e) = f^{-1}\left[G(e)\right], \forall e \in E$.

**Definition 2.11** ([20]). Let $\tau$ be a soft topology on $X$. Then a soft set $(F, E)$ is said to be a $\tau-$ soft neighbourhood (shortly soft nbd) of the soft element $e_x$ if there exists a soft set $(G, E) \in \tau$ such that $e_x \tilde{\in} (G, E) \tilde{\subseteq} (F, E)$.

The soft nbd system of a soft element $e_x$ in $(X, \tau, E)$ is denoted by $N_\tau(e_x)$.

**Proposition 2.12** ([20]). *If $\{N_\tau(e_x) : e_x \in SE(X, E)\}$ be the system of soft nbds then*
$(i)$ $N_\tau(e_x) \neq \phi$, $\forall e_x \in SE(X, E)$
$(ii)$ $e_x \tilde{\in} (F, E), \forall (F, E) \in N_\tau(e_x)$
$(iii)$ $(F, E) \in N_\tau(e_x), (F, E) \tilde{\subseteq} (G, E) \Rightarrow (G, E) \in N_\tau(e_x)$
$(iv)$ $(F, E), (G, E) \in N_\tau(e_x) \Rightarrow (F, E) \tilde{\cap} (G, E) \in N_\tau(e_x)$
$(v)$ $(F, E) \in N_\tau(e_x) \Rightarrow \exists (G, E) \in N_\tau(e_x)$ *such that* $(G, E) \tilde{\subseteq} (F, E)$ *and* $(G, E) \in N_\tau(e'_y), \forall e'_y \tilde{\in} (G, E)$.

**Proposition 2.13** ([21]). *Let $\tau$ be a soft topology over $X$. A soft set $(F, E)$ over $X$ is a $\tau-$open soft set iff for $e_x \tilde{\in} (F, E)$, $\exists (G, E) \in \tau$ such that $e_x \tilde{\in} (G, E) \tilde{\subseteq} (F, E)$.*

**Definition 2.14** ([20]). Let $(X, \tau, E)$ be a soft topological space. A sub-collection $\mathscr{B}$ of $\tau$ is said to be an open base of $\tau$ if every member of $\tau$ can be expressed as the union of some members of $\mathscr{B}$.

**Proposition 2.15** ([20]). *A collection $\mathscr{B}$ of soft open sets of a soft topological space $(X, \tau, E)$ forms an open base of $\tau$ iff $\forall (F, E) \in \tau$ and $\forall e_x \tilde{\in} (F, E)$, $\exists (G, E) \in \mathscr{B}$ such that $e_x \tilde{\in} (G, E) \tilde{\subseteq} (F, E)$.*

**Definition 2.16** ([20]). Let $(X, \tau, E)$ be a soft topological space. A soft element $e_x \in SE(X, E)$ is said to be a limiting soft element of a soft set $(F, E)$ over $X$ if every soft open set containing $e_x$ contains at least one soft element of $(F, E)$ other than $e_x$, i.e. if $\forall (G, E) \in \tau$ with $e_x \tilde{\in} (G, E)$, $(F, E) \tilde{\cap} [(G, E) \setminus e_x] \neq (\tilde{\Phi}, E)$. The union of all limiting soft elements of $(F, E)$ is called the derived soft set of $(F, E)$ and denoted by $(F, E)'$.

**Proposition 2.17** ([20]). *A soft set $(F, E)$ in a soft topological space $(X, \tau, E)$ is closed iff $(F, E)$ contains all its limiting soft elements.*

**Definition 2.18** ([20]). The closure of a soft set $(F, E)$ in $(X, \tau, E)$ denoted by $\overline{(F, E)}$ is defined by $\overline{(F, E)} = (F, E) \tilde{\cup} (F, E)'$.

**Proposition 2.19** ([20]). *A soft set $(F, E)$ over $X$ is soft closed iff $(F, E) = \overline{(F, E)}$.*

**Proposition 2.20** ([20]). *$\overline{(F, E)}$ is the smallest closed soft set containing $(F, E)$, i.e. it is the meet of all closed soft sets containing $(F, E)$.*

**Proposition 2.21** ([20]). *For any two soft sets $(F, E)$, $(G, E)$ in a soft topological space $(X, \tau, E)$, the following conditions hold:*

*(i)* $\overline{(\tilde{\Phi}, E)} = (\tilde{\Phi}, E)$;
*(ii)* $(F, E) \tilde{\subseteq} \overline{(F, E)}$;
*(iii)* $(F, E) \tilde{\subseteq} (G, E) \Rightarrow \overline{(F, E)} \tilde{\subseteq} \overline{(G, E)}$;
*(iii)* $\overline{(F, E) \tilde{\cup} (G, E)} = \overline{(F, E)} \tilde{\cup} \overline{(G, E)}$;
*(iv)* $\overline{\overline{(F, E)}} = \overline{(F, E)}$.

**Definition 2.22** ([20]). Let $(X, \tau, E)$ be a soft topological space. For any soft set $(F, E)$ over $X$, define $\tau_{(F,E)} = \{(G, E) \tilde{\cap} (F, E) : (G, E) \in \tau\}$. Then $\tau_{(F,E)}$ is a soft topology on $(F, E)$ and $\left[(F, E), \tau_{(F,E)}\right]$ is called subspace of $(X, \tau, E)$. We denote this subspace as $(F, E, \tau_F)$.

**Definition 2.23** ([26]). A soft topological space $(X, \tau, E)$ is said to be normal if for any two disjoint soft closed sets $(F, E)$ and $(G, E)$, there exist soft open sets $(F_1, E)$ and $(F_2, E)$ such that $(F_1, E) \tilde{\supset} (F, E)$ and $(F_2, E) \tilde{\supset} (G, E)$ and $(F_1, E) \tilde{\cap} (F_2, E) = (\tilde{\Phi}, E)$.

**Lemma 2.24** ([22]). *A soft topological space $(X, \tau, E)$ is soft normal iff given a soft closed set $(F, E)$ and a soft open set $(U, E)$ containing $(F, E)$, there exists a soft open set $(H_1, E)$ such that $(F, E) \tilde{\subseteq} (H_1, E) \tilde{\subseteq} \overline{(H_1, E)} \tilde{\subseteq} (U, E)$.*

**Definition 2.25** ([5])**.** Let $R$ be the set of real numbers and $\mathscr{B}(R)$ be the collection of all non-empty bounded subsets of $R$ where $A$ is taken as a set of parameters. Then a mapping $F : A \rightarrow \mathscr{B}(\mathrm{R})$ is called a soft real set. It is denoted by $(F, A)$. If specifically $(F, A)$ is a singleton soft set, then after identifying $(F, A)$ with the corresponding soft element, it will be called a soft real number.
The set of all soft real sets is denoted by $\mathbb{R}(A)$ and the set of all non-negative soft real sets by $\mathbb{R}(A)^*$.
The notations $\tilde{r}$,$\tilde{s}$,$\tilde{t}$ are used to denote soft real numbers whereas $\bar{r}$,$\bar{s}$,$\bar{t}$ will denote a particular type of soft real numbers such that $\bar{r}(\lambda) = r$, for all $\lambda \in A$ etc. For example $\bar{0}, \bar{1}$ are the soft real numbers where $\bar{0}(\lambda) = 0, \bar{1}(\lambda) = 1$ for all $\lambda \in A$.

## 3. Continuous soft mappings and some of their properties

**Definition 3.1.** Let $X$ and $Y$ be two non-empty sets and $E$ be the parameter set. Let $\{f_e : X \rightarrow Y, e \in E\}$ be a collection of functions. Then a mapping $\tilde{f} : SE(X, E) \rightarrow SE(Y, E)$ defined by $\tilde{f}(e_x) = e_{f_e(x)}$ is called a soft mapping, where $SE(X, E)$ and $SE(Y, E)$ are sets of all soft elements of the soft sets $(\tilde{X}, E)$ and ($\tilde{Y}$,$E$) respectively.

**Definition 3.2.** Let $X$ and $Y$ be two non-empty sets and $E$ be the parameter set. Further let $\tilde{f} : SE(X, E) \rightarrow SE(Y, E)$ be a soft mapping. Then

($i$) image of a soft set $(F, E)$ over $X$ under the soft mapping $\tilde{f}$ is denoted by $\tilde{f}[(F, E)]$ and defined as $\tilde{f}[(F, E)] = (G, E)$, where $(G, E) = \tilde{\cup}\{\tilde{f}(e_x) : e_x \tilde{\in} (F, E)\}$ i.e. $G(e) = f_e(F(e)), \forall e \in E$.

($ii$) inverse image of a soft set $(G, E)$ over $Y$ under the soft mapping $\tilde{f}$ is denoted by $\tilde{f}^{-1}[(G, E)]$ and defined as $\tilde{f}^{-1}[(G, E)] = (H, E)$, where $(H, E) = \tilde{\cup}\{\tilde{f}^{-1}(e_y) : e_y \tilde{\in} (G, E)\}$ i.e. $H(e) = f_e^{-1}(G(e)), \forall e \in E$.

**Definition 3.3.** Let $(X, \tau, E)$ and $(Y, \tau', E)$ be two soft topological spaces. Then a soft mapping $\tilde{f} : SE(X, E) \rightarrow SE(Y, E)$ is said to be continuous if for each soft open set $(G, E)$ in $(Y, \tau', E)$, $\tilde{f}^{-1}[(G, E)]$ is a soft open set in $(X, \tau, E)$.

**Proposition 3.4.** *Let $(X, \tau, E)$ and $(Y, \tau', E)$ be two soft topological spaces and $\tilde{f} : SE(X, E) \rightarrow SE(Y, E)$ be a soft mapping. Then the followings are equivalent:*
($i$) *$\tilde{f}$ is continuous.*
($ii$) *For every soft set $(F, E)$ over $X$, $\tilde{f}[\overline{(F, E)}] \tilde{\subset} \overline{\tilde{f}[(F, E)]}$.*
($iii$) *For every soft closed set $(G, E)$ in $(Y, \tau', E)$, $\tilde{f}^{-1}[(G, E)]$ is soft closed set in $(X, \tau, E)$.*
($iv$) *For each $e_x \tilde{\in} (\tilde{X}, E)$ and each soft nbd $(V, E)$ of $\tilde{f}(e_x)$ there is a soft nbd $(U, E)$ of $e_x$ such that $\tilde{f}[(U, E)] \tilde{\subset} (V, E)$.*

*Proof.* ($i$) $\Rightarrow$ ($ii$) : Let us assume that the soft mapping $\tilde{f}$ is continuous. Let $(F, E)$ be any soft set over $X$. We show that if $e_x \tilde{\in} \overline{(F, E)}$ then $\tilde{f}(e_x) \tilde{\in} \overline{\tilde{f}[(F, E)]}$. Let $e_x \tilde{\in} \overline{(F, E)}$ and $(V, E)$ be a soft nbd of $\tilde{f}(e_x)$. Then $\tilde{f}^{-1}[(V, E)]$ is a soft nbd of $e_x$ in $(X, \tau, E)$.
Then $\tilde{f}^{-1}[(V, E)] \tilde{\cap} (F, E) \neq (\tilde{\Phi}, E)$
i.e. $(V, E) \tilde{\cap} \tilde{f}[(F, E)] \neq (\tilde{\Phi}, E)$

i.e. $\tilde{f}(e_x)\tilde{\in}\overline{\tilde{f}[(F,E)]}$. Hence $\tilde{f}[\overline{(F,E)}]\tilde{\subset}\overline{\tilde{f}[(F,E)]}$.

$(ii) \Rightarrow (iii)$ : Let $(G,E)$ be any soft closed set in $(Y,\tau^{'},E)$ and $(F,E) = \tilde{f}^{-1}[(G,E)]$. We are to prove $(F,E)$ is soft closed. We show that $\overline{(F,E)} = (F,E)$. Now, $\overline{\tilde{f}[(F,E)]} = \overline{\tilde{f}(\tilde{f}^{-1}[(G,E)])}\tilde{\subset}\overline{(G,E)}$. Therefore if $e_x\tilde{\in}\overline{(F,E)}$, then $\tilde{f}(e_x)\tilde{\in}\tilde{f}[\overline{(F,E)}]\tilde{\subset}\overline{\tilde{f}[(F,E)]}$ [by $(ii)$] $\tilde{\subset}\overline{(G,E)} = (G,E)[\because (G,E)$ is soft closed]. Then $e_x\tilde{\in}\tilde{f}^{-1}[(G,E)] = (F,E)$. So, $e_x\tilde{\in}\overline{(F,E)}$ implies $e_x\tilde{\in}(F,E)$. Hence $\overline{(F,E)} = (F,E)$. Therefore $(F,E)$ is soft closed.

$(iii) \Rightarrow (i)$ : Let $(G,E)$ be any soft open set in $(Y,\tau^{'},E)$. then $(G,E)^C$ is soft closed set in $(Y,\tau^{'},E)$. By $(iii)$, $\tilde{f}^{-1}[(G,E)^C]$ is soft closed in $(X,\tau,E)$.
Again, $\tilde{f}^{-1}[(G,E)^C] = \tilde{f}^{-1}[(Y,E)\setminus(G,E)] = (X,E)\setminus\tilde{f}^{-1}[(G,E)] = \Big[\tilde{f}^{-1}(G,E)\Big]^C$.
$\therefore \tilde{f}^{-1}[(G,E)]$ is soft open in $(X,\tau,E)$ and hence the soft mapping $\tilde{f}$ is continuous.

$(iv) \Leftrightarrow (i)$ is similar. □

**Lemma 3.5.** *Let $(X,\tau,E)$ and $(Y,\nu,E)$ be two soft topological spaces and $\tilde{f} : SE(X,E) \to SE(Y,E)$ be a soft mapping. If $\tilde{f}$ is continuous, then $f_e : (X,\tau^e) \to (Y,\nu^e)$ is continuous, $\forall e \in E$. If further $\tau$ is enriched, then the converse is true.*

*Proof.* Let $\tilde{f} : SE(X,E) \to SE(Y,E)$ be continuous.
Let $e \in E$ and $V$ be any open set in $(Y,\nu^e)$. Then there exists a soft open set $(U,E)$ in $(Y,\nu,E)$ such that $U(e) = V$. Since $\tilde{f}$ is continuous soft mapping, $\tilde{f}^{-1}[(U,E)]$ is soft open in $(X,\tau,E)$. Then $f_e^{-1}(U(e))$ is open in $(X,\tau^e)$. Therefore $f_e$ is continuous.
Conversely, let $\tau$ be enriched soft topology and $f_e : (X,\tau^e) \to (Y,\nu^e)$ is continuous, $\forall e \in E$. Let $(F,E)$ be soft open set in $(Y,\nu,E)$. Then $F(e)$ is open in $(Y,\nu^e), \forall e \in E$.
Now, since $f_e$ is continuous, $f_e^{-1}(F(e))$ is open in $(X,\tau^e), \forall e \in E$.
Fix $e \in E$ and then there exists a soft open set $(G,E)$ in $(X,\tau,E)$ such that $f_e^{-1}(F(e)) = G(e)$. Let $(W_e,E)(e) = V(e)$ and $(W_e,E)(e') = \phi, \forall e'(\neq e) \in E$. Since $\tau$ is enriched, then $(W_e,E)$ is soft open $\forall e \in E$. Let $(W,E) = \tilde{\bigcup}_{e\in E}(W_e,E)$.
Then $(W,E)$ is soft open in $(X,\tau,E)$ and $\tilde{f}^{-1}[(F,E)] = (W,E)$.
Therefore $\tilde{f}$ is continuous soft mapping. □

**Example 3.6.** Let $X = \{x,y,z\}$ , $Y = \{a,b,c\}$ be two sets and $E = \{e,e'\}$ be the parameter set.
Let $\tau = \{\{\tilde{\phi},E\},\{\tilde{X},E\},\{\{x,y\},\{x\}\},\{\{y\},\{x,y\}\},\{\{x,y\},\{x,y\}\},$
$\{\{y,z\},\{x,y,z\}\},\{\{y,z\},\{x,z\}\},\{\{x,y,z\},\{x,z\}\},\{\{y\},\{x\}\}\}$.
and $\nu = \{\{\tilde{\phi},E\},\{\tilde{Y},E\},\{\{b\},\{a,c\}\},\{\{a,b\},\{a,c\}\},\{\{b\},\{c\}\},\{\{a,b\},\{b,c\}\},$
$\{\{a,b\},\{a,b,c\}\},\{\{b,c\},\{a,b,c\}\},\{\{b\},\{b,c\}\},\{\{b\},\{a,b,c\}\}\}$ be soft topologies on $X$ and $Y$ respectively.
Then $\tau^e = \{\phi,X,\{y\},\{x,y\},\{y,z\}\}$, $\tau^{e'} = \{\phi,X,\{x\},\{x,z\},\{x,y\}\}$,
$\nu^e = \{\phi,Y,\{a,b\},\{b,c\},\{b\}\}$ and $\nu^{e'} = \{\phi,Y,\{b,c\},\{c\},\{a,c\}\}$.
Let $f_e : X \to Y$ be defined as $f_e(x) = a$, $f_e(y) = b$, $f_e(z) = c$ and
$f_{e'} : X \to Y$ be defined as $f_{e'}(x) = c$, $f_{e'}(y) = b$, $f_{e'}(z) = a$ .
Then $f_e$ and $f_{e'}$ are continuous. Let $\tilde{f} : SE(X,E) \to SE(Y,E)$ be a soft mapping defined as $\tilde{f}(e_x) = e_a$ $\tilde{f}(e_y) = e_b$ $\tilde{f}(e_z) = e_c$ $\tilde{f}(e'_x) = e'_c$ $\tilde{f}(e'_y) = e'_b$ and $\tilde{f}(e'_z) = e'_a$.

Then $\tilde{f}$ is not continuous soft mapping. Because if we consider the soft open set $(G, E) = \{\{a, b\}, \{a, c\}\}$ (say) in $\nu$, then $\tilde{f}^{-1}[(G, E)] = \{\{x, y\}, \{x, z\}\}$ is not soft open in $(X, \tau, E)$.

## 4. Urysohn's lemma in soft topology

**Definition 4.1.** For $a, b \in \mathbb{R}$, let $(F_{(a,b)\cap[0,1]}, E)$ be a soft set $F_{(a,b)\cap[0,1]} : E \to P([0, 1])$ defined by $F_{(a,b)\cap[0,1]}(e) = (a, b) \cap [0, 1], \forall e \in E$.
Let $\mathscr{B} = \{(F_{(a,b)\cap[0,1]}, E); a, b(> a) \in \mathbb{R}\}$. Consider the soft topology on $[0, 1]$ generated by $\mathscr{B}$ as a base. Denote this soft topological space by $([0, 1], \nu, E)$.
Let $\wp$ be the collection of soft sets $(F, E)$ and $(G, E)$ such that $F : E \to P([0, 1])$ is defined as $F(e) = [0, t), \forall e \in E$ where $t \in [0, 1]$ is fixed but arbitrary and $G : E \to P([0, 1])$ is defined as $G(e) = (s, 1], \forall e \in E$ where $s \in [0, 1]$ is fixed but arbitrary. Then $\wp$ forms subbase for the soft topology $\nu$ on $[0, 1]$.

**Proposition 4.2.** *(Urysohn's lemma) Let $(X, \tau, E)$ be a soft topological space and consider the unit interval $[0, 1]$ with soft topology as Definition 4.1. Then $(X, \tau, E)$ is soft normal iff for any two disjoint soft closed sets $(F, E)$ and $(G, E)$ in $(X, \tau, E)$ there exists a continuous soft mapping $\tilde{f} : SE(X, E) \to SE([0, 1], E)$ such that $\tilde{f}(e_x) = e_0, \forall e_x \tilde{\in} (F, E)$ and $\tilde{f}(e_x) = e_1, \forall e_x \tilde{\in} (G, E), \forall e \in E$ i.e. $\tilde{f}(F, E) \tilde{\subseteq} \{\overline{0}\}$ and $\tilde{f}(G, E) \tilde{\subseteq} \{\overline{1}\}$, where $e_0$ and $e_1$ are soft elements of $[0, 1]$.*

*Proof.* Suppose the condition holds. We have to show $(X, \tau, E)$ is soft normal. Let $(F, E)$ and $(G, E)$ be two disjoint soft closed sets in $(X, \tau, E)$. Then by the assumption there exists a continuous soft mapping $\tilde{f} : SE(X, E) \to SE([0, 1], E)$ such that $\tilde{f}(F, E) \tilde{\subseteq} \{\overline{0}\}$ and $\tilde{f}(G, E) \tilde{\subseteq} \{\overline{1}\}$. Then $\tilde{f}^{-1}\left[(F_{[0,\frac{1}{2})}, E)\right]$ and $\tilde{f}^{-1}\left[(F_{(\frac{1}{2},1]}, E)\right]$ are disjoint soft open sets such that $(F, E) \tilde{\subset} \tilde{f}^{-1}\left[(F_{[0,\frac{1}{2})}, E)\right]$ and $(G, E) \tilde{\subset} \tilde{f}^{-1}\left[(F_{(\frac{1}{2},1]}, E)\right]$. So, $(X, \tau, E)$ is soft normal.
Conversely, let $(X, \tau, E)$ be soft normal space and $(F, E)$ and $(G, E)$ be two disjoint soft closed sets in $(X, \tau, E)$. Then $(G, E)^C$ is soft open and contains the soft closed set $(F, E)$. So by Lemma 2.24 there exists a soft open set $(H, E)$ in $(X, \tau, E)$ containing $(F, E)$ such that $(F, E) \tilde{\subseteq} (H, E) \tilde{\subseteq} \overline{(H, E)} \tilde{\subseteq} (G, E)^C$.
The set $\mathbb{Q} \cap [0, 1]$ is countable. Let $\{q_0, q_1, .., q_n, ..\}$ be an enumeration of $\mathbb{Q} \cap [0, 1]$ such that $q_0 = 0$ and $q_1 = 1$. Now we construct a sequence of soft open sets in $(X, \tau, E)$. Set $(F_{q_0}, E) = (H, E)$ and $(F_{q_1}, E) = (G, E)^C$. So, $(F_{q_0}, E) \tilde{\subseteq} \overline{(F_{q_0}, E)} \tilde{\subseteq} (F_{q_1}, E)$.
Again by normality of $(X, \tau, E)$, $\exists$ $(H_1, E) \in \tau$ such that
$\overline{(F_{q_0}, E)} \tilde{\subseteq} (H_1, E) \tilde{\subseteq} \overline{(H_1, E)} \tilde{\subseteq} (F_{q_1}, E)$. Set $(F_{q_2}, E) = (H_1, E)$. Suppose we have got soft sets $(F_{q_0}, E)$, $(F_{q_1}, E)$, $(F_{q_2}, E)$, .....,$(F_{q_n}, E)$ which are soft open and satisfies the property $\overline{(F_r, E)} \tilde{\subseteq} (F_s, E)$ if $r < s$, where $r, s \in \{q_0, q_1, ..., q_n\}$. Now consider the finite set $\{q_0, q_1, ..., q_n, q_{n+1}\}$ and suppose that, with respect to the ordering of rational numbers, $q_{n+1}$ fits in between $q_i$ and $q_j$ where $i, j \in \{0, 1, 2, ...., n\}$ and $i, j$ are so chosen that $q_i < q_{n+1} < q_j$ and for any other index $k \in \{0, 1, 2, ...., n\} \setminus \{i, j\}$ either $q_k < q_i$ or $q_j < q_k$. Such indices always exist since $q_0 = 0$ and $q_1 = 1$ and $q_{n+1} \in [0, 1]$. By the inductive assumption $\overline{(F_{q_i}, E)} \tilde{\subseteq} (F_{q_j}, E)$.
Therefore by the normality of $(X, \tau, E)$, $\exists$ $(K, E) \in \tau$ such that
$\overline{(F_{q_i}, E)} \tilde{\subseteq} (K, E) \tilde{\subseteq} \overline{(K, E)} \tilde{\subseteq} (F_{q_j}, E)$. Take $(F_{q_{n+1}}, E) = (K, E)$.

Thus by principle of finite induction, for each $r \in \mathbb{Q} \cap [0,1]$, $\exists$ $(F_r, E) \in \tau$ satisfying $\overline{(F_r, E)} \tilde{\subseteq} (F_s, E)$ if $r < s$, where $r, s \in \mathbb{Q} \cap [0,1]$. Extend this definition for all $t \in \mathbb{Q}$ by defining $(F_t, E) = (\tilde{\Phi}, E), t < 0$ and $(F_t, E) = (\tilde{X}, E), t > 1$.
Define a soft mapping $\tilde{f} : SE(X,E) \to SE([0,1],E)$ by $\tilde{f}(e_x) = e_{f_e(x)}, \forall e_x \tilde{\in} (X,E)$ where $f_e : X \to [0,1]$ is defined by $f_e(x) = inf\{r : x \in F_r(e)\}, \forall e \in E$. Clearly $f_e(x) = 0, \forall e_x \tilde{\in} (F,E)$ and $f_e(x) = 1, \forall e_x \tilde{\in} (G,E)$. So, $\tilde{f}(F,E) \tilde{\subseteq} \{\overline{0}\}$ and $\tilde{f}(G,E) \tilde{\subseteq} \{\overline{1}\}$.
To prove the continuity of the soft mapping $\tilde{f}$, we show that the inverse image of all subbasic soft open sets in $([0,1], \nu, E)$ are soft open in $(X, \tau, E)$.
First we show that for $t \in [0,1]$, $\tilde{f}^{-1}(F_{[0,t)}, E) = \tilde{\cup}\{(F_s, E) : s \in \mathbb{Q}\, and\, s < t\}$.
In fact, $e_x \tilde{\in} \tilde{f}^{-1}(F_{[0,t)}, E)$
$\Leftrightarrow \tilde{f}(e_x) \tilde{\in} (F_{[0,t)}, E)$
$\Leftrightarrow e_{f_e(x)} \tilde{\in} (F_{[0,t)}, E)$
$\Leftrightarrow f_e(x) \in [0,t)$
$\Leftrightarrow f_e(x) < t$
$\Leftrightarrow f_e(x) < s < t$, for some $s(< t) \in \mathbb{Q} \cap [0,1]$
$\Leftrightarrow e_x \tilde{\in} (F_s, E)$, for some $s(< t) \in \mathbb{Q} \cap [0,1]$
$\Leftrightarrow e_x \tilde{\in} \tilde{\cup}\{(F_s, E) : s \in \mathbb{Q} \cap [0,1]\, and\, s < t\}$.
Therefore, $\tilde{f}^{-1}(F_{[0,t)}, E) = \tilde{\cup}\{(F_s, E) : s \in \mathbb{Q}\, and\, s < t\}$.
Next, $e_x \tilde{\in} \tilde{f}^{-1}(F_{[0,t]}, E)$
$\Leftrightarrow \tilde{f}(e_x) \tilde{\in} (F_{[0,t]}, E)$
$\Leftrightarrow e_{f_e(x)} \tilde{\in} (F_{[0,t]}, E)$
$\Leftrightarrow f_e(x) \in [0,t]$
$\Leftrightarrow x \in F_s(e)$, for any $s(> t) \in \mathbb{Q}$
$\Leftrightarrow e_x \tilde{\in} (F_s, E)$, for any $s(> t) \in \mathbb{Q}$.
Again for any $s \in \mathbb{Q}$ with $s > t$ there exists $s_1 \in \mathbb{Q}$ with $s > s_1 > t$ and consequently $\overline{(F_{s_1}, E)} \tilde{\subset} (F_s, E)$. Thus $e_x \tilde{\in} (F_s, E)$, for any $s(> t) \in \mathbb{Q}$ iff $e_x \tilde{\in} \overline{(F_s, E)}$, for any $s(> t) \in \mathbb{Q}$.
Therefore, $\tilde{f}^{-1}(F_{[0,t]}, E) = \tilde{\cap}\{\overline{(F_s, E)} : s \in \mathbb{Q}\, and\, s > t\}$.
Then $\tilde{f}^{-1}(F_{[0,t]}, E)$ is soft closed in $(X, \tau, E)$.
Now, $\left[\tilde{f}^{-1}(F_{[0,t]}, E)\right]^C (e) = X \setminus f_e^{-1}[0,t] = f_e^{-1}(t,1] = f_e^{-1}\left[(F_{(t,1]}, E)(e)\right]$
$= \left[\tilde{f}^{-1}(F_{(t,1]}, E)\right](e), \forall e \in E$.
Therefore $\left[\tilde{f}^{-1}(F_{[0,t]}, E)\right]^C = \left[\tilde{f}^{-1}(F_{(t,1]}, E)\right]$. So, $\left[\tilde{f}^{-1}(F_{(t,1]}, E)\right]$ is soft open in $(X, \tau, E)$.
Hence $\tilde{f}$ is a continuous soft mapping.

□

## 5. Tietze's extension theorem in soft topology

**Proposition 5.1.** *(Tietze's extension theorem) Let $(X, \tau, E)$ be a soft topological space with enriched soft topology $\tau$ and $([-1,1], \nu, E)$ be the soft topological space as in Definition 4.1. Then $(X, \tau, E)$ is soft normal iff for any soft closed $(F,E)$ in $(X, \tau, E)$ and for any continuous soft mapping $\tilde{f} : SE(F,E) \to SE([-1,1],E)$, there* *is a continuous soft mapping* $\tilde{f}' : SE(X,E) \to SE([-1,1],E)$ *such that* $\tilde{f}'(e_x) = \tilde{f}(e_x)$, $\forall e_x \tilde{\in} (F,E)$, *i.e.* $e_{f'_e(x)} = e_{f_e(x)}$, $\forall x \in F(e)$,$\forall e \in E$.

*Proof.* Let $(X,\tau,E)$ be soft normal. First divide the closed interval $[-1,1]$ into three parts namely $[-1,-\frac{1}{3}]$, $[-\frac{1}{3},\frac{1}{3}]$ and $[\frac{1}{3},1]$ and consider the soft closed set $(F_{[-1,-\frac{1}{3}]},E)$ and $(F_{[\frac{1}{3},1]},E)$ in $([-1,1],\nu,E)$.

Let $(F_1,E) = \tilde{\cup}\{e_x \tilde{\in} (F,E) : \tilde{f}(e_x) \tilde{\in} (F_{[-1,-\frac{1}{3}]},E)\} = \tilde{f}^{-1}\left[(F_{[-1,-\frac{1}{3}]},E)\right]$ and $(G_1,E) = \tilde{f}^{-1}\left[(F_{[\frac{1}{3},1]},E)\right]$. Since $(F_{[-1,-\frac{1}{3}]},E)$ and $(F_{[\frac{1}{3},1]},E)$ are soft closed in $([-1,1],\nu,E)$ and $\tilde{f} : SE(F,E) \to SE([-1,1],E)$ is a continuous soft mapping $(F_1,E)$ and $(G_1,E)$ are disjoint soft closed sets in $(F,\tau_F,E)$. Since $(F,E)$ is soft closed in $(X,\tau,E)$, $(F_1,E)$ and $(G_1,E)$ are disjoint soft closed sets in $(X,\tau,E)$. Since $(X,\tau,E)$ is soft normal, by Urysohn's lemma, there exists a continuous soft mapping $\tilde{f}_1 : SE(X,E) \to SE([-1,1],E)$ such that $\tilde{f}_1(F_1,E) \tilde{\subseteq} \{\overline{-\frac{1}{3}}\}$ and $\tilde{f}_1(G_1,E) \tilde{\subseteq} \{\overline{\frac{1}{3}}\}$ i.e. $f_{1e}(x) = -\frac{1}{3}, \forall x \in F_1(e), \forall e \in E$. and $f_{1e}(x) = \frac{1}{3}, \forall x \in G_1(e), \forall e \in E$.

Let us construct a soft mapping $\tilde{g}_1$ on $(F,\tau_F,E)$ by

$\tilde{g}_1(e_x) = \tilde{f}(e_x) - \tilde{f}_1(e_x), \forall e_x \tilde{\in} (F,E), \forall e \in E$

i.e. $g_{1e}(x) = f_e(x) - f_{1e}(x) = (f_e - f_{1e})(x), \forall x \in F(e), \forall e \in E$.

Then $g_{1e}(x) \in [-\frac{2}{3},\frac{2}{3}], \forall x \in F(e), \forall e \in E$ and hence $\tilde{g}_1(e_x) \tilde{\in} (F_{[-\frac{2}{3},\frac{2}{3}]},E), \forall e_x \tilde{\in} (F,E)$. So, $\tilde{g}_1 : SE(F,E) \to SE([-\frac{2}{3},\frac{2}{3}],E)$ and since $\tau$ is enriched by Lemma 3.5, $\tilde{g}_1$ is continuous soft mapping since $(f_e - f_{1e})$ is continuous in $(F(e),\tau_F^e), \forall e \in E$. Let $(F_2,E) = \tilde{g}_1^{-1}\left[(F_{[-\frac{2}{3},-\frac{2}{9}]},E)\right]$ and $(G_2,E) = \tilde{g}_1^{-1}\left[(F_{[\frac{2}{9},\frac{2}{3}]},E)\right]$. By similar argument as above $(F_2,E)$ and $(G_2,E)$ are disjoint soft closed sets in $(X,\tau,E)$. Since $(X,\tau,E)$ is soft normal, by Urysohn's lemma, there exists a continuous soft mapping $\tilde{f}_2 : SE(X,E) \to SE([-\frac{2}{9},\frac{2}{9}],E)$ such that $\tilde{f}_2(F_2,E) \tilde{\subseteq} \{\overline{-\frac{2}{9}}\}$ and $\tilde{f}_2(G_2,E) \tilde{\subseteq} \{\overline{\frac{2}{9}}\}$ i.e. $f_{2e}(x) = -\frac{2}{9}, \forall x \in F_2(e), \forall e \in E$. and $f_{2e}(x) = \frac{2}{9}, \forall x \in G_2(e), \forall e \in E$.

Set $\tilde{g}_2(e_x) = \tilde{g}_1(e_x) - \tilde{f}_2(e_x) = \left(\tilde{f} - (\tilde{f}_1 + \tilde{f}_2)\right)(e_x), \forall e_x \tilde{\in} (F,E), \forall e \in E$,

where $g_{2e}(x) = (f_e - (f_{1e} + f_{2e}))(x), \forall x \in F(e), \forall e \in E$.

Then $\tilde{g}_2 : SE(F,E) \to SE([-\frac{4}{9},\frac{4}{9}],E)$ is continuous soft mapping, by similar arguments. Continuing the process at the n-th stage we get continuous soft mappings,

$\tilde{f}_n : SE(X,E) \to SE([-\frac{2^{n-1}}{3^n},\frac{2^{n-1}}{3^n}],E)$,

where $f_{ne}(x) \in [-\frac{2^{n-1}}{3^n},\frac{2^{n-1}}{3^n}], \forall x \in X, \forall e \in E, \forall n = 1,2,........$

and $\tilde{g}_n : SE(F,E) \to SE([-\frac{2^n}{3^n},\frac{2^n}{3^n}],E)$ defined by

$\tilde{g}_n(e_x) = \left(\tilde{f} - (\tilde{f}_1 + \tilde{f}_2 + .... + \tilde{f}_n)\right)(e_x), \forall e_x \tilde{\in} (F,E)$

where $g_{ne}(x) = (f_e - (f_{1e} + f_{2e} + ....f_{ne}))(x), \forall x \in F(e), \forall e \in E$

i.e. $g_{ne}(x) \in [-\frac{2^{n-1}}{3^n},\frac{2^{n-1}}{3^n}], \forall x \in F(e), \forall e \in E, \forall n = 1,2,........$

Suppose $\tilde{S}_n(e_x) = \sum_{i=1}^{n} \tilde{f}_i(e_x), \forall e_x \tilde{\in} (X,E)$ where $S_{ne}(x) = \sum_{i=1}^{n} f_{ie}(x), \forall x \in X, \forall e \in E$.

Now the continuity of the soft mappings $\tilde{f}_i, i = 1,2,...,n$ on $(X,\tau,E)$ implies the continuity of the mappings $f_{ie}, i = 1,2,...,n$ on $(X,\tau^e), \forall e \in E$.

Again, $\mid S_{ne}(x) \mid \leq \mid \sum_{i=1}^{n} f_{ie}(x) \mid \leq \sum_{i=1}^{n} \frac{2^{i-1}}{3^i} \leq \frac{1}{2}\sum_{i=1}^{\infty} \frac{2^i}{3^i}$.

Since the series $\frac{1}{2}\sum_{i=1}^{\infty} \frac{2^i}{3^i}$ converges to 1, so by comparison test the series $\sum_{n=1}^{\infty} f_{ne}$ converges uniformly on $(X,\tau^e), \forall e \in E$.

Since $f_{ne}$'s are continuous on $(X, \tau^e), \forall e \in E, \forall n \in \mathbb{N}$, the sum function $f'_e(x) = \sum_{n=1}^{\infty} f_{ne}(x)$ is continuous on $(X, \tau^e), \forall e \in E$.
Let $\tilde{f}'(e_x) = e_{f'_e(x)}, \forall e_x \tilde{\in} (X, E)$. Then $\tilde{f}'(e_x) = \sum_{n=1}^{\infty} \tilde{f}_i(e_x)$.
Thus $\tilde{f}' : SE(X, E) \to SE([-1, 1], E)$ is a continuous soft mapping.
Again, $\mid g_{ne}(x) \mid = \mid f_e(x) - \sum_{n=1}^{\infty} f_{ne}(x) \mid \leq \left(\frac{2}{3}\right)^n$, $\forall x \in F(e), \forall e \in E$.
Letting $n \to \infty$ we get, $f_e(x) - \sum_{n=1}^{\infty} f_{ne}(x) = 0$, $\forall x \in F(e), \forall e \in E$. So, $f_e(x) = f'_e(x)$, $\forall x \in F(e), \forall e \in E$.
Which implies, $\tilde{f}'(e_x) = \tilde{f}(e_x)$, $\forall e_x \tilde{\in} (F, E)$.
Conversely, suppose that the condition holds. Let $(G, E)$ and $(H, E)$ be any two disjoint soft closed sets in $(X, \tau, E)$. Let $(K, E) = (G, E) \tilde{\cup} (H, E)$. Consider $(K, E, \tau_K)$. Define a soft mapping $\tilde{f} : SE(K, E) \to SE([-1, 1], E)$ by $\tilde{f}(G, E) = \overline{0}$ and $\tilde{f}(H, E) = \overline{1}$ i.e. $f_e(G(e)) = 0$ and $f_e(H(e)) = 1$, $\forall e \in E$. Let $e \in E$ and $W$ be any closed set in $([-1, 1], \nu^e)$. Then

$$f_e^{-1}(W) = \begin{cases} G(e) & if\, 0 \in W \, and \, 1 \notin W \\ H(e) & if\, 1 \in W \, and \, 0 \notin W \\ K(e) & if\, 0, 1 \in W \\ \phi & if\, 0, 1 \notin W \end{cases}.$$

Then $f_e^{-1}(W)$ is closed in $(K(e), \tau_K^e)$. Therefore $f_e$ is continuous in $(K(e), \tau_K^e)$. Since $\tau$ is enriched, the soft mapping $\tilde{f} : SE(K, E) \to SE([-1, 1], E)$ is continuous. So, by given condition, there is a continuous soft mapping $\tilde{f}' : SE(X, E) \to SE([-1, 1], E)$ such that $\tilde{f}'(e_x) = \tilde{f}(e_x)$, $\forall e_x \tilde{\in} (K, E)$. i.e. $e_{f'_e(x)} = e_{f_e(x)}$, $\forall x \in K(e)$, $\forall e \in E$.
Then $\tilde{f}'^{-1}\left[(F_{[-1, \frac{1}{2})}, E)\right]$ and $\tilde{f}'^{-1}\left[(F_{(\frac{1}{2}, 1]}, E)\right]$ are disjoint open sets and $(G, E) \tilde{\subseteq} \tilde{f}'^{-1}\left[(F_{[-1, \frac{1}{2})}, E)\right]$ and $(H, E) \tilde{\subseteq} \tilde{f}'^{-1}\left[(F_{(\frac{1}{2}, 1]}, E)\right]$. So, $(X, \tau, E)$ is soft normal. □

## 6. Conclusions

In this paper, we extend Urysohn's lemma and Tietze's extension theorem in soft topology. Based on these fundamental results, there are scopes for investigating many other important results in soft topology such as the embedding lemma, metrization theorems etc.

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