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Fuzzy I_w−closed sets

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Abstract. The purpose of this paper is to introduce and study the concept of fuzzy I_w−closed sets in fuzzy ideal topological spaces.

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1. Introduction

In 1945, R. Vaidyanathaswamy [16] introduced the concept of ideal topological spaces. Hayashi [6] defined the local function and studied some topological properties using local function in ideal topological spaces in 1964. Since then many mathematicians studied various topological concepts in ideal topological spaces. After the introduction of fuzzy sets by Zadeh [19] in 1965 and fuzzy topology by Chang [2] in 1968, several researches were conducted on the generalization of the notions of fuzzy sets and fuzzy topology. The hybridization of fuzzy topology and fuzzy ideal theory was initiated by Mahmoud [7] and Sarkar [12] independently in 1997. They [7, 12] introduced the concept of fuzzy ideal topological spaces as an extension of fuzzy topological spaces and ideal topological spaces. Recently the concepts of fuzzy semi−I−open sets [5], fuzzy $\alpha - I$−open sets [18], fuzzy $\gamma - I$− open sets [4], fuzzy pre−I−open sets [10] and fuzzy $\delta - I$−open sets [17] have been introduced and studied in fuzzy ideal topological spaces. Recently Thakur and Banafar [13] introduced the concept of generalized closed sets in fuzzy ideal topological space. In the present paper we introduce and study the concept of fuzzy I_w−closed sets in fuzzy ideal topological spaces.

2. Preliminaries

Let X be a nonempty set. A family τ of fuzzy sets of X is called a fuzzy topology [2] on X if the null fuzzy set 0 and the whole fuzzy set 1 belongs to τ and τ is closed with respect to any union and finite intersection. If τ is a fuzzy topology on X, then the pair (X, τ) is called a fuzzy topological space. The members of τ are called fuzzy open sets of X and their complements are called fuzzy closed sets. The closure of a fuzzy set A of X denoted by $Cl(A)$, is the intersection of all fuzzy closed sets which contains A. The interior [2] of a fuzzy set A of X denoted by $Int(A)$ is the union of all fuzzy subsets contained in A. A fuzzy set A in (X, τ) is said to be quasi-coincident with a fuzzy set B, denoted by AqB, if there exists a point $x \in X$ such that $A(x)+B(x) > 1$ [4]. A fuzzy set V in (X, τ) is called a $Q-$neighbourhood of a fuzzy point x_β if there exists a fuzzy open set U of X such that $x_\beta qU \leq V$ [4].

A nonempty collection of fuzzy sets I of a set X satisfying the conditions (i) if $A \in I$ and $B \leq A$, then $B \in I$(heredity), (ii) if $A \in I$ and $B \in I$ then $A \bigcup B \in I$ (finite additivity) is called a fuzzy ideal on X. The triplex (X, τ, I) denotes a fuzzy ideal topological space with a fuzzy ideal I and fuzzy topology τ [6, 9]. The local function for a fuzzy set A of X with respect to τ and I denoted by $A^*(\tau, I)$ (briefly A^*) in a fuzzy ideal topological space (X, τ, I) is the union of all fuzzy points x_β such that if U is a $Q-$neighbourhood of x_β and $E \in I$ then for at least one point $y \in X$ for which $U(y) + A(y) - 1 > E(y)$ [6]. The $*-$closure operator of a fuzzy set A denoted by $Cl^*(A)$ in (X, τ, I) defined as $Cl^*(A) = A \bigcup A^*$ [3]. In (X, τ, I), the collection $\tau^*(I)$ is an extension of fuzzy topological space than τ via fuzzy ideal which is constructed by considering the class $\beta = \{U - E : U \in \tau, E \in I\}$ as a base [10].

Definition 2.1. A fuzzy set A of a fuzzy topological space (X, τ) is called:
(i) Fuzzy semi-open if $U \leq A \leq Cl(U)$, where U is fuzzy open[1].
(ii) Fuzzy semi-closed if $Int(F) \leq A \leq F$, where F is fuzzy closed [1].
(iii) Fuzzy generalized closed written as fuzzy $g-$closed if $Cl(A) \leq O$ whenever $A \leq O$ and O is fuzzy open [15].
(iv) Fuzzy $g-$open if $1 - A$ is fuzzy $g-$closed [15].
(v) Fuzzy $w-$closed if $Cl(A) \leq U$ whenever $A \leq U$ and U is fuzzy semi-open [14].

Remark 2.2. Every fuzzy closed set is fuzzy $w-$closed but its converse may not be true [14].

Remark 2.3. Every fuzzy $w-$closed set is fuzzy $g-$closed but converse may not be true [14].

Definition 2.4. A mapping f from a fuzzy topological space (X, τ) to a topological space (Y, σ) is said to be fuzzy-irresolute if the inverse image of every fuzzy semi open set of Y is fuzzy semi open in X [8].

Definition 2.5. A fuzzy set A of a fuzzy ideal topological space (X, τ, I) is called:
(i) Fuzzy I_g-closed if $A^* \leq U$, whenever $A \leq U$ and U is fuzzy open [13].
(ii) Fuzzy I_g-open if its complement $1 - A$ is fuzzy I_g-closed [13].
(iii) Fuzzy $*-$closed (resp. fuzzy $*-$dense in itself if $A^* \leq A$ (resp. $A \leq A^*$) [3].
(iv) Fuzzy $I - T_{1/2}$ if every fuzzy I_g-closed set in X is fuzzy closed in X [13].

Remark 2.6. Every fuzzy $g-$closed (resp. fuzzy $*-$closed) set is fuzzy I_g-closed, but the converse may not be true [13].

Lemma 2.7. *$A \leq B \Leftrightarrow \rceil(Aq(1-B))$ for every pair of fuzzy sets A and B of X* [11].

3. Fuzzy I_w-Closed Sets

Definition 3.1. A fuzzy set A of a fuzzy ideal topological space (X, τ, I) is called fuzzy I_w-closed if $A^* \leq U$ whenever $A \leq U$ and U is fuzzy semi-open.

Remark 3.2. Every fuzzy closed set is fuzzy I_w-closed but its converse may not be true.

Example 3.3. Let $X = \{a, b\}$ and the fuzzy sets A and U are defined as follows:

$$A(a) = 0.5, A(b) = 0.5$$

$$U(a) = 0.5, U(b) = 0.4$$

Let $\tau = \{0, 1, U\}$ be a fuzzy topology and $I = \{0\}$ be the fuzzy ideal on X. Then the fuzzy set A is fuzzy I_w-closed but it is not fuzzy closed.

Remark 3.4. Every fuzzy I_w-closed set is fuzzy I_g-closed but the converse may not be true. For,

Example 3.5. Let $X = \{a, b\}$ and the fuzzy sets A and U are defined as follows:

$$U(a) = 0.7, U(b) = 0.6$$

$$A(a) = 0.6, A(b) = 0.7$$

Let $\tau = \{0, 1, U\}$ be a fuzzy topology and $I = \{0\}$ be the fuzzy ideal on X. Then the fuzzy set A is fuzzy I_g-closed but it is not fuzzy I_w-closed.

Theorem 3.6. *Let (X, τ, I) be a fuzzy ideal topological space. Then A^* is fuzzy I_w-closed for every fuzzy set A of X.*

Proof. Let A be a fuzzy set of X and U be any fuzzy semi-open set of X such that $A^* \leq U$. Since $(A^*)^* \leq A^*$ it follows that $(A^*)^* \leq U$. Hence A^* is fuzzy I_w-closed. □

Theorem 3.7. *Let (X, τ, I) be a fuzzy ideal topological space and A be a fuzzy I_w-closed and fuzzy semi-open in X. Then A is fuzzy $*-$closed.*

Proof. Since A is fuzzy I_w-closed and fuzzy semi-open in X and $A \leq A$. It follows that $A^* \leq A$ which implies $Cl^*(A) = A \cup A^* \leq A$. Hence A is fuzzy $*-$closed. □

Theorem 3.8. *Let (X, τ, I) be a fuzzy ideal topological space and A be a fuzzy set of X. Then the following are equivalent:*

(i) A is fuzzy I_w-closed.
(ii) $Cl^*(A) \leq U$ whenever $A \leq U$ and U is fuzzy semi-open in X.
(iii) $\rceil(AqF) \Rightarrow \rceil(Cl^*(A)qF)$ for every fuzzy semi-closed set F of X.
(iv) $\rceil(AqF) \Rightarrow \rceil(A^*qF)$ for every fuzzy semi-closed set F of X.

Proof. $(i) \Rightarrow (ii)$. Let A be a fuzzy I_w-closed set in X. Let $A \leq U$ where U is fuzzy semi-open set in X. Then $A^* \leq U$. Hence $Cl^*(A) = A \cup A^* \leq U$. Which implies that $Cl^*(A) \leq U$.

$(ii) \Rightarrow (i)$. Let A be a fuzzy set of X. By hypothesis $Cl^*(A) \leq U$. Which implies that $A^* \leq U$. Hence A is fuzzy I_w-closed.

$(ii) \Rightarrow (iii)$. Let F be a fuzzy semi-closed set of X and $\rceil(AqF)$. Then $1 - F$ is fuzzy semi-open in X and by Lemma 2.7, $A \leq 1 - F$. Therefore, $Cl^*(A) \leq 1 - F$, because A is fuzzy I_w-closed. Hence by Lemma 2.7, $\rceil(Cl^*(A)qF)$.

$(iii) \Rightarrow (ii)$. Let U be a fuzzy semi-open set of X such that $A \leq U$. Then by Lemma 2.7, $\rceil(Aq(1-U))$ and $1 - U$ is fuzzy semi-closed in X. Therefore by hypothesis $\rceil(Cl^*(A)q(1-U))$. Hence,$Cl^*(A) \leq U$.

$(i) \Rightarrow (iv)$. Let F be a fuzzy semi-closed set in X such that $\rceil(AqF)$. Then $A \leq 1 - F$ where $1 - F$ is fuzzy semi-open. Therefore by (i) $A^* \leq 1 - F$. Hence $\rceil(A^*qF)$.

$(iv) \Rightarrow (i)$.Let U be a fuzzy semi-open set in X such that $A \leq U$. Then by Lemma 2.7, $\rceil(Aq(1-U))$ and $1 - U$ is fuzzy semi-closed in X. Therefore by hypothesis $\rceil(A^*q(1-U))$. Hence $A^* \leq U$ and A is fuzzy I_w-closed set in X. □

Theorem 3.9. *Let A be a fuzzy I_w-closed set in a fuzzy ideal topological space (X, τ, I) and x_β be a fuzzy point of X such that $x_\beta q Cl^*(A)$, then $scl(x_\beta)qA$.*

Proof. If $\rceil(scl(x_\beta)qA)$ then by Lemma 2.7, $A \leq 1 - scl(x_\beta)$. And so by Theorem 3.8 (i),$Cl^*(A) \leq 1 - scl(x_\beta) \leq 1 - x_\beta$ because A is fuzzy I_w-closed in X. Hence $\rceil((x_\beta)q(Cl^*(A)))$, which is a contradiction. □

Theorem 3.10. *Let (X, τ, I) be a fuzzy semi T_1-ideal topological space and A be a fuzzy I_w-closed set in X. Then A is fuzzy $*-$closed.*

Proof. It follows from Theorem 3.9 and Lemma 2.7. □

Theorem 3.11. *Let (X, τ, I) be a fuzzy ideal topological space and A be fuzzy $*-$dense in itself fuzzy I_w-closed set of X. Then A is fuzzy $w-$closed.*

Proof. Let U be a fuzzy semi-open set in X such that $A \leq U$. Since A is fuzzy I_w-closed then $Cl^*(A) \leq U$. Therefore $Cl(A) \leq U$ because A is fuzzy $*-$dense in itself. Hence A is fuzzy $w-$closed. □

Theorem 3.12. *Let (X, τ, I) be a fuzzy ideal topological space where $I = \{0\}$ and A be a fuzzy set of X. Then A is fuzzy I_w-closed if and only if A is fuzzy $w-$closed.*

Proof. Since $I = \{0\}$, $A^* = Cl(A)$ for each set of X. Now the result can be easily proved. □

Theorem 3.13. *Let (X, τ, I) be a fuzzy ideal topological space and A and B are fuzzy I_w-closed sets of X. Then $A \bigcup B$ is fuzzy I_w-closed.*

Proof. Let U be a fuzzy semi-open set in X such that $A \bigcup B \leq U$. Then $A \leq U$ and $B \leq U$. Since A and B are fuzzy I_w-closed, $A^* \leq U$ and $B^* \leq U$. Therefore $(A \bigcup B)^* \leq U$. Hence $A \bigcup B$ is fuzzy I_w-closed. □

Remark 3.14. The intersection of any two fuzzy I_w-closed sets in a fuzzy ideal topological space (X, τ, I) may not be fuzzy I_w-closed. For,

Example 3.15. Let $X = \{a, b, c\}$ and the fuzzy sets U, A and B of X are defined as follows:

$$U(a) = 1, U(b) = 0, U(c) = 0$$
$$A(a) = 1, A(b) = 1, A(c) = 0$$
$$B(a) = 1, B(b) = 0, B(c) = 1$$

Let $\tau = \{0, U, 1\}$ and $I = \{0\}$ be the fuzzy topology and fuzzy ideal on X respectively. Then A and B are fuzzy I_w-closed sets in (X, τ, I) but $A \bigcap B$ is not fuzzy I_w-closed.

Theorem 3.16. *Let (X, τ, I) be a fuzzy ideal topological space and A, B are fuzzy sets on X such that $A \leq B \leq Cl^*(A)$ and A is fuzzy I_w-closed set in X then B is fuzzy I_w-closed.*

Proof. Let U be a fuzzy semi-open set in X such that $B \leq U$. Then $A \leq U$ and since A is fuzzy I_w-closed, $Cl^*(A) \leq U$. Now $B \leq Cl^*(A) \Rightarrow Cl^*(B) \leq Cl^*(A) \leq U$. Consequently B is fuzzy I_w-closed. □

Theorem 3.17. *Let (X, τ, I) be a fuzzy ideal topological space and A, B are fuzzy sets on X such that $A \leq B \leq A^*$. Then A and B are fuzzy $w-$closed.*

Proof. Obvious. □

Theorem 3.18. *Let (X, τ, I) be a fuzzy ideal topological space and A, B are fuzzy sets on X such that $A \leq B \leq A^*$ and A is fuzzy I_w-closed. Then $A^* = B^*$ and B is fuzzy $*-$open in itself.*

Proof. Obvious. □

Theorem 3.19. *Let (X, τ, I) be a fuzzy ideal topological space and $FSO(X)$ (resp. F)be the family of all fuzzy semi-open (resp. fuzzy $*-$closed)sets of X. Then $FSO(X) \subset F$ if and only if every fuzzy set of X is fuzzy I_w-closed.*

Proof. Necessity. Let $FSO(X) \subset F$ and U be a fuzzy semi-open set in X such that $A \leq U$. Now $U \in FSO(X) \Rightarrow U \in F$. And so $Cl^*(A) \leq Cl^*(U) = U$ and A is fuzzy I_w-closed set in X.

Sufficiency. Suppose that every fuzzy set of X is fuzzy I_w-closed. Let $U \in FSO(X)$. Since U is fuzzy I_w-closed and $U \leq U, Cl^*(U) \leq U$. Hence $Cl^*(U) = U$ and $U \in F$. Therefore $FSO(X) \subset F$. □

Definition 3.20. A fuzzy set A of a fuzzy ideal topological space (X, τ, I) is called fuzzy I_w-open if and only if $1 - A$ is fuzzy I_w-closed.

Remark 3.21. Every fuzzy open(resp. fuzzy $g-$open, fuzzy I_g-open)set is fuzzy I_w-open. But the converse may not be true. For, the fuzzy set B defined by

$$B(a) = 0.5, B(b) = 0.5$$

in the fuzzy ideal topological space (X, τ, I) of Example 3.3, is fuzzy I_w-open but it is not fuzzy open and the fuzzy set C defined by

$$C(a) = 0.4, C(b) = 0.3$$

in the fuzzy ideal topological space (X, τ, I) of Example 3.5, is fuzzy I_w-open but it is not fuzzy $g-$open.

Theorem 3.22. *Let (X, τ, I) be a fuzzy ideal topological space and A is fuzzy set on X. Then A is fuzzy I_w-open if and only if $F \leq Int^*(A)$ whenever $F \leq A$ and F is fuzzy semi-closed.*

Proof. Necessity.Let A be fuzzy I_w-open and F is fuzzy semi-closed such that $F \leq A$. Then $1 - A$ is fuzzy I_w-closed, $1 - A \leq 1 - F$ and $1 - F$ is fuzzy semi-open in X. Hence $Cl^*(1 - A) \leq 1 - F$. Which implies that $F \leq Int^*(A)$.

Sufficiency.Let U be fuzzy semi-open in X such that $1 - A \leq U$. Then $1 - U$ is fuzzy semi-closed in X such that $1-U \leq A$. And so by hypothesis, $1-U \leq Int^*(A)$. Which implies that $Cl^*(1-A) \leq U$ and $1 - A$ is fuzzy I_w-closed. Hence A is fuzzy I_w-open. □

Theorem 3.23. *Let (X, τ, I) be a fuzzy ideal topological space and A be a fuzzy set on X. If A is fuzzy I_w-open and $Int^*(A) \leq B \leq A$ then B is fuzzy I_w-open.*

Proof. Let A be fuzzy I_w-open in X and $Int^*(A) \leq B \leq A$ then $1 - A$ is fuzzy I_w-closed and $1 - A \leq 1 - B \leq Cl(1 - A)$. Therefore by Theorem 3.16, $1 - B$ is fuzzy I_w-closed in X. Hence B is fuzzy I_w-open in X. □

Definition 3.24. A mapping f from a fuzzy ideal topological space (X, τ, I_1) to another fuzzy ideal topological space (Y, σ, I_2) is said to be fuzzy $*-$closed if the image of every fuzzy $*-$closed set of X is fuzzy $*-$closed set in Y.

Theorem 3.25. *Let A be a fuzzy I_w-closed set in a fuzzy ideal topological space (X, τ, I) and $f : (X, \tau, I_1) \to (Y, \sigma, I_2)$ is a fuzzy irresolute and fuzzy $*-$closed mapping. Then $f(A)$ is fuzzy I_w-closed in Y.*

Proof. Let $f(A) \leq G$ where G is fuzzy semi-open in Y. Then $A \leq f^{-1}(G)$ and hence $Cl^*(A) \leq f^{-1}(G)$. Thus $f(Cl^*(A)) \leq G$ and $f(Cl^*(A))$ is a fuzzy semi-closed set. It follows that $Cl^*(f(A)) \leq Cl^*(f(Cl^*(A))) = f(Cl^*(A)) \leq G$. Thus $Cl^*(f(A)) \leq G$ and $f(A)$ is a fuzzy I_w-closed in Y. □

Definition 3.26. A collection $\{A_i : i \in \wedge\}$ of fuzzy I_w-open sets in a fuzzy ideal topological space (X, τ, I) is called a fuzzy I_w-open cover of a fuzzy set B of X if $B \leq \bigcup\{A_i : i \in \wedge\}$.

Definition 3.27. A fuzzy ideal topological space (X, τ, I) is said to be fuzzy $IWO-$compact if every fuzzy I_w-open cover of X has a finite subcover.

Definition 3.28. A fuzzy set B of a fuzzy ideal topological space (X, τ, I) is said to be fuzzy $IWO-$compact if for every collection $\{A_i : i \in \wedge\}$ of fuzzy I_w-open subsets of X such that $B \leq \bigcup\{A_i : i \in \wedge\}$ there exists a finite subset $\wedge_0$ and $\wedge$ such that $B \leq \{A_i : i \in \wedge_0\}$.

Definition 3.29. A crisp subset B of a fuzzy ideal topological space (X, τ, I) is said to be fuzzy $IWO-$compact if B is fuzzy $IWO-$compact as a fuzzy ideal subspace of X.

Theorem 3.30. *A fuzzy I_w-closed crisp subset of a fuzzy $IWO-$compact space (X, τ, I) is fuzzy $IWO-$compact relative to X.*

Proof. Let A be a fuzzy I_w-closed crisp set of fuzzy $IWO-$compact space (X, τ, I). Then $1 - A$ is fuzzy I_w-open in X. Let M be a cover of A by fuzzy I_w-open sets in X. Let M be a cover of A by fuzzy I_w-open sets in X. Then the family $\{M, 1 - A\}$ is a fuzzy I_w-open cover of X. Since X is fuzzy $IWO-$compact, it has a finite sub cover say $\{G_1, G_2, ..., G_n\}$. If this sub cover contains $1 - A$, we discards it. Otherwise leave the sub cover as it is. Thus we have obtained a finite fuzzy $IWO-$open sub cover of A. Therefore A is fuzzy $IWO-$compact relative to X. □

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