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# Note on "on multiset topologies"

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Abstract. We show that an alleged theorem stated in [4] is invalid in general, by giving an example.

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## 1. Introduction

A collection of elements containing duplicates is called multiset (mset [1] or bag [5], for short). Formally, if $X$ is a set of elements, a mset $M$ drawn from the set $X$ is represented by a function count $M$ or $C_M$ defined as $C_M : X \to \mathbb{N}$ where $\mathbb{N}$ represents the set of non negative integers.

Theorem 6.4 of [4] asserted that for any $M-$ topological space, $A$ be a submset of a mset $M$, and $A'$ be the mset of all limit points of $A$. Then $C_{cl(A)}(x) = \max\{C_A(x), C_{A'}(x)\}$. This theorem is wrong and we give an example showing the theorem is indeed false. As a consequence, this invalidates also the proof of Corollary 6.5 of [4]. As well as, S. A. El-Sheikh and A. Zakaria [2] discovered invalid statements in [3].

## 2. Preliminaries

**Definition 2.1** ([4]). A domain $X$, is defined as a set of elements from which msets are constructed. The mset space $[X]^m$ is the set of all msets whose elements are in $X$ such that no element in the mset occurs more than $m$ times. The set $[X]^\infty$ is the set of all msets over a domain $X$ such that there is no limit on the number of occurrences of an element in an mset.

**Definition 2.2** ([4]). Let $M \in [X]^m$. Then the complement $M^c$ of $M$ in $[X]^m$ is an element of $[X]^m$ such that

$$C_{M^c}(x) = m - C_M(x) \ \forall x \in X.$$

**Definition 2.3** ([4])**.** Let $M$ be a mset drawn from a set $X$ and $\tau \subseteq P^*(M)$. Then $\tau$ is called a mset topology if $\tau$ satisfies the following properties.

(1) $\phi$ and M are in $\tau$.
(2) The union of the elements of any sub collection of $\tau$ is in $\tau$.
(3) The intersection of the elements of any finite sub collection of $\tau$ is in $\tau$.

A mset topological space is an ordered pair $(M, \tau)$ consisting of a mset $M$ and a mset topology $\tau \subseteq P^*(M)$ on $M$. Note that $\tau$ is an ordinary set whose elements are msets and the mset topology is abbreviated as a $M$-topology. Also, a submset $U$ of $M$ is an open mset of $M$ if $U$ belongs to the collection $\tau$. Moreover, a submset $N$ of $M$ is closed mset $M \ominus N$ is open mset.

**Definition 2.4.** [4] Let $(M, \tau)$ be a $M-$topological space and $A$ be a submset of $M$. Then the closure of $A$ is defined as the mset intersection of all closed msets containing $A$ and is denoted by $cl(A)$; that is,

$$cl(A) = \cap\{K \subseteq M : K \text{ is a closed mset and } A \subseteq K\} \text{ and}$$
$$C_{cl(A)}(x) = \min\{C_K(x) : A \subseteq K\}.$$

**Definition 2.5** ([4])**.** Let $(M, \tau)$ be a $M-$topological space and $A$ be a submset of $M$. Then $k/x$ is a limit point of an mset $A$ when every neighborhood of $k/x$ intersects $A$ in some point (point with non zero multiplicity) other than $k/x$ itself. $A'$ denotes the mset of all limit points of $A$.

## 3. Example

The following example shows that Theorem 6.4 of [4] is wrong in general.

**Example 3.1.** Let $M = \{2/x, 3/y\}$, $\tau = \{M, \phi, \{1/x\}, \{1/y\}, \{2/y\}, \{1/x, 1/y\}, \{1/x, 2/y\}\}$, and $A = \{1/x, 1/y\}$. Then $cl(A) = \{1/x, 1/y\}$ and $A' = \{2/x, 3/y\}$. So, $C_{cl(A)}(y) = 1$ and $\max\{C_A(y), C_{A'}(y)\} = \max\{1, 3\} = 3$. Thus $C_{cl(A)}(y) \neq \max\{C_A(y), C_{A'}(y)\}$ in general.

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