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Some properties on intuitionistic fuzzy sets of third type

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Abstract. The aim of this paper is to introduce the Intuitionistic Fuzzy Sets of Third Type (IFSTT) and to define the basic operations and modal operators like Necessity and Possibility over IFSTT and to establish the relation between the operators.

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Intuitionistic Fuzzy Set (IFS), Intuitionistic Fuzzy Set of Second Type (IFSST), Intuitionistic Fuzzy Set of Root Type (IFSRT), Intuitionistic Fuzzy Set of Third Type (IFSTT)

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1. Introduction

Fuzzy sets were introduced by Lofti A. Zadeh [7] in 1965 as a generalization of Classical (crisp) sets. Further, the fuzzy sets are generalized by Krassimir T. Atanassov [1] in which he has taken non-membership values also into consideration and he introduced IFS. Following the definition of IFS, Atanassov [1] further introduced the extension of IFS, namely, interval valued IFSs, IFSs of second type and temporal IFSs. Srinivasan and Palaniappan [3] introduced IFSs of Root Type. The authors introduced the new extension of IFSs, nemely IFSTT and defined modal operators and established their relations. In section 2, we give the basic definitions related to IFS and their extensions. In section 3, we present the basic operations like union, intersection, subset and complement on IFSTT and defined the modal operators necessity and possibility with example. In section 4, we have proved the relations between the necessity and possibility operators on IFSTT. Finally, in section 5, conclusion is given.

2. Preliminaries

In this section, we give the basic definitions related to IFS and their extensions.

Definition 2.1 ([1])**.** Let X be a non-empty set. An Intuitionistic Fuzzy Set (IFS) A in X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the membership and non-membership functions of A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$, for each $x \in X$. The IFS can also be written in the form $A = \langle x, \mu_A(x), \nu_A(x)\rangle$ or simply $A = \langle \mu_A, \nu_A\rangle$.

Remark. An ordinary fuzzy set can also be written as $\{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$. That is, all the fuzzy sets are IFSs

Definition 2.2 ([1])**.** Let X be the non-empty set. An Intuitionistic Fuzzy Set of Second Type IFSST A in X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where $\mu_A : x \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership and non-membership functions of A respectively, and $0 \leq [\mu_A(x)]^2 + [\nu_A(x)]^2 \leq 1$, for each $x \in X$.

Remark. It is obvious that for all real numbers $a, b \in [0,1]$, if $0 \leq a + b \leq 1$, then $0 \leq a^2 + b^2 \leq 1$

Definition 2.3 ([3])**.** Let X be the non-empty set. An Intuitionistic fuzzy set of root type IFSRT A in X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the membership and non-membership functions of A respectively, and $0 \leq \frac{\sqrt{\mu_A(x)}}{2} + \frac{\sqrt{\nu_A(x)}}{2} \leq 1$, for each $x \in X$

Remark. It is obvious that for all real numbers $a, b \in [0,1]$, if $0 \leq a + b \leq 1$ then $0 \leq \frac{1}{2}\sqrt{a} + \frac{1}{2}\sqrt{b} \leq 1$.

Definition 2.4. Let X be the non-empty set. An Intuitionistic fuzzy set of third type(IFSTT) A in X is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$

where $\mu_A : X \to [0,1]$ and $\nu_A :\to [0,1]$ denote the membership and non-membership functions of A respectively, and $0 \leq [\mu_A(x)]^3 + [\nu_A(x)]^3 \leq 1$, for each $x \in X$.

Remark. It is obvious that for all real numbers $a, b \in [0,1]$, if $0 \leq a + b \leq 1$, then $0 \leq a^3 + b^3 \leq 1$. Hence, all the IFSs are IFSTTs.

Definition 2.5 ([2])**.** The degree of non-determinacy (uncertainty) of an element $x \in X$ to the IFSTT A is defined by

$$\pi_A(x) = \sqrt[3]{1 - \mu_A^3(x) - \nu_A^3(x)}.$$

In case of ordinary fuzzy sets, $\pi_A(x) = 0$, for every $x \in X$.

3. Operations on IFSTT

In this section, we present the basic operations like union, intersection, subset and complement on IFSTT and defined the modal operators necessity and possibility with example.

Definition 3.1 ([4]). Let A and B be two IFSTTs of the non-empty set X such that

$$\begin{aligned} A &= \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}, \\ B &= \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}. \end{aligned}$$

We define the following basic operations on A and B :

(i) $A \subset B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x),\ \forall x \in X$
(ii) $A \supset B$ iff $\mu_A(x) \geq \mu_B(x)$ and $\nu_A(x) \leq \nu_B(x), \forall x \in X$
(iii) $A = B$ iff $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x), \forall x \in X$
(iv) $A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)),\ \min(\nu_A(x), \nu_B(x))\rangle : x \in X\}$
(v) $A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)),\ \max(\nu_A(x), \nu_B(x))\rangle : x \in X\}$
(vi) $\bar{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$, where $\bar{A}$ is the complement of A.

Definition 3.2 ([5, 6]). For every IFSTT A, we define the following operators:

(i) The Necessity operator

$\Box A = \{\langle x, \mu_A(x), \sqrt[3]{1 - \mu_A^3(x)}\,\rangle : x \in X\}$

(ii) The Possibility operator

$\Diamond A = \{\langle x, \sqrt[3]{1 - \nu_A^3(x)}, \nu_A(x)\rangle : x \in X\}$

Remark. If A is an ordinary fuzzy set, then $\Box A = A = \Diamond A$.

Example 3.3. Let $X = \{a, b, c, d, e\}$ and let the IFSTT A have the form

$$A = \{\langle a, 0.5, 0.3\rangle,\ \langle b, 0.1, 0.7\rangle,\ \langle c, 1.0, 0.0\rangle,\ \langle d, 0.0, 0.0\rangle,\ \langle e, 0.0, 1.0\rangle\}$$

Then

$$\begin{aligned} \Box A &= \Big\{\langle a, 0.5, \sqrt[3]{1-(0.5)^3}\rangle,\ \langle b, 0.1, \sqrt[3]{1-(0.1)^3}\rangle,\ \langle c, 1.0, \sqrt[3]{1-(1.0)^3}\rangle, \\ &\qquad \langle d, 0.0, \sqrt[3]{1-(0.0)^3}\rangle,\ \langle e, 0.0, \sqrt[3]{1-(0.0)^3}\rangle\Big\} \\ &= \{\langle a, 0.5, 0.9564\cdots\rangle,\ \langle b, 0.1, 0.9996\cdots\rangle,\ \langle c, 1.0, 0.0\rangle,\ \langle d, 0.0, 1.0\rangle,\ \langle e, 0.0, 1.0\rangle\} \end{aligned}$$

and

$$\begin{aligned} \Diamond A &= \Big\{\langle a, \sqrt[3]{1-(0.3)^3}, 0.3\rangle,\ \langle b, \sqrt[3]{1-(0.7)^3}, 0.7\rangle,\ \langle c, \sqrt[3]{1-(0.0)^3}, 0.0\rangle, \\ &\qquad \langle d, \sqrt[3]{1-(0.0)^3}, 0.0\rangle,\ \langle e, \sqrt[3]{1-(1.0)^3}, 1.0\rangle\Big\} \\ &= \{\langle a, 0.9909\cdots, 0.3\rangle,\ \langle b, 0.8693\cdots, 0.7\rangle,\ \langle c, 1.0, 0.0\rangle,\ \langle d, 1.0, 0.0\rangle,\ \langle e, 0.0, 1.0\rangle\} \end{aligned}$$

Clearly, $\Box A$ and $\Diamond A$ are IFSTTs.

4. Some Properties

In this section, we prove the relations between the necessity and possibility operators on IFSTT.

Proposition 4.1. *The following operations hold for every intuitionistic fuzzy set of third type A :*

$$\begin{array}{ll}
(i) & \overline{\Box\overline{A}} = \Diamond A \\
(ii) & \overline{\Diamond\overline{A}} = \Box A \\
(iii) & \Box\Box A = \Box A \\
(iv) & \Diamond\Diamond A = \Diamond A \\
(v) & \Box\Diamond A = \Diamond A \\
(vi) & \Diamond\Box A = \Box A
\end{array}$$

Proof. we present the proofs of these relations.

$$\begin{aligned}
\text{(i)} \quad \overline{\Box\ \overline{A}} &= \overline{\Box\ \{\overline{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X}\}} \\
&= \overline{\Box\ \{\ \langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}} \\
&= \overline{\{\ \langle x, \nu_A(x),\ \sqrt[3]{1-\nu_A^3(x)}\rangle : x \in X\}} \\
&= \{\ \langle x,\ \sqrt[3]{1-\nu_A^3(x)}, \nu_A(x)\rangle : x \in X\} \\
&= \Diamond\ A.
\end{aligned}$$

$$\begin{aligned}
\text{(ii)} \quad \overline{\Diamond\ \overline{A}} &= \overline{\Box\ \{\overline{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X}\}} \\
&= \overline{\Diamond\ \{\ \langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}} \\
&= \overline{\{\ \langle x,\ \sqrt[3]{1-\mu_A^3(x)}, \mu_A(x)\rangle : x \in X\}} \\
&= \{\ \langle x, \mu_A(x),\ \sqrt[3]{1-\mu_A^3(x)}\rangle : x \in X\} \\
&= \Box\ A.
\end{aligned}$$

$$\begin{aligned}
\text{(iii)} \quad \Box\Box\ A &= \Box\Box\ \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\} \\
&= \Box\{\ \langle x, \mu_A(x),\ \sqrt[3]{1-\mu_A^3(x)}\rangle : x \in X\} \\
&= \{\ \langle x, \mu_A(x),\ \sqrt[3]{1-\mu_A^3(x)}\rangle : x \in X\} \\
&= \Box\ A.
\end{aligned}$$

$$\begin{aligned}
\text{(iv)} \quad \Diamond\Diamond\ A &= \Diamond\Diamond\ \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\} \\
&= \Diamond\{\ \langle x,\ \sqrt[3]{1-\nu_A^3(x)}, \nu_A(x)\rangle : x \in X\} \\
&= \{\ \langle x,\ \sqrt[3]{1-\nu_A^3(x)}, \nu_A(x)\rangle : x \in X\} \\
&= \Diamond\ A.
\end{aligned}$$

$$\begin{aligned}
\text{(v)} \quad \Box\Diamond\ A &= \Box\,\Diamond\ \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\} \\
&= \Box\{\ \langle x,\ \sqrt[3]{1-\nu_A^3(x)}, \nu_A(x)\ \rangle : x \in X\} \\
&= \{\ \langle x,\ \sqrt[3]{1-\nu_A^3(x)},\ \sqrt[3]{1-(\ \sqrt[3]{1-\nu_A^3(x)})^3}\ \rangle : x \in X\} \\
&= \{\ \langle x,\ \sqrt[3]{(1-\nu_A^3(x))},\ \sqrt[3]{1-(1-\nu_A^3(x))}\ \rangle : x \in X\}
\end{aligned}$$

$$
\begin{aligned}
& & &= \{\ \langle x, \sqrt[3]{1-\nu_A^3(x)}, \nu_A(x)\rangle : x \in X\} \\
& & &= \Diamond\ A. \\
\text{(vi)} & \quad \Diamond\Box\ A &&= \Diamond\Box\ \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\} \\
& & &= \Diamond\{\ \langle x, \mu_A(x), \sqrt[3]{1-\mu_A^3(x)}\rangle : x \in X\} \\
& & &= \{\ \langle x, \sqrt[3]{1-(\ \sqrt[3]{1-\mu_A^3(x)})^3}, \sqrt[3]{1-\mu_A^3(x)}\ \rangle : x \in X\} \\
& & &= \{\ \langle x, \sqrt[3]{1-(1-\mu_A^3(x))}, \sqrt[3]{(1-\mu_A^3(x))}\ \rangle : x \in X\} \\
& & &= \{\ \langle x, \mu_A(x), \sqrt[3]{1-\mu_A^3(x)}\rangle : x \in X\} \\
& & &= \Box\ A.
\end{aligned}
$$

□

Proposition 4.2. *For every IFSTT A, the following result hold good at the extreme values of the membership and non-membership functions of* A :

$$\Box A \subset A \subset \Diamond A.$$

Proof. We have

$$
\begin{aligned}
A &= \{\langle x, \mu_A(x), \nu_A(x)\rangle\ : x\ \in X\} \\
\Box A &= \{\langle x\ ,\ \mu_A(x)\ ,\ \sqrt[3]{1-\mu_A^3(x)}\ \rangle\ :\ x \in X\} \\
\Diamond A &= \{\langle x\ ,\ \sqrt[3]{1-\nu_A^3(x)}\ ,\ \nu_A(x)\rangle\ :\ x \in X\}
\end{aligned}
$$

$A \subset B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$, for all $x \in X$.

Since $\mu_A(x) \leq \mu_A(x)$, to prove $\Box A \subset A$, it is enough to prove $\sqrt[3]{1-\mu_A^3(x)} \geq \nu_A(x)$.

From the definition of IFSTT,

$$
\begin{aligned}
\mu_A^3(x) + \nu_A^3(x) &\leq 1 \\
\nu_A^3(x) &\leq 1 - \mu_A^3(x) \\
\text{and hence} \quad \nu_A(x) &\leq \sqrt[3]{1-\mu_A^3(x)}
\end{aligned}
$$

Therefore, $\Box A \subset A$.

To prove $A \subset \Diamond A$, it is enough to prove $\mu_A(x) \leq \sqrt[3]{1-\nu_A^3(x)}$.

From the definition of IFSTT,

$$
\begin{aligned}
\mu_A^3(x) + \nu_A^3(x) &\leq 1 \\
\mu_A^3(x) &\leq 1 - \nu_A^3(x) \\
\text{and hence} \qquad \mu_A(x) &\leq \sqrt[3]{1-\nu_A^3(x)}
\end{aligned}
$$

Therefore, $A \subset \Diamond A$.
Hence $\Box A \subset A \subset \Diamond A$. □

5. Conclusion

We have defined a new extension of IFS, namely, IFSTT and studied the various basic operations like union, intersection, subset and complement. We have defined the necessity and possibility operators over the IFSTT with example. We have established the relations between the necessity and possibility operators on IFSTT. It is still open to check whether there exist an IFSTT in case of the operators already defined on an IFS. The defined IFSTT is quiet interesting and useful in many application areas than the existing IFS and IFSST.

References

[1] K. T. Atanassov, Intuitionistic Fuzzy Sets - Theory and Applications, Springer Verlag, New York 1999.

[2] R. Parvathi and N. Palaniappan, Some operations on IFSs of Second Type, Notes on Intuitionistic Fuzzy Sets 10 (2) (2004) 1–19.

[3] R. Srinivasan and N. Palaniappan, Some Operations on Intuitionistic fuzzy sets of Root type, Notes on IFS 12 (3) (2006) 20–29.

[4] R. Srinivasan and N. Palaniappan, Some properties of Intituitionstic Fuzzy Sets of Root Type, International Journal of Computational and Applied Mathematics 4 (3) (2009) 383–390.

[5] R. Srinivasan and N. Palaniappan, Some Topological Operators on Intuitionistic Fuzzy Sets of Root Type, Research methods in Mathematical sciences, India. Edited by Dr. U.Rizwan 4 (2011) 23–28.

[6] R. Srinivasan and N. Palaniappan, Some Operators on Intuitionistic Fuzzy Sets of Root Type, Ann. Fuzzy Math. Inform. 4 (2) (2012) 377–383.

[7] L.A. Zadeh, Fuzzy Sets, Information and Control 8 (1965) 338–353.

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