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Complementary nil dominating set in intuitionistic fuzzy graph using effective edges

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Abstract. In this paper, the complementary nil dominating set and its number in an intuitionistic fuzzy graph is defined. The bounds on this number is obtained for some standard intuitionistic fuzzy graphs. Theorems related to the above concepts are derived. Relation between complementary nil domination number and domination numbers are also derived. In this paper only intuitionistic fuzzy graphs without isolated vertices but not complete are considered.

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1. Introduction

In 1965, Zadeh [16] introduced the concept of fuzzy set as a method of finding uncertainty. Rosenfeld [11] introduced the concept of fuzzy graphs in 1975. Yeh and Bang [15] also introduced fuzzy graphs independently. In 1986, the idea of intuitionistic fuzzy set was introduced by Atanassov [1, 2]. Shannon and Atanassov [12] introduced the concept of intuitionistic fuzzy graphs in 1994. Parvathi and Karunambigai [7] gave a definition for intuitionistic fuzzy graph as a special case of intuitionistic fuzzy graphs defined by Shannon and Atanassov [13]. The concept of domination in fuzzy graph was introduced by Somasundaram and Somasundaram [14] in 1998. Depnath [4] introduced the concept of domination in fuzzy graph with interval-valued membership in 2013. The concept of perfect dominating set in intuitionistic fuzzy graph was presented by Mahioub [6]. Parvathi and Thamizhendhi [9, 10] introduced cardinality of an intuitionistic fuzzy graph and also introduced dominating set, domination number, total dominating set and total domination number of an intuitionistic fuzzy graph. Ismayil and Mohideen[5] introduced the concept of complementary nil domination in fuzzy graphs. In this paper, the concept of complementary nil domination in intuitionistic fuzzy graphs is introduced. Here, intuitionistic fuzzy graphs without isolated vertices but not complete are considered.

2. Preliminaries

Throughout this paper, assume that G^* is a crisp graph and G is an intuitionistic fuzzy graph.

Definition 2.1 ([11])**.** A fuzzy relation on a set V is a fuzzy subset of $V \times V$, that is, a map $\mu : V \times V \to [0,1]$. A fuzzy graph with V as the underlying set is a pair $G = (\sigma, \mu)$ where $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ is a fuzzy relation on σ, that is, $\mu(u,v) \leq \sigma(u) \wedge \sigma(v), \forall (u,v) \in V \times V$.

Definition 2.2. An intuitionistic fuzzy graph is of the form $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ on $G^* = (V, E)$, where

(1) $V = \{v_1, v_2, ..., v_n\}$, where $\sigma_1 : V \to [0,1]$ and $\sigma_2 : V \to [0,1]$ denote the degree of membership and non membership of the element $v_i \in V$ respectively such that $\sigma_1(v_i) + \sigma_2(v_i) \leq 1$ for all $v_i \in V$.

(2) $E \subseteq V \times V$, where $\mu_1 : E \to [0,1]$ and $\mu_2 : E \to [0,1]$ are defined by $\mu_1(v_i, v_j) \leq \sigma_1(v_i) \wedge \sigma_1(v_j)$ and $\mu_2(v_i, v_j) \geq \sigma_1(v_i) \wedge \sigma_1(v_j) + \sigma_2(v_i) \vee \sigma_2(v_j) - \mu_1(v_i, v_j)$ such that $\mu_1(v_i, v_j) + \mu_2(v_i, v_j) \leq 1, \forall (v_i, v_j) \in E$.

Definition 2.3 ([7])**.** An intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ is called strong intuitionistic fuzzy graph if $\mu_1(v_i, v_j) = \sigma_1(v_i) \wedge \sigma_1(v_j)$ and $\mu_2(v_i, v_j) = \sigma_2(v_i) \vee \sigma_2(v_j), \forall (v_i, v_j) \in E$.

Definition 2.4 ([8])**.** An intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ is called complete intuitionistic fuzzy graph if $\mu_1(v_i, v_j) = \sigma_1(v_i) \wedge \sigma_1(v_j)$ and $\mu_2(v_i, v_j) = \sigma_2(v_i) \vee \sigma_2(v_j), \forall v_i, v_j \in V$, $i \neq j$.

Definition 2.5 ([7])**.** An intuitionistic fuzzy graph $H = ((\sigma'_1, \sigma'_2), (\mu'_1, \mu'_2))$ is said to be an intuitionistic fuzzy subgraph of $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ if $\sigma'_1 \leq \sigma_1$, $\sigma'_2 \geq \sigma_2$, $\mu'_1 \leq \mu_1$ and $\mu'_2 \geq \mu_2$.

Example 2.6. An intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ on $G^* = (V, E)$, where $V = \{\mathrm{v}_1/(0.6, 0.2), \mathrm{v}_2/(0.4, 0.1), \mathrm{v}_3/(0.2, 0.8), \mathrm{v}_4/(0.5, 0.3)\}$,
$E = \{(\mathrm{v}_1,\mathrm{v}_2)/(0.3, 0.4), (\mathrm{v}_2,\mathrm{v}_3)/(0.2, 0.8), (\mathrm{v}_3,\mathrm{v}_4)/(0.1, 0.9), (\mathrm{v}_4,\mathrm{v}_1)/(0.5, 0.3)\}$.

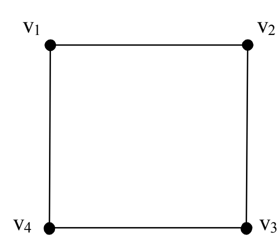

Figure 1. Intuitionistic Fuzzy Graph

Definition 2.7. In an intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$, the weight of a vertex $u \in V$ is defined by

$$w(u) = \frac{1 + \sigma_1(u) - \sigma_2(u)}{2}$$

and also the *weight of an edge* $e = (u, v) \in E$ is defined by

$$w(e) = \frac{1 + \mu_1(u, v) - \mu_2(u, v)}{2}$$

.

Definition 2.8 ([9]). Let $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ be an intuitionistic fuzzy graph, then the scalar cardinality of V defined by

$$|V| = \sum_{v_i \in V} \frac{1 + \sigma_1(v_i) - \sigma_2(v_i)}{2} = \sum_{v_i \in V} w(v_i)$$

is called order of G and is denoted by $O(G)$ or p.

Definition 2.9 ([9]). Let $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ be an intuitionistic fuzzy graph, then the scalar cardinality of E defined by

$$|E| = \sum_{(v_i, v_j) \in E} \frac{1 + \mu_1(v_i, v_j) - \mu_2(v_i, v_j)}{2} = \sum_{(v_i, v_j) \in E} w(v_i, v_j)$$

is called size of G and is denoted by $S(G)$ or q.

Example 2.10. In an intuitionistic fuzzy graph given in FIGURE 1, $O(G) = 2.15$, $S(G) = 1.35$.

Definition 2.11 ([6]). The two vertices are said to be adjacent in an intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ if $\mu_1(u, v) = \sigma_1(u) \wedge \sigma_1(v)$ and $\mu_2(u, v) = \sigma_2(u) \vee \sigma_2(v)$. In this case, u and v are said to be neighbours and (u, v) is incident at u and v also.

Definition 2.12 ([6]). An edge $e = (v_i, v_j)$ of an intuitionistic fuzzy graph is called an effective edge if $\mu_1(v_i, v_j) = \sigma_1(v_i) \wedge \sigma_1(v_j)$ and $\mu_2(v_i, v_j) = \sigma_2(v_i) \vee \sigma_2(v_j)$. In this case, e is incident with v_i and v_j.

Definition 2.13 ([6]). The effective neighbourhood degree of a vertex v in an intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ is defined to be the sum of the weights of the effective edges incident at v. It is denoted by $d_E(v)$.
The minimum effective neighbourhood degree of G is $\delta_E(G) = \min\{d_E(v) | v \in V\}$
The maximum effective neighbourhood degree of G is $\Delta_E(G) = max\{d_E(v) | v \in V\}$.

Definition 2.14 ([6]). A path in an intuitionistic fuzzy graph is a sequence of distinct vertices $v_0, v_1, ..., v_n$ such that $\mu_1(v_i, v_j) = \sigma_1(v_i) \wedge \sigma_1(v_j)$ and $\mu_2(v_i, v_j) = \sigma_2(v_i) \vee \sigma_2(v_j)$, $i = 0, 1, 2, ...n - 1$ and $j = i + 1$.

Definition 2.15 ([7]). The *length of the path* $v_0 v_1 ... v_n (n > 0)$ is n.

Definition 2.16 ([7]). An intuitionistic fuzzy graph is said to be *connected* if every pair of vertices contains at least a path.

Definition 2.17. The complement of an intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ is an intuitionistic fuzzy graph $\overline{G} = ((\sigma_1, \sigma_2), (\overline{\mu_1}, \overline{\mu_2}))$, where $\overline{\mu_1}(u, v) = \sigma_1(u) \wedge \sigma_1(v) - \mu_1(u, v)$ and $\overline{\mu_2}(u, v) = 1 + \sigma_2(u) \vee \sigma_2(v) - \mu_2(u, v)$.

Definition 2.18. An intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ on $G^* = (V, E)$ is said to be bipartite if the vertex set V can be partitioned into two non empty sets V_1 and V_2 such that

(1) $\mu_1(v_i, v_j) = 0, \mu_2(v_i, v_j) = 1$ if $v_i, v_j \in V_1$ or $v_i, v_j \in V_2$
(2) $\mu_1(v_i, v_j) \geq 0, \mu_2(v_i, v_j) > 0$ if $v_i \in V_1$ and $v_j \in V_2$ for some i and j (or) $\mu_1(v_i, v_j) > 0, \mu_2(v_i, v_j) \geq 0$ if $v_i \in V_1$ and $v_j \in V_2$ for some i and j.

Definition 2.19. A bipartite intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ is said to be complete bipartite if $\mu_1(v_i, v_j) = \sigma_1(v_i) \wedge \sigma_1(v_j)$ and $\mu_2(v_i, v_j) = \sigma_2(v_i) \vee \sigma_2(v_j)$ for all $v_i \in V_1$ and $v_j \in V_2$.It is denoted by K_{p_1,p_2}.

Definition 2.20. Let u be a vertex in an intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$, then $N(u) = \{v \in V : u$ is adjacent to $v\}$ is called the open neighbourhood set of u and $N[u] = N(u) \cup \{u\}$ is called the closed neighbourhood set of u.

Definition 2.21. A vertex $v \in V$ of an intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ is said to be an isolated vertex if there is no effective edge incident at v.

Definition 2.22. A vertex in an intuitionistic fuzzy graph having exactly one neighbour is called a pendent vertex. Otherwise, it is called non-pendent vertex. An edge in an intuitionistic fuzzy graph incident with a pendent vertex is called a pendent edge. Otherwise it is called non-pendent edge. A vertex in an intuitionistic fuzzy graph adjacent to the pendent vertex is called a support of the pendent edge.

Definition 2.23 ([6])**.** Let $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ be an intuitionistic fuzzy graph and $u, v \in V$, we say that u dominates v in G if $\mu_1(u, v) = \sigma_1(u) \wedge \sigma_1(v)$ and $\mu_2(u, v) = \sigma_2(u) \vee \sigma_2(v)$.

Definition 2.24 ([14])**.** A set $S \subset V$ is called a]dominating set in G if for every $v \in V - S$, there exists $u \in S$ such that u dominates v.

Definition 2.25 ([6])**.** A dominating set S of an intuitionistic fuzzy graph G is said to be minimal dominating set if there is no dominating set S' such that $S' \subset S$. Minimum scalar cardinality among all the minimal dominating set is called a domination number of G and is denoted by $\gamma(G)$. Maximum scalar cardinality among all minimal dominating set is called an upper domination number and is denoted by $\Gamma(G)$.

Definition 2.26. A set $S \subset V$ in an intuitionistic fuzzy graph G is said to be independent set if no two vertices of S are adjacent.

Definition 2.27 ([5])**.** Let $G = (\sigma, \mu)$ be a fuzzy graph on V. A set $S \subset V$ is said to be a complementary nil dominating set (or simply called cnd-set) of a fuzzy graph G if S is a dominating set and its complement $V - S$ is not a dominating set.

3. Complementary nil dominating set in an intuitionistic fuzzy graph

In this section, complementary nil dominating set and complementary nil domination number are defined with suitable example.

Definition 3.1. Let $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ on $G^* = (V, E)$ be an intuitionistic fuzzy graph on V. A set $S \subset V$ is said to be a complementary nil dominating set (or simply called cnd-set) of an intuitionistic fuzzy graph G if S is a dominating set and its complement $V - S$ is not a dominating set.

Example 3.2. Consider an intuitionsitic fuzzy $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ on $G^* = (V, E)$ given in FIGURE 2, where $V = \{(0.2, 0.7)/\text{v}_1, (0.5, 0.2)/\text{v}_2, (0.3, 0.6)/\text{v}_3,$
$(0.1, 0.9)/\text{v}_4, (0.7, 0.2)/\text{v}_5, (0.6, 0.2)/\text{v}_6\}$ and $E = \{(0.2, 0.7)/(\text{v}_1,\text{v}_2),$
$(0.2, 0.8)/(\text{v}_2,\text{v}_3), (0.1, 0.9)/(\text{v}_3,\text{v}_4), (0.1, 0.9)/(\text{v}_4,\text{v}_5), (0.4, 0.5)/(\text{v}_5,\text{v}_6),$
$(0.2, 0.7)/(\text{v}_6,\text{v}_1), (0.2, 0.7)/(\text{v}_1,\text{v}_3), (0.1, 0.9)/(\text{v}_1,\text{v}_4), (0.1, 0.9)/(\text{v}_4,\text{v}_6)\}$.
In this example, some of the complementary nil dominating sets are $\{\text{v}_1,\text{v}_3,\text{v}_4\}$, $\{\text{v}_1,\text{v}_2,\text{v}_4\}$,$\{\text{v}_1,\text{v}_4,\text{v}_6\}$,$\{\text{v}_1,\text{v}_4,\text{v}_5\}$

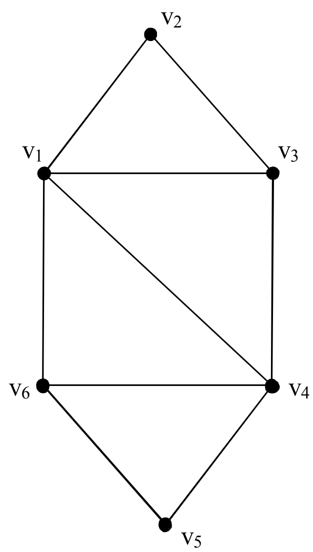

FIGURE 2. Complementary Nil Dominating Set

Definition 3.3. A cnd-set S of an intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ is called a minimal cnd-set if there is no cnd-set S' such that $S' \subset S$.

Definition 3.4. A cnd-set S of an intuitionsitic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ is called a minimum cnd-set if there is no cnd-set such that $|S'| < |S|$. The minimum scalar cardinality taken over all cnd-set is called a complementary nil domination number and is denoted by the symbol γ_{cnd}, the corresponding minimum cnd-set is denoted by γ_{cnd}-set.

Observation 3.5. For any intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$,

(1) Every super set of a cnd-set is also a cnd-set.
(2) Complement of a cnd-set is not a cnd-set.

Definition 3.6. The maximum scalar cardinality taken over all minimal cnd-set is called an upper complementary nil dominating number and is denoted by the symbol Γ_{cnd}.

Note 1. For any intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ on $G^* = (V, E)$, $\gamma_{cnd} \leq \Gamma_{cnd}$.

Example 3.7. Consider an intuitionistic fuzzy graph G given in FIGURE 3,
The only complementary nil dominating sets are $S_1 = \{\text{v}_1,\text{v}_2,\text{v}_4\}$ and $S_2 = \{\text{v}_2,\text{v}_3,\text{v}_4\}$.

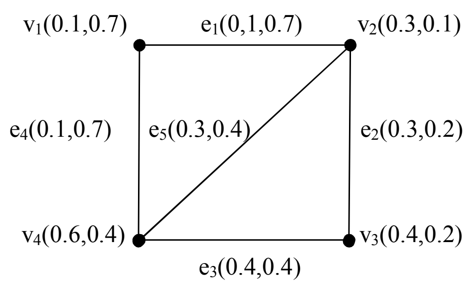

FIGURE 3. Illustration of complementary nil domination number

S_1 is a minimal as well as minimum cnd-set whereas S_2 is a minimal but not minimum cnd-set. The complementary nil domination number of G is $\gamma_{cnd} = 1.4$. The upper complementary nil domination number of G is $\Gamma_{cnd} = 1.8$.

Definition 3.8. A vertex $u \in S \subseteq V$ is said to be an enclave of S if $\mu_1(u, v) < \sigma_1(u) \wedge \sigma_1(v)$ or $\mu_2(u, v) > \sigma_2(u) \vee \sigma_2(v)$ for all $v \in V - S$ that is $N[u] \subseteq S$.

Example 3.9. In Example 3.7, v_1 and v_3 are enclaves of the cnd-sets S_1 and S_2 respectively.

4. Theorems related to complementary nil dominating set

In this section, also proved a dominating set of an intuitionistic fuzzy graph is a cnd-set iff it contains atleast one enclave and some other theorems related to complementary nil dominating sets are stated and proved.

Theorem 4.1. *A dominating set S of an intuitionistic fuzzy graph is a cnd-set if and only if it contains at least one enclave.*

Proof. Let S be a cnd-set of a intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$. Then $V - S$ is not a dominating set which implies that there exists a vertex $u \in S$ such that either $\mu_1(u, v) < \sigma_1(u) \wedge \sigma_1(v)$ or $\mu_2(u, v) > \sigma_2(u) \vee \sigma_2(v)$ for all $v \in V - S$. Therefore u is an enclave of S. Hence S contains at least one enclave.
Conversely, Suppose the dominating set S contains enclaves. Without loss of generality let us take u be the enclave of S. That is $\mu_1(u, v) < \sigma_1(u) \wedge \sigma_1(v)$ or $\mu_2(u, v) > \sigma_2(u) \vee \sigma_2(v)$, for all $v \in V - S$. Hence $V - S$ is not a dominating set. Hence the dominating set S is a cnd-set. □

Theorem 4.2. *If S is a cnd-set of a connected intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$, then there is a vertex $u \in S$ such that $S - \{u\}$ is a dominating set.*

Proof. Let S be a cnd-set of a connected intuitionistic fuzzy graph. By theorem 4.1, every cnd-set contains at least one enclave of S. Let $u \in S$ be an enclave of S. Then $\mu_1(u,v) < \sigma_1(u) \wedge \sigma_1(v)$ or $\mu_2(u,v) > \sigma_2(u) \vee \sigma_2(v)$ for all $v \in V - S$. Since G is a connected intuitionistic fuzzy graph, there exists at least a vertex $w \in S$ such that $\mu_1(u,w) = \sigma_1(u) \wedge \sigma_1(w)$ and $\mu_2(u,w) = \sigma_2(u) \vee \sigma_2(w)$. Hence $S - \{u\}$ is a dominating set. □

Theorem 4.3. *A cnd-set in an intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ is not a singleton.*

Proof. Let S be a cnd-set of an intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$. By theorem 4.1, every cnd-set contains at least one enclave of S. Let $u \in S$ be an enclave of S. Then

$$\mu_1(u,v) < \sigma_1(u) \wedge \sigma_1(v) \text{ or } \mu_2(u,v) > \sigma_2(u) \vee \sigma_2(v) \text{ for all } v \in V - S \text{ ——(1).}$$

Suppose S contains only one vertex u, (1) shows it must be isolated in G, which is a contradiction to connectedness. Hence cnd-set in an intuitionistic fuzzy graph is not a singleton. □

Theorem 4.4. *Let $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ be an intuitionistic fuzzy graph and S be a γ_{cnd}-set of G. If u and v are two enclaves of S. Then*

1. *$N[u] \cap N[v] \neq \phi$ and*
2. *$\mu_1(u,v) = \sigma_1(u) \wedge \sigma_1(v)$ and $\mu_2(u,v) = \sigma_2(u) \vee \sigma_2(v)$, that is u and v are adjacent.*

Proof. Let S be a minimum cnd-set of an intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ and let u and v are two enclaves of S.

(1) Suppose $N[u] \cap N[v] = \phi$. Then u is an enclave of $S - N(v)$ which implies that $V - (S - N(v))$ is not a dominating set. Therefore $S - N(v)$ is a cnd-set of G and $|S - N(v)| < |S| = \gamma_{cnd}(G)$. Which is a contradiction to the minimality of S. Hence $N[u] \cap N[v] \neq \phi$.

(2) Suppose $\mu_1(u,v) < \sigma_1(u) \wedge \sigma_1(v)$ or $\mu_2(u,v) > \sigma_2(u) \vee \sigma_2(v)$, that is u and v are non-adjacent. Then $u \notin N(v)$ and so u is an enclave of $S - \{v\}$ which implies that $V - (S - \{v\})$ is not a dominating set. Hence $S - \{v\}$ is a cnd-set, which is a contradiction to minimality of S. Hence u and v are adjacent. □

Theorem 4.5. *A cnd-set of an intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ is not independent.*

Proof. Let $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ be an intuitionistic fuzzy graph. Suppose a cnd-set S of G is independent. Then S is a minimal dominating set which implies that $V - S$ is a dominating set. Hence S is not a cnd-set, which is a contradiction. □

Theorem 4.6. *A cnd-set S of an intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ is minimal if and only if for each $u \in S$ at least one of the following condition is satisfied.*

1. *there exists $v \in V - S$ such that $N(v) \cap S = \{u\}$.*
2. *$V - (S - \{u\})$ is a dominating set of G.*

Proof. If S is a minimal cnd-set of an intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$. Suppose, if there exists $u \in S$ such that u does not satisfy both the given conditions (1) and (2). Then By theorem 4.1, S is not a minimal dominating set. Hence the proper subset $S_1 = S - \{u\}$ is a dominating set. By our assumption on (2) $V - (S - \{u\})$ is a dominating set. Hence $S_1 = S - \{u\}$ is a cnd-set. Which is a contradiction to the minimality of the cnd-set S.
Conversely, let S is a cnd-set and for each $u \in S$ at least one of the two conditions holds. Now we show that S is minimal cnd-set of G. Suppose S is not minimal, then there exists a vertex $u \in S$ such that $S - \{u\}$ is a cnd-set. Hence $\mu_1(u, v) = \sigma_1(u) \wedge \sigma_1(v)$ and $\mu_2(u, v) = \sigma_2(u) \vee \sigma_2(v)$ for at least one vertex $v \in S - \{u\}$. Also $S - \{u\}$ is a dominating set, every vertex in $V - (S - \{u\})$ is adjacent to at least one vertex in $S - \{u\}$. Therefore condition (1) does not hold. Since $S - \{u\}$ is a cnd-set, $V - (S - \{u\})$ not a dominating set. That is condition (2) does not hold.
Hence there exists a vertex $u \in S$ which does not satisfy conditions (1) and (2), a contradiction to our assumption. Hence S is a minimal cnd-set of G. □

Theorem 4.7. *For any intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ every γ_{cnd}-set of G intersects with every γ-set of G.*

Proof. Let S be a γ_{cnd}-set and D be a γ-set of $G = (V, E)$. Suppose $S \cap D = \phi$, then $D \subseteq V - S$, $V - S$ contains a dominating set D. Therefore $V - S$, a super set of D, is a dominating set. Which is a contradiction to our assumption. Hence $S \cap D \neq \phi$. □

Corollary 4.8. *For any intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ any two γ_{cnd}-sets intersects .*

5. Bounds for complementary nil domination number of an intuitionistic fuzzy graph

In this section, bounds for complementary nil domination number of some standard intuitionistic fuzzy graphs are determined.

Observation 5.1. .

(1) For any intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$, $\gamma < \gamma_{cnd} < p$.
(2) $\gamma_{cnd}(K_p - e) \leq p - w_0$, where $w_0 = \min_{v_i \in V} w(v_i)$
(3) $\gamma_{cnd}(K_p - e) = p - w(u)$,
where $w(u)$ is obtained from $w(e) < w(u) \wedge w(v) = w(u)$
(4) For any intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$,
$2w_0 \leq \gamma_{cnd} \leq p - w_0$.
(5) For a complete bipartite intuitionistic fuzzy graph,
$\gamma_{cnd}(K_{p_1,p_2}) \leq \min\{p_1, p_2\} + w_n$, where $w_n = \max_{v_i \in V} w(v_i)$
(6) For a complete bipartite intuitionistic fuzzy graph

$$\gamma_{cnd}(K_{p_1,p_2}) = \begin{cases} p_1 + w_{20} & , if|V_1| < |V_2| \\ p_2 + w_{10} & , if|V_1| > |V_2| \\ p_1 + w_0 & , if|V_1| = |V_2| \end{cases}$$

where w_{10} and w_{20} are the minimum weight of a vertex in V_1 and V_2 respectively.

(7) Let T_p be tree in an intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$, Then $\gamma_{cnd}(T_p) \leq \gamma(T_p) + w_n$.

Theorem 5.2. *For any intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ on $G = (V, E)$, $\delta_N + w_0 \leq \gamma_{cnd} \leq \gamma + \delta_N + w_n - w_0$.*

Proof. Let S be a cnd-set of an intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$. Since $V - S$ is not a dominating set, there exists $u \in S$ such that $\mu_1(u, v) < \sigma_1(u) \wedge \sigma_1(v)$ or $\mu_2(u, v) > \sigma_2(u) \vee \sigma_2(v)$ for all $v \in V - S$. Then $N[u] \subseteq S$ which implies $|N[u]| \leq |S|$. Hence $\delta_N + w_0 \leq \gamma_{cnd}$.
Let S_1 be a γ-set of G and let $u \in V$ such that $d_N(u) = \delta_N$. Then u is either in D or in $V - D$.
Case(i): If $u \in D$, then $D \cup N(u)$ contains an enclave. Therefore $D \cup N(u)$ is a cnd-set. Hence $\gamma_{cnd} \leq \gamma + \delta_N$.
Case(ii): If $u \in V - D$, then at least a vertex $v \in D$ such that $\mu_1(u, v) < \sigma_1(u) \wedge \sigma_1(v)$ or $\mu_2(u, v) > \sigma_2(u) \vee \sigma_2(v)$. Then $D \cup N[u]$ contains an enclave. Therefore $D \cup N[u]$ is a cnd-set and $D \cap N[u]$ is a non empty set because $v \in D \cap N[u]$. Hence $\gamma_{cnd} \leq |D \cup N[u]| = |D| + |N[u]| - |D \cap N[u]| = \gamma + \delta_N + w_n - w_0$.
In both the cases $\gamma_{cnd} \leq |D \cup N[u]| = |D| + |N[u]| - |D \cap N[u]| = \gamma + \delta_N + w_n - w_0$. □

Example 5.3. Consider the intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ in Example 3.7, the complementary nil dominating sets are $S_1 = \{v_1, v_2, v_4\}$ and $S_2 = \{v_2, v_3, v_4\}$, $\gamma_{cnd} = 1.4, \gamma = 0.6, \delta_N = 1.2, w_n = 0.6, w_0 = 0.2$. Hence $\delta_N + w_0 \leq \gamma_{cnd} \leq \gamma + \delta_N + w_n - w_0$.

Theorem 5.4. *Let T_p be an intuitionistic fuzzy tree , then $\gamma_{cnd}(T_p) \leq p - r + w_{p_0}$, where r is the scalar cardinality of set of all pendent vertices in T_p and w_{p_0} is a minimum membership grade of a pendent vertex.*

Proof. Let T_p be an intuitionistic fuzzy tree ,then the set of all non-pendent vertices together with a pendent vertex form a cnd-set. Hence $\gamma_{cnd}(T_p) \leq r + w_{p_0}$, where r is the scalar cardinality of set of all pendent vertices in T_p and w_{p_0} is a minimum value of a pendent vertex. □

Theorem 5.5. *For any intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ if $\gamma = \frac{p}{2}$, then $\gamma_{cnd}(G) = \frac{p}{2} + w_0$.*

Proof. Let $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ be an intuitionistic fuzzy graph and let S be γ-set of G with $\gamma(G) = \frac{p}{2}$. Hence $V - S$ is a γ-set with $|V - S| = \frac{p}{2}$. Choose a vertex $u \in V$ such that $w(u) = w_0$. Now either $u \in S$ or $u \in V - S$. Hence $[(V - S) \cup \{u\}]$ or $S - \{u\}$ is a cnd-set.Then either $D \cup \{u\}$ or $(V - S) - \{u\}$ is not a dominating set. Therefore either $D \cup \{u\}$ or $[(V - S) - \{u\}]$ is a cnd-set. Hence $\gamma_{cnd} = \frac{p}{2} + w_0$. □

Theorem 5.6. *For any intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$, $\Gamma + \gamma_{cnd} \leq p + w_n$.*

Proof. Let S be a Γ-set of G, then there exists a vertex $u \in S$ such that $S - \{v\}$ is not a dominating set of G. Since S is a minimal dominating set. Then $V - S$ is a dominating set and $[(V - S) \cup \{u\}]$ is also a dominating set. Complement of $[(V - S) \cup \{u\}]$ is $S - \{u\}$, but $S - \{u\}$ is not a dominating set. Therefore $(V - S) \cup \{u\}$ is a cnd-set. Hence $\gamma_{cnd} \leq |(V - S) \cup \{u\}| = p - \Gamma + w_n$. Thus $\Gamma + \gamma_{cnd} \leq p + w_n$. □

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