Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 10, No. 5, (November 2015), pp. 755–764
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

@FMI

http://www.kyungmoon.com

$\lambda-$Closed maps in intuitionistic fuzzy topological spaces

P. Rajarajeswari, G. Bagyalakshmi

Received 30 October 2014; Revised 6 February 2015; Accepted 19 March 2015

Abstract. In this paper we introduce the concept intuitionistic fuzzy λ-open maps and intuitionistic fuzzy λ-closed maps in intuitionistic fuzzy topological space and study some of their properties.

2010 AMS Classification: AMS subject classification (2000): 54A40, 03F55

Keywords: Intuitionistic fuzzy topology, Intuitionistic fuzzy $\lambda-$closed maps, intuitionistic fuzzy $\lambda-$open maps .

Corresponding Author: G. Bagyalakshmi (g_bagyalakshmi@yahoo.com)

1. Introduction

The concept of fuzzy set (FS) and fuzzy operations were first introduced by L.A Zadeh in 1965, in his classical paper [15]. Subsequently several authors have applied varies basic concepts from general topology to fuzzy sets and developed the theory of fuzzy topological space. After the introduction of fuzzy topology by Chang [2] in 1968, there have been several generalization of notions of fuzzy sets and fuzzy topology. The idea of "intuitionistic fuzzy sets " was introduced by Atanassov [1] as a generalization of fuzzy set in 1983. Coker [3] introduced the notion of intuitionistic fuzzy topology in 1997. This approach provides a wide field for investigation in the area of fuzzy topology and its application. The aim of this paper is to introduce intuitionistic fuzzy $\lambda-$open maps, intuitionistic fuzzy $\lambda-$closed maps and studied some of their properties.

2. Preliminaries

Definition 2.1 ([1])**.** Let X be a nonempty fixed set. An intuitionistic fuzzy set (IFS) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the function $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ denotes the degree of membership $\mu_A(x)$ and the degree of non membership $\nu_A(x)$ of each element $x \in X$ to the set A respectively and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$.

Definition 2.2 ([1]). Let A and B be intuitionistic fuzzy sets of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, and form $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle /x \in X\}$
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle /x \in X\}$
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle /x \in X\}$.

The intuitionistic fuzzy sets $\underset{\sim}{0} = \{\langle x, 0, 1\rangle /x \in X\}$ and $\underset{\sim}{1} = \{\langle x, 1, 0\rangle /x \in X\}$ are respectively the empty set and whole set of X.

Definition 2.3 ([5]).) An intuitionistic fuzzy topology (IFT) on X is a family of IFS which satisfying the following axioms.

(i) $\underset{\sim}{0}, \underset{\sim}{1} \in \tau$
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(iii) $\cup G_i \in \tau$ for any family $\{Gi/i \in I\} \subseteq \tau$

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS) and each intuitionistic fuzzy set in τ is known as an intuitionistic fuzzy open set (IFOS) in X. The complement A of an IFOS in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS) in (X, τ).

Definition 2.4 ([3]). Let (X, τ) be an intuitionistic fuzzy topology and $A = \{\langle x, \mu_A(x), \nu_B(x)\rangle : x \in X\}$, be an intuitionistic fuzzy set in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by
$int(A)$=$\cup\{G/G$ is an intuitionistic fuzzy open set in X and $G \subseteq A\}$
$cl(A)$=$\cap\{K/K$ in an intuitionistic fuzzy closed set in X and $A \subseteq K\}$

Definition 2.5 ([10]). Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an

(i) intuitionistic fuzzy open mapping (IF open mapping) if $f(A)$ is an IFOS in Y for every IFOS A in X.
(ii) intuitionistic fuzzy closed mapping (IF open mapping) if $f(A)$ is an IFCS in Y for every IFCS A in X.

Remark 2.6. For any intuitionistic fuzzy set A in (X, τ), we have

(i) $cl(A^c) = [int(A)]^c$,
(ii) $int(A^c) = [cl(A)]^c$,
(iii) A is an intuitionistic fuzzy closed set in $X \Leftrightarrow cl(A) = A$
(iv) A is an intuitionistic fuzzy open in $X \Leftrightarrow int(A) = A$

Definition 2.7 ([6]). An intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \nu_B(x)\rangle : x \in X\}$ in an intuitionistic fuzzy topology space (X, τ) is said to be

(i) intuitionistic fuzzy semi closed if $int(cl(A)) \subseteq A$.
(ii) intuitionistic fuzzy pre closed if $cl(int(A)) \subseteq A$.

Definition 2.8 ([5]). Let X and Y are nonempty sets and $f : X \to Y$ is a function

(a) If $B = \{\langle y, \mu_B(y), \nu_B(y)\rangle : y \in Y\}$ is an intuitionistic fuzzy set in Y, then the pre image of B under f denoted by $f^{-1}(B)$ is defined by
$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x))\rangle : x \in X\}$

(b) If $A = \{\langle x, \mu_A(x), \nu_B(x)\rangle : x \in X\}$ is an intuitionistic fuzzy set in X, the image of A under f denoted by $f(A)$ is the intuitionistic fuzzy set in Y defined by $f(A) = \{\langle y, f(\mu_A(y)), f(\nu_A(y))\rangle : y \in Y\}$ where $f(\nu_A) = 1 - f(1 - \nu_A)$.

Definition 2.9 ([8])**.** An intutionistic fuzzy set A of an intuitionistic topology space (X, τ) is called an

(i) intuitionistic fuzzy $\lambda-$closed set (IF $\lambda-$CS) if $A \supseteq cl(U)$ whenever $A \supseteq U$ and U is intuitionistic fuzzy open set in X.

(ii) intuitionistic fuzzy $\lambda-$open set (IF $\lambda-$OS) if the complement A^c is an intuitionistic fuzzy $\lambda-$closed set in A.

Definition 2.10. An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) called

(i) intuitionistic fuzzy generalized closed set [13](intuitionistic fuzzy $g-$closed) is $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic fuzzy semi open

(ii) intuitionistic fuzzy $g-$open set [12], if the complement of an intuitionistic fuzzy $g-$closed set is called intuitionistic fuzzy $g-$open set.

(iii) intuitionistic fuzzy semi open (resp. intuitionistic fuzzy semi closed) [6] if there exists an intuitionistic fuzzy open (resp. intuitionistic fuzzy closed) such that $U \subseteq A \subseteq cl(U)$ (resp. $int(U) \subseteq A \subseteq U$).

Definition 2.11. An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) s called

(i) an intuitionistic fuzzy $w-$closed [12] if $cl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy semi open in (X, τ).

(ii) an intuitionistic fuzzy generalized $\alpha-$closed set [7] (IF $\alpha-$CS) if $\alpha cl(A) \subseteq O$ whenever $A \subseteq O$ and O is IF $\alpha-$OS in (X, τ)

(iii) an intuitionistic fuzzy $\alpha-$generalized closed set [11] (IF $\alpha-$GCS) if $\alpha cl(A) \subseteq O$ whenever $A \subseteq O$ and O is IFOS in (X, τ)

(iv) an intuitionistic fuzzy regular closed set [4] (IFRCS in short) if $A = cl(int(A))$.

(v) an intuitionistic fuzzy regular open set [4] (IFROS in short) if $A = int(cl(A))$.

Definition 2.12 ([9])**.** A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be intutionistic fuzzy $\lambda-$continuous if the inverse image of every intutionistic fuzzy closed set of Y is intutionistic fuzzy $\lambda-$closed in X.

Definition 2.13 ([10])**.** A topological space (X, τ) is called intuitionistic fuzzy $\lambda - T_{1/2}$space (IF $\lambda - T_{1/2}$space in short) if every intutionistic fuzzy $\lambda-$closed set is intuitionistic fuzzy closed in X.

Definition 2.14. A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be

(i) an intuitionistic fuzzy $w-$closed [13] if image of every intuitionistic fuzzy closed set of X is intuitionistic fuzzy $w-$closed set in Y

(ii) an intuitionistic fuzzy regular closed [14] if image of every intuitionistic fuzzy closed set of X is intuitionistic fuzzy regular closed set in Y

(iii) an intuitionistic fuzzy generalized $\alpha-$closed [7] if image of every intuitionistic fuzzy closed set of X is intuitionistic fuzzy generalized $\alpha-$closed set in Y

(iv) an intuitionistic fuzzy $\alpha-$generalized closed [11] if image of every intuitionistic fuzzy closed set of X is intuitionistic fuzzy $\alpha-$generalized closed set in Y

Definition 2.15 ([9]). Let A be an IFS in an IFTS (X, τ). Then the intuitionistic fuzzy $\lambda-$interior and intuitionistic fuzzy $\lambda-$closure of A are defined as follows
$\lambda - int(A)$=$\cup\{G/G$ is an IF $\lambda-$OS in X and $G \subseteq A$ $\}$
$\lambda - cl(A)$=$\cap\{K/K$ is an IF $\lambda-$CS in X and $A \subseteq K\}$

3. Intuitionistic fuzzy $\lambda-$closed mappings

Definition 3.1. A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be intuitionistic fuzzy $\lambda-$closed map (IF $\lambda-$closed map) if $f(V)$ is $\lambda-$closed in (Y, σ) for every closed set V in (X, τ).

Theorem 3.2. *Every IF closed map is an IF-λ closed map but not conversely.*

Proof. Let $f : X \to Y$ be an IF closed map. Let A be an IFCS in X. Then $f(A)$ is an IFCS in Y. Since every IFCS is an IF $\lambda-$CS, $f(A)$ is an IF $\lambda-$CS in Y [9]. Hence, f is an IF $\lambda-$closed map. □

Remark 3.3. The converse of above theorem need not be true as seen from the following example.

Example 3.4. Let $X = \{a, b\}$ and $Y = \{u, v\}$ and intutionistic fuzzy sets U and V are defined as follows: $U = \{\langle a, 0.5, 0.5\rangle, \langle b, 0.2, 0.8\rangle\}$, $V = \{\langle u, 0.5, 0.5\rangle, \langle v, 0.3, 0.6\rangle\}$. Let $\tau = \{0, 1, U\}$ and $\sigma = \{0, 1, V\}$ be intuitionistic fuzzy topologies on X and Y respectively. Define a map $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then $f(\{\langle a, 0.5, 0.5\rangle, \langle b, 0.8, 0.2\rangle\}) = \{\langle u, 0.5, 0.5\rangle, \langle v, 0.8, 0.2\rangle\}$ is $\lambda-$closed set in Y. But $\{\langle u, 0.5, 0.5\rangle, \langle v, 0.8, 0.2\rangle\}$ is not closed set in Y. Hence, f is intuitionistic fuzzy $\lambda-$closed mapping in Y but not intuitionistic fuzzy closed in Y.

Theorem 3.5. *Let $f : X \to Y$ be an IF $\lambda-$closed map where Y is an IF $\lambda - T_{1/2}$ space, then f is an IF closed map if every IF $\lambda-$CS is an IFCS in Y.*

Proof. Let f be an IF $\lambda-$closed map. Then for every IFCS A in X, $f(A)$ is an IF $\lambda-$CS in Y. Since Y is an IF $\lambda - T_{1/2}$ space, $f(A)$ is an IF $\lambda-$CS in Y and by hypothesis $f(A)$ is an IFCS in Y. Hence, f is an IF closed map. □

Theorem 3.6. *Every IF pre closed map is IF $\lambda-$closed map.*

Proof. Let $f : X \to Y$ be an IF pre closed map. Let A be an IFCS in X. By assumption, $f(A)$ is an IF pre closed set in Y. Since every IF pre closed set is an IF $\lambda-$CS [6] $f(A)$ is an IF $\lambda-$CS in Y. Hence, f is an IF $\lambda-$closed map. □

Remark 3.7. The converse of above theorem need not be true as seen from the following example.

Example 3.8. Let $X = \{a, b\}$ and $Y = \{u, v\}$ and intutionistic fuzzy sets U and V are defined as follows: $U = \{\langle a, 0.5, 0.5\rangle, \langle b, 0.5, 0.4\rangle\}$, $V = \{\langle u, 0.5, 0.5\rangle, \langle v, 0.5, 0.2\rangle\}$. Let $\tau = \{0, 1, U\}$ and $\sigma = \{0, 1, V\}$ be intuitionistic fuzzy topologies on X and Y respectively. Define a map $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then

$f(\{\langle a,0.5,0.5\rangle,\langle b,0.4,0.5\rangle\}) = \{\langle u,0.5,0.5\rangle,\langle v,0.4,0.5\rangle\}$ is IF $\lambda-$closed set in Y. But not IF pre closed set in Y. Hence, f is intuitionistic fuzzy $\lambda-$closed map but not intuitionistic fuzzy pre closed map.

Remark 3.9. IF $\lambda-$closed map and IF $w-$closed map are independent to each other as seen from the following example.

Example 3.10. Let $X = \{a,b\}$ and $Y = \{u,v\}$ and intutionistic fuzzy sets U and V are defined as follows: $U = \{\langle a,0.5,0.5\rangle,\langle b,0.5,0.4\rangle\}$, $V = \{\langle u,0.5,0.5\rangle,\langle v,0.5,0.2\rangle\}$. Let $\tau = \{0,1,U\}$ and $\sigma = \{0,1,V\}$ be intuitionistic fuzzy topologies on X and Y respectively. Define a map $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Then $f(\{\langle a,0.5,0.5\rangle,\langle b,0.4,0.5\rangle\}) = \{\langle u,0.5,0.5\rangle,\langle v,0.4,0.5\rangle\}$ is intuitionistic fuzzy IF $\lambda-$closed set but not IF $w-$closed set. Hence, f is intuitionistic fuzzy $\lambda-$closed mapping but not intuitionistic fuzzy $w-$closed mapping.

Example 3.11. Let $X = \{a,b\}$ and $Y = \{u,v\}$ and intutionistic fuzzy sets U and V are defined as follows: $U = \{\langle a,0.5,0.5\rangle,\langle b,0.5,0.5\rangle\}$, $V = \{\langle u,0.5,0.5\rangle,\langle v,0.4,0.6\rangle\}$. Let $\tau = \{0,1,U\}$ and $\sigma = \{0,1,V\}$ be intuitionistic fuzzy topologies on X and Y respectively. Define a map $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Then $f(\{\langle a,0.5,0.5\rangle,\langle b,0.5,0.5\rangle\}) = \{\langle u,0.5,0.5\rangle,\langle v,0.5,0.5\rangle\}$ is IF $w-$closed set but not IF $\lambda-$closed set. Hence, f is intuitionistic fuzzy $w-$closed mapping but not intuitionistic fuzzy $\lambda-$closed mapping.

Remark 3.12. Intuitionistic fuzzy $g-$closed mappings and intuitionistic fuzzy $\lambda-$closed mappings are independent as seen from the following examples.

Example 3.13. Let $X = \{a,b\}$ and $Y = \{u,v\}$ and intutionistic fuzzy sets U and V are defined as follows: $U = \{\langle a,0.5,0.5\rangle,\langle b,0.6,0.3\rangle\}$, $V = \{\langle u,0.5,0.5\rangle,\langle v,0.2,0.6\rangle\}$. Let $\tau = \{\underset{\sim}{0},\underset{\sim}{1},U\}$ and $\sigma = \{\underset{\sim}{0},\underset{\sim}{1},V\}$ be intuitionistic fuzzy topologies on X and Y respectively. Define a map $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Then $f(\{\langle a,0.5,0.5\rangle,\langle b,0.3,0.6\rangle\}) = \{\langle u,0.5,0.5\rangle,\langle v,0.3,0.6\rangle\}$ is intuitionistic fuzzy $g-$closed set but not intuitionistic fuzzy $\lambda-$closed set. Hence, f is intuitionistic fuzzy $g-$closed mapping and not intuitionistic fuzzy $\lambda-$ mapping.

Example 3.14. Let $X = \{a,b\}$ and $Y = \{u,v\}$ and intutionistic fuzzy sets U and V are defined as follows: $U = \{\langle a,0.5,0.5\rangle,\langle b,0.4,0.5\rangle\}$, $V = \{\langle u,0.5,0.5\rangle,\langle v,0.5,0.2\rangle\}$. Let $\tau = \{\underset{\sim}{0},\underset{\sim}{1},U\}$ and $\sigma = \{\underset{\sim}{0},\underset{\sim}{1},V\}$ be intuitionistic fuzzy topologies on X and Y respectively. Define a map $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Then $f(\{\langle a,0.5,0.5\rangle,\langle b,0.5,0.4\rangle\}) = \{\langle u,0.5,0.5\rangle,\langle v,0.5,0.4\rangle\}$ is intuitionistic fuzzy $\lambda-$closed set but not intuitionistic fuzzy $g-$closed set. Hence, f is intuitionistic fuzzy $\lambda-$ mapping and not intuitionistic fuzzy $g-$closed mapping.

Remark 3.15. The concept of intuitionistic fuzzy $\lambda-$closed mappings and intuitionistic fuzzy semi closed mappings are independent as seen from the following examples.

Example 3.16. Let $X = \{a,b\}$ and $Y = \{u,v\}$ and intutionistic fuzzy sets U and V are defined as follows: $U = \{\langle a,0.5,0.5\rangle,\langle b,0.3,0.5\rangle\}$, $V = \{\langle u,0.5,0.5\rangle,\langle v,0.1,0.9\rangle\}$. Let $\tau = \{\underset{\sim}{0},\underset{\sim}{1},U\}$ and $\sigma = \{\underset{\sim}{0},\underset{\sim}{1},V\}$ be intuitionistic fuzzy topologies on X and Y respectively. Define a map $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Then the mapping $f(\{\langle a,0.5,0.5\rangle,\langle b,0.5,0.4\rangle\}) = \{\langle u,0.5,0.5\rangle,\langle v,0.5,0.4\rangle\}$ is intuitionistic fuzzy semi closed set and not intuitionistic fuzzy $\lambda-$closed set. Hence, f is intuitionstic fuzzy semi closed mapping but not intuitionistic fuzzy $\lambda-$closed.

Example 3.17. Let $X = \{a,b\}$, $Y = \{u,v\}$ and intuitionistic fuzzy sets U and V are defined as follows: $U = \{\langle a,0.5,0.5\rangle,\ \langle b,0.5,0.2\rangle\},\ V = \{\langle a,0.5,0.5\rangle,\ \langle b,0.5,0.4\rangle\}$. Let $\tau = \{\underset{\sim}{0},\underset{\sim}{1},U\}$ and $\sigma = \{\underset{\sim}{0},\underset{\sim}{1},V\}$ be intuitionistic fuzzy topologies on X and Y respectively. Define the map $f:(X,\tau)\to(Y,\sigma)$ by $f(a) = x$ and $f(b) = y$ then $f(U) = f(\{\langle a,0.5,0.5\rangle,\langle b,0.2,,0.5\rangle\}) = \{(\langle u,0.5,0.5\rangle,\langle v,0.2,0.5\rangle)\}$ is intuitionistic fuzzy $\lambda-$closed set but not intuitionistic fuzzy semi closed set. Hence f is intuitionnistic fuzzy λ-closed mapping but not intuitionistic fuzzy semi closed mapping.

Remark 3.18. The concept of intuitionistic fuzzy λ-closed mappings and intuitionistic fuzzy semi pre closed mappings are independent as seen from the following examples.

Example 3.19. Let $X = \{a,b\}, Y = \{u,v\}$ and intuitionistic fuzzy sets U and V are defined as follows: $U = \{\langle a,0.5,0.5\rangle,\langle b,0.5,0.3\rangle\}, V = \{\langle u,0.5,0.5\rangle,\langle v,0.5,0.3\rangle\}$. Let $\tau = \{\underset{\sim}{0},\underset{\sim}{1},U\}$ and $\sigma = \{\underset{\sim}{0},\underset{\sim}{1},V\}$ be intuitionistic fuzzy topologies on X and Y respectively. Define the map $f:(X,\tau)\to(Y,\sigma)$ by $f(a) = u$ and $f(b) = v$ then $f(U) = f(\{\langle a,0.5,0.5\rangle,\langle b,0.5,,0.3\rangle\}) = \{\langle u,0.5,0.5\rangle,\langle v,0.5,0.3\rangle\}$ is intuitionistic fuzzy $\lambda-$closed set but not intuitionistic fuzzy semi pre-closed set. Then f is intuitionnistic fuzzy $\lambda-$closed mapping but not intuitionistic fuzzy semi pre closed mapping.

Example 3.20. Let $X = \{a,b\}, Y = \{u,v\}$ and intuitionistic fuzzy sets U and V are defined as follows: $U = \{\langle a,0.5,0.5\rangle,\langle b,0.4,0.6\rangle\}, V = \{\langle u,0.5,0.5\rangle,\langle v,0.1,0.9\rangle\}$. Let $\tau = \{\underset{\sim}{0},\underset{\sim}{1},U\}$ and $\sigma = \{\underset{\sim}{0},\underset{\sim}{1},V\}$ be intuitionistic fuzzy topologies on X and Y respectively. Define the map $f:(X,\tau)\to(Y,\sigma)$ by $f(a) = u$ and $f(b) = v$ then $f(\{\langle a,0.5,0.5\rangle,\langle b,0.6,0.4\rangle\}) = \{\langle u,0.5,0.5\rangle,\langle v,0.6,0.4\rangle\}$ is intuitionistic fuzzy semi pre closed mapping but not intuitionistic fuzzy $\lambda-$closed mapping.

Remark 3.21. IFG$\alpha-$closed mapping and IF $\lambda-$closed mappings are independent to each other.

Example 3.22. Let $X = \{a,b\}, Y = \{u,v\}$ and intuitionistic fuzzy sets U and V are defined as follows: $U = \{\langle a,0.4,0.6\rangle,\langle b,0.3,0.7\rangle\}, V = \{\langle u,0.2,0.8\rangle,\langle v,0.3,0.7\rangle\}$. Let $\tau = \{\underset{\sim}{0},\underset{\sim}{1},U\}$ and $\sigma = \{\underset{\sim}{0},\underset{\sim}{1},V\}$ be intuitionistic fuzzy topologies on X and Y respectively. Define the map $f:(X,\tau)\to(Y,\sigma)$ by $f(a) = u$ and $f(b) = v$ then $f(\{\langle a,0.6,0.4\rangle,\langle b,0.7,0.3\rangle\}) = \{\langle a,0.6,0.4\rangle,\langle b,0.7,0.3\rangle\}$ is IFG$\alpha-$closed set but not IF-closed set. Hence f is intuitionistic fuzzy λ mapping, but not intuitionistic fuzzy IFG$\alpha-$closed mapping.

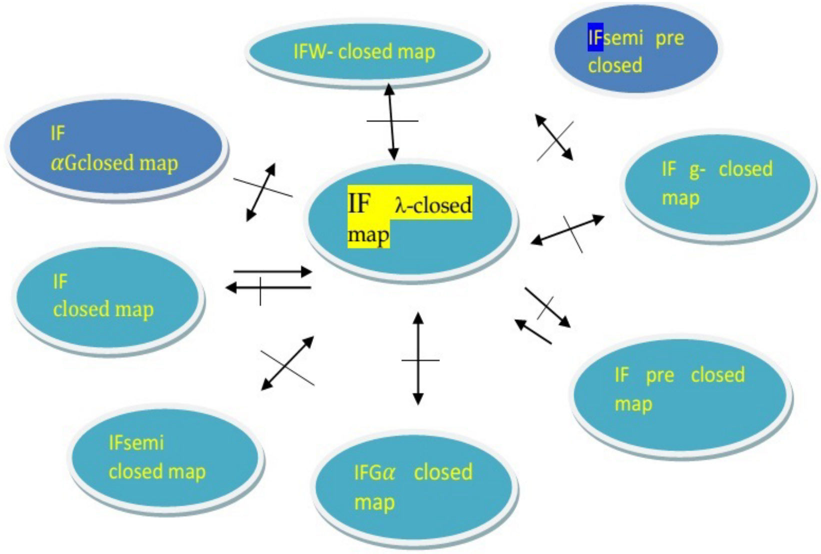
Example 3.23. Let $X = \{a,b\}, Y = \{u,v\}$ and intuitionistic fuzzy sets U and V are defined as follows: $U = \{\langle a,0.1,0.9\rangle,\langle b,0.3,0.7\rangle\}, V = \{\langle a,0.8,0.2\rangle,\langle b,0.8,0.1\rangle\}$. Let $\tau = \{\underset{\sim}{0},\underset{\sim}{1},U\}$ and $\sigma = \{\underset{\sim}{0},\underset{\sim}{1},V\}$ be intuitionistic fuzzy topologies on X and Y respectively. Define the map $f:(X,\tau)\to(Y,\sigma)$ by $f(a) = u$ and $f(b) = v$ then $f(\{\langle a,0.9,0.1\rangle,\langle b,0.7,0.3\rangle\}) = \{\langle a,0.9,0.1\rangle,\langle b,0.7,0.3\rangle\}$ is IF $\lambda-$closed set in Y but not IFG$\alpha-$closed set. Hence f is intuitionistic fuzzy λ mapping, but not intuitionistic fuzzy IFG$\alpha-$closed mapping.

Remark 3.24. If IFαG-closed mapping and IF $\lambda-$closed mapping are independent to each other as seen from the following example:

Example 3.25. Let $X = \{a, b\}, Y = \{u, v\}$ and intuitionistic fuzzy sets U and V are defined as follows: $U = \{\langle a, 0.6, 0.4\rangle, \langle b, 0.7, 0.2\rangle\}, V = \{\langle u, 0.2, 0.6\rangle, \langle b, 0.2, 0.7\rangle\}$. Let $\tau = \{\underset{\sim}{0}, \underset{\sim}{1}, U\}$ and $\sigma = \{\underset{\sim}{0}, \underset{\sim}{1}, V\}$ be intuitionistic fuzzy topologies on X and Y respectively. Define the map $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$ then $f(\{\langle a, 0.4, 0.6\rangle, \langle b, 0.2, 0.7\rangle\}) = \{\langle a, 0.4, 0.6\rangle, \langle b, 0.2, 0.7\rangle\}$ is αG-closed set in Y but not $\lambda-$closed set in Y. Hence f is intuitionistic fuzzy IFG$\alpha-$closed mapping, but not IF$\lambda-$closed mapping.

Example 3.26. Let $X = \{a, b\}, Y = \{u, v\}$ and intuitionistic fuzzy sets U and V are defined as follows: $U = \{\langle a, 0.1, 0.5\rangle, \langle b, 0.2, 0.6\rangle\}, V = \{\langle u, 0.2, 0.4\rangle, \langle b, 0.3, 0.5\rangle\}$. Let $\tau = \{\underset{\sim}{0}, \underset{\sim}{1}, U\}$ and $\sigma = \{\underset{\sim}{0}, \underset{\sim}{1}, V\}$ be intuitionistic fuzzy topologies on X and Y respectively. Define the map $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$ then $f(\{\langle a, 0.5, 0.1\rangle, \langle b, 0.6, 0.2\rangle\}) = \{\langle a, 0.5, 0.1\rangle, \langle b, 0.6, 0.2\rangle\}$ is $\lambda-$closed set Y but not αG-closed set in Y. Hence f is intuitionistic fuzzy $\lambda-$closed mapping, but not intuitionistic fuzzy αG-closed mappings.

Remark 3.27. From above examples and remarks we get following diagram of implications.



In this Diagram $A \to B$ means that A implies B. $A \not\leftarrow B$ means that B does not implies A.

$A \not\leftrightarrow B$ means that A and B are independent to each other.

Theorem 3.28. *Let $f : X \to Y$ be a mapping. Then the following are equivalent if Y is an IF$\lambda - T_{\frac{1}{2}}$ space.*

(i) *f is an IF$\lambda-$closed map.*
(ii) *$\lambda - cl(f(A)) \subseteq f(cl(A))$ for each IFS A of X.*

Proof. (i) $\Rightarrow$ (ii) Let A be an IFS in X. Then $cl(A)$ is an IFCS in X. (i) implies that $f(cl(A))$ is an IFλ-CS in Y. Since Y is an IF$\lambda - T_{\frac{1}{2}}$ space, $f(cl(A))$ is an IFCS in Y. Therefore λ-$cl(f(cl(A))) = f(cl(A))$. Now $\lambda - cl(f(A)) \subseteq \lambda$-$cl(f(cl(A))) = f(cl(A))$. Hence $\lambda - cl(f(A)) \subseteq f(cl(A))$ for each IFS A of X.
(ii) $\Rightarrow$ (i) Let A be any IFCS in X. Then $cl(A) = A$. (ii) implies that $\lambda - cl(f(A)) \subseteq f(cl(A)) = f(A)$. But $f(A) \subseteq \lambda - cl(f(A))$. Therefore $\lambda - cl(f(A)) = f(A)$. This implies $f(A)$ is an IF$\lambda-$CS in Y. Since every IF$\lambda-$CS is an IFCS, $f(A)$ is an IF$\lambda-$CS in Y. Hence f is an IF$\lambda-$closed map. □

Theorem 3.29. *Let $f : X \to Y$ be a bijection. Then the following are equivalent if Y is an IF$\lambda - T_{\frac{1}{2}}$ space.*

(i) *f is an IF$\lambda-$closed map.*
(ii) *$\lambda - cl(f(A)) \subseteq f(cl(A))$ for each IFS A of X.*
(iii) *$f^{-1}(\lambda - cl(B)) \subseteq cl(f^{-1}(B))$ for every IFS B of Y.*

Proof. (i) $\Rightarrow$ (ii) is obvious from theorem 3.15.
(ii) $\Rightarrow$ (iii) Let B be an IFS in Y. Then $f^{-1}(B)$ is an IFS in X. Since f is onto, λ-$cl(B) = \lambda - cl(f(f^{-1}(B)))$ and (ii) implies $\lambda - cl(f(f^{-1}(B))) \subseteq f(cl(f^{-1}(B)))$. Therefore $\lambda - cl(B) \subseteq f(cl(f^{-1}(B)))$. Now $f^{-1}(\lambda - cl(B)) \subseteq f^{-1}(f(cl(f^{-1}(B))))$. Since f is one to one $f^{-1}(\lambda - cl(B)) \subseteq cl(f^{-1}(B))$.
(iii) $\Rightarrow$ (ii) Let A be any IFS of X. Then $f(A)$ is an IFS of Y. Since f is one to one, (iii) implies that $f^{-1}(\lambda - cl(f(A))) \subseteq cl(f^{-1}(A)) = cl(A)$. Therefore $f(f^{-1}(\lambda - cl(f(A)))) \subseteq f(cl(A))$. Since f is onto, $\lambda - cl(f(A)) = f(f^{-1}(\lambda - cl(f(A)))) \subseteq f(cl(A))$. □

Theorem 3.30. *Let $f : X \to Y$ be an IFλ-closed map. Then for every IFS A of $X, f(cl(A))$ is an IF $\lambda-CS$ in Y.*

Proof. Let A be any IFS in X. Then $cl(A)$ is an IFCS in X. By hypothesis, $f(cl(A))$ is an IF $\lambda-$CS in X. □

Theorem 3.31. *Let $f : X \to Y$ be an IFλ-closed map where Y is an IF$\lambda - T_{\frac{1}{2}}$ space. Then f is a IF regular closed map if every IF$\lambda-CS$ is an IFRCS in Y.*

Proof. Let A be an IFRCS in X. Since every IFRCS is an IFCS [5], A is an IFCS in X. By hypothesis $f(A)$ is an IFλ-CS in Y. Since Y is an IF$\lambda - T_{\frac{1}{2}}$ space, $f(A)$ is an IF$\lambda-$CS in Y and hence is an IFCS in Y, by hypothesis. This implies that $f(A)$ is an IF regular closed map. □

Theorem 3.32. *If every IFS is an IFCS, then an IF$\lambda-$closed mapping is an IFλ-continuous mapping.*

Proof. Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFS in X. Therefore $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IF$\lambda-$CS, $f^{-1}(A)$ is an IF$\lambda-$CS in X. This implies that f is an IF$\lambda-$continuous mapping. □

Theorem 3.33. *A mapping $f : X \to Y$ is an IF$\lambda-$closed mapping if and only if for every IFS B of Y and for every IFOS U containing $f^{-1}(B)$, there is an IF$\lambda-OS$ A of Y such that $B \subseteq A$ and $f^{-1}(A) \subseteq U$.*

Proof. **Necessity:** Let B be any IFS in Y. Let U be an IFOS in X such that $f^{-1}(B) \subseteq U$, then U^c is an IFCS in X. By hypothesis $f(U^c)$ is an IF$\lambda-$CS in Y. Let $A = (f(U^c))^c$, then A is an IF$\lambda-$OS in Y and $B \subseteq A$. Now $f^{-1}(A) = f^{-1}(f(U^c))^c = (f^{-1}(f(U^c)))^c \subseteq U$.

Sufficiency: Let A be an IFCS in X, then A^c is an IFOS in X and $f^{-1}(f(A^c))^c \subseteq A^c$. By hypothesis, there exists an IF$\lambda-$OS B in Y such that $f(A^c) \subseteq B$ and $f^{-1}(B) \subseteq A^c$. Therefore $A \subseteq (f^{-1}(B))^c$. Hence $B^c \subseteq f(A) \subseteq f(f^{-1}(B))^c \subseteq B^c$. This implies that $f(A) = B^c$. Since B^c is an IF$\lambda-$CS in Y, $f(A)$ is an IF$\lambda-$CS in Y. Hence f is an IF$\lambda-$closed mapping. $\square$

Theorem 3.34. *If $f : X \to Y$ is an IF closed map and $g : Y \to Z$ is an IF$\lambda-$closed map, then $g \circ f$ is an IF$\lambda-$closed map.*

Proof. Let A be an IFCS in X, then $f(A)$ is an IFCS in Y, since f is an IF closed map. Since g is an IF$\lambda-$closed map, $g(f(A))$ is an IF$\lambda-$CS in Z. Therefore $g \circ f$ is an IF$\lambda-$closed map. $\square$

Remark 3.35. The composition of two IF closed maps are not IF closed map as seen from the following example.

Example 3.36. Let $X = \{a, b\}, Y = \{c, d\}$ and $Z = \{u, v\}$ and intuitionistic fuzzy sets U, V and W are defined as follows: $U = \{\langle a, 0.5, 0.5\rangle, \langle b, 0.5, 0.2\rangle\}, V = \{\langle c, 0.5, 0.5\rangle, \langle d, 0.5, 0.4\rangle\}$ and $W = \{\langle u, 0.5, 0.5\rangle, \langle v, 0.6, 0.4\rangle\}$ Let $\tau = \{\underset{\sim}{0}, \underset{\sim}{1}, U\}, \sigma = \{\underset{\sim}{0}, \underset{\sim}{1}, V\}$ and $\delta = \{\underset{\sim}{0}, \underset{\sim}{1}, W\}$ be intuitionistic fuzzy topologies on X, Y and Z respectively. Define a map $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c$ and $f(b) = d$ and $g : (Y, \sigma) \to (Z, \delta)$ by $g(c) = u$ and $g(d) = v$ then $f(\{\langle a, 0.5, 0.5\rangle, \langle b, 0.2, 0.5\rangle\}) = \{\langle c, 0.5, 0.5\rangle, \langle d, 0.2, 0.5\rangle\}$ is $\lambda-$closed set in (Y, σ) and hence f is $\lambda-$closed map in (Y, σ) and $g(\{\langle c, 0.5, 0.5\rangle, \langle d, 0.5, 0.4\rangle\}) = \{\langle u, 0.5, 0.5\rangle, \langle b, 0.5, 0.4\rangle\}$ is λ-closed set in (Z, δ) and hence g is $\lambda-$closed map. But their composition $g \circ f : X \to Z$ is not $\lambda-$closed map in (Z, δ). Since $g(f(U)) = g(f(\{\langle a, 0.5, 0.5\rangle, \langle b, 0.2, 0.5\rangle\})) = g(\{\langle c, 0.5, 0.5\rangle, \langle d, 0.2, 0.5\rangle\}) = \{\langle u, 0.5, 0.5\rangle, \langle v, 0.2, 0.5\rangle\}$ is not λ-closed set in (Z, δ). Hence $g(f(A))$ is not intuitionistic fuzzy $\lambda-$closed mapping in (Z, δ). Therefore $g \circ f$ is not an intuitionistic fuzzy $\lambda-$closed mapping.

Theorem 3.37. *Let $f : X \to Y$ be a bijective map where Y is an IF$\lambda - T_{\frac{1}{2}}$ space. Then the following are equivalent.*

(i) *f is an IF$\lambda-$closed map.*
(ii) *$f(B)$ is an IF$\lambda-OS$ in Y for every IFOS B in X.*

Proof. (i) $\Leftrightarrow$ (ii) is obvious. $\square$

Definition 3.38. A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be intuitionistic fuzzy $\lambda-$open map (IF $\lambda-$open map) if $f(V)$ is $\lambda-$open set in (Y, σ) for every closed set in X.

4. Conclusions

In this paper we have introduced intuitionistic fuzzy $\lambda-$open mappings, intuitionistic fuzzy $\lambda-$closed mappings and studied some of their properties.

References

[1] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87–96.
[2] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24 (1968) 182–190.
[3] D. Coker, An introduction to intuitionistic fuzzy topological space, Fuzzy Sets and Systems 88 (1997) 81–89.
[4] H. Gurcay, Es. A. Haydar and D. Coker, On fuzzy continuity in intuitionistic fuzzy topological spaces, J. Fuzzy Math. 5 (1997) 365–378.
[5] J. K. Jeon, Y. B. Jun, and J. H. Park, Intuitionistic fuzzy alpha-continuity and intuitionistic fuzzy pre continuity, International Journal of Mathematics and Mathematical Sciences 19 (2005) 3091–3101.
[6] R. Santhi and D. Jayanthi, Intuitionistic fuzzy generalized semi-preclosed sets (accepted).
[7] P. Rajarajeswari and G. Bagyalakshmi, $\lambda-$closed sets in intuitionistic fuzzy topological space, International Journal of Computer Applications 34 (2011) 25–27.
[8] P. Rajarajeswari and G. Bagyalakshmi, $\lambda-$continuous in intuitionistic fuzzy topological space, International Journal of Applied Information Systems 1 (2012) 6–9.
[9] P. Rajarajeswari and G. Bagyalakshmi, Application of $\lambda-$continuous in intuitionistic fuzzy topological space, International Journal of Computer Applications 94 (2014) 39–43.
[10] R. Santhi and K. Sakthivel, Alpha generalized closed mappings in intuitionistic fuzzy topological spaces, Far East Journal of Mathematical Sciences 43 (2010) 265–275.
[11] R. Santhi and K. Sakthivel, Intuitionistic fuzzy generalized semi continuous Mappings, Advances in Theoretical and Applied Mathematics 5 (2010) 11–20.
[12] S. S. Thakur and R. Chaturvedi, Regular generalized closed sets in intuitionistic fuzzy topological spaces, Universitatea Din Bacau Studii Si Cetari Stiintifice 6 (2006) 257–272.
[13] M. Thirumalaiswamy and K. M. Arifmohammed, Semipre generalized closed mappings in intuitionistic fuzzy topological spaces, International Journal of Mathematics Trends and Technology 4 (1) (2013) 1–5.
[14] Y. B. Jun and S.-Z. Song, Intuitionistic fuzzy semi-pre open sets and intuitionistic fuzzy semi-pre continuous mappings, Journal of Applied Mathematics and Computing 19 (2005) 464–474.
[15] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.

P. Rajeswari (p.rajeswari29@gmail.com)
Department of Mathematics, Chikkanna Government Arts College, Tiruppur - 641 602, Tamil Nadu, India

G. Bagyalakshmi (g_bagyalakshmi@yahoo.com)
Department of Mathematics, AJK College of Arts and Science, Coimbatore - 641 105, Tamil Nadu, India