Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 10, No. 5, (November 2015), pp. 727–736
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

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On fuzzy almost GP-spaces

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Received 23 October 2014; Revised 10 February 2014

Abstract. In this paper, the concepts of fuzzy almost GP-spaces are introduced and several characterizations of fuzzy almost GP-spaces are studied. The conditions under which fuzzy topological spaces become fuzzy almost GP-spaces, are investigated.

2010 AMS Classification: 54 A 40, 03 E 72.

Keywords: Fuzzy G_δ-set, Fuzzy F_σ-set, Fuzzy dense set, Fuzzy nowhere dense set, Fuzzy submaximal space, Fuzzy second category space, Fuzzy weakly Volterra space.

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1. Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by L. A. Zadeh in his classical paper [21] in the year 1965. Thereafter the paper of C. L. Chang [5] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed.

L. Gillman and M. Henriksen [7] defined and characterized the classes of P-spaces in 1954. A. K. Mishra [9] introduced the concepts of P-spaces as a generalization of ω_α-additive spaces of Sikorski [10] and L. W. Cohen and C. Goffman [6]. The concept of P-spaces in fuzzy setting was introduced by G. Balasubramanian in [11]. Almost P-spaces in classical topology was introduced by A. I. Veksler [19] and was also studied further by R. Levy [8]. The concept of almost GP-spaces in classical topology was introduced by M. R. Ahmadi Zand [1]. The concept of almost P-spaces in fuzzy setting was introduced by the authors in [18]. In this paper, in section 3, the concepts of fuzzy almost GP-spaces are introduced and several characterizations of fuzzy almost GP-spaces are studied. In section 4, the conditions under which fuzzy topological spaces become fuzzy almost GP-spaces, are investigated. In section 5, some results concerning functions that preserve fuzzy almost GP-spaces in the context of images and preimages are obtained. Examples are given to illustrate the concepts introduced in this paper.

2. Preliminaries

Now we introduce some basic notions and results used in the sequel. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang.

Definition 2.1. Let λ and μ be any two fuzzy sets in (X, T). Then we define $\lambda \vee \mu : X \to [0, 1]$ as follows : $(\lambda \vee \mu)(x) = Max\{\lambda(x), \mu(x)\}$. Also we define $\lambda \wedge \mu : X \to [0, 1]$ as follows : $(\lambda \wedge \mu)(x) = Min\{\lambda(x), \mu(x)\}$.

For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in (X, T), the union $\psi = \vee_i \lambda_i$ and intersection $\delta = \wedge_i \lambda_i$ are defined respectively as $\psi(x) = sup_i\{\lambda_i(x), x \in X\}$ and $\delta(x) = inf_i\{\lambda_i(x), x \in X\}$.

Definition 2.2 ([2])**.** Let (X, T) be a fuzzy topological space. For a fuzzy set λ of X, the interior $int(\lambda)$ and the closure $cl(\lambda)$ of (X, T) are defined respectively as $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$ and $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$.

Lemma 2.3 ([2])**.** *Let λ be any fuzzy set in a fuzzy topological space (X, T). Then* $1 - cl(\lambda) = int(1 - \lambda)$ *and* $1 - int(\lambda) = cl(1 - \lambda)$.

Definition 2.4 ([12])**.** A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy dense if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$. That is, $cl(\lambda) = 1$.

Definition 2.5 ([12])**.** A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X, T) such that $\mu < cl(\lambda)$. That is, $intcl(\lambda) = 0$.

Definition 2.6 ([3])**.** A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy G_δ-set in (X, T) if $\lambda = \wedge_{i=1}^{\infty} \lambda_i$ *where* $\lambda_i \in T$, *for* $i \in I$.

Definition 2.7 ([3])**.** A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy F_σ-set in (X, T) if $\lambda = \vee_{i=1}^{\infty} \lambda_i$ *where* $1 - \lambda_i \in T$, *for* $i \in I$.

Lemma 2.8 ([2])**.** *For a family $\mathscr{A} = \{\lambda_\alpha\}$ of fuzzy sets of a fuzzy space X, $\vee\big(cl(\lambda_\alpha)\big) \leq cl\big(\vee(\lambda_\alpha)\big)$. In case $\mathscr{A}$ is a finite set, $\vee\big(cl(\lambda_\alpha)\big) = cl\big(\vee(\lambda_\alpha)\big)$. Also $\vee\big(int(\lambda_\alpha)\big) \leq int\big(\vee(\lambda_\alpha)\big)$.*

Definition 2.9 ([14])**.** A fuzzy topological space (X, T) is called a fuzzy Baire space if $int\big(\vee_{i=1}^{\infty}(\lambda_i)\big) = 0$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T).

Definition 2.10 ([12])**.** A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of fuzzy second category.

Definition 2.11 ([12])**.** A fuzzy topological space (X, T) is called a fuzzy first category space if $1_X = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T). A fuzzy topological space (X, T) which is not of fuzzy first category, is said to be of fuzzy second category.

Definition 2.12 ([11]). A fuzzy topological space (X, T) is called a fuzzy P-space if countable intersection of fuzzy open sets in (X, T) is fuzzy open. That is, every non-zero fuzzy G_δ-set in (X, T) is fuzzy open in (X, T).

Definition 2.13 ([17]). A fuzzy topological space (X, T) is called a fuzzy almost P-space if for every non-zero fuzzy G_δ-set in (X, T), $int(\lambda) \neq 0$ in (X, T).

Lemma 2.14 ([2]). *Let $f : (X, T) \to (Y, S)$ be a mapping and $\{\lambda_\alpha\}$ be a family of fuzzy sets of Y. Then*

(a) $f^{-1}(\cup_\alpha \lambda_j) = \cup_\alpha f^{-1}(\lambda_j)$,
(b) $f^{-1}(\cap_\alpha \lambda_j) = \cap_\alpha f^{-1}(\lambda_j)$.

Definition 2.15 ([13]). A fuzzy topological space (X, T) is called a fuzzy resolvable space if there exists a fuzzy dense set λ in (X, T) such that $cl(1 - \lambda) = 1$. Otherwise (X, T) is called a fuzzy irresolvable space.

3. Fuzzy almost GP-spaces

Definition 3.1. A fuzzy topological space (X, T) is called a fuzzy almost GP-space if $int(\lambda) \neq 0$, for each non-zero fuzzy dense and fuzzy G_δ-set λ in (X, T). That is, (X, T) is a fuzzy almost GP-space if for each non-zero fuzzy G_δ-set λ in (X, T) with $cl(\lambda) = 1$, $int(\lambda) \neq 0$.

Example 3.2. Let $X = \{a, b, c\}$. The fuzzy sets λ, μ and γ are defined on X as follows :

$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.8$; $\lambda(b) = 0.9$; $\lambda(c) = 0.7$
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.9$; $\mu(b) = 0.8$; $\mu(c) = 0.7$
$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 0.7$; $\gamma(b) = 0.6$; $\gamma(c) = 0.8$

Then $T = \{0, \lambda, \mu, \gamma, \lambda \vee \mu, \lambda \vee \gamma, \mu \vee \gamma, \lambda \wedge \mu, \lambda \wedge \gamma, \gamma \vee (\lambda \wedge \mu), \lambda \vee \mu \vee \gamma, 1\}$. Clearly T is a fuzzy topology on X. Now $\lambda \wedge \mu = \lambda \wedge (\lambda \vee \gamma) \wedge [\gamma \vee (\lambda \wedge \mu)]$ and $\lambda \wedge \gamma = \mu \wedge (\lambda \vee \mu) \wedge (\mu \vee \gamma) \wedge (\lambda \vee \mu \vee \gamma)$ are fuzzy G_δ-sets in (X, T). Also $cl(\lambda \wedge \mu) = 1$ and $cl(\lambda \wedge \gamma) = 1$ and $int(\lambda \wedge \mu) = \lambda \wedge \mu \neq 0$ and $int(\lambda \wedge \gamma) = \lambda \wedge \gamma \neq 0$. Hence the topological space (X, T) is a fuzzy almost GP-space.

Proposition 3.3. *If λ is a fuzzy dense and fuzzy G_δ-set in a fuzzy almost GP-space (X, T), then $1 - \lambda$ is not a fuzzy dense set in (X, T).*

Proof. Let λ be a fuzzy dense and fuzzy G_δ-set in (X, T). Since (X, T) is a fuzzy almost GP-space, $int(\lambda) \neq 0$. Now $cl(1 - \lambda) = 1 - int(\lambda) \neq 1$. Hence, $1 - \lambda$ is not a fuzzy dense set in (X, T). □

Proposition 3.4. *If λ is a fuzzy F_σ-set in a fuzzy almost GP-space (X, T) such that $int(\lambda) = 0$, then λ is not a fuzzy dense set in (X, T).*

Proof. Let λ be a fuzzy F_σ-set in (X, T) such that $int(\lambda) = 0$. Now $cl(1 - \lambda) = 1 - int(\lambda) = 1 - 0 = 1$. Then $1 - \lambda$ is a fuzzy dense set in (X, T). Since λ is a fuzzy F_σ-set in (X, T), $1 - \lambda$ is a fuzzy G_δ-set in (X, T). Thus, $1 - \lambda$ is a fuzzy dense and fuzzy G_δ-set in (X, T). Then, by proposition 3.3, $cl(1 - [1 - \lambda]) \neq 1$ and hence λ is not a fuzzy dense set in (X, T). □

Proposition 3.5. *If λ is a fuzzy F_σ-set and fuzzy nowhere dense set in a fuzzy almost GP-space (X, T), then λ is not a fuzzy dense set in (X, T).*

Proof. Let λ be a fuzzy F_σ-set in (X, T) and fuzzy nowhere dense set in (X, T). Since λ is a fuzzy nowhere dense set in (X, T), $intcl(\lambda) = 0$. Now $int(\lambda) \leq intcl(\lambda)$, implies that $int(\lambda) = 0$. Hence λ is a fuzzy F_σ-set in (X, T) such that $int(\lambda) = 0$. Then, by proposition 3.4, λ is not a fuzzy dense set in (X, T). □

Theorem 3.6 ([15]). *If λ is a fuzzy dense and fuzzy G_δ-set in a fuzzy topological space (X, T), then $1 - \lambda$ is a fuzzy first category set in (X, T).*

Proposition 3.7. *If the fuzzy first category set μ is formed from the fuzzy dense and fuzzy G_δ-set λ in a fuzzy almost GP-space (X, T), then the fuzzy first category set μ is not a fuzzy dense set in (X, T).*

Proof. Let λ be a fuzzy dense and fuzzy G_δ-set in a fuzzy almost GP-space (X, T). Then, $int(\lambda) \neq 0$. Since λ is a fuzzy dense and fuzzy G_δ-set in (X, T), by theorem 3.6, $1 - \lambda$ is a fuzzy first category set in (X, T). Let $\mu = 1 - \lambda$. Now $cl(1 - \lambda) = 1 - int(\lambda) \neq 1$ and hence the fuzzy first category set μ is not a fuzzy dense set in (X, T). □

Proposition 3.8. *If $int(\mu) = 0$, where μ is a fuzzy F_σ-set in a fuzzy almost GP-space (X, T), then each fuzzy first category set is not a fuzzy dense set in (X, T).*

Proof. Assume the contrary. Suppose that the fuzzy first category set λ in (X, T) is a fuzzy dense set in (X, T). Since λ is a fuzzy first category set in (X, T), $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T). Now $1 - cl(\lambda_i)$ is a fuzzy open set in (X, T). Let $\delta = \wedge_{i=1}^{\infty}[1 - cl(\lambda_i)]$. Then δ is a fuzzy G_δ-set in (X, T). Let $\mu = \vee_{i=1}^{\infty}(cl(\lambda_i))$. Then μ is a fuzzy F_σ-set in (X, T). By hypothesis, $int(\mu) = 0$. This implies that $int(\vee_{i=1}^{\infty}(cl(\lambda_i)) = 0$. Then $1 - int(\vee_{i=1}^{\infty}(cl(\lambda_i)) = 1 - 0 = 1$ and therefore we have $cl(1 - (\vee_{i=1}^{\infty}(cl\lambda_i)) = 1$. This implies that $cl(\wedge_{i=1}^{\infty}[1 - cl(\lambda_i)] = 1$ and hence $cl(\delta) = 1$. Now δ is a non-zero fuzzy dense and fuzzy G_δ-set in a fuzzy almost GP-space (X, T). Then $int(\delta) \neq 0$. Also $\wedge_{i=1}^{\infty}[1 - cl(\lambda_i)] = 1 - \vee_{i=1}^{\infty}(cl\lambda_i) \leq 1 - \vee_{i=1}^{\infty}(\lambda_i) = 1 - \lambda$. Hence $\delta \leq 1 - \lambda$. Then $int(\delta) \leq int(1 - \lambda)$, implies that $int(\delta) \leq 1 - cl(\lambda) = 1 - 1 = 0$. This will imply that $int(\delta) = 0$, a contradiction and hence we must have $cl(\lambda) \neq 1$. Therefore the first category set λ is not a fuzzy dense set in (X, T). □

Proposition 3.9. *If $int(\wedge_{i=1}^{\infty}(\lambda_i)) \neq 0$, where (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in a fuzzy topological space (X, T), then (X, T) is a fuzzy almost GP-space.*

Proof. Let (λ_i)'s $(i = 1 \; to \; \infty)$ be fuzzy dense and fuzzy G_δ-sets in a fuzzy topological space (X, T) such that $int(\wedge_{i=1}^{\infty}(\lambda_i)) \neq 0$. Then $int(\wedge_{i=1}^{\infty}(\lambda_i)) \leq \wedge_{i=1}^{\infty}int(\lambda_i)$ implies that $0 \leq \wedge_{i=1}^{\infty}int(\lambda_i)$. This will imply that $int(\lambda_i) \neq 0$. Hence, for the fuzzy dense and fuzzy G_δ-sets λ_i in (X, T), we have $int(\lambda_i) \neq 0$. Therefore (X, T) is a fuzzy almost GP-space. □

Proposition 3.10. *If $cl(\vee_{i=1}^{\infty}(\mu_i)) \neq 1$, where (μ_i)'s are fuzzy F_σ-sets with $int(\mu_i) = 0$ in a fuzzy topological space (X, T), then (X, T) is a fuzzy almost GP-space.*

Proof. Let (μ_i)'s $(i = 1 \; to \; \infty)$ be fuzzy F_σ-sets with $int(\mu_i) = 0$ and $cl(\vee_{i=1}^{\infty}(\mu_i)) \neq 1$. Then $(1 - \mu_i)$'s are fuzzy dense and fuzzy G_δ-sets in (X, T). Now $cl(\vee_{i=1}^{\infty}(\mu_i)) \neq 1$ implies that $1 - cl(\vee_{i=1}^{\infty}(\mu_i)) \neq 0$. Then $int[\wedge_{i=1}^{\infty}(1 - \mu_i)] \neq 0$. Let $\lambda_i = 1 - \mu_i$. Then we have that $int[\wedge_{i=1}^{\infty}(\lambda_i)] \neq 0$, where (λ_i)'s fuzzy dense and fuzzy G_δ-sets in

(X, T). By proposition 3.9, $int(\lambda_i) \neq 0$ for the fuzzy dense and fuzzy G_δ-sets λ_i in (X, T). Therefore (X, T) is a fuzzy almost GP-space. □

4. Fuzzy almost GP-spaces and other fuzzy topological spaces

Proposition 4.1. *If the fuzzy topological space (X, T) is a fuzzy irresolvable space, then (X, T) is a fuzzy almost GP-space.*

Proof. Let λ be a fuzzy dense and fuzzy G_δ-set in a fuzzy irresolvable space (X, T). Since (X, T) is a fuzzy irresolvable space, for the fuzzy dense set λ in (X, T), we have $cl(1 - \lambda) \neq 1$. But $1 - int(\lambda) = cl(1 - \lambda) \neq 1$, implies that $int(\lambda) \neq 0$ and hence (X, T) is a fuzzy almost GP-space. □

Proposition 4.2. *If the fuzzy topological space (X, T) is a fuzzy P-space, then (X, T) is a fuzzy almost GP-space.*

Proof. Let λ be a non-zero fuzzy dense and fuzzy G_δ-set in a fuzzy P-space (X, T). Since (X, T) is a fuzzy P-space, the fuzzy G_δ-set λ in (X, T), is fuzzy open in (X, T). Then, we have $int(\lambda) = \lambda \neq 0$ and hence (X, T) is a fuzzy almost GP-space. □

Remark 4.3. : The converse of the above proposition need not be true. For, consider the following example :

Example 4.4. Let $X = \{a, b, c\}$. The fuzzy sets λ, μ and γ are defined on X as follows :

$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.8$; $\lambda(b) = 0.9$; $\lambda(c) = 0.7$
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.7$; $\mu(b) = 0.8$; $\mu(c) = 0.9$
$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 0.9$; $\gamma(b) = 0.7$; $\gamma(c) = 0.8$

Then $T = \{0, \lambda, \mu, \gamma, \lambda\vee\mu, \lambda\vee\gamma, \mu\vee\gamma, \lambda\wedge\mu, \lambda\wedge\gamma, \mu\wedge\gamma, \lambda\vee(\mu\wedge\gamma), \mu\vee(\lambda\wedge\gamma), \gamma\vee(\lambda\wedge\mu), \lambda\wedge(\mu\vee\gamma), \mu\wedge(\lambda\vee\gamma), \gamma\wedge(\lambda\vee\mu), \lambda\wedge\mu\wedge\gamma, \lambda\vee\mu\vee\gamma, 1\}$. Clearly T is a fuzzy topology on X. Now $\lambda\wedge\mu = \lambda\wedge(\lambda\vee\mu)\wedge(\lambda\vee[\mu\wedge\gamma])\wedge(\mu\vee[\lambda\wedge\gamma])\wedge(\lambda\wedge[\mu\vee\gamma])\wedge(\gamma\wedge[\lambda\vee\mu])$, $\gamma\vee(\lambda\wedge\mu) = (\lambda\vee\gamma)\wedge(\mu\vee\gamma)\wedge(\lambda\vee\mu\vee\gamma)$ and $\lambda\wedge\mu\wedge\gamma = \mu\wedge(\lambda\wedge\mu)\wedge(\mu\wedge[\lambda\vee\gamma])$ are fuzzy G_δ-sets in (X, T). Also, $cl(\lambda\wedge\mu) = 1$, $cl(\gamma\vee[\lambda\wedge\mu]) = 1$ and $cl(\lambda\wedge\mu\wedge\gamma) = 1$. Then $\lambda\wedge\mu$, $\gamma\vee[\lambda\wedge\mu]$ and $\lambda\wedge\mu\wedge\gamma$ are fuzzy dense and fuzzy G_δ-sets in (X, T). Since $int(\lambda\wedge\mu) = \lambda\wedge\mu \neq 0$, $int(\gamma\vee[\lambda\wedge\mu]) = \gamma\vee[\lambda\wedge\mu] \neq 0$ and $int(\lambda\wedge\mu\wedge\gamma) = \lambda\wedge\mu\wedge\gamma \neq 0$, (X, T) is a fuzzy almost GP-space. Consider the fuzzy set $\alpha = (\lambda\vee\mu)\wedge(\lambda\vee\gamma)\wedge(\mu\vee\gamma)$. Then α is a fuzzy dense and fuzzy G_δ-set in (X, T) with $int(\alpha) = \lambda\wedge\mu\wedge\gamma \neq 0$. But α is not a fuzzy open set in (X, T). Therefore (X, T) is not a fuzzy P-space.

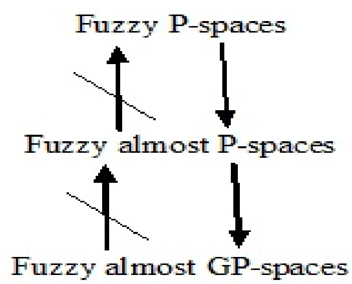
Proposition 4.5. *If the fuzzy topological space (X, T) is a fuzzy almost P-space, then (X, T) is a fuzzy almost GP-space.*

Proof. Let λ be a non-zero fuzzy dense and fuzzy G_δ-set in a fuzzy almost P-space (X, T). Since (X, T) is a fuzzy almost P-space, for the fuzzy G_δ-set λ in (X, T), we have $int(\lambda) \neq 0$ and hence (X, T) is a fuzzy almost GP-space. □

Proposition 4.6. *If each fuzzy G_δ-set is a fuzzy dense set in a fuzzy almost GP-space (X, T), then (X, T) is a fuzzy almost P-space.*

Proof. Let λ be a non-zero fuzzy G_δ-set in a fuzzy almost GP-space (X, T). By hypothesis, λ is a fuzzy dense set. Then λ is a non-zero fuzzy dense and fuzzy G_δ-set in a fuzzy almost GP- space (X, T). Hence we have $int(\lambda) \neq 0$ and thus, for every non-zero fuzzy G_δ-set in (X,T), we have $int(\lambda) \neq 0$ in (X,T). Therefore (X,T) is a fuzzy almost P-space. □

The relationship among the classes of fuzzy P-spaces, fuzzy almost P-spaces and fuzzy almost GP-spaces can be summarized as follows :



Proposition 4.7. *If $int(\mu) = 0$, where μ is a fuzzy F_σ-set in a fuzzy almost GP-space (X,T), then (X,T) is a fuzzy second category space.*

Proof. Let $int(\mu) = 0$, where μ is a fuzzy F_σ-set in a fuzzy almost GP-space (X,T). Suppose that (X,T) is a fuzzy first category space. Then $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T). Now $1 - cl(\lambda_i)$ is a fuzzy open set in (X,T). Let $\delta = \wedge_{i=1}^{\infty}[1 - cl(\lambda_i)]$. Then δ is a fuzzy G_δ-set in (X,T). Now consider the fuzzy set $\mu = \vee_{i=1}^{\infty}(cl(\lambda_i))$. Then μ is a fuzzy F_σ-set in (X,T). By hypothesis, $int(\mu) = 0$. This implies that $int(\vee_{i=1}^{\infty}(cl(\lambda_i)) = 0$. Then $1 - int(\vee_{i=1}^{\infty}(cl(\lambda_i)) = 1 - 0 = 1$ and therefore we have $cl(1 - \vee_{i=1}^{\infty}(cl(\lambda_i)) = 1$. This implies that $cl(\wedge_{i=1}^{\infty}[1 - cl(\lambda_i)] = 1$ and hence $cl(\delta) = 1$. Now δ is a non-zero fuzzy dense and fuzzy G_δ-set in a fuzzy almost GP-space (X,T). Then $int(\delta) \neq 0$. Now $\delta = \wedge_{i=1}^{\infty}[1 - cl(\lambda_i)] = 1 - \vee_{i=1)}^{\infty}(cl\lambda_i) \leq 1 - \wedge_{i=1}^{\infty}(\lambda_i) = 1 - 1 = 0$. Hence $\delta \leq 0$. That is, $\delta = 0$. This will imply that $int(\delta) = 0$, a contradiction to $int(\delta) \neq 0$. Thus $\vee_{i=1}^{\infty}(\lambda_i) \neq 1$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T) and therefore (X,T) is a fuzzy second category space. □

The class of submaximal spaces was introduced by N. Bourbaki in Topologie Generale [4]. This concept in fuzzy setting was introduced by G. Balasubramanian in [3].

Definition 4.8 ([3])**.** A fuzzy topological space (X,T) is called a fuzzy submaximal space if for each fuzzy set λ in (X,T) such that $cl(\lambda) = 1$, then $\lambda \in T$ in (X,T).

Proposition 4.9. *If the fuzzy topological space (X,T) is a fuzzy submaximal space, then (X,T) is a fuzzy almost GP-space.*

Proof. Let λ be a non-zero fuzzy dense and fuzzy G_δ-set in a fuzzy submaximal space (X,T). Since (X,T) is a fuzzy submaximal space, the fuzzy dense set λ in (X,T), is fuzzy open in (X,T) and thus we have $int(\lambda) = \lambda \neq 0$. Hence (X,T) is a fuzzy almost GP-space. □

Theorem 4.10 ([14])**.** *Let (X,T) be a fuzzy topological space. Then the following are equivalent :*

(1) (X, T) *is a fuzzy Baire space.*
(2) $int(\lambda) = 0$ *for every fuzzy first category set* λ *in* (X, T).
(3) $cl(\mu) = 1$ *for every fuzzy residual set* μ *in* (X, T).

Theorem 4.11 ([18])**.** *If* λ *is a fuzzy residual set in a fuzzy submaximal space* (X, T)*, then* λ *is a fuzzy* G_δ*-set in* (X, T).

The following proposition ensures the existence of fuzzy almost GP-spaces.

Proposition 4.12. *If the fuzzy topological space* (X, T) *is a fuzzy Baire and fuzzy submaximal space, then* (X, T) *is a fuzzy almost* GP*-space.*

Proof. Let (X, T) be a fuzzy Baire and fuzzy submaximal space and λ be a fuzzy residual set in (X, T). Since (X, T) is a fuzzy Baire space, by theorem 4.10, $cl(\lambda) = 1$ in (X, T). Also, since λ is a fuzzy residual set in the fuzzy submaximal space (X, T), by theorem 4.11, λ is a fuzzy G_δ-set in (X, T). Since (X, T) is a fuzzy submaximal space, the fuzzy dense set λ in (X, T), is fuzzy open in (X, T). That is, $int(\lambda) = \lambda \neq 0$ and thus we have $int(\lambda) \neq 0$, for a non-zero fuzzy dense and fuzzy G_δ-set λ in (X, T). Hence (X, T) is a fuzzy almost GP-space. □

Definition 4.13 ([16])**.** Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy weakly Volterra space if $cl\Big(\wedge_{i=1}^{N} (\lambda_i)\Big) \neq 0$, where (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in (X, T).

Proposition 4.14. *If the fuzzy topological space* (X, T) *is a fuzzy almost* GP*-space, then* (X, T) *is a fuzzy weakly Volterra space.*

Proof. Let (λ_i)'s $(i = 1 \ to \ N)$ be fuzzy dense and G_δ-sets in a fuzzy almost GP-space (X, T). Then we have $int(\lambda_i) \neq 0$ in (X, T). Then, $1 - int(\lambda_i) \neq 1$ and hence $cl(1 - \lambda_i) \neq 1$. Now $cl\big(\vee_{i=1}^{N} (1 - \lambda_i)\big) = \vee_{i=1}^{N}\big(cl(1 - \lambda_i)\big)$, implies that $cl\big(\vee_{i=1}^{N} (1 - \lambda_i)\big) \neq 1$. Then $cl\big(1 - \wedge_{i=1}^{N}(\lambda_i)\big) \neq 1$, implies that $int\big(\wedge_{i=1}^{N} (\lambda_i)\big) \neq 0$. Now $int\big(\wedge_{i=1}^{N} (\lambda_i)\big) \leq \wedge_{i=1}^{N}(\lambda_i) < cl\big(\wedge_{i=1}^{N} (\lambda_i)\big)$ implies that $cl\big(\wedge_{i=1}^{N} (\lambda_i)\big) \neq 0$ and hence (X, T) is a fuzzy weakly Volterra space. □

Definition 4.15 ([17])**.** A fuzzy topological space (X, T) is said to be a fuzzy strongly irresolvable space if $clint(\lambda) = 1$ for each fuzzy dense set λ in (X, T).

Proposition 4.16. *If the fuzzy topological space* (X, T) *is a fuzzy strongly irresolvable space, then* (X, T) *is a fuzzy almost* GP*-space.*

Proof. Let λ be a non-zero fuzzy dense and fuzzy G_δ-set in a fuzzy strongly irresolvable space (X, T). Since (X, T) is a fuzzy strongly irresolvable space, for the fuzzy dense set λ in (X, T), we have $clint(\lambda) = 1$ in (X, T) and thus we have $int(\lambda) \neq 0$. (otherwise, if $int(\lambda) = 0$, then $clint(\lambda) = cl(0) \neq 1$, a contradiction). Thus, for a non-zero fuzzy dense and fuzzy G_δ-set λ in (X, T), we have $int(\lambda) \neq 0$. Hence (X, T) is a fuzzy almost GP-space. □

5. Fuzzy almost GP-spaces and functions

Let f be a function from the fuzzy topological space (X, T) to the fuzzy topological space (Y, S). Underwhat conditions on "f "may we assert that if (X, T) is a fuzzy almost GP-space, then (Y, S) is a fuzzy almost GP-space. The following propositions establish the desired conditions.

Theorem 5.1 ([20]). *Let $f : (X, T) \to (Y, S)$ be a fuzzy open function. Then, for every fuzzy set β in (Y, S), $f^{-1}\big(cl(\beta)\big) \leq cl\big(f^{-1}(\beta)\big)$.*

Definition 5.2 ([2]). A function $f : (X, T) \to (Y, S)$ from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S) is called fuzzy continuous if $f^{-1}(\lambda)$ is fuzzy open in (X, T) for each fuzzy open set λ in (Y, S).

Definition 5.3 ([2]). A function $f : (X, T) \to (Y, S)$ from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S) is said to be fuzzy open if the image of every fuzzy open set in (X, T) is fuzzy open in (Y, S).

Proposition 5.4. *If the function $f : (X, T) \to (Y, S)$ from a fuzzy topological space (X, T) onto another fuzzy topological space (Y, S) is a fuzzy continuous and fuzzy open function and if (X, T) is a fuzzy almost GP-space, then (Y, S) is a fuzzy almost GP-space.*

Proof. Let λ be a non-zero fuzzy dense and fuzzy G_δ-set in a fuzzy topological space (Y, S). Then $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$ where $\lambda_i \in S$ and $cl(\lambda) = 1$ in (X, T). Then, by theorem 5.1, we have $f^{-1}\big(cl(\lambda)\big) \leq cl\big(f^{-1}(\lambda)\big)$. This implies that $f^{-1}(1) \leq cl\big(f^{-1}(\lambda)\big)$ and hence $1 \leq cl\big(f^{-1}(\lambda)\big)$. That is, $cl\big(f^{-1}(\lambda)\big) = 1$. Therefore $f^{-1}(\lambda)$ is a fuzzy dense set in (X, T).

Now $f^{-1}(\lambda) = f^{-1}\big(\wedge_{i=1}^{\infty}(\lambda_i)\big) = \wedge_{i=1}^{\infty}\big(f^{-1}(\lambda_i)\big)$. Since f is a fuzzy continuous function from (X, T) onto (Y, S) and (λ_i)'s are fuzzy open sets in (Y, S), $f^{-1}(\lambda_i)$'s are fuzzy open sets in (X, T). Then $f^{-1}(\lambda)$ is a fuzzy G_δ-set in (X, T). Thus $f^{-1}(\lambda)$ is a fuzzy dense and fuzzy G_δ-set in a fuzzy almost GP-space (X, T). Then, $int[f^{-1}(\lambda)] \neq 0$ and hence there exists a fuzzy open set μ in (X, T) such that $\mu \neq 0$ and $\mu \leq f^{-1}(\lambda)$. Then $f(\mu) \leq f\big(f^{-1}(\lambda)\big)$. But $f\big(f^{-1}(\lambda)\big) \leq \lambda$. Hence $f(\mu) \leq \lambda$. Again since f is a fuzzy open function from (X, T) onto (Y, S) and μ is a fuzzy open set in (X, T), $f(\mu)$ is a fuzzy open set in (Y, S). Thus $f(\mu) \leq \lambda$, implies that $int(\lambda) \neq 0$. Hence, for a fuzzy dense and fuzzy G_δ-set in (Y, S), we have $int(\lambda) \neq 0$. Therefore (Y, S) is a fuzzy almost GP-space. □

Definition 5.5 ([12]). A function $f : (X, T) \to (Y, S)$ from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S) is called somewhat fuzzy open if for every $\lambda \in T$ and $\lambda \neq 0$ there exists a fuzzy open set μ in (Y, S) such that $\mu \neq 0$ and $\mu \leq f(\lambda)$. That is, $int[f(\lambda)] \neq 0$.

Theorem 5.6 ([12]). *Suppose (X, T) and (Y, S) are two fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be an onto function. Then the following conditions are equivalent:*

(1) *f is somewhat fuzzy open.*
(2) *If λ is a fuzzy dense set in (Y, S), then $f^{-1}(\lambda)$ is a fuzzy dense set in (X, T).*

Proposition 5.7. *If the function $f : (X, T) \to (Y, S)$ from a fuzzy topological space (X, T) onto another fuzzy topological space (Y, S) is a fuzzy continuous and somewhat fuzzy open function and if (X, T) is a fuzzy almost GP-space, then (Y, S) is a fuzzy almost GP-space.*

Proof. Let λ be a non-zero fuzzy dense and fuzzy G_δ-set in a fuzzy topological space (Y, S). Then $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$ where $\lambda_i \in S$ and $cl(\lambda) = 1$ in (X, T). Since f is a somewhat fuzzy open from (X, T) onto (Y, S) and λ is a fuzzy dense set in (Y, S), by theorem 5.1, $f^{-1}(\lambda)$ is a fuzzy dense set in (X, T). Now $f^{-1}(\lambda) = f^{-1}\big(\wedge_{i=1}^{\infty}(\lambda_i)\big) = \wedge_{i=1}^{\infty}\big(f^{-1}(\lambda_i)\big)$. Since f is a fuzzy continuous function from (X, T) onto (Y, S) and (λ_i)'s are fuzzy open sets in (Y, S), $f^{-1}(\lambda_i)$'s are fuzzy open sets in (X, T). Then $f^{-1}(\lambda)$ is a fuzzy G_δ-set in (X, T). Thus $f^{-1}(\lambda)$ is a fuzzy dense and fuzzy G_δ-set in a fuzzy almost GP-space (X, T). Then, $int[f^{-1}(\lambda)] \neq 0$ and hence there exists a fuzzy open set μ in (X, T) such that $\mu \neq 0$ and $\mu \leq f^{-1}(\lambda)$. Then $f(\mu) \leq f\big(f^{-1}(\lambda)\big)$. But $f\big(f^{-1}(\lambda)\big) \leq \lambda$. Hence $f(\mu) \leq \lambda$. Again since f is a fuzzy open function from (X, T) onto (Y, S) and μ is a fuzzy open set in (X, T), there exists a fuzzy open set δ in (Y, S) such that $\delta \neq 0$ and $\delta \leq f(\mu)$. Thus we have $\delta \leq f(\mu) \leq \lambda$ and hence $int(\lambda) \neq 0$, for a fuzzy dense and fuzzy G_δ-set in (Y, S). Therefore (Y, S) is a fuzzy almost GP-space. □

6. Conclusions

The concepts of fuzzy almost GP-spaces are introduced and studied. The conditions under which fuzzy topological spaces become fuzzy almost GP-spaces, are established and also some results concerning functions that preserve fuzzy almost GP-spaces in the context of images and preimages are obtained.

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