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L - locally uniform spaces(II)

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Abstract. In this paper we take up the problem of completeness in the context of L-locally uniform spaces[11]. The primary object of this paper is to study about the comparison of compactness and completeness in a totally bounded L-locally uniform space. For this purpose, we have developed the notion of strong completeness in the context of L-locally uniform spaces. The problem of its hereditary property, unimorphic invariance and productivity in an L-local uniformity are then discussed. In one of our future paper we will consider the problem of compactifications and completions in L-locally uniform spaces.

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1. Introduction

To study about the uniform properties (such as completeness, uniform continuity and uniform convergence) in the setting of general topological spaces, uniform spaces were developed. Further, many generalizations of uniform spaces have been developed leading to a broad spectrum of theory and applications in related fields. One of the generalization of uniform spaces namely locally uniform spaces were developed by James Williams [13] via localization of the triangle axiom. In [13], a topological space was shown to have a compatible local uniformity if and only if it is regular. Following this generalization of uniform spaces many interesting and useful results on compactness, completeness and pseudo-metrizability were obtained in [13].

Consequent to the development of the theory fuzzy topology, many spectacular and creative work about the theory of uniformities on various categories of fuzzy topological spaces have been accomplished by several authors including Hutton, Katsarsas, Lowen, Hu Cheng-Ming et. al. [4, 7, 9, 10].

However, the localization aspect of uniformity has not been considered in any of the above settings. In one of our previous work [11], we developed locally uniform spaces in the category L**-TOP**. Subsequently, many interesting results on compactness, pseudo-metrization and weakly uniform continuity were obtained in L-locally uniform spaces.

The problem of completeness has occupied an important place in the study of uniform spaces. Having developed the theory of L-locally uniform spaces[11] and the problem of completeness in L-semi-uniform spaces[3], we now consider the problem of completeness in the context of L-local uniformity $\mathscr{U}$ on L^X, for which $x_\alpha \in U(x_\alpha)$, instead of our earlier assumption $x_\alpha \subseteq U(x_\alpha),\ \forall\, U \in \mathscr{U},\ \forall\, x_\alpha \in L^X$ in [11]. In the process we have obtained a subclass of the class of the L-local uniformities developed in [11]. This change has been necessitated in order to accommodate the system of Q-nbd at an L-fuzzy point x_α. The Q-nbd system, as may be noted, plays an important role in the theory of convergence. We shall continue to call this subclass of L-locally uniform spaces as L-locally uniform spaces for sake of convenience. With this assumption, we have introduced the notion of strong completeness for an L-locally uniform space. Its hereditary property with respect to a closed subspace and its unimorphic invariance are then discussed. Our main achievement is to establish that in an L-locally uniform space, compactness in the sense of Hutton [5] can be decomposed into strong completeness and totally boundedness. The productivity of this notion is also taken up towards the end of this paper.

Throughout the paper $(L, \leq, \bigwedge, \bigvee)$ denotes a fuzzy lattice with order reversing involution $'$; 0_L and 1_L are respectively inf and sup in L. X is an arbitrary (ordinary) set and L^X denotes the collection of all mappings $A : X \to L$. Any member of L^X is an L-fuzzy set. The L-fuzzy sets $x_\alpha : X \to L$ defined by $x_\alpha(y) = 0_L$ if $x \neq y$ and $x_\alpha(y) = \alpha$ if $x = y$ are the L-fuzzy points. The mappings $A : X \to L$ and $B : X \to L$ defined by $A(x) = 1_L,\ \forall x \in X$ and $B(x) = 0_L,\ \forall x \in X$ are denoted by $\underline{1}$ and $\underline{0}$ respectively. For any $A,\ B \in L^X$, the union and intersection of A and B are defined as $A \cup B(x) = A(x) \vee B(x)$ and $A \bigcap B(x) = A(x) \wedge B(x)$ respectively. Further, we say that $A \subseteq B$ iff $A(x) \leq B(x)$ and $x_\alpha \in A$ iff $\alpha < A(x)$, where x_α is an L-fuzzy point; complement A' of A is defined as $A'(x) = A(x)'$. An L-topology $\mathbb{F}$ on L^X is a subset of L^X closed under finite intersection and arbitrary union. The elements of $\mathbb{F}$ are called open sets and their complements are the closed sets. For basic fuzzy topological definitions we refer to Chang [2]. Uniformity referred to in this paper is in the sense of Hutton [4].

2. Preliminaries

This section includes basic definitions and results that are used in the subsequent sections.

Definition 2.1. Let X be a nonempty ordinary set and L be a fuzzy lattice.

Then a mapping $i : L^X \to L^X$ is called an interior operator on L^X, if it fulfills the following conditions:

(IO1) $i(\underline{1}) = \underline{1}$.

(IO2) $i(A) \subseteq A,\ \forall\, A \in L^X$.

(IO3) $i(A \bigcap B) = i(A) \bigcap i(B),\ \forall\, A, B \in L^X$.

L^X together with an interior operator 'i' shall be called an interior space.

For any $A \in L^X$, we shall call $(i(A'))'$ is the closure of A with respect to the interior operator 'i' [denoted by $c(A)$] and A is called closed or open with respect to that interior operator according as $A = c(A)$ or $A = i(A)$ respectively.

Obviously, for any interior operator 'i' and $A \in L^X$, we have A is open with respect to 'i' iff A' is closed with respect to that interior operator.

An interior operator 'i' shall be called an L-topological interior operator if in addition it satisfies the axiom

(IO4) $i(i(A)) = i(A), \ \forall A \in L^X$.

Definition 2.2 ([8])**.** For any ordinary mapping $f : X \to Y$, the induced L-fuzzy mapping $f^{\to} : L^X \to L^Y$ and its L-fuzzy reverse mapping $f^{\leftarrow} : L^Y \to L^X$ are respectively defined as:

$f^{\to}(A)(y) = \bigvee\{A(x) \mid x \in X, \ f(x) = y\}, \ \forall A \in L^X, \ \forall y \in Y$

and $f^{\leftarrow}(B)(x) = B(f(x)), \ \forall B \in L^Y, \ \forall x \in X$.

It has stated in [8], that $f^{\to}$ is bijective iff f is bijective.

Definition 2.3 ([8])**.** For any x_α, A, $B \in L^X$, x_α is said to be quasi coincide with A, denoted by $x_\alpha \ll A$ if $x_\alpha \nsubseteq A'$ i.e., $\alpha \nleq A'(x)$.

A is said to be quasi coincides with B at y if $A(y) \nleq B'(y)$. A is said to be quasi coincides with B, denoted by $A\widehat{q}B$, if A quasi coincides with B at some $y \in X$.

Definition 2.4. For any L-topological space $(L^X, \mathbb{F})$ and $x_\alpha \in L^X$, in our discussion N is said to be neighborhood (nbd) at x_α if there is $G \in \mathbb{F}$ such that $G \nsubseteq x_\alpha$ and $G \subseteq N$.

Definition 2.5 ([8])**.** Let $(L^X, \mathbb{F})$ be any L-topological space and $x_\alpha \in L^X$. Then, $U \in \mathbb{F}$ is said to be a quasi-coincident neighborhood (Q-nbd) at x_α if $x_\alpha \ll U$.

The family of all Q-nbd at x_α in $(L^X, \mathbb{F})$ is denoted by $\mathscr{Q}(x_\alpha)$.

Definition 2.6 ([8])**.** A subfamily $\mathscr{A} \subseteq \mathscr{Q}(x_\alpha)$ is called a Q-nbd base of x_α, if for every $U \in \mathscr{Q}(x_\alpha)$, $\exists \ V \in \mathscr{A}$ s.t. $V \subseteq U$.

Definition 2.7. For any $x_\alpha \in L^X$ we define its dual point as an L-fuzzy point x^*_α such that

$$x^*_\alpha(y) = \begin{cases} \alpha', & \text{if } y = x, \\ 0_L, & \text{if } y \neq x. \end{cases}$$

Hence, in this framework the Q-nbd system and the nbd system are dual to each other generalizing the similar notion in [10].

Definition 2.8 ([8])**.** Let $(L^X, \mathbb{F}_1)$ be a L-topological space. Then for any $A \in L^X$, the interior and closure of A, denoted by A^o and $\overline{A}$ respectively, defined respectively as the largest open set contained in A and smallest closed set containing A.

Theorem 2.9 ([8])**.** *Let $(L^X, \mathbb{F}_1)$ be a L-topological space. Then for any A, $B \in L^X$, we have the following:*

(1) $\overline{A} = ((A^{'})^o)^{'}$.

(2) $\overline{\overline{A}} = A$.

(3) $A \subseteq B \Rightarrow \overline{A} \subseteq \overline{B}$.
(4) $\overline{(A \bigcup B)} = \overline{A} \bigcup \overline{B}$.

Definition 2.10 ([8]). Let $(L^X, \mathbb{F}_1)$ and $(L^Y, \mathbb{F}_2)$ be two L- topological spaces. Then $f^{\rightarrow} : L^X \rightarrow L^Y$ is called continuous, if $f^{\leftarrow} : L^Y \rightarrow L^X$ maps every open set in $(L^Y, \mathbb{F}_2)$ as an open set in $(L^X, \mathbb{F}_1)$.

Definition 2.11 ([8]). Let $(L^X, \mathbb{F}_1)$ and $(L^Y, \mathbb{F}_2)$ be two L- topological spaces. Then $f^{\rightarrow} : L^X \rightarrow L^Y$ is called an open, if it maps every open set in $(L^X, \mathbb{F}_1)$ as an open set in $(L^Y, \mathbb{F}_2)$.

Definition 2.12 ([8]). Let $(L^X, \mathbb{F}_1)$ and $(L^Y, \mathbb{F}_2)$ be two L- topological spaces. Then $f^{\rightarrow} : L^X \rightarrow L^Y$ is called an L-fuzzy homeomorphism, if it is bijective, continuous and open.

Definition 2.13 ([8, 6]). Let $(L^X, \mathbb{F})$ be an L-topological space. A non empty sub collection $\mathscr{F}$ of L^X is said to be a filter if:
(F1) $\underline{0} \notin \mathscr{F}$.
(F2) $U, V \in \mathscr{F} \Rightarrow U \bigcap V \in \mathscr{F}$.
(F3) $U \in \mathscr{F}$ and $G \in L^X$ such that $U \subseteq G$ then $G \in \mathscr{F}$.

Definition 2.14 ([5]). Let $(L^X, \mathbb{F})$ be a fuzzy topological space. Then a filter $\mathscr{F}$ is said to be closed if for any $F \in \mathscr{F}$ implies $F' \in \mathbb{F}$.

Definition 2.15 ([8, 6]). Let $(L^X, \mathbb{F})$ be an L-topological space. A subfamily $\mathscr{B}$ of L^X is called a filter base in $(L^X, \mathbb{F})$ if
(B1) $\underline{0} \notin \mathscr{B}$.
(B2) For any $U, V \in \mathscr{B}$, there exists $W \in \mathscr{B}$ such that $W \subseteq U \bigcap V$.

Definition 2.16 ([5]). Let $A \in L^X$ be any L-fuzzy set. Then a filter $\mathscr{F}$ is said to be relative to A, if for any $F \subseteq A$ implies $F \notin \mathscr{F}$.

Obviously every filter $\mathscr{F}$ is a filter relative to $\underline{0}$.

Definition 2.17 ([8]). Let $x_\alpha \in L^X$ and $\mathscr{F}$ be a filter. Then $\mathscr{F}$ is said to be convergent to x_α, denoted by $\mathscr{F} \rightarrow x_\alpha$, if for any $U \in \mathscr{Q}(x_\alpha)$ there exists $F \in \mathscr{F}$ such that $F \subseteq U$, that is, $\mathscr{Q}(x_\alpha) \subseteq \mathscr{F}$. The cluster set of $\mathscr{F}$ is given by $\bigcap\{\overline{F} \mid F \in \mathscr{F}\}$.

For any $x_\alpha \in L^X$, if x_α is in the cluster set of $\mathscr{F}$, then we denote it by, $\mathscr{F} \rightsquigarrow x_\alpha$.

Definition 2.18. Let $A \in L^X$. We shall call the maximal filter (partially ordered by set inclusion) $\mathscr{F}_и$ relative to A as an ultrafilter relative to A. If $A = \underline{0}$, then we simply call $\mathscr{F}_и$ as an ultrafilter.

By Proposition 4.3(2) in [6], we have the following:

Lemma 2.19. *Let $\mathscr{F}_и$ be an ultrafilter relative to $C \in L^X$ and $A, B \in L^X$ such that $A \bigcup B \in \mathscr{F}_и$. Then either $A \in \mathscr{F}_и$ or $B \in \mathscr{F}_и$.*

We now adopt all the necessary definitions and results of L- locally uniform spaces in the setting of our discussion.

Definition 2.20. Let $\mathscr{U}^*$ be the collection of all maps $U : L^X \rightarrow L^X$ which satisfy:
(s1) $x_\alpha \in U(x_\alpha)$.

(s2) $U(\bigcup_\lambda V_\lambda) = \bigcup_\lambda U(V_\lambda), \quad V_\lambda \in L^X$.
For any $U \in \mathscr{U}^*$, we say $(x_\alpha, y_\beta) \in U \Leftrightarrow y_\beta \in U(x_\alpha)$, where $x_\alpha,\ y_\beta \in L^X$.
For any $U,\ V \in \mathscr{U}^*$, $U \circ V$ is the composition of functions.

Definition 2.21 ([4]). For any $U \in \mathscr{U}^*$, $U^r(x_\alpha) = \bigcap\{y_\beta \mid U(y'_\beta) \subseteq x'_\alpha\}$.

Obviously, $U^r \in \mathscr{U}^*$ and by Proposition 10.2 in [4], $(U^r)^r = U$.
If $U = U^r$, then U is said to be symmetric.

Lemma 2.22. *For any $U \in \mathscr{U}^*$ and $x_\alpha, y_\beta \in L^X$, $y_\beta \subseteq U(x_\alpha)$ iff $x_\alpha \subseteq U^r(y_\beta)$.*

Proof. Since $(U^r)^r = U$, therefore, we need to prove only one way implication.
Here, $U^r(y_\beta) = \bigcap\{z_\gamma \mid U(z'_\gamma) \subseteq y'_\beta\}$.
Let $y_\beta \subseteq U(x_\alpha)$, $A = \{z_\gamma \mid U(z'_\gamma) \subseteq y'_\beta\}$ and $B = \{w_\eta \mid U(w_\eta) \subseteq [U(x_\alpha)]'\}$.
Since $w_\eta \in B'$, we have, $w'_\eta \in B$. Then, $U(w'_\eta) \subseteq [U(x_\alpha)]'$. This implies that $U(w'_\eta) \subseteq y'_\beta$ as $y_\beta \in U(x_\alpha)$, which implies $[U(x_\alpha)]' \subseteq y'_\beta$. Therefore, $B' \subseteq A$ and consequently, $\bigcup A' \subseteq \bigcup B$. Also, $a_\mu \in B$ gives $U(a_\mu) \subseteq [U(x_\alpha)]'$. Then, $U(a_\mu) \subseteq x'_\alpha$ as $x_\alpha \in U(x_\alpha)$. Therefore, $a_\mu \subseteq x'_\alpha$ implying $\bigcup B \subseteq x'_\alpha$, and hence, $\bigcup A' \subseteq x'_\alpha$. Therefore, $x_\alpha \subseteq \bigcap A$ and the conclusion follows. ☐

Lemma 2.23. *For any $U \in \mathscr{U}^*$, $U \circ U = \bigcup_{z_\gamma} U^r(z_\gamma) \times U(z_\gamma)$.*

Proof. For any $(x_\alpha, y_\beta) \in U \circ U$, we have

$$\begin{aligned}(x_\alpha, y_\beta) \in U \circ U &\Leftrightarrow (x_\alpha, z_\gamma) \in U,\ \ (z_\gamma, y_\beta) \in U \ \text{ for some } z_\gamma \in L^X\\ &\Leftrightarrow (z_\gamma, x_\alpha) \in U^r,\ \ (z_\gamma, y_\beta) \in U \ \text{ for some } z_\gamma \in L^X \text{ by Lemma 2.22}\\ &\Leftrightarrow (x_\alpha, y_\beta) \in U^r(z_\gamma) \times U(z_\gamma) \text{ for some } z_\gamma \in L^X\\ &\Leftrightarrow (x_\alpha, y_\beta) \in \bigcup_{z_\gamma} U^r(z_\gamma) \times U(z_\gamma).\end{aligned}$$

Hence, $U \circ U = \bigcup_{z_\gamma} U^r(z_\gamma) \times U(z_\gamma),\ \forall\, U \in \mathscr{U}^*$. ☐

Definition 2.24. An L-semi uniformity on L^X is a non void subset $\mathscr{U}$ of $\mathscr{U}^*$ such that,
(S_1) If $U \in \mathscr{U}$, then $U^r \in \mathscr{U}$.
(S_2) $U \bigcap V \in \mathscr{U}$ whenever $U, V \in \mathscr{U}$.
(S_3) If $V \in \mathscr{U}^*$ such that $U \subseteq V$, for some $U \in \mathscr{U}$, then $V \in \mathscr{U}$.
The pair $(L^X, \mathscr{U})$ is then called an $L-$semi uniform space.

Definition 2.25. The non void subfamily $\mathscr{B}$ of $\mathscr{U}^*$ is called a base for some L-semi uniformity if it satisfies axiom (S_1) and if $U,\ V \in \mathscr{B}$, then there is $W \in \mathscr{B}$ such that $W \subseteq U \bigcap V$.

Collection of all symmetric members of an L-semi uniformity $\mathscr{U}$ is a base for $\mathscr{U}$.

Definition 2.26. A base $\mathscr{B}$ for a L-semi uniformity on L^X is called L-local uniformity iff for each $U \in \mathscr{B}$ and $x_\alpha \in L^X$, $\exists\, V \in \mathscr{B}$ such that $V \circ V(x_\alpha) \subseteq U(x_\alpha)$. If $\mathscr{U}$ is an L-semi uniformity which is L-local uniformity, we call $\mathscr{U}$ to be an L-local uniformity and $(L^X, \mathscr{U})$ to be an L-locally uniform space.

Lemma 2.27. *Let* $(L^X, \mathscr{U})$ *be an* L*-locally uniform space. Define* $int : L^X \to L^X$ *as* $int(G) = \bigcup\{y_\beta \mid \exists\, U \in \mathscr{U} \text{ s.t. } U(y_\beta) \subseteq G\}$*. Then 'int' is an* L*-topological interior operator.*

Proof. Axioms (IO1) $int(\underline{1}) = \underline{1}$ and (IO2) $int(A) \subseteq A$ for $A \in L^X$ are trivially satisfied.
Also, if G is an L- fuzzy set and $U \in \mathscr{U}$ is such that $U(x_\alpha) \subseteq G$, then we can find $V \in \mathscr{U}$ such that $V \circ V(x_\alpha) \subseteq U(x_\alpha)$. So, in particular, $V(V(x_\alpha)) \subseteq G$.
Thus, $V(x_\alpha) \subseteq int(G)$, which implies $x_\alpha \in int(int(G))$ and since the other inclusion follows by (IO2), so we have (IO3) $int(G) = int(int(G))$.
(IO4) $int(A \bigcap B) = int(A) \bigcap int(B)$ follows by (S2).
Thus, the given mapping '*int*' is an L-topological interior operator. □

Now, by Proposition 4 in [4], we conclude the following:

Theorem 2.28. *Every* L*-locally uniform space is* L*-topological.*

We shall use $\mathbb{F}(\mathscr{U})$ to denote the L-topology induced by an L-local uniformity $\mathscr{U}$ on L^X.

Then by Theorem 2.9(1), we may conclude that in the L-topological space $(L^X, \mathbb{F}(\mathscr{U}))$, for any $A \in L^X$, $\overline{A} = c(A)$.

Definition 2.29. If $\mathscr{U}$ and $\mathscr{V}$ are $L-$semi uniformities on L^X, $\mathscr{V}$ is said to be coarser than $\mathscr{U}$ iff $\mathscr{V} \subseteq \mathscr{U}$.

We have the following notations:
For any $V \in \mathscr{U}^*$, $V^2 = V \circ V$; $V^{2r} = V^r \circ V^r$; $V^{n+1} = V^n \circ V$; $V^{(n+1)r} = V^{nr} \circ V^r$; $\mathscr{U}^n = \{U : L^X \to L^X \mid \exists\, V \in \mathscr{U} \text{ such that } V^n \subseteq U\}$ for each $n \in \mathbb{N}$, where $\mathscr{U}$ is an L-local uniformity.

Theorem 2.30. *If* $\mathscr{U}$ *is an* L*-local uniformity, then for each* $n \in \mathbb{N}$*,* $\mathscr{U}^n$ *is an* L*-local uniformity with the same* $L-$*topology generated by* $\mathscr{U}$*.*

Proof. It suffices to prove the theorem for $\mathscr{U}^2$. Clearly $\mathscr{U}^2$ is an L-semi uniformity. Now, for $W \in \mathscr{U}^2$ and $x_\alpha \in L^X$, there exists $U,\ V \in \mathscr{U}$ such that $U^4(x_\alpha) \subseteq V(x_\alpha)$ and $V^2 \subseteq W$. But $U^2 \in \mathscr{U}^2$ as $U \subseteq U^2$ for any $U \in \mathscr{U}$. Also, $(U^2 \circ U^2)(x_\alpha) \subseteq V(x_\alpha) \subseteq V^2(x_\alpha) \subseteq W(x_\alpha)$. Hence, $\mathscr{U}^2$ is an $L-$local uniformity.
Also by the definition of $\mathscr{U}^2$, the relative $L-$topology of $\mathscr{U}^2$ is weaker than that of $\mathscr{U}$. Again since, $U^2 \in \mathscr{U}^2$, so it is also stronger. Thus $\mathscr{U}^2$ and $\mathscr{U}$ generated the same L-topology. Hence the theorem follows. □

Definition 2.31 ([4])**.** Let $(L^X, \mathscr{U})$ and $(L^Y, \mathscr{V})$ be uniform spaces. A map $f^{\to} : L^X \to L^Y$ is said to be uniformly continuous if for every $V \in \mathscr{V}$, there exists $U \in \mathscr{U}$ such that $\widehat{f}^{\to}(U) \subseteq V$, where $\widehat{f}^{\to}(x_\alpha, y_\beta) = (f^{\to}(x_\alpha), f^{\to}(y_\beta))$, that is, for $A \in L^X,\ U(A) \subseteq f^{\leftarrow}(V)(f^{\to}(A))$.

Definition 2.32 ([11])**.** Two L-local uniformities $\mathscr{V}$ and $\mathscr{U}$ are said to be weakly equivalent if for some $n, m \in \mathbb{N}$, $\mathscr{V}^n \subseteq \mathscr{U}$ and $\mathscr{U}^m \subseteq \mathscr{V}$.

In view of Theorem 2.30, it is observed that two weakly equivalent L-local uniformities generate the same $L-$topology.

Definition 2.33 ([11]). Let $(L^X, \mathscr{U})$ and $(L^Y, \mathscr{V})$ be $L-$locally uniform spaces, a function $f^{\rightarrow} : (L^X, \mathscr{U}) \rightarrow (L^Y, \mathscr{V})$ is $L-$weakly uniformly continuous iff for some $n \in \mathbb{N}$, $f^{\rightarrow} : (L^X, \mathscr{U}) \rightarrow (L^Y, \mathscr{V}^n)$ is uniformly continuous.

Definition 2.34. Let $f^{\rightarrow} : L^X \rightarrow L^Y$ be a mapping, then $f^{\rightarrow}$ is said to be L-weakly uniform isomorphism iff $f^{\rightarrow}$ is bijective and both $f^{\rightarrow}$ and $f^{\leftarrow}$ are L-weakly uniformly continuous.

Theorem 2.35. *L-weakly uniformly continuous functions on $L-$locally uniform spaces are continuous with respect to the relative $L-$topologies.*

Proof. Let $(L^X, \mathscr{U})$ and $(L^Y, \mathscr{V})$ be $L-$locally uniform spaces. Let for some $n \in \mathbb{N}$, $f^{\rightarrow} : (L^X, \mathscr{U}) \rightarrow (L^Y, \mathscr{V}^n)$ be uniformly continuous. For each x_α and each open set N containing $f^{\rightarrow}(x_\alpha)$, we may choose $V \in \mathscr{V}$ and $U \in \mathscr{U}$ so that $V^n(f(x_\alpha)) \subseteq N$ and $\widehat{f}^{\rightarrow}(U) \subseteq V^n$. This implies, $f^{\rightarrow}(U(x_\alpha)) = \widehat{f}^{\rightarrow}(U)(f^{\rightarrow}(x_\alpha)) \subseteq V^n(f^{\rightarrow}(x_\alpha)) \subseteq N$ and consequently, $f^{\rightarrow} : (L^X, \mathbb{F}(\mathscr{U})) \rightarrow (L^Y, \mathbb{F}(\mathscr{V}))$ is continuous.
Hence the theorem follows. □

3. Completeness and strong completeness

In this section we introduce the notion of strong completeness, and study about its hereditary property and unimorphic invariance.

Definition 3.1. Let $(L^X, \mathscr{U})$ be an L-locally uniform space, then a Cauchy filter $\mathscr{F}$ is a filter s.t. for any $U \in \mathscr{U}$, there exists $F \in \mathscr{F}$ such that $F \times F \subseteq U$.

Definition 3.2. Let $(L^X, \mathscr{U})$ be an L-locally uniform space and $n \in \mathbb{N}$, then a weak Cauchy filter of degree 'n' is a filter $\mathscr{F}$ such that for any $U \in \mathscr{U}$, there exists $F \in \mathscr{F}$ such that $F \times F \subseteq U^n$, and $F \times F \nsubseteq U^m$, $\forall m < n$ and $\forall F \in \mathscr{F}$.

Clearly, Cauchy filters are weak Cauchy filters of degree 1.

Definition 3.3. An L-locally uniform space $(L^X, \mathscr{U})$ is said to be (strongly) complete if every (weak) Cauchy filter relative to an $\mathbb{F}(\mathscr{U})$-open set is convergent.

Theorem 3.4. *Every convergent filter in an L-locally uniform space is a weak Cauchy filter of degree 2.*

Proof. Let $(L^X, \mathscr{U})$ be an L-locally uniform space and $\mathscr{F}$ be a filter such that for some $x_\alpha \in L^X$, $\mathscr{F} \rightarrow x_\alpha$ in $(L^X, \mathbb{F}(\mathscr{U}))$. Let $\mathscr{Q}(x_\alpha) = \{U(x^*_\alpha) \mid U \in \mathscr{U}\}$ and $\mathscr{Q}_{\mathscr{B}}(x_\alpha) = \{V(x^*_\alpha) \mid V \in \mathscr{B}\}$, where $\mathscr{B} = \{U \in \mathscr{U} \mid U^r = U\}$. Then, $\mathscr{Q}(x_\alpha)$ is Q-nbd system at x_α in $\mathbb{F}(\mathscr{U})$. Since for any $U \in \mathscr{U}$, there exists $V \in \mathscr{B}$ s.t. $V \subseteq U$. This implies that $V(x^*_\alpha) \subseteq U(x^*_\alpha)$ and hence, $\mathscr{Q}_{\mathscr{B}}(x_\alpha)$ is a base for $\mathscr{Q}(x_\alpha)$.
Now since $\mathscr{F} \rightarrow x_\alpha$, therefore for $V(x^*_\alpha) \in \mathscr{Q}_{\mathscr{B}}(x_\alpha)$, there exists $F \in \mathscr{F}$ s.t. $F \subseteq V(x^*_\alpha)$. Now, if $(y_\beta, z_\gamma) \subseteq F \times F$, then, $y_\beta \subseteq F$, $z_\gamma \subseteq F$.
Therefore, $y_\beta \subseteq V(x^*_\alpha), z_\gamma \subseteq V(x^*_\alpha)$ and hence, $x^*_\alpha \subseteq V^r(y_\beta)$. (By Lemma 2.22.)
Then, $x^*_\alpha \subseteq V(y_\beta)$ and $z_\gamma \subseteq V(x^*_\alpha)$. (since $V = V^r$.)
Now, $z_\gamma \subseteq V^2(y_\beta)$ and consequently, $(y_\beta, z_\gamma) \subseteq V^2 \subseteq U^2$.
Thus, for any $U \in \mathscr{U}$, there exists $F \in \mathscr{F}$ such that $F \times F \subseteq U^2$.
Hence F be a Cauchy filter of degree 2.
This completes the proof. □

Definition 3.5. Let $(L^X, \mathscr{U})$ be an L-locally uniform space and $A \in L^X$.
Let for any $U \in \mathscr{U}$, $U_A : L^X \to L^X$ be a mapping such that

$$U_A(x_\alpha) = \begin{cases} U(x_\alpha) & \text{if } x_\alpha \subseteq A \\ \underline{0} & \text{if } x_\alpha \nsubseteq A \end{cases}$$

Then, $\mathscr{U}_A = \{U_A \mid U \in \mathscr{U}\}$ is an L-local uniformity on A, which we call a sub L-local uniformity on A and $(A, \mathscr{U}_A)$ to be the subspace.
$\mathscr{U}_A$ is called open or closed sub L-local uniformity provided $A \in \mathbb{F}(\mathscr{U})$ or $A' \in \mathbb{F}(\mathscr{U})$ respectively.

Theorem 3.6. *Every closed subspace in a strongly complete L-locally uniform space is strongly complete.*

Proof. Let $(L^X, \mathscr{U})$ be a strongly complete L-locally uniform space and $A \in L^X$ such that $A' \in \mathbb{F}(\mathscr{U})$.
Let $\mathscr{F} = \{F \mid F \subseteq A\}$ be a weak Cauchy filter relative to an open set B in $\mathbb{F}(\mathscr{U}_A)$, where $\mathbb{F}(\mathscr{U}_A)$ is the L-topology on A induced by $\mathscr{U}_A$. Then, $B \notin \mathscr{F}$. If $B' \in \mathscr{F}$, then $B' \subseteq A$, as $\mathscr{F}$ is a filter in $(L^A, \mathbb{F}(\mathscr{U}_A))$. Then, $A' \subseteq B$ and consequently, $A' \notin \mathscr{F}$. Also if, $B' \notin \mathscr{F}$, then as $B \subseteq A$ implies $A' \subseteq B'$, so $A' \notin \mathbb{F}$. Now since for any $U \in \mathscr{U}$ we have ${U_A}^n \subseteq U^n$, so in either case $\mathscr{F}$ is a weak Cauchy filter in L^X relative to the open set A' and consequently there exists $x_\alpha \in L^X$ such that $\mathscr{F} \to x_\alpha$. But as $A' \in \mathbb{F}(\mathscr{U})$, so by Theorem 5.2.19 in [8], we have $x_\alpha \in A$ and hence $(A, \mathscr{U}_A)$ is strongly complete. □

Theorem 3.7. *Let $(L^X, \mathscr{U})$ and $(L^Y, \mathscr{V})$ be L-locally uniform spaces and let $f^\to : L^X \to L^Y$ be L-weakly uniformly continuous. If $\mathscr{F}$ is a weak Cauchy filter in $(L^X, \mathscr{U})$, then $f^\to(\mathscr{F})$ is weak Cauchy filter in $(L^Y, \mathscr{V})$.*

Proof. Let $\mathscr{F}$ be a weak Cauchy filter of degree n on L^X. Let $V \in \mathscr{V}$. Since $f^\to : L^X \to L^Y$ is L-weakly uniformly continuous, therefore there exists $U \in \mathscr{U}$ such that $\widehat{f}^\to(U) \subseteq V^m$ for some $m \in \mathbb{N}$. Now $\mathscr{F}$ is a weak Cauchy filter of degree n on L^X. Hence, there exists $F \in \mathscr{F}$ such that $F \times F \subseteq U^n$. Therefore, $f^\to(F) \times f^\to(F) \subseteq V^{mn}$. Hence, $f^\to(\mathscr{F})$ is a weak Cauchy filter of degree nm on $(L^Y, \mathscr{V})$. □

Theorem 3.8. *Let $(L^X, \mathscr{U})$ and $(L^Y, \mathscr{V})$ be two L-locally uniform spaces and $f^\to : L^X \to L^Y$ be a L-weakly uniformly isomorphism, then $(L^X, \mathscr{U})$ is strongly complete iff $(L^Y, \mathscr{V})$ is so.*

Proof. Let $(L^Y, \mathscr{V})$ be L-strongly complete and $\mathscr{F}$ be a weak Cauchy filter of degree n on L^X relative to an open set G. Let $V \in \mathscr{V}$, then by Theorem 3.7, $f^\to(\mathscr{F})$ is a weak Cauchy filter on $(L^Y, \mathscr{V})$. Again, since $f^\leftarrow$ is L-weakly uniform continuous, therefore by Theorem 2.35, $f^\leftarrow$ is continuous and so $f^\to$ is open. This implies $f^\to(G)$ is open in L^Y. Also as $G \subseteq f^\leftarrow(f^\to(G))$ and $G \notin \mathscr{F}$, therefore $f^\to(\mathscr{F})$ is a weak Cauchy filter relative to the open set $f^\to(G)$. Thus $f^\to(\mathscr{F})$ is convergent on $(L^Y, \mathscr{V})$, being strongly complete. But $f^\to$ is an L- fuzzy homeomorphism being an L-weakly uniform isomorphism and consequently, $\mathscr{F}$ converges on $(L^X, \mathscr{U})$. This implies $(L^X, \mathscr{U})$ is strongly complete. In a similar way, one can show the other way implication. Hence the theorem. □

4. Compactness and totally boundedness

In this section we establish the equivalence of the compactness and completeness in an L-locally uniform space.

Definition 4.1 ([5]). A subset $\mathscr{F}$ of L^X is said to satisfy the finite intersection property or the F. I. P. relative to an L-fuzzy set G if $F_1, ..., F_n \in \mathscr{F} \Rightarrow \bigcap_{i=1}^{n} F_i \nsubseteq G$.

Obviously by Lemma 4.2 in [6], we may conclude that, every subset $\mathscr{F}$ of L^X which satisfy the F. I. P. relative to G is contained in a filter relative to G.

Definition 4.2. An L-locally uniform space $(L^X, \mathscr{U})$ is said to be totally bounded if $\forall\ U \in \mathscr{U}$ there is finite $A \subseteq Pt(L^X)$ such that $\underline{1} = U(A) = \bigcup\{U(x_\alpha) \mid x_\alpha \in A\}$.

This is a generalization of the same notion in the sense of G. Artico and R. Moresco[1].

In the sequel, we refer to the definitions of compactness in L-topology in the sense of Hutton, that is,

Definition 4.3 ([5]). Let $(L^X, \mathbb{F})$ be an L-topological spaces. Then $(L^X, \mathbb{F})$ is said to be compact if it satisfies any of following equivalent statements:

(1) Every open cover $\mathscr{C}$ of each closed set A has a finite subcover.
(2) Every collection of closed sets $\mathscr{F}$ satisfying the F. I. P. relative to an open set G satisfies $\bigcap_{F\in\mathscr{F}} F \nsubseteq G$.

Theorem 4.4. *Every compact L-locally uniform space $(L^X, \mathscr{U})$ is totally bounded.*

Proof. Let $(L^X, \mathscr{U})$ be a compact space. Then for any $U \in \mathscr{U}$, the collection
$\{\text{int}\, U(x_\alpha) \mid x_\alpha \in \text{Pt}(L^X)\}$ is an open cover of $\underline{1}$.
Since $\underline{1}$ is closed. Therefore, by compactness there exists a collection of finite L-fuzzy points $\mu_i,\ 1 \le i \le n,\ n \in \mathbb{N}$ s.t. $\underline{1} = \bigcup_{i=1}^{n} \text{int}U(\mu_i)$.
Therefore, $\underline{1} = \bigcup_{i=1}^{n} U(\mu_i)$. Hence, $(L^X, \mathscr{U})$ is totally bounded. □

Theorem 4.5. *In a totally bounded L-locally uniform space $(L^X, \mathscr{U})$, every ultrafilter relative to an $\mathbb{F}(\mathscr{U})$-open set G is a weak Cauchy filter relative to G.*

Proof. Let $\mathscr{F}_{\scriptscriptstyle\mathsf{M}}$ be an ultrafilter relative to an $\mathbb{F}(\mathscr{U})$-open set G.
Let $\mathscr{B} = \{U \in \mathscr{U} \mid U^r = U\}$. Then $\mathscr{B}$ is a base for $\mathscr{U}$.
Let $U \in \mathscr{B}$. By totally boundedness there is a finite $A \subseteq Pt(L^X)$ such that
$\underline{1} = U(A) = \bigcup\{U(x_\alpha) \mid x_\alpha \in A\}$.
But $\underline{1} \in \mathscr{F}_{\scriptscriptstyle\mathsf{M}}$. So, by Lemma 2.19, there is $F \in \mathscr{F}_{\scriptscriptstyle\mathsf{M}}$ such that
$F \subseteq U(x_\alpha)$ for some $x_\alpha \in A$.
This implies $F \times F \subseteq U(x_\alpha) \times U(x_\alpha) = U^r(x_\alpha) \times U(x_\alpha)$, since $U = U^r$.
Now by Lemma 2.23, we get $F \times F \subseteq U^2$.
Hence $\mathscr{F}_{\scriptscriptstyle\mathsf{M}}$ is a weak Cauchy filter relative to G. □

Corollary 4.6. *In a totally bounded L-locally uniform space $(L^X, \mathscr{U})$, every ultrafilter is a weak Cauchy filter.*

Theorem 4.7. *In a compact L-locally uniform space $(L^X, \mathbb{F}(\mathscr{U}))$ every filter relative to an $\mathbb{F}(\mathscr{U})$-open set has a cluster point.*

Proof. Let $(L^X, \mathbb{F}(\mathscr{U}))$ be a compact L-locally uniform space and $\mathscr{F}$ be a filter relative to the $\mathbb{F}(\mathscr{U})$-open set G. Then, $\mathscr{G} = \{\overline{F} \mid F \in \mathscr{F}\}$ is a closed-filter relative to the $\mathbb{F}(\mathscr{U})$-open set G. Now, by the Proposition 9 in [5] we have, $\bigcap \mathscr{G} \nleq G$ which implies $\exists\, x_\alpha \in L^X$ s.t. $x_\alpha \in \bigcap \mathscr{G}$. Hence, x_α is a cluster point of $\mathscr{F}$ as $\bigcap \mathscr{G} = \bigcap \{\overline{F} \mid F \in \mathscr{F}\}$. □

Theorem 4.8. *Every compact L-locally uniform space is a strongly complete.*

Proof. Let $(L^X, \mathbb{F}(\mathscr{U}))$ be a compact L-locally uniform space and $\mathscr{F}$ be a weak Cauchy filter of degree n relative to the $\mathbb{F}(\mathscr{U})$-open set G. Then, for any $U \in \mathscr{U}$, there exists $F \in \mathscr{F}$ such that $F \times F \subseteq V^{n-1}$. Now by Theorem 4.7, there exists $x_\alpha \in L^X$ such that $x_\alpha \in \cap \overline{F}$ for all $F \in \mathbb{F}$. For any $U \in \mathscr{U}$, choose $V \in \mathscr{U}$ such that $V^n(x^*_\alpha) \subseteq U(x^*_\alpha)$ and $F \times F \subseteq V^{n-1}$.
Also, since for any A, $B \in L^X$, $A \widehat{q} B$ implies $A \cap B \neq \underline{0}$, therefore for $F \in \mathscr{F}$, there exists $y_\beta \in L^X$ such that $y_\beta \in F \cap V(x^*_\alpha)$. This implies $y_\beta \in F$ and $y_\beta \in V(x^*_\alpha)$. But $y_\beta \in F$ implies $F \subseteq V^{n-1}(y_\beta)$, which further implies that $F \subseteq U(x^*_\alpha)$ (as $V^{n-1}(y_\beta) \subseteq V^n(x^*_\alpha)$). Thus, $U(x^*_\alpha) \in \mathscr{F}$, $\forall\, U \in \mathscr{U}$ and consequently, $\mathscr{F} \to x_\alpha$. Hence, the given space is strongly complete. □

Theorem 4.9. *Let $(L^X, \mathscr{U})$ be an L-locally uniform space, then the space is compact iff*

(i) $(L^X, \mathscr{U})$ *is totally bounded, and*
(ii) $(L^X, \mathscr{U})$ *is strongly complete.*

Proof. Let $(L^X, \mathscr{U})$ be an L-locally uniform space which is compact then by Theorem 4.4, (i) $(L^X, \mathscr{U})$ is totally bounded.

Also, by Theorem 4.8, (ii) $(L^X, \mathscr{U})$ is strongly complete.
Conversely, let $(L^X, \mathscr{U})$ be totally bounded and strongly complete.
Let $\mathscr{F}$ be a collection of closed sets satisfying F. I. P. relative to an $\mathbb{F}(\mathscr{U})$-open set G. Then $\mathscr{F}$ is contained in a filter $\mathscr{F}^*$(say) relative to G. This implies $\mathscr{F}^*$ is contained in a ultrafilter $\mathscr{F}_{\text{и}}$(say) relative to G. Then, by Theorem 4.5, $\mathscr{F}_{\text{и}}$ is a weak Cauchy filter relative to G. So, by strong completeness, $\mathscr{F}_{\text{и}}$ is convergent to x_α(say).
This implies $\mathscr{Q}(x_\alpha) \subseteq \mathscr{F}_{\text{и}}$. This implies $F \bigcap Q \neq \underline{0}$, $\ \forall\, F \in \mathscr{F}_{\text{и}}$, $\forall\, Q \in \mathscr{Q}(x_\alpha)$. This implies $F \bigcap Q \neq \underline{0}$, $\ \forall\, F \in \mathscr{F}^*$, $\forall\, Q \in \mathscr{Q}(x_\alpha)$. [Since $\mathscr{F}^* \subseteq \mathscr{F}_{\text{и}}$].
This implies $F \bigcap Q \neq \underline{0}$, $\ \forall\, F \in \mathscr{F}$, $\forall\, Q \in \mathscr{Q}(x_\alpha)$. [Since $\mathscr{F} \subseteq \mathscr{F}^*$].
This implies $(\bigcap_{F \in \mathscr{F}} F) \bigcap Q \neq \underline{0}$, $\ \forall\, Q \in \mathscr{Q}(x_\alpha)$.
This implies $(\bigcap_{F \in \mathscr{F}} F) \bigcap (\bigcap_{Q \in \mathscr{Q}(x_\alpha)} Q) \neq \underline{0}$.
But Since $\mathscr{F}_{\text{и}}$ is relative to G. Therefore by Definition 2.16, $G \notin \mathscr{F}_{\text{и}}$.
This implies $Q \nsubseteq G$, $\ \forall\, Q \in \mathscr{Q}(x_\alpha)$. [Since $\mathscr{Q}(x_\alpha) \subseteq \mathscr{F}_{\text{и}}$.]
This implies $(\bigcap_{Q \in \mathscr{Q}(x_\alpha)} Q) \nsubseteq G$.
This implies $(\bigcap_{F \in \mathscr{F}} F) \nsubseteq G$. [Since $(\bigcap_{F \in \mathscr{F}} F) \bigcap (\bigcap_{Q \in \mathscr{Q}(x_\alpha)} Q) \neq \underline{0}$.]
Hence the space is compact. □

Thus in a totally bounded L- locally uniform space, the notions of compactness and strong completeness are equivalent.

5. Product Space and Strong Complete

We proceed to establish the productivity of strongly complete spaces in an L-locally uniform space.

Definition 5.1. Let $\{(L^{X^\alpha}, \mathscr{U}^\alpha) \mid \alpha \in \Lambda\}$ be an indexed collection of L-locally uniform spaces. Then the product L-local uniformity on $L^X = \Pi_{\alpha\in\Lambda} L^{X^\alpha}$ is the coarsest L-local uniformity such that projections $\pi_\alpha^{\rightarrow} : L^X \rightarrow L^{X^\alpha}$ are L-weakly uniformly continuous.

Theorem 5.2. *A filter $\mathscr{F}$ converges to (x_β^α) in ΠL^{X^α} iff $\pi_\alpha^{\rightarrow}(\mathscr{F})$ converges to $\pi_\alpha^{\rightarrow}((x_\beta^\alpha))$ in L^{X^α}, $\forall\ \alpha$.*

Proof. By Theorem 2.35, we have L-weakly uniformly continuity implies continuity. Therefore for any α, $\pi_\alpha^{\rightarrow}$ is continuous and consequently by Theorem 5.2.27 in [8], one part follows.
By equivalence of convergence of nets and filters, the second part follows from the Theorem 5.2.27 in [8] and the Theorem 2.4 in [12]. □

Theorem 5.3. *Product of strongly complete spaces is strongly complete.*

Proof. Let $\{(L^{X^\alpha}, \mathscr{U}^\alpha) \mid \alpha \in \Lambda\}$ be a collection of strongly complete spaces and $\mathscr{U}$ be the product L-local uniformity on $L^X = \Pi_{\alpha\in\Lambda} L^{X^\alpha}$. Let $\mathbb{F}(\mathscr{U})$ be the L-topology on L^X induced by $\mathscr{U}$ and $\mathscr{F}$ be a weak Cauchy filter relative to an $\mathbb{F}(\mathscr{U})$-open set G'. Then for any

$$B \in L^X \text{ with } B \subseteq G' \text{ implies } B \notin \mathscr{F} \tag{1}$$

For any $\alpha \in \Lambda$, $\pi_\alpha^{\rightarrow}(\mathscr{F})$ is a weak Cauchy filter on L^{X^α}. Let $C = \pi_\alpha^{\rightarrow}(G)$. By Theorem 2.1.25(i) in [8] we have, $G \subseteq \pi_\alpha^{\leftarrow}(\pi_\alpha^{\rightarrow}(G))$. Then, $[\pi_\alpha^{\leftarrow}(\pi_\alpha^{\rightarrow}(G))]' \subseteq G'$, which implies $[\pi_\alpha^{\leftarrow}(C)]' \subseteq G'$ and hence, $[\pi_\alpha^{\leftarrow}(C)]' \notin \mathscr{F}$ [By 1]. Again by Proposition 2.2.5 in [8], since, $[\pi_\alpha^{\leftarrow}(C)]' = \pi_\alpha^{\leftarrow}(C')$, therefore $\pi_\alpha^{\leftarrow}(C') \notin \mathscr{F}$. Then, $C' \notin \pi_\alpha^{\rightarrow}(\mathscr{F})$. For if $C' \in \pi_\alpha^{\rightarrow}(\mathscr{F})$, then $C \in [\pi_\alpha^{\rightarrow}(\mathscr{F})]'$, which implies $\pi_\alpha^{\leftarrow}(C) \in \pi_\alpha^{\leftarrow}([\pi_\alpha^{\rightarrow}(\mathscr{F})]') = [\pi_\alpha^{\leftarrow}(\pi_\alpha^{\rightarrow}(\mathscr{F}))]'$. But as $\mathscr{F} \subseteq \pi_\alpha^{\leftarrow}(\pi_\alpha^{\rightarrow}(\mathscr{F}))$, we have, $[\pi_\alpha^{\leftarrow}(\pi_\alpha^{\rightarrow}(\mathscr{F}))]' \subseteq \mathscr{F}'$, and therefore, $\pi_\alpha^{\leftarrow}(C) \in \mathscr{F}'$, which implies $[\pi_\alpha^{\leftarrow}(C)]' \in \mathscr{F}$, contradicting the fact that $[\pi_\alpha^{\leftarrow}(C)]' = \pi_\alpha^{\leftarrow}(C') \notin \mathscr{F}$.
Now $C' = [\pi_\alpha^{\rightarrow}(G)]' \notin \pi_\alpha^{\rightarrow}(\mathscr{F})$. Hence, $[[\pi_\alpha^{\rightarrow}(G)]']^o \notin \pi_\alpha^{\rightarrow}(\mathscr{F})$, as $[[\pi_\alpha^{\rightarrow}(G)]']^o \subseteq [\pi_\alpha^{\rightarrow}(G)]'$. Hence for any $F_\alpha \subseteq [[\pi_\alpha^{\rightarrow}(G)]']^o$, we have $F_\alpha \notin \pi_\alpha^{\rightarrow}(\mathscr{F})$.
Thus, $\pi_\alpha^{\rightarrow}(\mathscr{F})$ is a weak Cauchy filter relative to the open set $[[\pi_\alpha^{\rightarrow}(G)]']^o$. So $\exists\ x_\beta^\alpha \in X^\alpha$ such that $\pi_\alpha^{\rightarrow}(\mathscr{F}) \rightarrow x_\beta^\alpha$. Therefore, by Theorem 5.2, $\mathscr{F} \rightarrow (x_\beta^\alpha)$.
Hence $(L^X, \mathscr{U})$ is strongly complete. □

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