Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 10, No. 5, (November 2015), pp. 691–704
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

On soft multi semi-continuous functions

Deniz Tokat, İsmail Osmanoğlu, Yaşar Üçok

Received 14 February 2015; Accepted 08 April 2015

Abstract. In this work, first, we recall the concepts of soft multiset, soft multi function, and soft multi topology. Then we introduce and examine the concepts of soft semi-open set, soft semi-closed set, soft semi-interior, and soft semi-closure. Finally, we define soft multi semi-continuous function, soft multi semi-open function, and soft multi semi-closed function on soft multi topological spaces and give basic theorems about them.

2010 AMS Classification: 03E70, 54C05, 54C10

Keywords: Soft multiset, Soft multi function, Soft multi topology, Soft multi semi-open set, Soft multi semi-continuous.

Corresponding Author: Deniz Tokat (dtokat@nevsehir.edu.tr)

1. Introduction

Classical mathematical methods are not enough to solve the problems of daily life and also are not enough to meet the new requirements. Therefore, some theories such as Fuzzy set theory [20], Rough set theory [11], Soft set theory [8, 9] and Multiset (or Bag) theory [19] has been developed to solve these problems.

Applications of these theories appear in topology and many areas of mathematics. Shabir and Naz [13] defined the soft topological space and studied the concepts of soft open set, soft multi interior point, soft neighborhood of a point, soft separation axioms and subspace of a soft topological space. There are some other studies on the structure of soft topological spaces [2, 3, 18, 21]. Maji et al. [7] also initiated the more generalized concept of fuzzy soft sets which is a combination of fuzzy set and soft set. Tanay and Kandemir introduced topological structure of fuzzy soft set in [14] and gave a introductory theoretical base to carry further study on this concept. Following this study, some others [1, 6, 12, 17] studied on the concept of fuzzy soft topological spaces.

The concept of soft multisets which is a combination of soft sets and multisets can be used to solve some real life problems. Also this concept can be used in many areas, such as data storage, computer science, information science, medicine, engineering, etc. The concept of soft multisets was introduced in [15]. Moreover, in [15] and [10] soft multi topology and its some properties was given. Also [16] soft multi continuous function was given.

In this work, first we recall the concepts of soft multiset, soft multi function and soft multi topology. Then we introduce and examine the concepts of soft semi-open set, soft semi-closed set, soft semi-interior and soft semi-closure. Finally we define soft multi semi-continuous function, soft multi semi-open function and soft multi semi-closed function on soft multi topological spaces and give basic theorems about them.

2. Preliminaries

2.1. **Soft set, multiset and soft multiset.** In this section, we present the basic definitions of soft set, multiset and soft multiset which may be found in earlier studies [5, 9, 15].

Definition 2.1 ([9], Soft set)**.** Let U be an initial universe set and E be set of parameters. Let $P(U)$ denotes the power set of U and $A \subseteq U$.

A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.

Definition 2.2 ([5], Multiset)**.** An multiset M drawn from the set X is represented by a function *Count* M or C_M defined as $C_M : X \to \mathbb{N}$.

Let M be an multiset from X with x appearing n times in M. It is denoted by $x \in^n M$. $M = \{k_1/x_1, k_2/x_2, ..., k_n/x_n\}$ where M is an multiset with x_1 appearing k_1 times, x_2 appearing k_2 times and so on.

Definition 2.3 ([4])**.** Let M be an multiset drawn from a set X. The support set of M denoted by M^* is a subset of X which is $M^* = \{x \in X : C_M(x) > 0\}$. i.e., M^* is an ordinary set and it is also called root set.

The power set of an multiset is the support set of the power multiset and is denoted by $P^*(M)$.

Example 2.4 ([4])**.** Let $M = \{2/x, 3/y\}$ be an multiset. Then $M^* = \{x, y\}$ is the support set of M. The collection

$P(M) = \{3/\{2/x, 1/y\}, 3/\{2/x, 2/y\}, 6/\{1/x, 1/y\}, 6/\{1/x, 2/y\}, 2/\{1/x, 3/y\}, 1/\{2/x\}, 2/\{1/x\}, 1/\{3/y\}, 3/\{2/y\}, 3/\{1/y\}, M, \varnothing\}$

is the power multiset of M. The collection

$P^*(M) = \{\{2/x, 1/y\}, \{2/x, 2/y\}, \{1/x, 1/y\}, \{1/x, 2/y\}, \{1/x, 3/y\}, \{2/x\}, \{1/x\}, \{3/y\}, \{2/y\}, \{1/y\}, M, \varnothing\}$

is the support set of $P(M)$.

Definition 2.5 ([15], Soft multiset)**.** Let U be a universal multiset, E be a set of parameters and $A \subseteq E$. Then a pair (F, A) is called a soft multiset where F is a mapping given by $F : A \to P^*(U)$. For all $e \in A$, multiset $F(e)$ represent by a count function $C_{F(e)} : U^* \to \mathbb{N}$.

Example 2.6 ([15])**.** Let multiset and the parameter set be $U = \{1/x, 5/y, 3/z, 4/w\}$ and $E = \{p, q, r\}$. Define a mapping $F : E \to P^*(U)$ as follows:

$$F(p) = \{1/x, 2/y, 3/z\}, F(q) = \{4/w\} \text{ and } F(r) = \{3/y, 1/z, 2/w\}.$$

Then (F, A) is a soft multiset where for all $e \in A$, $F(e)$ multiset represent by count function $C_{F(e)} : U^* \to N$, which are defined as follows:

$C_{F(p)}(x) = 1,\quad C_{F(p)}(y) = 2,\quad C_{F(p)}(z) = 3,\quad C_{F(p)}(w) = 0,$
$C_{F(q)}(x) = 0,\quad C_{F(q)}(y) = 0,\quad C_{F(q)}(z) = 0,\quad C_{F(q)}(w) = 4,$
$C_{F(r)}(x) = 0,\quad C_{F(r)}(y) = 3,\quad C_{F(r)}(z) = 1,\quad C_{F(r)}(w) = 2.$

Then $(F, A) = \{F(p), F(q), F(r)\} = \{\{1/x, 2/y, 3/z\}, \{4/w\}, \{3/y, 1/z, 2/w\}\}$.

Definition 2.7 ([15]). For two soft multisets (F, A) and (G, B) over U, we say that (F, A) is a soft submultiset of (G, B) if

i. $A \subseteq B$
ii. $C_{F(e)}(x) \leq C_{G(e)}(x), \forall x \in U^*, \forall e \in A$

We write $(F, A)\tilde{\subset}(G, B)$.

In addition to (F, A) is a whole soft submultiset of (G, B) if $C_{F(e)}(x) = C_{G(e)}(x), \forall x \in U^*, \forall e \in A$.

Definition 2.8 ([15]). Let (F, A) and (G, B) be two soft multisets over U.

Equality: $(F, A) = (G, B) \Leftrightarrow (F, A) \tilde{\subseteq} (G, B)$ and $(F, A) \tilde{\supseteq} (G, B)$.

Union: $(H, C) = (F, A)\tilde{\cup}(G, B)$ where $C = A \cup B$ and $C_{H(e)}(x) = \max\{C_{F(e)}(x), C_{G(e)}(x)\}, \forall e \in A \cup B, \forall x \in U^*$.

Intersection: $(H, C) = (F, A)\tilde{\cap}(G, B)$ where $C = A \cap B$ and $C_{H(e)}(x) = \min\{C_{F(e)}(x), C_{G(e)}(x)\}, \forall e \in A \cap B, \forall x \in U^*$. We write $(F, A)\tilde{\cap}(G, B)$.

Difference: $(H, E) = (F, E)\backslash(G, E)$ where $C_{H(e)}(x) = \max\left\{C_{F(e)}(x) - C_{G(e)}(x), 0\right\}, \forall x \in U^*$.

Null: A soft multiset (F, A) is said to be a NULL soft multiset denoted by Φ if for all $e \in A$, $F(e) = \varnothing$.

Complement: The complement of a soft multiset (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \to P^*(U)$ is a mapping given by $F^c(e) = U\backslash F(e)$ for all $e \in A$ where $C_{F^c(e)}(x) = C_U(x) - C_{F(e)}(x), \forall x \in U^*$.

Definition 2.9 ([15]). Let V be a non-empty submultiset of U, then $\tilde{V}$ denotes the soft multiset (V, E) over U for which $V(e) = V$, for all $e \in E$.

In particular, (U, E) will be denoted by $\tilde{U}$.

2.2. **Soft multi function.** In this section, we recall soft multi function which was given in [10].

Definition 2.10 ([10]). Let X be universal multiset and E be set of parameters. Then the collection of all soft multisets over X with parameters from E is called a soft multi class and is denoted as X_E.

Definition 2.11 ([10]). Let X_E and Y_K be two soft multi class. Let $\varphi : X^* \to Y^*$ and $\psi : E \to K$ be two function. Then the pair (φ, ψ) is called a soft multi function and denoted by $f = (\varphi, \psi) : X_E \to Y_K$ is defined as follows:

Let (F, E) be a soft multiset in X_E. Then the image of (F, E) under soft multi function f is soft multiset in Y_K defined by $f(F, E)$, where for $k \in \psi(E) \subseteq K$ and $y \in Y^*$,

$$C_{f(F,E)(k)}(y)=\begin{cases}\sup\limits_{e\in\psi^{-1}(k)\cap E,x\in\varphi^{-1}(y)} C_{F(e)}\left(x\right), & \text{if } \psi^{-1}(k)\neq\varnothing,\varphi^{-1}(y)\neq\varnothing;\\ 0, & \text{otherwise.}\end{cases}$$

Let (G,K) be a soft multiset in Y_K. Then the inverse image of (G,K) under soft multi function f is soft multiset in X_E defined by $f^{-1}(G,K)$, where for $e\in\psi^{-1}(K)\subseteq E$ and $x\in X^*$,

$$C_{f^{-1}(G,K)(e)}(x)=C_{G(\psi(e))}\left(\varphi(x)\right).$$

f is said to be injective (onto or surjective) if both $\varphi: X^*\to Y^*$ and $\psi: E\to K$ are injective (onto or surjective) mappings. If f is both injective as well as surjective, then f is said to be a soft multi bijective function.

Example 2.12. [10] Let $X=\{2/a,3/b,4/c,5/d\}$, $Y=\{5/x,4/y,3/z,2/w\}$, $E=\{e_1,e_2,e_3,e_4\}$, $K=\{k_1,k_2,k_3\}$ and X_E, Y_K, classes of soft multisets. Let $\varphi: X^*\to Y^*$ and $\psi: E\to K$ be two function defined as

$$\varphi(a)=z,\quad \varphi(b)=y,\quad \varphi(c)=y,\quad \varphi(d)=x,$$
$$\psi(e_1)=k_1,\quad \psi(e_2)=k_3,\quad \psi(e_3)=k_2,\quad \psi(e_4)=k_1.$$

Choose two soft multisets in X_E and Y_K, respectively, as

$$(F,A)=\{e_1=\{1/a,2/b,1/d\},e_3=\{3/b,2/c,1/d\},e_4=\{2/a,5/d\}\},$$
$$(G,B)=\{k_1=\{4/x,2/w\},k_2=\{1/x,1/y,2/z,2/w\}\}$$

Then soft multiset image of (F,A) under $f: X_E\to Y_K$ is obtained as

$$\begin{aligned}C_{f(F,A)(k_1)}(x)&=\begin{cases}\sup\limits_{e\in\psi^{-1}(k_1)\cap A,a\in\varphi^{-1}(x)} C_{F(e)}\left(a\right), & \text{if } \psi^{-1}(k_1)\neq\varnothing,\varphi^{-1}(x)\neq\varnothing;\\ 0, & \text{otherwise.}\end{cases}\\ &=\begin{cases}\sup\limits_{e\in\{e_1,e_4\},a\in\{d\}} C_{F(e)}\left(a\right), & \text{if } \psi^{-1}(k_1)\neq\varnothing,\varphi^{-1}(x)\neq\varnothing;\\ 0, & \text{otherwise.}\end{cases}\\ &=\sup\left\{C_{F(e_1)}\left(d\right),C_{F(e_4)}\left(d\right)\right\}\\ &=1,\end{aligned}$$

$$\begin{aligned}&C_{f(F,A)(k_1)}(y)=\sup\left\{C_{F(e_1)}\left(b\right),C_{F(e_4)}\left(b\right),C_{F(e_1)}\left(c\right),C_{F(e_4)}\left(c\right)\right\}=2,\\ &C_{f(F,A)(k_1)}(z)=\sup\left\{C_{F(e_1)}\left(a\right),C_{F(e_4)}\left(a\right)\right\}=1,\\ &C_{f(F,A)(k_1)}(w)=0 \text{ (since } \varphi^{-1}(w)=\varnothing\text{)},\\ &C_{f(F,A)(k_2)}(x)=\sup\left\{C_{F(e_3)}\left(d\right)\right\}=5,\\ &C_{f(F,A)(k_2)}(y)=\sup\left\{C_{F(e_3)}\left(b\right),C_{F(e_3)}\left(c\right)\right\}=0,\\ &C_{f(F,A)(k_2)}(z)=\sup\left\{C_{F(e_3)}\left(a\right)\right\}=2,\\ &C_{f(F,A)(k_2)}(w)=0 \text{ (since } \varphi^{-1}(w)=\varnothing\text{)},\\ &C_{f(F,A)(k_3)}(x)=\sup\left\{C_{F(e_2)}\left(d\right)\right\}=1,\\ &C_{f(F,A)(k_3)}(y)=\sup\left\{C_{F(e_2)}\left(b\right),C_{F(e_2)}\left(c\right)\right\}=3,\\ &C_{f(F,A)(k_3)}(z)=\sup\left\{C_{F(e_2)}\left(a\right)\right\}=0,\\ &C_{f(F,A)(k_3)}(w)=0 \text{ (since } \varphi^{-1}(w)=\varnothing\text{)}.\end{aligned}$$

Consequently, we have

$$(f(F,A),B)=\{k_1=\{1/x,2/y,1/z\},k_2=\{5/x,2/y\},k_3=\{1/x,3/y\}\}.$$

Soft multiset inverse image of (G,B) under $f: X_E\to Y_K$ is obtained as

$C_{f^{-1}(G,B)(e_1)}(a) = C_{G(\psi(e_1))}(\varphi(a)) = C_{G(k_1)}(z) = 0,$
$C_{f^{-1}(G,B)(e_1)}(b) = C_{G(\psi(e_1))}(\varphi(b)) = C_{G(k_1)}(y) = 0,$
$C_{f^{-1}(G,B)(e_1)}(c) = C_{G(\psi(e_1))}(\varphi(c)) = C_{G(k_1)}(y) = 0,$
$C_{f^{-1}(G,B)(e_1)}(d) = C_{G(\psi(e_1))}(\varphi(d)) = C_{G(k_1)}(x) = 4,$
$C_{f^{-1}(G,B)(e_3)}(a) = C_{G(\psi(e_3))}(\varphi(a)) = C_{G(k_2)}(z) = 2,$
$C_{f^{-1}(G,B)(e_3)}(b) = C_{G(\psi(e_3))}(\varphi(b)) = C_{G(k_2)}(y) = 1,$
$C_{f^{-1}(G,B)(e_3)}(c) = C_{G(\psi(e_3))}(\varphi(c)) = C_{G(k_2)}(y) = 1,$
$C_{f^{-1}(G,B)(e_3)}(d) = C_{G(\psi(e_3))}(\varphi(d)) = C_{G(k_2)}(x) = 1,$
$C_{f^{-1}(G,B)(e_4)}(a) = C_{G(\psi(e_4))}(\varphi(a)) = C_{G(k_1)}(z) = 0,$
$C_{f^{-1}(G,B)(e_4)}(b) = C_{G(\psi(e_4))}(\varphi(b)) = C_{G(k_1)}(y) = 0,$
$C_{f^{-1}(G,B)(e_4)}(c) = C_{G(\psi(e_4))}(\varphi(c)) = C_{G(k_1)}(y) = 0,$
$C_{f^{-1}(G,B)(e_4)}(d) = C_{G(\psi(e_4))}(\varphi(d)) = C_{G(k_1)}(x) = 4.$

Consequently, we have
$(f^{-1}(G,B), D) = \{e_1 = \{4/d\}, e_3 = \{2/a, 1/b, 1/c, 1/d\}, e_4 = \{4/d\}\}.$

Theorem 2.13 ([10]). *Let $f : X_E \to Y_K$ be a soft multi function, $(F,A),(F_i,A)$ soft multisets in X_E and $(G,B),(G_i,B)$ soft multisets in Y_K.*

(1) $f(\Phi) = \Phi$, $f(\tilde{X})\tilde{\subseteq}\tilde{Y}$,
(2) $f^{-1}(\Phi) = \Phi$, $f^{-1}(\tilde{Y}) = \tilde{X}$,
(3) $f((F_1,A_1)\tilde{\cup}(F_2,A_2)) = f(F_1,A_1)\tilde{\cup}f(F_2,A_2)$.
In general, $f(\tilde{\cup}_{i\in I}(F_i,A_i)) = \tilde{\cup}_{i\in I}\ f(F_i,A_i)$,
(4) $f^{-1}((G_1,B)\tilde{\cup}(G_2,B)) = f^{-1}(G_1,B)\tilde{\cup}f^{-1}(G_2,B)$.
In general, $f^{-1}(\tilde{\cup}_{i\in I}(G_i,B)) = \tilde{\cup}_{i\in I}\ f^{-1}(G_i,B)$,
(5) $f((F_1,A)\tilde{\cap}(F_2,A))\tilde{\subseteq}f(F_1,A)\tilde{\cap}f(F_2,A)$.
In general, $f(\tilde{\cap}_{i\in I}(F_i,A))\tilde{\subseteq}\tilde{\cap}_{i\in I}\ f(F_i,A)$,
(6) $f^{-1}((G_1,B)\tilde{\cap}(G_2,B)) = f^{-1}(G_1,B)\tilde{\cap}f^{-1}(G_2,B)$.
In general, $f^{-1}(\tilde{\cap}_{i\in I}(G_i,B)) = \tilde{\cap}_{i\in I}\ f^{-1}(G_i,B)$,
(7) *If* $(F_1,A)\tilde{\subseteq}(F_2,A)$, *then* $f(F_1,A)\tilde{\subseteq}f(F_2,A)$,
(8) *If* $(G_1,B)\tilde{\subseteq}(G_2,B)$, *then* $f^{-1}(G_1,B)\tilde{\subseteq}f^{-1}(G_2,B)$.

2.3. **Soft multi topology.** In this section, we recall soft multi topology which was given in [15].

Definition 2.14 ([15]). Let $\tau \subseteq X_E$, then τ is said to be a soft multi topology on X if the following conditions hold.

i. Φ,$\tilde{X}$ belong to τ.
ii. The union of any number of soft multisets in τ belongs to τ.
iii. The intersection of any two soft multisets in τ belongs to τ.

τ is called a soft multi topology over X and the binary (X_E, τ) is called a soft multi topological space over X.

The members of τ are said to be soft multi open sets in X.

A soft multiset (F,E) over X is said to be a soft multi closed set in X, if its complement $(F,E)^c$ belongs to τ .

Example 2.15 ([15]). Let $X = \{2/x, 3/y, 4/z, 5/w\}$, $E = \{p,q\}$ and $\tau = \{\Phi, \tilde{X}, (F_1,E),$ $(F_2,E),(F_3,E)\}$ where $(F_1,E),(F_2,E),(F_3,E)$ are soft multisets over X, defined as follows

$F_1(p) = \{1/x, 2/y, 3/z\}$, $\quad F_1(q) = \{4/w\}$
$F_2(p) = X$, $\quad F_2(q) = \{1/x, 3/y, 4/z, 5/w\}$
$F_3(p) = \{2/x, 3/y, 3/z, 1/w\}$, $\quad F_3(q) = \{1/x, 4/w\}$.

Then τ defines a soft multi topology on X and hence (X_E, τ) is a soft multi topological space over X.

Remark 2.16. In the example above, without allowing the repetitions for the elements of X, we obtain soft topology on X. Thus the concept of soft multi topology is more general than that of soft topology.

3. Soft multi semi-open and soft multi semi-closed sets

In this section, we introduce and examine the concepts of soft semi-open set, soft semi-closed set, soft semi-interior and soft semi-closure on soft multi topological space. Also we give basic theorems about them.

Definition 3.1. A soft multi set (A, E) in a soft multi topological space (X_E, τ) will be termed soft multi semi-open if and only if there exists a soft multi open set (O, E) such that $(O, E)\tilde{\subset}(A, E)\tilde{\subset}\overline{(O, E)}$.

From the above definition, we say that every soft multi open set in a soft multi topological space (X_E, τ) is soft multi semi-open. Now we give an example to show that the converse of above remark does not hold.

Example 3.2. Let $X = \{1/x, 3/y, 2/z\}$, $E = \{p, q\}$ and $\tau = \{\Phi, \tilde{X}, (F_1, E), (F_2, E),$ $(F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E)\}$ where $(F_1, E), (F_2, E), ..., (F_7, E)$ are soft multisets over X, defined as follows

$F_1(p) = \{1/x, 3/y\}$, $\quad F_1(q) = \{1/x, 3/y\}$,
$F_2(p) = \{3/y\}$, $\quad F_2(q) = \{1/x, 2/z\}$,
$F_3(p) = \{3/y, 2/z\}$, $\quad F_3(q) = \{1/x\}$,
$F_4(p) = \{3/y\}$, $\quad F_4(q) = \{1/x\}$,
$F_5(p) = \{1/x, 3/y\}$, $\quad F_5(q) = X$,
$F_6(p) = X$, $\quad F_6(q) = \{1/x, 3/y\}$
$F_7(p) = \{3/y, 2/z\}$, $\quad F_7(q) = \{1/x, 2/z\}$.

Then τ defines a soft multi multi topology on X and hence (X_E, τ) is a soft multi topological space over X. (G, E) are soft multisets over X, defined as follows

$G(p) = \{3/y, 2/z\}$, $\quad G(q) = \{1/x, 3/y\}$.

Then, for the soft multi open set (F_3, E), we have $(F_3, E) \subset (G, E) \subset \overline{(F_3, E)} = \tilde{X}$. Thus the soft multi set (G, E) is soft multi semi-open in the soft multi topological space (X_E, τ) but not soft multi open since $(G, E) \notin \tau$.

Theorem 3.3. *Any soft multiset (A, E) in (X_E, τ) is soft multi semi-open if and only if $(A, E)\tilde{\subset}\overline{((A, E)^\circ)}$*

Proof. Let (A, E) be soft multi semi-open. Then $(O, E)\tilde{\subset}(A, E)\tilde{\subset}\overline{(O, E)}$ for some soft multi open set (O, E). Thus $(O, E)\tilde{\subset}(A, E)^\circ$ and so $\overline{(O, E)}\tilde{\subset}\overline{((A, E)^\circ)}$. Hence $(A, E)\tilde{\subset}\overline{(O, E)}\tilde{\subset}\overline{((A, E)^\circ)}$.

Let $(A, E)\tilde{\subset}\overline{((A, E)^\circ)}$. Then for $(O, E) = (A, E)^\circ$, we have $(O, E)\tilde{\subset}(A, E)\tilde{\subset}\overline{(O, E)}$. □

Theorem 3.4. *Let* $\{(A_\alpha, E) : \alpha \in I\}$ *be a collection of soft multi semi-open sets in* (X_E, τ). *Then* $\tilde{\cup}_{\alpha\in I}(A_\alpha, E)$ *is soft multi semi-open.*

Proof. For each $\alpha \in I$, we have a soft multi open set (O_α, E) such that $(O_\alpha, E)\tilde{\subset}(A_\alpha, E)$ $\tilde{\subset}\overline{(O_\alpha, E)}$. Then $\tilde{\cup}_{\alpha\in I}(O_\alpha, E)\tilde{\subset}\tilde{\cup}_{\alpha\in I}(A_\alpha, E)\tilde{\subset}\tilde{\cup}_{\alpha\in I}\overline{(O_\alpha, E)}\tilde{\subset}\overline{\tilde{\cup}_{\alpha\in I}(O_\alpha, E)}$.Thus $\tilde{\cup}_{\alpha\in I}(A_\alpha, E)$ is soft multi semi-open. □

Theorem 3.5. *Let* (A, E) *be soft multi semi-open in* (X_E, τ) *and suppose* $(A, E)\tilde{\subset}$ $(B, E)\tilde{\subset}\overline{(A, E)}$. *Then* (B, E) *is soft multi semi-open.*

Proof. Since (A, E) is soft multi semi-open. there exists a soft multi open set (O, E) such that $(O, E)\tilde{\subset}(A, E)\tilde{\subset}\overline{(O, E)}$. Then $(O, E)\tilde{\subset}(B, E)$. Also $\overline{(A, E)}\tilde{\subset}\overline{(O, E)}$ and thus $(B, E)\tilde{\subset}\overline{(O, E)}$. Hence $(O, E)\tilde{\subset}(B, E)\tilde{\subset}\overline{(O, E)}$ and (B, E) is soft multi semi-open. □

Definition 3.6. A soft multi set (B, E) in a soft multi topological space (X_E, τ) will be termed soft multi semi-closed if its relative complement is soft multi semi-open, i.e., there exists a soft multi closed set (F, E) such that $(F, E)^\circ\tilde{\subset}(B, E)\tilde{\subset}(F, E)$.

From the above definition, we say that every soft multi closed set in a soft multi topological space (X_E, τ) is soft multi semi-closed. Now we give an example to show that the converse of above remark does not hold.

Example 3.7. Let us consider the soft multi topological space that is given in Example 3.2. Since (G, E) is soft multi semi-open, then $(G, E)^c = (B, E)$ is soft multi semi-closed where $B(p) = \{1/x\}, B(q) = \{2/z\}$ .soft multi closed sets in (X_E, τ) are $(H_1, E), (H_2, E), ..., (H_7, E)$ defined as follows:

$H_1(p) = \{2/z\}$, $H_1(q) = \{2/z\}$,
$H_2(p) = \{1/x, 2/z\}$, $H_2(q) = \{3/y\}$,
$H_3(p) = \{1/x\}$, $H_3(q) = \{3/y, 2/z\}$,
$H_4(p) = \{1/x, 2/z\}$, $H_4(q) = \{3/y, 2/z\}$,
$H_5(p) = \{2/z\}$, $H_5(q) = \varnothing$,
$H_6(p) = \varnothing$, $H_6(q) = \{2/z\}$
$H_7(p) = \{1/x\}$, $H_7(q) = \{3/y\}$.

So (B, E) is a soft multi semi-closed set but not a soft multi closed set.

Theorem 3.8. *Any soft multiset* (B, E) *in* (X_E, τ) *is soft multi semi-closed if and only if* $(\overline{(B, E)})^\circ\tilde{\subset}(B, E)$.

Proof. Let (B, E) be soft multi semi-closed. Then $(F, E)^\circ\tilde{\subset}(B, E)\tilde{\subset}(F, E)$ for some soft multi closed set (F, E). Thus $\overline{(B, E)}\tilde{\subset}\overline{(F, E)} = (F, E)$ and so $(\overline{(B, E)})^\circ\tilde{\subset}(F, E)^\circ$. Hence $(\overline{(B, E)})^\circ\tilde{\subset}(F, E)^\circ\tilde{\subset}(F, E)$.

Let $(\overline{(B, E)})^\circ\tilde{\subset}(B, E)$. Then for $(F, E) = \overline{(B, E)}$, we have $(F, E)^\circ\tilde{\subset}(B, E)\tilde{\subset}(F, E)$. □

Theorem 3.9. *Let* $\{(B_\alpha, E) : \alpha \in I\}$ *be a collection of soft multi semi-closed sets in* (X_E, τ). *Then* $\tilde{\cap}_{\alpha\in I}(B_\alpha, E)$ *is soft multi semi-closed.*

Proof. For each $\alpha \in I$, we have a soft multi closed set (F_α, E) such that $(F_\alpha, E)^\circ$ $\tilde{\subset}(B_\alpha, E)\tilde{\subset}(F_\alpha, E)$. Then $(\tilde{\cap}_{\alpha\in I}(F_\alpha, E))^\circ\tilde{\subset}\tilde{\cap}_{\alpha\in I}(F_\alpha, E)^\circ\tilde{\subset}\tilde{\cap}_{\alpha\in I}(B_\alpha, E)\tilde{\subset}\tilde{\cap}_{\alpha\in I}(F_\alpha, E)$. Thus $\tilde{\cap}_{\alpha\in I}(B_\alpha, E)$ is soft multi semi-closed. □

Theorem 3.10. *Let (B,E) be soft multi semi-closed in (X_E,τ) and suppose $(B,E)^\circ \tilde{\subset}(A,E)\tilde{\subset}(B,E)$. Then (A,E) is soft multi semi-closed.*

Proof. There exists a soft multi closed set (F,E) such that $(F,E)^\circ\tilde{\subset}(B,E)\tilde{\subset}(F,E)$. Then $(A,E)\tilde{\subset}(F,E)$ and $((F,E)^\circ)^\circ = (F,E)^\circ\tilde{\subset}(B,E)^\circ$ Thus $(F,E)^\circ\tilde{\subset}(A,E)$. Hence $(F,E)^\circ\tilde{\subset}(A,E)\tilde{\subset}(F,E)$ and (A,E) is soft multi semi-closed. □

Definition 3.11. Let (X_E,τ) be a soft multi topological space and (A,E) be a soft multi set over X.

- The soft multi semi-interior of (A,E) is the soft multi set $(A,E)_\circ = \tilde{\cup}\{(O,E) : (O,E)$ is soft multi semi-open and $(O,E)\tilde{\subset}(A,E)\}$.
- The soft multi semi-closure of (A,E) is the soft multi set $\underline{(A,E)} = \tilde{\cap}\{(F,E) : (F,E)$ is soft multi semi-closed and $(A,E)\tilde{\subset}(F,E)\}$.

Example 3.12. Let us consider the semi-open set (G,E) that is given in Example 3.2. It is seen that $(G,E)_\circ = (G,E)$.

Example 3.13. Let us consider the semi-closed set (B,E) that is given in Example 3.7. It is seen that $\underline{(B,E)} = (B,E)$.

Theorem 3.14. *Let (X_E,τ) be a soft multi topological space and (A,E) be a soft multi set over X. We have $(A,E)^\circ\tilde{\subset}(A,E)_\circ\tilde{\subset}(A,E)\tilde{\subset}\underline{(A,E)}\tilde{\subset}\overline{(A,E)}$.*

Proof. It can be proved easily using Definition 3.1, Definition 3.6 and Definition 3.11. □

Theorem 3.15. *Let (X_E,τ) be a soft multi topological space and let (F,E) and (G,E) be soft multi sets over X. Then*

i. $\Phi_\circ = \Phi$ *and* $\tilde{X}_\circ = \tilde{X}$,
ii. (F,E) *is soft multi semi-open if and only if* $(F,E) = (F,E)_\circ$,
iii. $((F,E)_\circ)_\circ = (F,E)_\circ$,
iv. $(F,E)\ \tilde{\subset}(G,E)$ *implies* $(F,E)_\circ\ \tilde{\subset}(G,E)_\circ$,
v. $(F,E)_\circ\ \tilde{\cup}(G,E)_\circ\tilde{\subset}((F,E)\ \tilde{\cup}(G,E))_\circ$.

Proof.
i. It is obvious.
ii. If (F,E) is a soft multi semi-open set over X then $(F,E)\tilde{\subset}(F,E)_\circ$ and so $(F,E) = (F,E)_\circ$. Conversely, suppose that $(F,E) = (F,E)_\circ$. Since $(F,E)_\circ$ is a soft multi semi-open set, so (F,E) is a soft multi semi-open set over X.
iii. Since $(F,E)_\circ$ is a soft multi semi-open set and by ii. we have $((F,E)_\circ)_\circ = (F,E)_\circ$.
iv. Suppose that $(F,E)\ \tilde{\subset}(G,E)$. Since $(F,E)_\circ\tilde{\subset}(F,E)\ \tilde{\subset}(G,E)$. $(F,E)_\circ$ is a soft multi semi-open subset of (G,E), so by definition of $(G,E)_\circ$, then $(F,E)_\circ\ \tilde{\subset}(G,E)_\circ$.
v. Since $(F,E)\ \tilde{\subset}((F,E)\ \tilde{\cup}(G,E))$ and $(G,E)\ \tilde{\subset}((F,E)\ \tilde{\cup}(G,E))$, so by iv. , $(F,E)_\circ\ \tilde{\subset}((F,E)\ \tilde{\cup}(G,E))_\circ$ and $(G,E)_\circ\ \tilde{\subset}((F,E)\ \tilde{\cup}(G,E))_\circ$. Thus $(F,E)_\circ\ \tilde{\cup}(G,E)_\circ\tilde{\subset}((F,E)\ \tilde{\cup}(G,E))_\circ$.

□

Theorem 3.16. *Let (X_E,τ) be a soft multi topological space and let (F,E) and (G,E) be soft multi sets over X. Then*

i. $\underline{\Phi} = \Phi$ *and* $\underline{\tilde{X}} = \tilde{X}$,
ii. (F,E) *is soft multi semi-closed if and only if* $(F,E) = \underline{(F,E)}$,
iii. $\underline{(\underline{(F,E)})} = \underline{(F,E)}$,
iv. $(F,E)\ \tilde{\subset}(G,E)$ *implies* $\underline{(F,E)}\tilde{\subset}\underline{(G,E)}$,
v. $\underline{((F,E)\tilde{\cap}(G,E))}\tilde{\subset}\underline{(F,E)}\tilde{\cap}\underline{(G,E)}$.

Proof. i. It is obvious.

ii. If (F,E) is a soft multi semi-closed set over X then $\underline{(F,E)}\tilde{\subset}(F,E)$ and so $(F,E) = \underline{(F,E)}$. Conversely, suppose that $(F,E) = \underline{(F,E)}$. Since $\underline{(F,E)}$ is a soft multi semi-closed set, so (F,E) is a soft multi semi-closed set over X.

iii. Since $\underline{(F,E)}$ is a soft multi semi-open set and by ii. we have $\underline{(\underline{(F,E)})} = \underline{(F,E)}$.

iv. Suppose that $(F,E)\ \tilde{\subset}(G,E)$. Since $(F,E)\tilde{\subset}(G,E)\ \tilde{\subset}\underline{(G,E)}$. $\underline{(G,E)}$ is a soft multi semi-closed super set of (F,E), so by definition of $\underline{(F,E)}$, then $\underline{(F,E)}\tilde{\subset}\underline{(G,E)}$.

v. Since $((F,E)\ \tilde{\cap}(G,E))\tilde{\subset}(F,E)$ and $((F,E)\ \tilde{\cap}(G,E))\tilde{\subset}(G,E)$, so by iv. , $\underline{((F,E)\tilde{\cap}(G,E))}\tilde{\subset}\underline{(F,E)}$ and $\underline{((F,E)\tilde{\cap}(G,E))}\tilde{\subset}\underline{(G,E)}$. Thus $\underline{((F,E)\tilde{\cap}(G,E))}\ \tilde{\subset}\underline{(F,E)}\tilde{\cap}\underline{(G,E)}$.

□

Theorem 3.17. *Let (X_E,τ) be a soft multi topological space and (A,E) be a soft multi set over X. Then*

i. $(\underline{(A,E)})^c = ((A,E)^c)_\circ$,
ii. $((A,E)_\circ)^c = \underline{((A,E)^c)}$,
iii. $(A,E)_\circ = (\underline{((A,E)^c)})^c$.

Proof.

i.
$$\begin{aligned}(\underline{(A,E)})^c &= (\tilde{\cap}\{(F,E) : (F,E) \text{ is soft multi semi-closed and } (A,E)\tilde{\subset}(F,E)\})^c\\ &= \tilde{\cup}\{(F,E)^c : (F,E) \text{ is soft multi semi-closed and } (A,E)\tilde{\subset}(F,E)\}\\ &= \tilde{\cup}\{(F,E)^c : (F,E)^c \text{ is soft multi semi-open and } (F,E)^c\tilde{\subset}(A,E)^c\}\\ &= ((A,E)^c)_\circ.\end{aligned}$$

ii. It can be proved similar to i.
iii. It is clear from ii.

□

Theorem 3.18. *Let (X_E,τ) be a soft multi topological space and (A,E) be a soft multi set over X. Then*

i. $((A,E)^\circ)_\circ = ((A,E)_\circ)^\circ = (A,E)^\circ$,
ii. $\underline{(\overline{(A,E)})} = \overline{(\underline{(A,E)})} = \overline{(A,E)}$.

Proof. i. Since $(A,E)^\circ$ is soft multi open, we have $(A,E)^\circ$ is soft multi semi-open. Thus $((A,E)^\circ)_\circ = (A,E)^\circ$ by Theorem 3.16(ii.).

By Theorem 3.14, we have $(A,E)^\circ\tilde{\subset}(A,E)_\circ\tilde{\subset}(A,E)$, then $(A,E)^\circ\tilde{\subset}((A,E)_\circ)^\circ\ \tilde{\subset}(A,E)^\circ$ and so $((A,E)_\circ)^\circ = (A,E)^\circ$.

ii. Since $\overline{(A,E)}$ is soft multi closed, we have $\overline{(A,E)}$ is soft multi semi-closed. Thus $\underline{(\overline{(A,E)})} = \overline{(A,E)}$ by Theorem 3.15(ii.).

By Theorem 3.14, we have $(A,E)\tilde{\subset}\underline{(A,E)}\tilde{\subset}\overline{(A,E)}$, then $\overline{(A,E)}\tilde{\subset}\ \overline{(\underline{(A,E)})}$ $\tilde{\subset}\overline{(A,E)}$ and so $\overline{(\underline{(A,E)})}=\overline{(A,E)}$.

□

4. Soft multi semi-continuous, soft multi semi-open, soft multi semi-closed and soft multi irresolute functions

In this section, we define notion of soft multi semi-continuous function and examine its basic theorems.

Definition 4.1. Let (X_E,τ) and (Y_K,σ) be two soft multi topological spaces, $f:(X_E,\tau)\to(Y_K,\sigma)$ be a soft multi function. Then f is said to be soft multi semi-continuous function if $f^{-1}((G,B))$ is a soft multi semi-open set in X_E, for each soft multi open set (G,B) in Y_K.

Theorem 4.2. *Let (X_E,τ) and (Y_K,σ) be two soft multi topological spaces, $f:(X_E,\tau)\to(Y_K,\sigma)$ be a soft multi function. Then f is soft multi semi-continuous function if and only if $f^{-1}((G,B))$ is a soft multi semi-closed set in X_E, for each soft multi closed set (G,B) in Y_K.*

Proof. It is clear from Definition 4.1. □

Theorem 4.3. *Let (X_E,τ) and (Y_K,σ) be two soft multi topological spaces, $f:(X_E,\tau)\to(Y_K,\sigma)$ be a soft multi function. Then f is soft multi semi-continuous function if and only if for each soft multiset (F,A) in X_E, $f(\underline{(F,A)})\tilde{\subseteq}\overline{f((F,A))}$.*

Proof. Let f be soft multi semi-continuous function. Since $\overline{f((F,A))}$ is soft multi closed set in Y_K and f is soft multi semi-continuous, then $f^{-1}(\overline{f((F,A))})$ is soft multi semi-closed set in X_E and $(F,A)\tilde{\subseteq}f^{-1}(\overline{f((F,A))})$. Thus $\underline{(F,A)}\tilde{\subseteq}\underline{f^{-1}(\overline{f((F,A))})}=f^{-1}(\overline{f((F,A))})$ and so $f(\underline{(F,A)})\tilde{\subseteq}\overline{f((F,A))}$.

Conversely, let (G,B) is soft multi closed set in Y_K. Then $f^{-1}((G,B))$ is soft multiset in X_E.Then $f(\underline{f^{-1}((G,B))})\tilde{\subseteq}\overline{f(f^{-1}((G,B)))}$ and so $f(\underline{f^{-1}((G,B))})\tilde{\subseteq}\overline{(G,B)}=(G,\dot{B})$. Thus $\underline{f^{-1}((G,B))}=f^{-1}((G,\dot{B}))$ and it is a soft multi semi-closed set in X_E. □

Theorem 4.4. *Let (X_E,τ) and (Y_K,σ) be two soft multi topological spaces, $f:(X_E,\tau)\to(Y_K,\sigma)$ be a soft multi function. Then f is soft multi semi-continuous function if and only if for each soft multiset (G,B) in Y_K, $f^{-1}((G,B)^{\circ})\tilde{\subseteq}(f^{-1}((G,B)))_{\circ}$.*

Proof. Let f be soft multi semi-continuous function. Since $(G,B)^{\circ}$ is soft multi open set in Y_K and f is soft multi semi-continuous, then $f^{-1}((G,B)^{\circ})$ is soft multi semi-open set in X_E and $f^{-1}((G,B)^{\circ})\tilde{\subseteq}\ f^{-1}(G,B)$. Thus $(f^{-1}((G,B)^{\circ}))_{\circ}=f^{-1}((G,B)^{\circ})\tilde{\subseteq}(f^{-1}(G,B))_{\circ}$.

Conversely, let (G,B) is soft multi open set in Y_K. Then $f^{-1}((G,B)^{\circ})\tilde{\subseteq}(f^{-1}((G,B)))_{\circ}$ and so $f^{-1}((G,B))\tilde{\subseteq}(f^{-1}((G,B)))_{\circ}$. Thus $(f^{-1}((G,B)))_{\circ}=f^{-1}((G,B))$ and it is a soft multi semi-open set in X_E. □

Definition 4.5. Let (X_E, τ) and (Y_K, σ) be two soft multi topological spaces, $f : (X_E, \tau) \to (Y_K, \sigma)$ be a soft multi function.

- Soft multi function f is called soft multi semi-open if $f((F, A))$ is a soft multi semi-open set in Y_K, for each soft multi semi-open set (F, A) in X_E.
- Soft multi function f is called soft multi semi-closed if $f((F, A))$ is a soft multi semi-closed set in Y_K, for each soft multi semi-closed set (F, A) in X_E.

Theorem 4.6. *Let (X_E, τ) and (Y_K, σ) be two soft multi topological spaces, $f : (X_E, \tau) \to (Y_K, \sigma)$ be a soft multi function.*

i. *f is a soft multi semi-open function if and only if for each soft multi set (F, A) in X_E, $f((F, A)_\circ)\tilde{\subseteq}(f((F, A)))_\circ$ is satisfied.*
ii. *f is a soft multi semi-closed function if and only if for each soft multi set (F, A) in X_E, $\underline{(f((F, A)))}\tilde{\subseteq}f(\underline{(F, A)})$ is satisfied.*

Proof. i. Let f be a soft multi semi-open function and (F, A) be a soft multiset in X_E. Since $(F, A)_\circ\tilde{\subseteq}(F, A)$ and f is soft multi semi-open function, then $f((F, A)_\circ)\tilde{\subseteq}f((F, A))$ and $f((F, A)_\circ)$ is a soft multi semi-open set in Y_K. Thus $f((F, A)_\circ)\tilde{\subseteq}(f((F, A)))_\circ$.

Conversely, let (F, A) be any soft semi-open multiset in X_E. Using $(F, A) = (F, A)_\circ$, we have $f((F, A)) = f((F, A)_\circ)\tilde{\subseteq}(f((F, A)))_\circ$. Then $f((F, A)) = (f((F, A)))_\circ$ and so f is a soft multi semi-open function.

ii. Let f be a soft multi semi-closed function and (F, A) be a soft multiset in X_E. Since $(F, A)\tilde{\subseteq}\underline{(F, A)}$ and f is soft multi semi-closed function, then $f((F, A))\tilde{\subseteq}f(\underline{(F, A)})$ and $f(\underline{(F, A)})$ is a soft multi semi-closed set in Y_K. Thus $\underline{(f((F, A)))}\tilde{\subseteq}f(\underline{(F, A)})$.

Conversely, let (F, A) be any soft semi-closed multiset in X_E. Using $(F, A) = \underline{(F, A)}$, we have $\underline{(f((F, A)))}\tilde{\subseteq}f(\underline{(F, A)}) = f((F, A))$. Then $f((F, A)) = \underline{(f((F, A)))}$ and so f is a soft multi semi-closed function.

□

Definition 4.7. Let (X_E, τ) and (Y_K, σ) be two soft multi topological spaces, $f : (X_E, \tau) \to (Y_K, \sigma)$ be a soft multi function. Then f is said to be soft multi irresolute function if $f^{-1}((G, B))$ is a soft multi semi-open set in X_E, for each soft multi semi-open set (G, B) in Y_K.

Concepts of soft multi continuous function and soft multi irresolute function are independent. That means that both do not require each other. This can easily be seen from the following examples.

Example 4.8. Let $X = \{7/a, 8/b\}$ and $E = \{e_1, e_2\}$. Let $\varphi : X^* \to X^*$ and $\psi : E \to E$ be two identity functions. Then f is identity function. Two soft multisets in X_E defined as follows

$(F, E) = \{F(e_1), F(e_2)\} = \{\{5/a, 3/b\}, \{4/a, 7/b\}\}$,
$(G, E) = \{G(e_1), G(e_2)\} = \{\{5/a, 6/b\}, \{6/a, 8/b\}\}$.

Let us consider the following soft multi topology $\tau_1 = \{\Phi, \tilde{X}, (F, E)\}$ and $\tau_2 = \{\Phi, \tilde{X}, (G, E)\}$. Now we show that $f : (X_E, \tau_1) \to (X_E, \tau_2)$ is soft multi irresolute function, but not soft multi continuous function.

Let (A, E) be soft multi semi-open in (X_E, τ_2). Then $(G, E)\tilde{\subseteq}(A, E)\tilde{\subseteq}\overline{(G, E)}$, for soft multi open set (G, E). Soft multiset inverse image of (A, E) under f is $f^{-1}((A, E)) = (A, E)\tilde{\supseteq}\ (G, E)$. Since $(G, E)\tilde{\supseteq}(F, E)$ and $\overline{(F, E)} = \tilde{X}$, then $(F, E)\tilde{\subseteq}f^{-1}((A, E))\tilde{\subseteq}\overline{(F, E)}$. Hence f is soft multi irresolute function. But since $f^{-1}((G, E)) = (G, E)$ is not soft multi open set in (X_E, τ_1) then f is not soft multi continuous function.

Example 4.9. Let $X = \{6/a, 6/b\}$ and $E = \{e_1, e_2\}$. Let $\varphi : X^* \to X^*$ and $\psi : E \to E$ be be two function defined as

$$\varphi(a) = a, \quad \varphi(b) = b,$$
$$\psi(e_1) = e_2, \quad \psi(e_2) = e_2.$$

Two soft multisets in X_E defined as follows

$(F, E) = \{F(e_1), F(e_2)\} = \{\{4/a, 3/b\}, \{4/a, 3/b\}\}$,

$(G, E) = \{G(e_1), G(e_2)\} = \{\{5/a, 6/b\}, \{4/a, 3/b\}\}$.

Let us consider the following soft multi topology $\tau_1 = \{\Phi, \tilde{X}, (F, E)\}$ and $\tau_2 = \{\Phi, \tilde{X}, (G, E)\}$. Now we show that $f : (X_E, \tau_1) \to (X_E, \tau_2)$ is soft multi continuous function, but not soft multi irresolute function.

Since $f^{-1}((G, E)) = (F, E)$, then f is soft multi continuous function.

Let $(A, E) = \{A(e_1), A(e_2)\} = \{\{8/a, 7/b\}, \{9/a, 10/b\}\}$ be a soft multiset. Since $(G, E)\tilde{\subseteq}(A, E)\tilde{\subseteq}\overline{(G, E)} = \tilde{X}$, then (A, E) is soft multi semi-open in (X_E, τ_2). But $f^{-1}((A, E)) = \{\{9/a, 10/b\}, \{9/a, 10/b\}\}$ soft multiset is not soft multi semi-open in (X_E, τ_1) since $(F, E)\tilde{\subseteq}\ f^{-1}((A, E))$ and $f^{-1}((A, E))\tilde{\nsubseteq}\overline{(F, E)} = (F, E)^c$. Hence f is not soft multi irresolute function.

Theorem 4.10. *Let (X_E, τ) and (Y_K, σ) be two soft multi topological spaces, $f : (X_E, \tau) \to (Y_K, \sigma)$ be a soft multi function. Then f is soft multi irresolute function if and only if $f^{-1}((G, B))$ is a soft multi semi-closed set in X_E, for each soft multi semi-closed set (G, B) in Y_K.*

Proof. It is clear from Definition 4.7. □

Theorem 4.11. *Let (X_E, τ) and (Y_K, σ) be two soft multi topological spaces, $f : (X_E, \tau) \to (Y_K, \sigma)$ be a soft multi function. Then f is soft multi irresolute function if and only if for each soft multiset (F, A) in X_E, $f(\underline{(F, A)})\tilde{\subseteq}\underline{(f((F, A)))}$.*

Proof. Let f be soft multi irresolute function. Since $\underline{(f((F, A)))}$ is soft multi semi-closed set in Y_K and f is soft multi irresolute, then $f^{-1}((\underline{f((F, A))}))$ is soft multi semi-closed set in X_E and $(F, A)\tilde{\subseteq}f^{-1}((\underline{f((F, A))}))$. Thus $\underline{(F, A)}\tilde{\subseteq}\underline{f^{-1}((\underline{f((F, A))}))} = f^{-1}((\underline{f((F, A))}))$ and so $f(\underline{(F, A)})\tilde{\subseteq}\underline{(f((F, A)))}$.

Conversely, let (G, B) is soft multi semi-closed set in Y_K. Then $f^{-1}((G, B))$ is soft multiset in X_E.Then $f(\underline{f^{-1}((G, B))})\tilde{\subseteq}\underline{(f(f^{-1}((G, B))))}$ and so $f(\underline{f^{-1}((G, B))})\tilde{\subseteq}\underline{(G, B)} = (G, B)$. Thus $\underline{f^{-1}((G, B))} = f^{-1}((G, B))$, hence is soft multi semi-closed set in X_E. □

Theorem 4.12. *Let (X_E, τ) and (Y_K, σ) be two soft multi topological spaces, $f : (X_E, \tau) \to (Y_K, \sigma)$ be a soft multi semi-open and soft multi bijection function. Then $f^{-1} : (Y_K, \sigma) \to (X_E, \tau)$ is a soft multi irresolute function.*

Proof. Let (F,A) be any soft semi-open multiset in X_E. Since f is soft multi semi-open and soft multi bijection function, then $g^{-1}((F,A)) = f((F,A))$ where $g = f^{-1}$ and $f((F,A))$ is soft semi-open multiset in Y_K. Thus f^{-1} is a soft multi irresolute function. □

Definition 4.13. Let (X_E,τ) and (Y_K,σ) be two soft multi topological spaces, $f : (X_E,\tau) \to (Y_K,\sigma)$ be a soft multi function. If f is soft multi bijection, soft multi irresolute and soft multi semi-open function, then f is said to be soft multi semi-homeomorphism from X to Y . When a homeomorphism f exists between X and Y , we say that X is soft multi semi-homeomorphic to Y.

Theorem 4.14. *Let (X_E,τ) and (Y_K,σ) be two soft multi topological spaces, $f : (X_E,\tau) \to (Y_K,\sigma)$ be a soft multi function. If f is soft multi continuous and multi open function, then f is soft multi irresolute and soft multi semi-open function.*

Proof. Definition of soft multi continuous and multi open function was given in [16].

Let f be soft multi continuous, multi open function and (G,B) be soft multi semi-open set in Y_K. There exists a soft multi open set (O,B) in Y_K such that $(O,B)\tilde{\subset}(G,B)\tilde{\subset}\overline{(O,B)}$. Then $f^{-1}((O,B))\tilde{\subset}f^{-1}((G,B))\tilde{\subset}f^{-1}(\overline{(O,B)})$. Since f is soft multi continuous and multi open function, then $f^{-1}(\overline{(O,B)}) = \overline{(f^{-1}((O,B)))}$ and so $f^{-1}((O,B))\tilde{\subset}f^{-1}((G,B))\tilde{\subset}\overline{(f^{-1}((O,B)))}$. Hence $f^{-1}((G,B))$ soft multi semi-open set in X_E and thus f is soft multi irresolute function.

Let (F,A) be soft multi semi-open set in X_E. There exists a soft multi open set (U,A) in X_E such that $(U,A)\tilde{\subset}(F,A)\tilde{\subset}\overline{(U,A)}$. Then $f((U,A))\tilde{\subset}f((F,A))\tilde{\subset}f(\overline{(U,A)})$. Since f is soft multi continuous and multi open function, then then $f(\overline{(U,A)})\tilde{\subset}\overline{(f((U,A)))}$ and so $f((U,A))\tilde{\subset}f((F,A))\tilde{\subset}\overline{(f((U,A)))}$. Hence $f((F,A))$ soft multi semi-open set in Y_K and thus f is soft multi semi-open function. □

Theorem 4.15. *Let (X_E,τ) and (Y_K,σ) be two soft multi topological spaces, $f : (X_E,\tau) \to (Y_K,\sigma)$ be a soft multi homeomorphism. Then f is soft multi semi-homeomorphism.*

Proof. Definition of soft multi homeomorphism was given in [16].

Let f be soft multi homeomorphism. Then f is soft multi bijection, soft multi continuous and f^{-1} is soft multi continuous function. Thus f is soft multi continuous and multi open function by Theorem 3.14. of [16]. Hence f is soft multi irresolute and soft multi semi-open function by Theorem 4.14. and so f is soft multi semi-homeomorphism. □

5. Conclusion

In this study first we extended the notions of soft multi set. Then we made an extension to the concept of soft multi continuous function.

References

[1] S. Atmaca and I. Zorlutuna, On fuzzy soft topological spaces, Ann. Fuzzy Math. Inform. 5 (1) (2013) 377–386.

[2] A. Aygunoglu and H. Aygun, Some note on soft topological spaces, Neural Comput. Appl. 21 (2012) 113–119.

[3] N. Cagman, S. Karatas and S. Enginoglu, Soft topology, Comput. Math. Appl. 62 (2011) 351–358.
[4] K. P. Girish and S. J. John, Multiset topologies induced by multiset relations, Inform. Sci. 188 (2012) 298–313.
[5] S. P. Jena and S. K. Ghosh, B.K. Tripathy, On the theory of bags and lists, Inform. Sci. 132 (2001) 241–254.
[6] J. Mahanta and P. K. Das, Results on fuzzy soft topological spaces, arXiv:1203.0634, 2012.
[7] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, J. Fuzzy Math. 203 (2) (2001) 589–602.
[8] P. K. Maji, R. Biswas and A. R. Roy. Soft set theory. Comput. Math. Appl. 45 (2003) 555–562.
[9] D. Molodtsov, Soft set theory-First results, Comput. Math. Appl. 37 (4/5) (1999) 19–31.
[10] I. Osmanoglu and D. Tokat, Compact Soft Multi Spaces, Eur. J. Pure Appl. Math. 7 (2014) 97–108.
[11] Z. Pawlak, Rough sets, Int. J. Inf. Comp. Sci. 11 (1982) 341–356.
[12] S. Roy and T. K. Samanta, A note on fuzzy soft topological spaces, Ann. Fuzzy Math. Inform. 3 (2) (2011) 305–311.
[13] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl. 61 (2011) 1786–1799.
[14] B. Tanay and M. B. Kandemir, Topological structures of fuzzy soft sets, Comput. Math. Appl. 61 (2011) 412–418.
[15] D. Tokat and I. Osmanoglu, Connectedness on Soft Multi Topological Spaces, J. New Results Sci. 2 (2013) 8–18.
[16] D. Tokat, I. Osmanoglu and Y. Ucok, On Soft Multi Continuous Functions, Journal of New Theory 1 (2015) 50–58.
[17] B. P. Varol and H. Aygun, Fuzzy soft topology, Hacet. J. Math. Stat. 41 (3) (2012) 407–419.
[18] B. P. Varol and H. Aygun, On soft Hausdorff spaces, Ann. Fuzzy Math. Inform. 5 (1) (2012) 15–24.
[19] R. R. Yager, On the theory of bags, Int. J. General Systems 13 (1981) 23–37.
[20] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.
[21] I. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, Remarks on soft topological spaces, Ann. Fuzzy Math. Inform. 3 (2) (2012) 171–185.

Deniz Tokat (dtokat@nevsehir.edu.tr)
Department of Mathematics, Faculty of Arts and Sciences, Nevşehir Hacı Bektaş Veli University, 50300 Nevşehir, Turkey

İsmail Osmanoğlu (ismailosmanoglu@yahoo.com)
Department of Mathematics, Faculty of Arts and Sciences, Nevşehir Hacı Bektaş Veli University, 50300 Nevşehir, Turkey

Yaşar Üçok (yasarucok@hotmail.com)
Department of Mathematics, Faculty of Arts and Sciences, Nevşehir Hacı Bektaş Veli University, 50300 Nevşehir, Turkey